Monopoles of the Dirac type and color confinement inQCD

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1. Introduction
2. Abelian monopoles of the Dirac type in QCD $J_\mu = k_\mu$
3. Perfect Abelian and monopole dominance of the string tension
4. Existence of the continuum limit
5. Summary and Outlook

T. Suzuki, arXiv:1402.1294 (2014),
T. Suzuki et al, P.R. D80, 054504 (2009),
T. Suzuki, K.Ishiguro, V.Bornyakov, P.R. D97, 034501, 099905(erratum) (2018),
T. Suzuki, P.R. D97, 034509 (2018)
T. Suzuki, K. Ishiguro and A. Hiraguchi, Talks at APLAT 2020

1. Introduction

Color confinement problem not yet solved.

Almost half a century history !!!

1. 1963: Quark model (Gell-Mann and Zweig): fractionally charged quarks are searched, but not observed.
2. 1974-75: Idea of dual superconductor (electric $\leftrightarrow$ magnetic) as the color-confinement mechanism ('tHooft-Mandelstam): Something color magnetic must be condensed.
3. 1981: 'tHooft idea of monopole in QCD: A partial gauge-fixing $SU(3) \to U(1) \times U(1)$ and Abelian projection: Monopoles appear as a topological object coming from the singularity of the gauge-fixing matrix. Numerical data supporting this idea are shown especially on the basis of maximally Abelian gauge. But this idea has serious problems:(1) gauge dependence, (2) Abelian charge confinement, not non-Abelian color confinement, (3) asymmetry among eight gluons (diagonal: photon like and off-diagonal: massive matters), (4) in Polyakov-loop gauge, monopoles are predicted to run only time-like, but actually space-like monopoles are important.

The key point is to find
a gauge-independent color magnetic quantity, a magnetic monopole in QCD
without any additional artificial assumption like a special partial gauge-fixing to a subgroup.

2. Abelian magnetic monopoles of the Dirac type in QCD

Note the Jacobi identities:

$$\epsilon_{\mu\nu\rho\sigma}[D_\nu, [D_\rho, D_\sigma]] = 0,$$

where $D_\mu \equiv \partial_\mu - igA_\mu$. Calculate explicitly:

$$\begin{aligned}
[D_\rho, D_\sigma] &= [\partial_\rho - igA_\rho, \partial_\sigma - igA_\sigma] \\
&= -ig(\partial_\rho A_\sigma - \partial_\sigma A_\rho - ig[A_\rho, A_\sigma]) + [\partial_\rho, \partial_\sigma] \\
&= -igG_{\rho\sigma} + [\partial_\rho, \partial_\sigma]
\end{aligned}$$

If $[\partial_\rho, \partial_\sigma]$ is neglected, we get $D_\nu G^*_{\mu\nu} = 0 \rightarrow$ Non-Abelian Bianchi identity (NABI):

When define an Abelian-like field strength:

$$\begin{aligned}
f_{\mu\nu} &\equiv \partial_\mu A_\nu - \partial_\nu A_\mu \\
&= (\partial_\mu A^a_\nu - \partial_\nu A^a_\mu)\sigma^a/2,
\end{aligned}$$

if A^a_μ are regular $\rightarrow$ $\partial_\nu f^*_{\mu\nu} = 0$: Abelian-like Bianchi identity:

What happens if $[\partial_\rho, \partial_\sigma]$ is not neglected?

Jacobi identity + $[D_\nu, G_{\rho\sigma}] = D_\nu G_{\rho\sigma}$

$$
\begin{aligned}
\Longrightarrow D_\nu G^*_{\mu\nu} &= \frac{1}{2}\epsilon_{\mu\nu\rho\sigma} D_\nu G_{\rho\sigma} \\
&= -\frac{i}{2g}\epsilon_{\mu\nu\rho\sigma}[D_\nu, [\partial_\rho, \partial_\sigma]] \\
&= \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}[\partial_\rho, \partial_\sigma] A_\nu = \partial_\nu f^*_{\mu\nu} \\
J_\mu = \frac{1}{2}J^a_\mu \sigma^a = D_\nu G^*_{\mu\nu} &= \partial_\nu f^*_{\mu\nu} = \frac{1}{2}k^a_\mu \sigma^a = k_\mu
\end{aligned}
$$

$k^a_\mu \neq 0 \rightarrow$ color magnetic Abelian-like monopole: $\partial_\mu k_\mu = 0$
$J^a_\mu \neq 0 \rightarrow$ Violation of NABI

Color magnetic monopoles= Violation of non-Abelian Bianchi identity (VNABI) :Reference C. Bonati et al,, P.R.D81, 085022 (2010)

$$[\partial_\rho, \partial_\sigma] A_\nu \neq 0$$
$$\Downarrow$$

Line singularities existing in gauge fields $A_\mu(x)$ themselves!!! are the origin of Abelian monopoles in QCD.

