Excited States of Isolated Fermions in the Higgs phase of gauge Higgs theories

Jeff Greensite and Kazue Matsuyama
San Francisco State University

Virtual Tribute to Quark Confinement and the Hadron Spectrum
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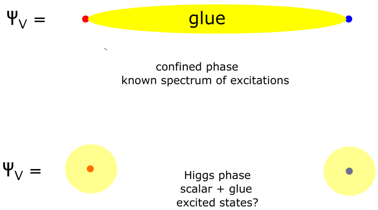
Next year in Norway, inshallah.

The question we address

Composite systems (molocules, atoms, nuclei, hadrons...) generally have a spectrum of excitations. What about non-composite systems: charged "elementary" particles like quarks and leptons?

If the particle is charged, then by Gauss's Law it is accompanied by a surrounding gauge (and possibly other) fields. If these surrounding fields interact with themselves, could they not also exhibit a spectrum of excitations? This would *look like a mass spectrum* of the isolated elementary particle.



This doesn't happen in pure QED. Any energy eigenstate containing a static $\pm$ charge pair is just the Coulomb field plus some number of photons. *Gauge Higgs theories could be different.*

Motivation: superconductivity, electroweak sector.

Are all physical states gauge invariant?

No, not quite. The Gauss law constraint only requires invariance under infinitesimal gauge transformations. In QED, in an infinite volume, a physical state containing a single static charge *transforms under a global subgroup of the gauge group.*

The ground state of pure QED containing a single static electric charge at point $\mathbf{x}$ is the Dirac state

$$|\Psi_{\mathbf{x}}\rangle = \overline{\psi}^{+}(\mathbf{x})\rho_C(\mathbf{x};A)|\Psi_0\rangle$$

where

$$\boxed{\rho_C(\mathbf{x};A) = \exp\left[-i\frac{e}{4\pi}\int d^3z\, A_i(\mathbf{z})\frac{\partial}{\partial z_i}\frac{1}{|\mathbf{x}-\mathbf{z}|}\right]}$$

It is easy to check that $|\Psi_{\mathbf{x}}\rangle$ satisfies the Gauss Law. However. Let $g(x) = e^{i\theta(x)}$ be an arbitrary U(1) gauge transformation, and we separate out the zero mode $\theta(x) = \theta_0 + \tilde{\theta}(x)$. Then

$$\psi(\mathbf{x}) \rightarrow e^{i\theta(\mathbf{x})}\psi(\mathbf{x})$$

but

$$\boxed{\rho_C(\mathbf{x};A) \rightarrow e^{i\tilde{\theta}(x)}\rho_C(\mathbf{x};A)}$$

Pseudomatter fields

It follows that

$$|\Psi_{\mathbf{x}}\rangle \to e^{-i\theta_0}|\Psi_{\mathbf{x}}\rangle$$

Local symmetries cannot break spontaneously, of course (Elitzur). But global symmetries can.

The operator

$$\rho_C(\mathbf{x};A) \quad = \quad \exp\left[-i\frac{e}{4\pi}\int d^3z\, A_i(\mathbf{z})\frac{\partial}{\partial z_i}\frac{1}{|\mathbf{x}-\mathbf{z}|}\right]$$

is a first example of a **pseudomatter field**. It is responsible for "dressing" the bare charge with a Coulomb electric field.

Definition

A pseudomatter field $\rho(\mathbf{x};A)$ is a non-local functional of the gauge field which transforms like a matter field in the fundamental representation of the gauge group, *except* under the global center subgroup of the gauge group.

We combine the scalar field and pseudomatter fields with the static charge operator to create physical states in gauge Higgs theories.

more examples of pseudomatter fields

1. Any SU(N) gauge transformation $g_F(\mathbf{x};A)$ to a physical gauge $F(A)=0$ can be decomposed into N pseudomatter fields $\{\rho_n\}$, and vice-versa:

$$\rho_n^a(\mathbf{x};A) = g_F^{\dagger an}(\mathbf{x};A)$$

In particular, the operator $\rho_C^*(\mathbf{x};A)$ defined earlier is precisely the gauge transformation to Coulomb gauge in an abelian theory. This operator dresses a static charge with a surrounding Coulomb field: $\overline{\psi}(\mathbf{x})\rho_C(\mathbf{x};A)\Psi_0$.

2. In an SU(N) lattice gauge theory, any eigenstate $\xi_n(\mathbf{x};U)$ of the covariant Laplacian operator,

$$-D^2\xi_n = \kappa_n\xi_n$$

where

$$(-D^2)^{ab}_{\mathbf{xy}} = \sum_{k=1}^{3}\left[2\delta^{ab}\delta_{\mathbf{xy}} - U_k^{ab}(\mathbf{x})\delta_{\mathbf{y},\mathbf{x}+\hat{k}} - U_k^{\dagger ab}(\mathbf{x}-\hat{k})\delta_{\mathbf{y},\mathbf{x}-\hat{k}}\right]$$

is a pseudomatter field

$$\boxed{\rho^a(\mathbf{x};U) = \xi_n^a(\mathbf{x};U)}$$

(This is the idea behind the Laplacian gauges of Vink and Wiese.)

