Effective string description of the confining potential in the $(2+1)$ dimensional $SU(2)$ Lattice Gauge Theory.[1]

Michele Caselle

Universitá degli Studi di Torino

Confinement21 06/08/2021

[1] F. Caristo, M. Caselle, N. Magnoli, A. Nada, M. Panero, A. Smecca in preparation.

Summary:

1. Introduction and motivations
2. Main Results
3. Interquark Potential and Deconfinement Transition
4. Conclusions

Introduction and motivations

- The long distance behaviour of the interquark potential at zero temperature in pure LGTs is well described by the Nambu-Goto Effective String Theory (EST)
- At shorter distances and/or higher temperature corrections beyond the Nambu-Goto action are expected, their study is of great importance to understand confinement: Nambu-Goto can be considered as a sort of mean field model for the interquark potential, the fine details of the gauge theory (the gauge group, the physical mechanisms behind confinement, the physical degrees of freedoms which originate the EST) are encoded in these higher order terms.
- At zero temperature these corrections are very difficult to identify since they are shadowed by the "boundary term". This problem can be circumvented by studying the behaviour of the interquark potential at high temperature, in the vicinity of the deconfinement transition (but still in the confining phase). It can be shown that in this regime the boundary term becomes subleading and the EST corrections beyond Nambu-Goto can be detected with high precision Montecarlo simulations.

Introduction and motivations

- A major advantage of studying the interquark potential in the high T regime is that, thanks to the Svetitsky-Yaffe correspondence we have also another independent way to model the intequark potential, as the spin-spin correlator of the spin model in one dimension less and symmetry group the center of the original gauge group
- A perfect laboratory to address this issue is the (2+1) dimensional SU(2) model, which can be simulated at a reasonable cost and is mapped by the Svetitsky-Yaffe correspondence into the 2d Ising model, which is solved exactly.
- In particular the Polyakov loop correlator from which the interquark potential is extracted is mapped into the spin-spin correlator which is exactly known for the Ising model in d=2.

Main Results

- The Svetitsky-Yaffe correspondence works also outside the critical point, at least down to $T \sim 0.8Tc$, both for $R < \xi$ and for $R > \xi$
- For $R > \xi$ the 2d Ising correlator coincides with the EST prediction (but not with Nambu-Goto) and both agree with the data
- For $R < \xi$ the 2d Ising correlator coincides with the Conformal Perturbation Theory (CPT) predictions and perfectly agrees with the data. The deviation from the Nambu-Goto prediction increases as R decreases.
- The deviations with respect to Nambu-Goto can be better appreciated looking at the T dependence of the energy levels and a reliable estimate of the first correction to Nambu-Goto can be extracted from the data. As expected from consistency constraints of the EST, this correction is proportional to $1/(\sigma^3 N_t^7)$.

Interquark Potential and Deconfinement Transition

- For some choices of the gauge group the finite temperature Deconfinement Transition of a pure Yang-Mills Theory is critical and is described by a CFT. This is the case, in particular for the (2+1) dimensional SU(2) LGT whose deconfinement transition belongs to the same universality class of the 2d Ising model
- The order parameter of the transition is the Polyakov loop

$$P(x,y) = \mathrm{Tr} \prod_{0 \le t < N_t} U_0(x,y,t\,a)\ .$$

where $N_t a = 1/T$ is the length of the lattice in the compactified direction and is associated to the inverse temperature of the system.

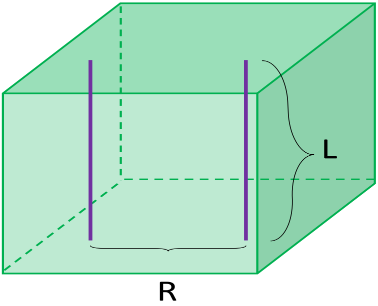
- The interquark potential can be extracted from the correlator of two Polyakov loops:

$$V(R,T) = -\frac{1}{aN_t} \log < P(0)P(R)^\dagger >$$

- This correlator in the vicinity of the critical point is equivalent to the spin-spin correlator of the 2d Ising model $\langle \sigma(r)\sigma(0) \rangle_t$.
- The confining phase is the analogous of the *symmetric* phase of the perturbed CFT (i.e the high T phase of the Ising realization, even if for the Yang-Mills theory it corresponds to the low T regime!!).

Polyakov loop correlator.