Since the monopoles defined here comes from the (line) singularities of the gauge field themselves, they are much the same as those discussed by Dirac in QED with magnetic monopoles in 1931.

$N^2 - 1$ monopoles exist in $SU(N)$.

Hence, The Abelian monopoles defined here are completely different from those discussed by 'tHooft using additional partial gauge fixing.

Comparison between the 'tHooft Abelian projection studies and the present work in $SU(3)$ QCD.

	The 'tHooft scheme	This work.
Origin of k_μ	Singularity of gauge fixing matrix	Singularity in gauge fields.
No. of conserved k_μ	2	8
Flux squeezing	Only two electric fields	Eight electric fields

3. Lattice studies of the new QCD magnetic monopoles

Are these monopoles of the Dirac type important in QCD?

Consider one-colored monopole $k^1(s,\mu)$ among eight ($a = 1 \sim 8$ in $SU(3)$) monopoles and define them following DeGrand-Toussait in the framework of lattice QCD.

Lattice monopole is not gauge-invariant. But Elitzer's theorem says that gauge-invariant contents, if exist, can be extracted by Monte-Carlo average of gauge-variant quantities.
S. Elitzur, P.R. D12 (1975) 3978.

Abelian link fields on lattice without any additional gauge-fixing

Maximize $R = \sum_{s,\mu} Re\,\mathrm{Tr}\, e^{i\theta_1(s,\mu)\lambda_1} U^\dagger(s,\mu)$

$$\Downarrow$$

$$\begin{aligned}
\theta_1(s,\mu) &= \tan^{-1}\frac{U_1(s,\mu)}{U_0(s,\mu)}, \quad (\mathrm{SU2}: \quad U(s,\mu) = U_0(s,\mu) + i\vec{\sigma}\cdot\vec{U}(s\mu)) \\
&= \tan^{-1}\frac{Im(U_{12}(s,\mu) + U_{21}(s,\mu))}{Re(U_{11}(s,\mu) + U_{22}(s,\mu))}, \quad (\mathrm{SU3})
\end{aligned}$$

Abelian monopoles on lattice

Calculate Abelian plaquette variables:

$$\begin{aligned}\theta_1(s,\mu\nu) &= \partial_\mu\theta_1(s,\nu)-\partial_\nu\theta_1(s,\mu)\\ &= \bar{\theta}_1(s,\mu\nu)+2\pi n_1(s,\mu\nu)\quad (|\bar{\theta}_1(s,\mu\nu)|<\pi)\end{aligned}$$

Since $n_1(s,\mu\nu)$ can be regarded as the number of the Dirac string, Abelian monopoles are defined following DeGrand-Toussaint:

$$\begin{aligned}k_\mu^1(s) &= -(1/2)\epsilon_{\mu\alpha\beta\gamma}\partial_\alpha\bar{\theta}_1(s+\hat{\mu},\beta\gamma)\\ &= (1/2)\epsilon_{\mu\alpha\beta\gamma}\partial_\alpha n_1(s+\hat{\mu},\beta\gamma)\end{aligned}$$

Note $\epsilon_{\mu\alpha\beta\gamma}\partial_\alpha\theta_1(s+\hat{\mu},\beta\gamma)=0$ trivially.

let us evaluate each static potential through Polyakov-loop correlators.

$$V(R) = -\frac{1}{aN_t} \ln\langle P(0)P^*(R)\rangle \ .$$

$$P_{\mathrm{F}} = \mathrm{Tr}\, \Pi_{k=0}^{N_t-1} U(s+k\hat{4},4) \ ,$$

$$P_{\mathrm{A}} = \exp[i \sum_{k=0}^{N_t-1} \theta_1(s+k\hat{4},4)] = P_{\mathrm{ph}} \cdot P_{\mathrm{mon}} \ ,$$

$$P_{\mathrm{ph}} = \exp\{-i \sum_{k=0}^{N_t-1} \sum_{s'} D(s+k\hat{4}-s')\partial'_\nu \bar{\Theta}_1(s',\nu 4)\} \ ,$$

$$P_{\mathrm{mon}} = \exp\{-2\pi i \sum_{k=0}^{N_t-1} \sum_{s'} D(s+k\hat{4}-s')\partial'_\nu n_1(s',\nu 4)\}$$

(1). Perfect Abelian dominance can be proved using the Lüscher's multilevel method.