An aside: Higgs and confinement phases

We consider excitations of static charges in the *Higgs phase* of gauge Higgs theories, where the Higgs field is in the fundamental representation of the gauge group. The word *"phase"* is used deliberately.

It is true (Fradkin-Shenker-Osterwalder-Seiler) that there is no thermodynamic transition line which isolates the Higgs from the confinement phase.

However, there are examples of physically distinct phases which are *not* separated by such transition lines, e.g.

1. The roughening transition in lattice pure gauge theories. Inside the rough phase, the width of the confining flux tube grows logarithmically with quark separation. Outside, it has a finite limit.

2. In the Ising model in an external magnetic field, there is a percolation transition of spin clusters, which is not a thermodynamic transition. The transition line between percolating and non-percolating phases is known as the Kertesz line.

K. Matsuyama and I claim that there is likewise a sharp transition between physically distinct Higgs and confinement phases.

An aside II

Intuitively, the confinement phase has metastable flux tubes and linear Regge trajectories. The Higgs phase does not have these features. More precisely, the two phases are distinguished by

- **Symmetry**. The Higgs phase is a **spin glass phase**, in which the global center symmetry of the gauge group is spontaneously broken. The gauge invariant order parameter is a generalization of the Edwards-Anderson order parameter for spin glasses.

- **Confinement type**. Color **("C") confinement** in the Higgs phase; i.e. a spectrum of color neutral particles, and a stronger type of confinement – separation-of-charge **("S_c") confinement** – in the confinement phase, which is related to the existence of metastable flux tubes.

The transition lines between unbroken/broken symmetry and between confinement types coincide.

See my article with Kazue Matsuyama: PRD 101 (2020) 5, 054508, arXiv: 2001.03068 [hep-th]

Excitations of elementary fermions in gauge Higgs theories

For static quarks in a pure gauge theory there is a tower of metastable states

$$\Psi_n(R) = \overline{q}(\mathbf{x}) V_n(\mathbf{x}, \mathbf{y}; U) q(\mathbf{y}) \Psi_0$$

corresponding to string excitations. This has been observed in computer simulations.
Juge, Kuti, and Morningstar, (2003), Brandt and Meineri (2016)

For light quarks, the excited gluonic states lie on Regge trajectories. Should also exist in the confinement region of a gauge Higgs theory.

In the spin glass (aka Higgs) phase, is there a similar tower of metastable states of the form

$$\Psi_n(R) = \overline{q}^a(\mathbf{x}) \left[\sum_m c_m^{(n)} \rho_m^a(\mathbf{x}) \rho_m^{\dagger b}(\mathbf{y}) \right] q^b(\mathbf{y}) \Psi_0$$

where the $\{\rho_m(\mathbf{x})\}$ are pseudo-matter fields?

The Models

We investigate:

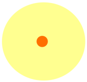
1. SU(3) gauge Higgs theory. The Higgs scalar is in the fundamental representation.
   J.G., PRD 102 (2020) 5, 054504 , arXiv: 2007.11616 [hep-lat]
2. The $q = 2$ abelian Higgs model. The Higgs scalar has charge $q = 2$.
   K. Matsuyama, PRD 103 (2021) 7, 074508 , arXiv: 2012.13991 [hep-lat]
3. Landau-Ginzburg effective action for superconductivity.
   K. Matsuyama and J.G., in progress
4. Chiral U(1) gauge Higgs theory (Smit-Swift formulation). The Higgs scalar has charge $q = 1$.
   J.G., arXiv: 2104.12237 [hep-lat]

In each of these models we impose a unimodular constraint $|\phi| = 1$ for simplicity.

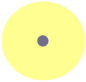
In the study of static fermion excitations, we find that each model has its own special features which must be taken into account.

$\Psi_V =$ 

confined phase
known spectrum of excitations

$\Psi_V =$ 

Higgs phase
scalar + glue
excited states?



general idea II

If $|\Psi(R)\rangle$ is some arbitrary physical state containing a static fermion-antifermion pair with separation R, and $E_1(R)$ is the lowest energy of such states above the vacuum energy $\mathcal{E}_0$, then on general grounds

$$\langle\Psi|\mathcal{T}^T|\Psi\rangle = \sum_n c_n e^{-E_n(R)T} \to c_1 e^{-E_1(R)T} \quad \text{as } T \to \infty$$

where $\mathcal{T}$ is the transfer matrix multiplied by $e^{\mathcal{E}_0}$. Drawback: We get the ground state, not easy to find excited states.