Expectation value of two Polyakov loops at distance R and Temperature $T = 1/L$



$$V(R,T) = -\frac{1}{L}\log < P(0)P(R)^{\dagger} >$$

$\langle s(0)s(R)\rangle$ correlator in the 2d Ising model

For the 2d Ising model, thanks to the exact integrability, the $\langle s(0)s(R)\rangle$ correlator is known exactly (apart from the non-universal normalization constants k_1 and k_2):

- For $R < \xi$ it coincides with the Conformal Perturbation result:

$$\langle s(0)s(R)\rangle = \frac{k_1}{R^{\frac{1}{4}}}\left[1 + \frac{t}{2}\ln\left(\frac{e^{\gamma_E}t}{8}\right) + \frac{1}{16}t^2 + \frac{1}{32}t^3\ln\left(\frac{e^{\gamma_E}t}{8}\right) + O(t^4)\right]$$

where $t = \frac{R}{\xi}$

- For $R > \xi$ is the typical expression for a free isolated particle in $d = 2$

$$\langle s(0)s(r)\rangle = k_2 K_0(R/\xi)$$

where K_0 denotes the modified Bessel function: $K_0(x) \sim \frac{e^{-x}}{\sqrt{x}}$ for large x

- Notice the shift from $R^{-\frac{1}{4}}$ at short distance to $R^{-\frac{1}{2}}$ at large distance

Effective String Theory

- For a generic Poincaré invariant EST in $D = 2+1$, we have[1] for $R >> N_t$

$$\langle P(0)P^\dagger(R)\rangle = \sum_{n=0}^{\infty} |v_n(N_t)|^2 \frac{E_n}{\pi} K_0(E_n R)$$

- In the large R limit the sum is dominated by the lowest state and from the exponential decay of the modified Bessel function we may extract the correlation length $E_0 = \frac{1}{\xi}$

- For the Nambu-Goto string

$$\langle P(0)P^\dagger(R)\rangle = \sum_{n=0}^{\infty} \omega_n N_t \sqrt{\frac{\sigma}{\pi}} K_0(E_n R)$$

with $E_n = \sigma N_t \sqrt{1 + \frac{8\pi}{\sigma N_t^2}\left(n - \frac{1}{24}\right)} \Longrightarrow E_0 = \sigma N_t \sqrt{1 - \frac{\pi}{3\sigma N_t^2}}$

- Corrections to the Nambu-Goto action can appear only at the order $\frac{1}{\sigma^3 N_t^7}$.

[1] M. Lüscher and P. Weisz, *JHEP* 2004.07(2004), p. 014

Geometrical description.

An intuitive geometrical description of this result is obtained writing the effective action as the most general mapping fulfilling Poincare' invariance in the target manifold:

$$X^\mu : \mathcal{M} \to \mathbb{R}^D, \qquad \mu = 0, \cdots, D-1$$

- $\mathcal{M}$: two-dimensional surface describing the worldsheet of the string
- $\mathbb{R}^D$: (flat) D dimensional target space $\mathbb{R}^D$ of the gauge theory.

Main Result [1] : "**Low energy universality**"

- The first few terms of the action compatible with Poincaré and parity invariance are suitable combinations of geometric invariants constructed from the induced metric $g_{\alpha\beta} = \partial_\alpha X^\mu \partial_\beta X_\mu$.
- These terms can be classified according to their weight, i.e. the difference between the number of derivatives minus the number of fields X^μ.

[1] M. Lüscher and P. Weisz, JHEP **0407** (2004) 014
O. Aharony and Z. Komargodski, JHEP **1305** (2013) 118
S. Dubovsky, R. Flauger and V. Gorbenko, JHEP **1209** (2012) 044

Geometrical description.

- The only term of weight zero is the Nambu-Goto action

$$S_{\text{NG}} = \sigma \int d^2\xi \sqrt{g} \ ,$$

where $g \equiv \det(g_{\alpha\beta})$.
- This term has a natural geometric interpretation: it measures the area swept out by the worldsheet in space-time.
- Fixing the physical gauge one finds (choosing an euclidean metric)

$$S = \sigma \int d^2\xi \sqrt{det(\eta_{\alpha\beta} + \partial_\alpha X \cdot \partial_\beta X)}$$

$$\sim \sigma RT + \frac{\sigma}{2} \int d^2\xi \left[\partial_\alpha X \cdot \partial^\alpha X + \frac{1}{8}(\partial_\alpha X \cdot \partial^\alpha X)^2 - \frac{1}{4}(\partial_\alpha X \cdot \partial_\beta X)^2 + \dots \right] ,$$

Geometrical description.

- At weight two, two new contributions appear:

$$S_{2,\mathcal{R}} = \gamma \int d^2\xi \sqrt{g}\mathcal{R} \ ,$$
$$S_{2,K} = \alpha \int d^2\xi \sqrt{g} K^2,$$

where $\mathcal{R}$ denotes the Ricci scalar constructed from the induced metric, and $K \equiv \Delta(g)X$ is the extrinsic curvature, where $\Delta(g)$ is the Laplacian in the space with metric $g_{\alpha\beta}$.

However both these terms can be neglected!