Note that the Abelian Polyakov loop operator without any additional gauge-fixing can be defined locally!!

Table 2: Simulation parameters for the measurement of static potential using multilevel method. N_{sub} is the sublattice size divided and N_{iup} is the number of internal updates in the multilevel method .

β	$N_s^3 \times N_t$	$a(\beta)$ [fm]	N_{conf}	N_{sub}	N_{iup}
5.60	$12^3 \times 12$	0.2235	6	2	5,000,000
5.60	$16^3 \times 16$	0.2235	6	2	10,000,000
5.70	$12^3 \times 12$	0.17016	6	2	5,000,000
5.80	$12^3 \times 12$	0.13642	6	3	5,000,000

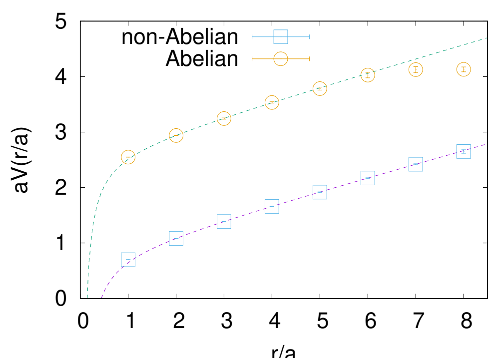


Figure 1: The static-quark potentials from non-Abelian and Abelian PLCF at $\beta = 5.6$ on $16^3 \times 16$ lattice.

Table 3: Best fitted values of the string tension σa^2, the Coulombic coefficient c, and the constant μa for the potentials V_{NA}, V_{A}.

$\beta = 5.6, 12^3 \times 12$	σa^2	c	μa
V_{NA}	0.2368(1)	-0.384(1)	0.8415(7)
V_{A}	0.21(5)	-0.6(6)	2.7(4)
$\beta = 5.6, 16^3 \times 16$			
V_{NA}	0.239(2)	-0.39(4)	0.79(2)
V_{A}	0.25(2)	-0.3(1)	2.6(1)
$\beta = 5.7, 12^3 \times 12$			
V_{NA}	0.159(3)	-0.272(8)	0.79(1)
V_{A}	0.145(9)	-0.32(2)	2.64(3)
$\beta = 5.8, 12^3 \times 12$			
V_{NA}	0.101(3)	-0.28(1)	0.82(1)
V_{A}	0.102(9)	-0.27(2)	2.60(3)

Perfect Abelian dominance is proved in pure $SU(3)$ QCD without any additional assumption in compatible with the theoretical studies done by Ogilvie and Faber et al.. .

(2). Perfect monopole dominance:

$$P_{\rm mon} = \exp\{-2\pi i \sum_{k=0}^{N_t-1}\sum_{s'} D(s+k\hat{4}-s')\partial'_\nu n_1(s',\nu 4)\}$$

$$\uparrow$$

$D(s-s')$: a non-local operator.

The Lüscher's multilevel method does not work.

To evaluate $< P_{\rm mon}P^*_{\rm mon} >$, we need tremendous number of vacuum configurations. We also perform random gauge-transformation a few thousand times for each one. Additional random gauge fixings are done to increase S/N ratio.

Table 4: Simulation parameters for the measurement of the static potential and the force from $P_{\rm A}$, $P_{\rm ph}$ and $P_{\rm mon}$. $N_{\rm RGT}$ is the number of random gauge transformations.

β	$N_s^3 \times N_t$	$a(\beta)$ [fm]	$N_{\rm conf}$	$N_{\rm RGT}$
$SU2$, 2.20	$24^3 \times 4$	0.211(7)	6,000	1,000
2.35	$24^3 \times 6$	0.137(9)	4,000	2,000
2.35	$36^3 \times 6$	0.137(9)	5,000	1,000
2.43	$24^3 \times 8$	0.1029(4)	7,000	4,000
$SU3$, 5.6	$24^3 \times 4$	0.2235	60,000	4,000

Table 5: Best fitted values of the string tension σa^2, the Coulombic coefficient c, and the constant μa for the potentials $V_{\rm NA}$, $V_{\rm A}$, $V_{\rm mon}$ and $V_{\rm ph}$.