Alternatively, let $\{|\Phi_\alpha(R)\rangle\}$ span a subspace of the full Hilbert space with the two static charges. Then one could get an approximate spectrum by diagonalizing $\mathcal{T}$ in this subspace. This approach is followed in some lattice QCD spectrum calculations. Drawback: This requires a pretty big set $\sim$ hundreds of states. Not practical for our purposes, where it is *expensive* to generate the $|\Phi_\alpha(R)\rangle$.

general idea III

Generate a small set of states $\{|\Phi_\alpha(R)\rangle\}$, and diagonalize either $\mathcal{T}$ or $\mathcal{T}^p$ in the small subspace spanned by these states. The hope is that one or more of the eigenstates $|\Psi_n\rangle$ in the subspace will be orthogonal (nearly) to the true ground state. If $|\Psi\rangle$ is such a state, then

$$
\begin{aligned}
\langle\Psi|\mathcal{T}^T|\Psi\rangle &= \sum_n c_n e^{-E_n(R)T} \\
&\to c_{ex} e^{-E_{ex}(R)T} \quad \text{at large } T
\end{aligned}
$$

There are no guarantees, it just has to be tried.

Let ξ_n denote the eigenstates of the lattice Laplacian operator (no time derivatives)

$$-D^2\xi_n = \kappa_n\xi_n$$

Let $\mathcal{T} = e^{-(H-\mathcal{E}_0)}$ be the (rescaled) transfer matrix, and consider at each quark separation $R = |\mathbf{x} - \mathbf{y}|$, the 4-dimensional subspace spanned by three quark-pseudomatter states, and one quark-scalar state

$$\begin{aligned} \Phi_n(R) &= [\overline{q}^a(\mathbf{x})\xi_n^a(\mathbf{x})] \times [\xi_n^{\dagger b}(\mathbf{y})q^b(\mathbf{y})]\ \Psi_0 \quad (n = 1, 2, 3) \\ \Phi_4(R) &= [\overline{q}^a(\mathbf{x})\phi^a(\mathbf{x})] \times [\phi^{\dagger b}(\mathbf{y})q^b(\mathbf{y})]\ \Psi_0 \end{aligned}$$

We calculate numerically the matrix elements and overlaps

$$\begin{aligned} \mathcal{T}_{mn}(R) &= \langle\Phi_m|\mathcal{T}|\Phi_n\rangle \\ O_{mn}(R) &= \langle\Phi_m|\Phi_n\rangle \end{aligned}$$

The eigenvalues of $\mathcal{T}$ in the subspace are obtained by solving the generalized eigenvalue problem

$$[\mathcal{T}]\vec{\upsilon}_n = \lambda_n [O]\vec{\upsilon}^{(n)}$$

and we have eigenstates of $\mathcal{T}$ in the subspace

$$|\Psi_n(R)\rangle = \sum_{i=1}^{4} \upsilon_i^{(n)} |\Phi_i(R)\rangle$$

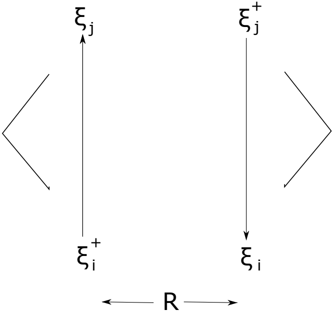
Likewise we consider evolving states for Euclidean time T, and compute

$$\begin{aligned} \mathcal{T}^T_{nn}(R) &= \langle\Psi_n|\mathcal{T}^T|\Psi_n\rangle \\ &= \upsilon_i^{(n)*}\langle\Phi_i|\mathcal{T}^T|\Phi_j\rangle\upsilon_j^{(n)} \\ \\ E_n(R,T) &= -\log\left[\frac{\mathcal{T}^T_{nn}(R)}{\mathcal{T}^{T-1}_{nn}(R)}\right] \end{aligned}$$

Integrating out the massive (i.e. static) fermion fields generates a pair of Wilson lines.

The numerical computation of $\langle \Phi_i | \mathcal{T}^T | \Phi_j \rangle$ involves expectation values of products of Wilson lines, terminated by matter or pseudomatter fields:

$$\langle \Phi_i | \mathcal{T}^T | \Phi_j \rangle =$$



Three possibilities:

1. $\Psi_n(R)$ is an *eigenstate* in the full Hilbert space. Then $E_n(R) = E(R,T)$ is time independent.

2. $\Psi_n(R)$ evolves to the ground state. Then $E_n(R,T)$ drops steadily to the lowest energy with increasing T.

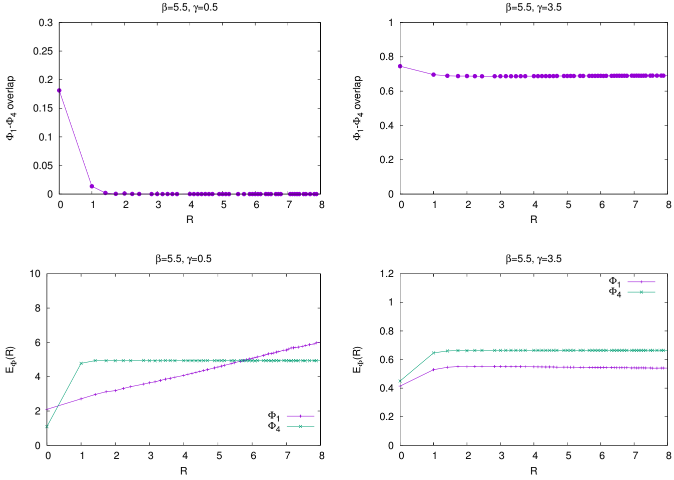
3. $\Psi_n(R)$ evolves in Euclidean time to a stable or metastable *excited* state. Then $E_n(R,T)$ converges to a value which is almost constant, over some range of Euclidean time. Analogous to string excitations in the confining phase.