$SU(2)$		σa^2	c	μa	FR(R/a)	$\chi^2/N_{\rm df}$
$24^3 \times 4$	$V_{\rm NA}$	0.181(8)	0.25(15)	0.54(7)	3.9 - 8.5	1.00
	$V_{\rm A}$	0.183(8)	0.20(15)	0.98(7)	3.9 - 8.2	1.00
	$V_{\rm mon}$	0.183(6)	0.25(11)	1.31(5)	3.9 - 6.7	0.98
	$V_{\rm ph}$	$-2(1) \times 10^{-4}$	0.010(1)	0.48(1)	4.9 - 9.4	1.02
$24^3 \times 6$	$V_{\rm NA}$	0.072(3)	0.49(6)	0.53(3)	4.0 - 9.0	0.99
	$V_{\rm A}$	0.073(4)	0.41(7)	1.09(3)	3.7 - 10.9	1.00
	$V_{\rm mon}$	0.073(4)	0.44(10)	1.41(4)	3.9 - 9.3	1.00
	$V_{\rm ph}$	$-1.7(3) \times 10^{-4}$	0.0131(1)	0.4717(3)	5.1 - 9.4	0.99
$36^3 \times 6$	$V_{\rm NA}$	0.072(3)	0.48(9)	0.53(3)	4.6 - 12.1	1.03
	$V_{\rm A}$	0.073(2)	0.47(6)	1.10(2)	4.3 - 11.2	1.03
	$V_{\rm mon}$	0.073(3)	0.46(7)	1.43(3)	4.0 - 11.8	1.01
	$V_{\rm ph}$	$-1.0(1) \times 10^{-4}$	0.0132(1)	0.4770(2)	6.4 - 11.5	1.03
$24^3 \times 8$	$V_{\rm NA}$	0.0415(9)	0.47(2)	0.46(8)	4.1 - 7.8	0.99
	$V_{\rm A}$	0.041(2)	0.47(6)	1.10(3)	4.5 - 8.5	1.00
	$V_{\rm mon}$	0.043(3)	0.37(4)	1.39(2)	2.1 - 7.5	0.99
	$V_{\rm ph}$	$-6.0(3) \times 10^{-5}$	0.0059(3)	0.46649(6)	7.7 - 11.5	1.02
$SU(3)$	$24^3 \times 4 V_{\rm NA}$	0.193(4)	0.422(3)	1.146(20)	1 - 10	0.99
	$V_{\rm A}$	0.184(15)	0.458(97)	2.912(80)	1 - 9	1.10
	$V_{\rm mon}$	0.188(16)	0.453(99)	2.906(82)	1 - 8	0.97
	$V_{\rm ph}$	-0.0014(2)	0.073(5)	1.521(3)	1 - 11	1.00

Perfect Abelian and monopole dominance are obtained beautifully.

The existence of the perfect Abelian and monopole dominance suggests that the Abelian dual Meissner effect can be seen coming from this new-type Abelian monopoles.

The Abelian dual Meissner effect coming from this new-type monopoles in SU2 QCD, see T. Suzuki et al.,P.R.D80 (2009)054504.

In SU3, see the talk by Dr.Atsuki Hiraguchi in this conference on August 5.

4. Existence of the continuum limit

Does the continuum limit of $k^a(s,\mu)$ exist?

(1). The monopole density in the continuum limit in pure SU2 QCD.

The lattice vacuum is contaminated with large amount of lattice artifact monopoles. To reduce lattice artifacts, various techniques smoothing the vacuum are introduced.

1. Tadpole improved action:
48^4 at $\beta = 3.0 \sim 3.9$ and 24^4 at $\beta = 3.0 \sim 3.7$

2. Introduction of various smooth gauge-fixings
1) Maximal center gauge(MCG): Maximization of $R = \sum_{s,\mu}(\mathrm{Tr}U(s,\mu))^2$
SU(2) $\to$ Z(2)

2) Direct Laplacian center gauge (DLCG)

3) Maximal Abelian Wilson loop gauge (AWL): Maximization of
$R = \sum_{s,\mu\neq\nu}\sum_a(cos(\theta^a_{\mu\nu}(s))$

4) Maximal Abelian and $U(1)$ Landau gauge (MAU1):