We have computed $E_n(R,T)$ for SU(3) gauge theory with a unimodular Higgs field on a $14^3 \times 32$ lattice volume, at $\beta = 5.5$ with $\gamma = 0.5$ and $\gamma = 3.5$, in the confinement and Higgs phases respectively. The action is

$$
\begin{aligned}
S &= -\frac{\beta}{3} \sum_{plaq} \mathrm{ReTr}[U_\mu(x) U_\nu(x+\hat{\mu}) U_\mu^\dagger(x+\hat{\nu}) U_\nu^\dagger(x)] \\
&\quad -\gamma \sum_{x,\mu} \mathrm{Re}[\phi^\dagger(x) U_\mu(x) \phi(x+\widehat{\mu})]
\end{aligned}
$$

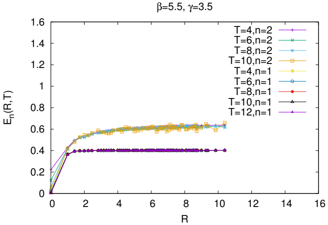
confinement vs Higgs phase

Look at $E(R,1)$ for the Higgs $\Phi_4(R)$ and pseudomatter $\Phi_1(R)$ states, as well as the overlap of these (normalized) states at $\beta = 5.5$ in the confinement phase ($\gamma = 0.5$) and Higgs phase ($\gamma = 3.5$) respectively.

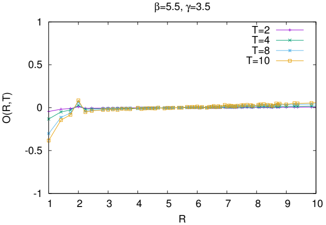


Higgs phase

Now we show $E_n(R,T)$ and the overlap for $\Psi_1(R)$, $\Psi_2(R)$ and $T = 4 - 12$.



(a) Energies



(b) Overlap

There seems to be clear evidence of a metastable excited state in the spectrum, orthogonal to the ground state.

The energy gap is far smaller than the threshold for vector boson creation.

$q = 2$ abelian Higgs model

A similar result has recently been found in the $q = 2$ abelian Higgs model by
K. Matsuyama, arXiv: 2012.13991 [hep-lat].

This is a version of the abelian Higgs model in which the scalar field (like Cooper pairs) carries two units of electric charge:

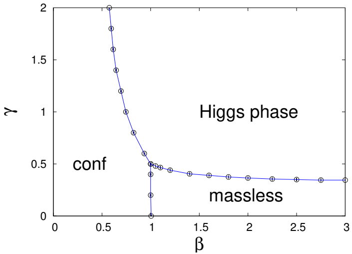
$$\begin{aligned} S &= -\beta \sum_{plaq} \mathrm{Re}[U_\mu(x) U_\nu(x+\hat{\mu}) U^*_\mu(x+\hat{\nu}) U^*_\nu(x)] \\ &\quad -\gamma \sum_{x,\mu} \mathrm{Re}[\phi^*(x) U^2_\mu(x) \phi(x+\widehat{\mu})] \end{aligned}$$

The phase diagram is on the right. Matsuyama works just inside the Higgs phase, at $\beta = 3.0, \gamma = 0.5$.

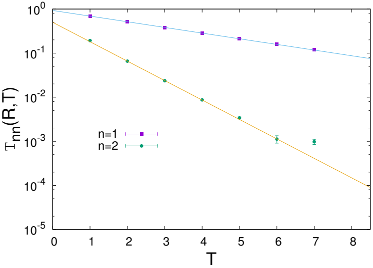
The photon mass is determined, from the plaquette-plaquette correlator, to be

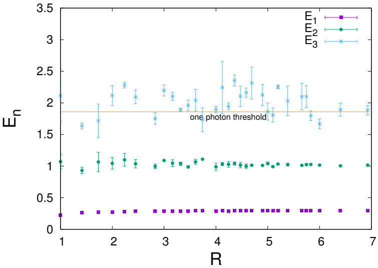
$$m_\gamma = 1.57(1)$$

in lattice units.



K.M. considers charge $q = 2$ sources. A surprise is that, following along the same lines of diagonalizing $\mathcal{T}$ in a small subspace, and judging by the fit to $\mathcal{T}_{11}(R,T)$ and $T_{22}(R,T)$, the lowest two states $\Psi_{1,2}$ seem to be very nearly exact eigenstates of the system, even at small T:





Once again, the first excitation is well below the threshold, and is therefore stable against massive photon emission.

What about real superconductors?

Effective Landau-Ginzburg model

This is work in progress (*K. Matsuyama and J.G.*)

The effective Landau-Ginzburg model for ordinary superconductivity is a non-relativistic $q = 2$ abelian Higgs model of this form:

$$\begin{aligned} S \quad = \quad & -\beta \sum_{plaq} \mathrm{Re}[UUU^* U^*] - \gamma \sum_x \sum_{k=1}^{3} \phi^*(x) U_k^2(x) \phi(x + \hat{k}) \\ & - \frac{\gamma}{\upsilon^2} \sum_x \phi^*(x) U_0^2(x) \phi(x + \hat{t}) \end{aligned}$$

where $\upsilon \sim 10^{-2}$ in natural units, is on the order of the Fermi velocity in a metal, and $\beta = 1/e^2 = 10.9$ for ordinary electrodynamics. Go to unitary gauge, so that $U_0(x) \approx \pm 1$. We then compute the excitations around a pair of static $q = \pm 1$ (e) charges, having electrons and holes in mind.

γ, β determine the photon mass (inverse to the penetration depth) in lattice units, so for a given γ the penetration depth fixes the lattice spacing in physical units.

But this time things are not so simple, and diagonalizing $\mathcal{T}$ in a small subspace doesn't work. Eigenstates in the subspace flow in Euclidean time to the ground state.

Landau-Ginzburg II

Let us instead (at each separation R) diagonalize $\mathcal{T}^{2t_0}$ in the basis Φ_α, so that

$$\langle \Psi_m | \mathcal{T}^{2t_0} | \Psi_n \rangle = \lambda_n(t_0)\delta_{mn}$$

and define

$$\Psi_n(t) = \mathcal{T}^t \Psi_n$$

Suppose, after evolving Ψ_1 by t_0 units of Euclidean time, that $\Psi_1(t_0)$ is approximately the true ground state in the full Hilbert space. It follows that $\Psi_{n>1}(t_0)$ is orthogonal to the ground state, because

$$\langle \Psi_m(t_0) | \Psi_n(t_0) \rangle \propto \delta_{mn}$$

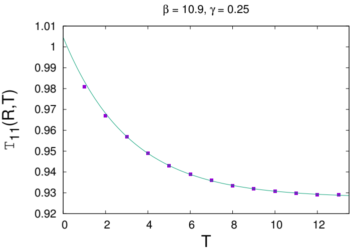
and therefore, at large $T > 2t_0$

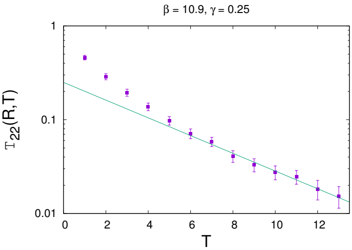
$$\begin{aligned} \mathcal{T}_{22}(R,T) &= \langle \Psi_2 | \mathcal{T}^T | \Psi_2 \rangle \\ &= \langle \Psi_2(t_0) | \mathcal{T}^{T-2t_0} | \Psi_2(t_0) \rangle \\ &\rightarrow \text{const} \times e^{-E_{ex}T} \quad \text{where} \quad E_{ex} > E_1 \end{aligned}$$

So we try that.

Landau-Ginzburg III

At $R = 5.385, \gamma = 0.25,$





Choose $2t_0 = 9$. We fit $\mathcal{T}_{11}$ to

$$f_1(T) = a_1 \exp(-b_1 T) + c_1$$

The fact that $c_1 \neq 0$ means that the ground state energy $E_1 \approx 0$.
b_1 gives an excited state energy.

Then we fit T_{22} in the range $T > 6$ to a single exponential

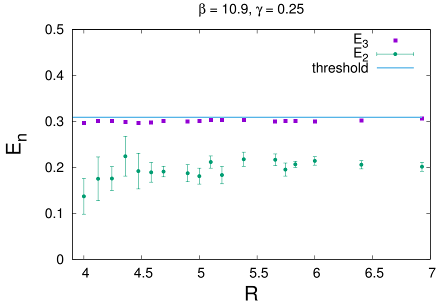
$$f_2(T) = a_2 \exp(-b_2 T)$$

The coefficient $b_2 < b_1$ gives another excitation energy.

Landau-Ginzburg IV

The data at $R < 4.0$ are rather noisy, with large χ^2. Here are the results for $R \geq 4.0$.

At these couplings, $E_1(R) \approx 0$.



Once again, the first excited state is stable. The next excited state, which is right on the threshold, is presumably the ground state plus a massive photon.

Can such excitations be detected experimentally? E.g. by ARPES (angle-resolved photoemission spectrum) data? We don't yet know...

Excitations in chiral gauge theories

No known lattice formulation of chiral non-abelian gauge theories with a continuum limit. There is a formulation for U(1) gauge theories due to Lüscher, involving overlap fermions. Difficult to implement numerically.

In this exploratory work, we chose a simpler option.