3. The blockspin transformation of monopoles

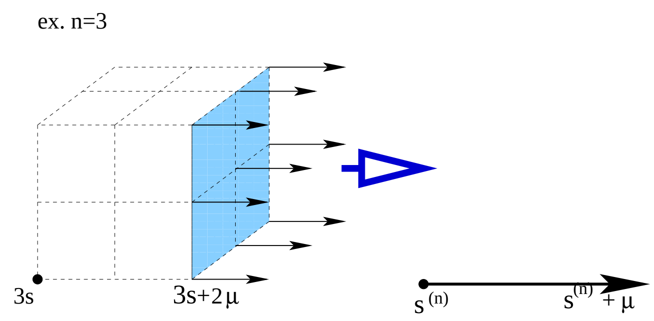


Figure 2: Blockspin definition of monopoles:
T.L. Ivanenko et al., Phys. Lett. **B252**, (1990) 631

Monopole is defined on a a^3 cube and
the n-blocked monopole is defined on a cube
with a lattice spacing $b = na$

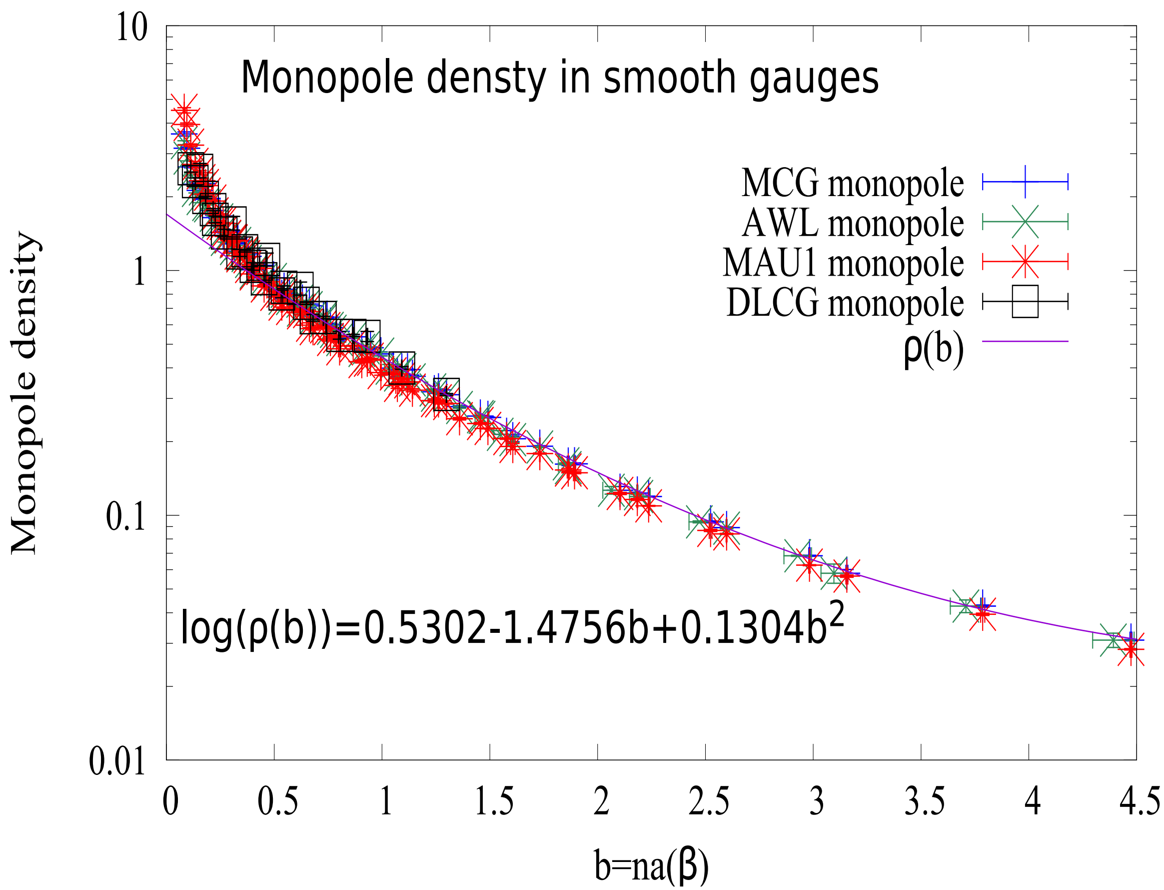
$$k_\mu^{(n)}(s_n) = \sum_{i,j,l=0}^{n-1} k_\mu(ns_n + (n-1)\hat{\mu} + i\hat{\nu} + j\hat{\rho} + l\hat{\sigma})$$

$n = 1, 2, 3, 4, 6, 8, 12$ blockings are adopted on 48^4 lattice.