For *static* fermions, work instead with a quenched version, at fixed lattice spacing, of the Smit-Swift lattice action, U(1) gauge group, with oppositely charged right and left-handed fermions.

Doublers restore chiral symmetry, so the idea was to use a Wilson-style non-local mass term so that the mass of the doublers is infinite in the continuum limit.

The continuum limit doesn't work...Smit-Swift is not a true chiral gauge theory. Moreover, the positivity of the transfer matrix is unproven. But at least there is a mass asymmetry, between the desired states and the doublers, in part of the phase diagram. We can try it.

Smit-Swift model

The action of the Smit-Swift model with a U(1) gauge group and opposite charged right and left-handed fermions:

$$
\begin{aligned}
S \quad = \quad & -\beta \sum_x \sum_{\mu<\nu} \mathrm{Re}[U_\mu(x) U_\nu(x+\hat{\nu}) U_\mu^*(x+\hat{\mu}) U_\nu^*(x)] - \gamma \sum_x \sum_\mu \mathrm{Re}[\phi^*(x) U_\mu(x) \phi(x+\hat{\mu})] \\
& + M \sum_x [\overline{\psi}_L(x) \varphi(x) \psi_R(x) + \overline{\psi}_R(x) \varphi^*(x) \psi_L(x)] \\
& - \frac{1}{2} \sum_x \sum_\mu \left[\overline{\psi}_R(x), \overline{\psi}_L(x)\right] \mathbf{D}_{\mu+}(x) \begin{bmatrix} \psi_L(x+\hat{\mu}) \\ \psi_R(x+\hat{\mu}) \end{bmatrix} \\
& - \frac{1}{2} \sum_x \sum_\mu \left[\overline{\psi}_R(x), \overline{\psi}_L(x)\right] \mathbf{D}_{\mu-}(x) \begin{bmatrix} \psi_L(x-\hat{\mu}) \\ \psi_R(x-\hat{\mu}) \end{bmatrix}
\end{aligned}
$$

with

$$
|\phi(x)| = 1 \quad , \quad \varphi(x) = \phi^2(x)
$$

Chiral I

$$\mathbf{D}_{\mu+}(x) = \begin{bmatrix} \frac{1}{2}r[\varphi^*(x)U_\mu(x) + U^*_\mu(x)\varphi^*(x+\hat{\mu})] & -\eta^R_\mu U^*_\mu(x) \\ -\eta^L_\mu U_\mu(x) & \frac{1}{2}r[\varphi(x)U^*_\mu(x) + U_\mu(x)\varphi(x+\hat{\mu})] \end{bmatrix}$$

$$\mathbf{D}_{\mu-}(x) = \begin{bmatrix} \frac{1}{2}r[\varphi^*(x)U^*_\mu(x-\hat{\mu}) + U_\mu(x-\hat{\mu})\varphi^*(x-\hat{\mu})] & \eta^R_\mu U_\mu(x-\hat{\mu}) \\ \eta^L_\mu U^*_\mu(x-\hat{\mu}) & \frac{1}{2}r[\varphi(x)U_\mu(x-\hat{\mu}) + U^*_\mu(x-\hat{\mu})\varphi(x-\hat{\mu})] \end{bmatrix}$$

Here we have defined

$$\begin{aligned} \eta^R_k &= -\eta^L_k = -i\sigma_k \quad (k = 1, 2, 3) \\ \eta^R_4 &= \eta^L_4 = \mathbb{1}_2 \end{aligned}$$

The diagonal terms in $\mathbf{D}_{\mu\pm}(x)$ are analogous to Wilson non-local mass terms. This particular choice is not unique, e.g. in a different construction one can dispense with link variable.

The left and right-handed fermion operators transform differently under a U(1) gauge transformations $g(x) = \exp(i\theta(x))$, which transform fields according to

$$\begin{aligned} \psi_L(x) &\rightarrow g(x)\psi_L(x) \quad , \quad \psi_R(x) \rightarrow g^*(x)\psi_R(x) \\ \overline{\psi}_L(x) &\rightarrow g^*(x)\overline{\psi}_L(x) \quad , \quad \overline{\psi}_R(x) \rightarrow g(x)\overline{\psi}_R(x) \\ \phi(x) &\rightarrow g(x)\phi(x) \quad , \quad \varphi(x) \rightarrow g^2(x)\varphi(x) \\ U_\mu(x) &\rightarrow g(x)U_\mu(x)g^*(x+\hat{\mu}) \end{aligned}$$

Chiral II

Regarding the local "mass" term in the action

$$S_M = M \sum_x [\overline{\psi}_L(x)\varphi(x)\psi_R(x) + \overline{\psi}_R(x)\varphi^*(x)\psi_L(x)]$$

as a vertex between, e.g., a right-handed fermion and a composite left-handed fermion + Higgs state of the same U(1) charge, then we may construct $q = \pm 1$ massive fermions from a combination of the corresponding local operators,

$$\begin{aligned} a^\dagger(x) &= \frac{1}{\sqrt{2m}}(\overline{\psi}_L(x)\varphi(x) + \overline{\psi}_R(x)) \quad , \quad a(x) = \frac{1}{\sqrt{2m}}(\psi_R(x) + \psi_L(x)\varphi^\dagger(x)) \\ b^\dagger(x) &= \frac{1}{\sqrt{2m}}(\psi_L(x)\varphi^\dagger(x) - \psi_R(x)) \quad , \quad b(x) = \frac{1}{\sqrt{2m}}(\overline{\psi}_R(x) - \overline{\psi}_L(x)\varphi(x)) \end{aligned}$$

In the same way one can construct operators transforming covariantly with opposite charge, by combining the right instead of left-handed fermion operators with the squared Higgs field.

Chiral III

Construct a set of states which span a small subspace of the Hilbert space containing a static fermion-antifermion pair:

$$|\Phi_i(R)\rangle = \{a^\dagger(\mathbf{x})\zeta_i^*(\mathbf{x};U)\}\,\{b^\dagger(\mathbf{y})\zeta_i(\mathbf{y};U)\}\,|\Psi_0\rangle$$

with

$$\zeta_i(\mathbf{x};U) = \left\{ \begin{array}{cl} \xi_i(\mathbf{x};U) & i \le n_{ev} \\ \varphi(x)\xi^*_{i-n_{ev}}(\mathbf{x};U) & n_{ev}+1 \le i \le 2n_{ev} \\ \phi(\mathbf{x}) & i = 2n_{ev}+1 \end{array} \right. .$$

where the $\{\xi_i,\ i=1,2,..,n_{ev}\}$ are eigenstates of the covariant Laplacian.

Then proceed as before, looking for the ground and excited states. This time we compute the expectation value of products of $\mathbf{D}_\pm$, rather than simply Wilson lines at $\mathbf{x}$, $\mathbf{y}$.

Chiral IV

Explicitly, we compute numerically

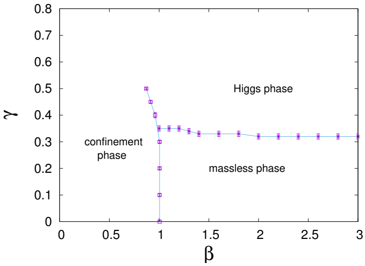
$$[\mathcal{T}^T]_{ji}(R) \quad = \quad \langle \Phi_j | \mathcal{T}^T | \Phi_i \rangle = \langle Q_{ji}^{+T}(\mathbf{x}, t) Q_{ji}^{-T}(\mathbf{y}, t) \rangle$$

where

$$\begin{aligned}
Q_{ji}^{+T}(\mathbf{x}, t) \quad &= \quad \Big[\zeta_i(\mathbf{x}, t), -\varphi^*(\mathbf{x}, t) \zeta_i(\mathbf{x}, t) \Big] \, \mathbf{D}_{4+}(\mathbf{x}, t) \\
&\quad \left(\prod_{\tau=1}^{T-1} \mathbf{F}(\mathbf{x}, t+\tau) \mathbf{D}_{4+}(\mathbf{x}, t+\tau) \right) \begin{bmatrix} \varphi(\mathbf{x}, t+T) \zeta_j^*(\mathbf{x}, t+T) \\ -\zeta_j^*(\mathbf{x}, t+T) \end{bmatrix} \\
Q_{ji}^{-T}(\mathbf{x}, t) \quad &= \quad \Big[\zeta_j(\mathbf{x}, t+T), \varphi^*(\mathbf{x}, t+T) \zeta_j(\mathbf{x}, t+T) \Big] [\, \mathbf{D}_{4-}(\mathbf{x}, t+T) \\
&\quad \left(\prod_{\tau=1}^{T-1} \mathbf{F}(\mathbf{x}, t+T-\tau) \mathbf{D}_{4-}(\mathbf{x}, t+T-\tau) \right) \begin{bmatrix} \varphi(\mathbf{x}, t) \zeta_i^*(\mathbf{x}, t) \\ \zeta_i^*(\mathbf{x}, t) \end{bmatrix} \\
\mathbf{F}(x) \quad &= \quad \begin{bmatrix} \varphi(x) & 0 \\ 0 & \varphi^*(x) \end{bmatrix}
\end{aligned}$$

Chiral V

The (thermodynamic) phase diagram of U(1) gauge Higgs theory with a $q = 1$ Higgs field

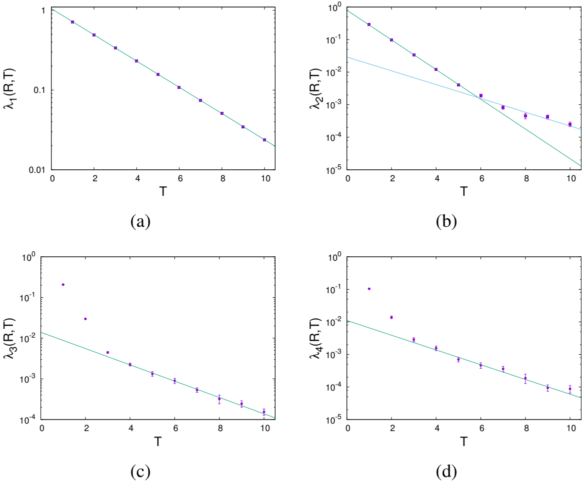


The numerical simulation is carried out on a $14^3 \times 32$ lattice volume at $\beta = 3.0,\ \gamma = 1.0$ using $n_{ev} = 4$ Laplacian eigenstates.