Evaluate a gauge-invariant density of the n-blocked monopole:

$$\boldsymbol{\rho}(a(\beta), n) = \frac{\sum_{\mu, s_n} \sqrt{\sum_a (k_\mu^{(n)a}(s_n))^2}}{4\sqrt{3} V_n b^3}$$

Figure 3: Comparison of the VNABI (Abelian-like monopoles) densities versus $b = na(\beta)$ in MCG, AWL, DLCG and MAU1 cases. A uniform curve is obtained for all gauges.



Summary

1. Clear scaling behaviors are observed up to the 12-step blockspin transformations for $\beta = 3.0 \sim 3.9$. The density $\rho(a(\beta), n)$ is a function of $b = na(\beta)$ alone, i.e. $\rho(b)$. $n \to \infty$ means $a(\beta) \to 0$ for fixed $b = na$. Existence of the continuum limit!

2. When the vacuum becomes smooth enough shown here in MCG, DLCG, AWL, MAU1, the same $\rho(b)$ is obtained. Gauge independence!
   This is naturally expected in the continuum limit.

(2). The infrared effective monopole action in the continuum limit in pure SU2 QCD.

The effective monopole action is defined as follows:

$$
\begin{aligned}
e^{-\mathcal{S}[k]} &= \int DU(s,\mu)e^{-S(U)} \\
&\times \prod_a \delta(k_\mu^a(s) - \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}\partial_\nu n_{\rho\sigma}^a(s+\hat{\mu})),
\end{aligned}
$$

where $S(U)$ is the gauge-field action.

The monopole action is written as a linear combination of the interaction operators between monopole currents:

$$
\mathcal{S}[k] = \sum_i F(i)\mathcal{S}_i[k],
$$

where $F(i)$ are coupling constants.