Results

In the figures, $\lambda_n(R,T) \equiv \mathcal{T}_{nn}(R,T)$.

- E_1 is derived from λ_1.
- E_2 is derived from either λ_3 or λ_4 in the large $T \geq 4$ range.
- E_3 is derived from λ_2 in the small $T \leq 5$ range.



Nature of the ground state

The fact that $\mathcal{T}_{11}(R,T)$ fits, almost exactly, a single exponential, means that $\Psi_1(R,T)$ is very nearly an exact eigenstate if $\mathcal{T}$ in the *full* Hilbert space.

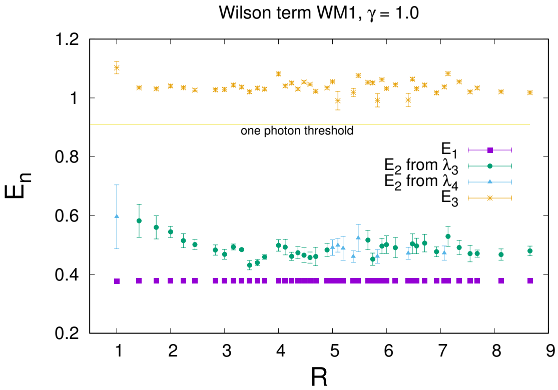
Recall that

$$\Phi_9(R;\psi,\overline{\psi},U) = \{a^\dagger(\mathbf{x})\phi_i^*(\mathbf{x};U)\}\ \{b^\dagger(\mathbf{y})\phi(\mathbf{y};U)\}\ \Psi_0(U)\ ,$$

We can compute the overlap

$$f = \frac{|\langle\Phi_9(R)|\Psi_1(R,T)\rangle|^2}{\langle\Phi_9(R)|\Phi_9(R)\rangle}$$

It is found that this is with 0.1% of unity. The ground state is the state envisaged by Frohlich, Morcio, and Marchetti, i.e. a simple (and neutral) combination of the quark and Higgs fields. What is new is the excitations.



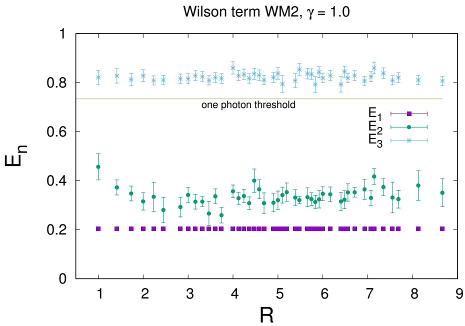
Energies E_1, E_2, E_3 vs. R at $\beta = 3, \gamma = 1$, shown together with the one photon threshold.

Altermate non-local mass terms

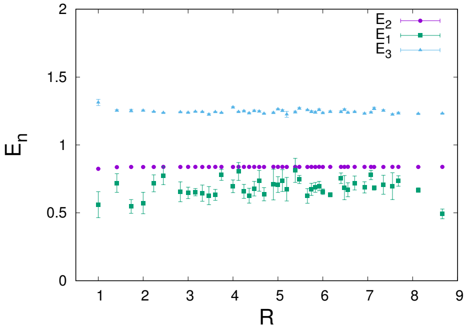
There are many (infinite) other choices of the non-local mass term. One choice, which avoids the use of a gauge link variable, it to replace the diagonal parts of the $D_{\mu\pm}$ term by

$$\begin{bmatrix} r\phi^*(x)\phi^*(x\pm\hat{\mu}) & \\ & r\phi(x)\phi(x\pm\hat{\mu}) \end{bmatrix} .$$

We get about the same result:



At $\beta = 3.0$, the transition to the massless phase is at $\gamma = 0.32$. A surprise is that if we reduce γ to $\gamma = 0.5$, the energies derived from T_{11} and T_{44} "change places'."



Also the overlap f between Ψ_1 and Φ_9 falls to ≈ 0.89.

Conclusions

- The gauge+Higgs fields surrounded a charged static fermion have a spectrum of localized excitations, which cannot be interpreted as simply the ground state plus some massive bosons.
- *This means that charged "elementary" particles can have a mass spectrum in gauge Higgs theories.*
- This conclusion seems robust. We see it in SU(3), abelian Higgs, Landau-Ginzburg, and chiral U(1) models.
- Observable? Maybe in ARPES studies of conventional superconductors? Core electron spectra above and below the transition temperature?
- Electroweak theory? Excitations of quarks and leptons?? (we'll see...)