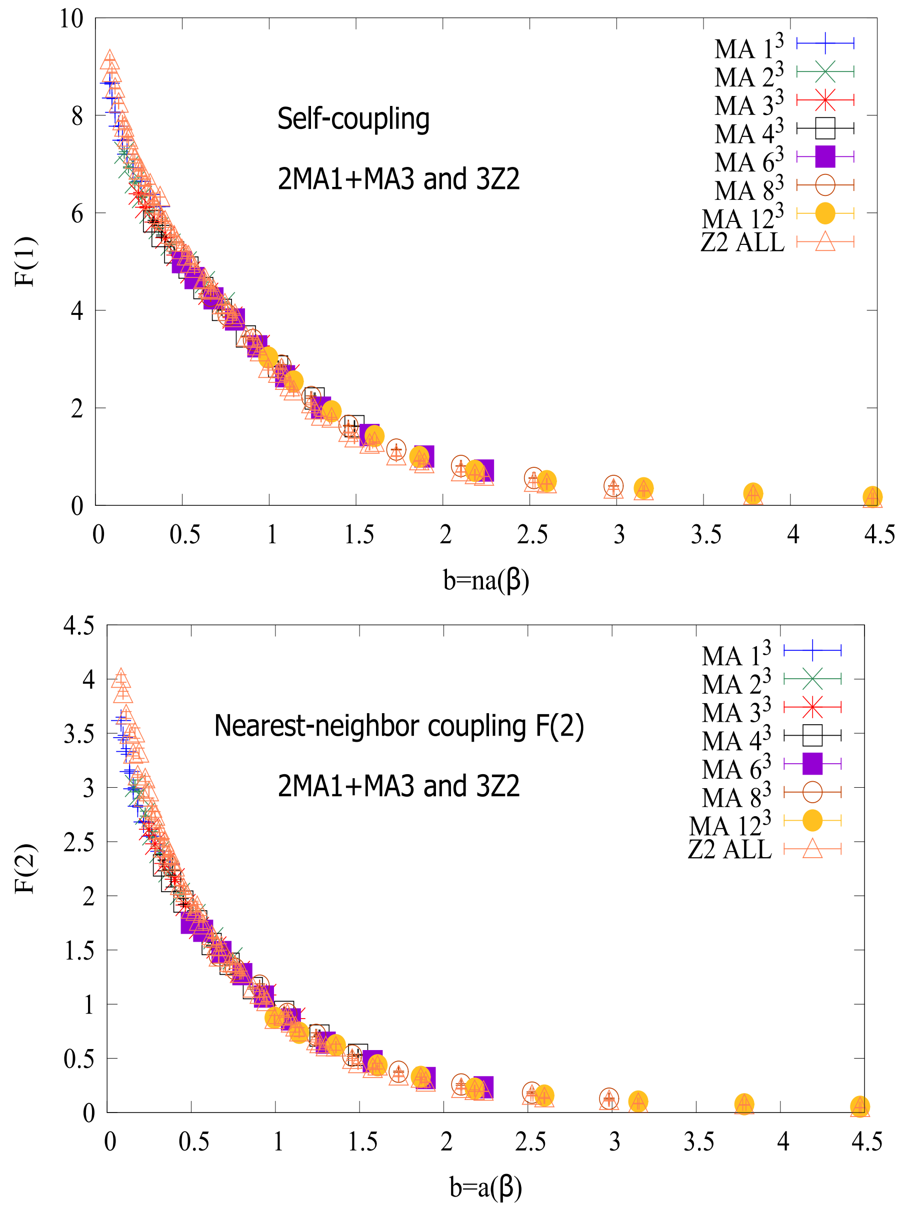
For example,

$$
\begin{aligned}
\mathcal{S}_1[k] &= \sum_{s,\mu,a} (k^a_\mu(s))^2 \\
\mathcal{S}_2[k] &= \sum_{s,\mu,a} k^a_\mu(s) k^a_\mu(s+\mu)
\end{aligned}
$$

We determine the monopole action, that is, the set of couplings $F(i)$ from the monopole current ensemble $\left\{k^a_\mu(s)\right\}$ with the aid of an inverse Monte-Carlo method first developed by Swendsen and extended to closed monopole currents by Shiba and Suzuki.

Techniques:
1. Tadpole improved action on 48^4 lattice at $\beta = 3.0 \sim 3.9$
2. Smooth gauge-fixings: Direct Maximal center gauge (MCG), Direct Laplacia center gauge (DLCG), Maximal Abelian Wilson loop gauge (AWL) and Maximally Abelian gauge+U(1) (MA).
3. Block spin transformation of monopole currents. $n = 1, 2, 3, 4, 6, 8, 12$
The inverse M-C method determines the coupling constants F_i. In general they are the function of β and n. $F_i(a(\beta), n)$

Figure 4: The coupling constants of the self and the nearest-neighbor interactions in the effective monopole action versus $b = na(\beta)$ in MAU1 and MCG on 48^4. The sum of each coupling constants with respect to three color components are shown.



Summary:

1. $F(i)$ satisfy a beautiful scaling, that is, they are a function of the product $b = na(\beta)$ alone for lattice coupling constants $3.0 \leq \beta \leq 3.9$ and the steps of blocking $1 \leq n \leq 12$. The effective action showing the scaling behavior can be regarded as an almost perfect action corresponding to the continuum limit, since $a \to 0$ as $n \to \infty$ for fixed b.

2. The almost perfect action showing the scaling is found to be independent of the smooth gauges adopted here as naturally expected from the gauge invariance of the continuum theory.

From the scaling results of the monopole density and the infrared monopole action, we can say that the new monopoles of the Dirac type have the continuum limit.

5. Future outlook

1. To study the new monopoles in the case of string tensions of higher representations is intersting. In progress.

2. There is in principle no problem concerning the existence of this new color magnetic monopoles in full QCD. To study these Abelian new monopoles of the Dirac type in full QCD is important.

   - What is the scaling behavior with respect to monopole dnsity when small dynamical quarks exist?
   - Could they explain all mass generation in QCD such as hadron masses?
   - What is an infrared effective monopole action in full QCD
   - Is it rewritten by a kind of the dual Abelian Higgs model?
   - Could the monopoles explain also chiral symmetry breaking?

3. In usual axiomatic field theory, a field operator is regarded as an operator-valued distribution $\hat{\phi}(f)$ where $f(x)$ is a regular function. Since the derivative of the operator is defined by that of the test function, no singularity is assumed to exist leading to the violation of non-Abelian Bianchi identity. How to formulate a field theory containing line singularities mathematically is not known yet and interesting.