- $\mathcal{R}$ is topological in two dimensions and, since in the long string limit in which we are interested we do not expect topologically changing fluctuations, its contribution is constant and can be neglected.
- K^2 is proportional to the equation of motion of the Nambu-Goto Lagrangian and can be eliminated by a suitable field redefinition.

Geometrical description.

Thus the first non trivial terms appear at level four and contribute to the interquark potential with terms proportional to $1/N_t^7 \rightarrow$ **Low Energy Universality**

The coefficient of this term: γ_4 is a new non-universal constant which together with σ defines the EST and likely depends on the peculiar features of the model: gauge group, space time dimensions ...

A strong consistency check of the whole picture is that γ_4 is a dimensionful quantity and should scale as σ^3 thus, the adimensional ratio $k_c \equiv \gamma_4 \sigma^3$ should not change with β as we approach the continuum limit.

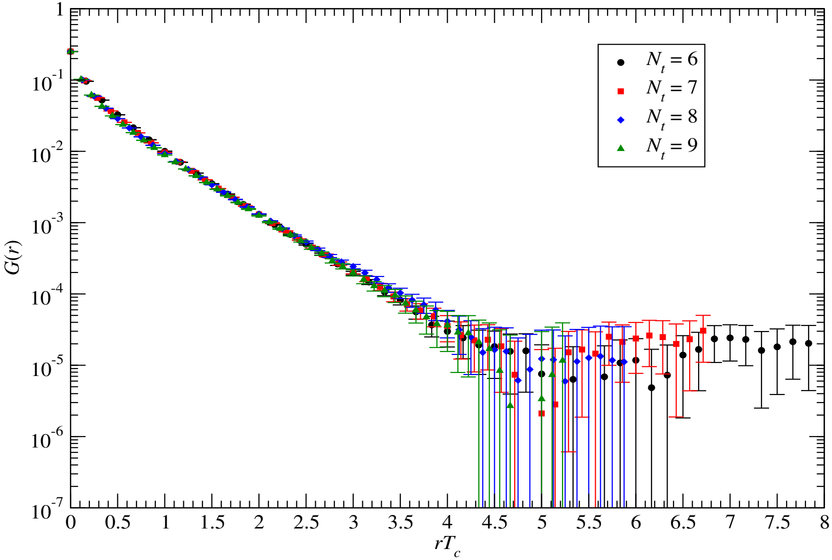
SU(2) LGT in (2+1) dimensions.

Summary of the simulations

$N_t \times N_s^2$	β	T/T_c	n_{conf}
9×96^2	11.3048	0.80	2.5×10^5
	11.72873	0.83	2.5×10^5
	12.15266	0.86	2.5×10^5
	12.57659	0.89	2.5×10^5
	13.00052	0.92	2.5×10^5
	13.42445	0.95	2.5×10^5
	13.84838	0.98	2.5×10^5
	14.131	1.00	1.0×10^5
7×96^2	9.228023	0.83	2.5×10^5
	9.561566	0.86	2.5×10^5
	9.895109	0.89	2.5×10^5
	10.562195	0.95	2.5×10^5
	10.895738	0.98	2.5×10^5
	11.1181	1.00	1.0×10^5

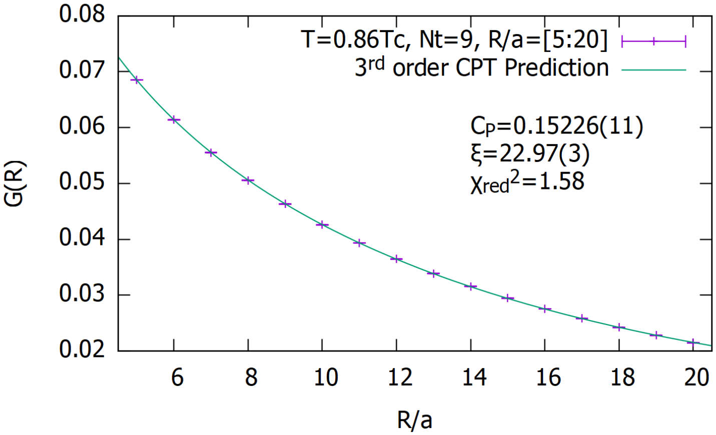
$N_t \times N_s^2$	β	T/T_c	n_{conf}
8×96^2	10.10736	0.80	2.5×10^5
	10.486386	0.83	2.5×10^5
	10.865412	0.86	2.5×10^5
	11.244438	0.89	2.5×10^5
	11.623462	0.92	2.5×10^5
	12.00249	0.95	2.5×10^5
	12.381516	0.98	2.5×10^5
	12.6342	1.00	1.0×10^5
6×96^2	8.258494	0.86	2.5×10^5
	8.546581	0.89	2.5×10^5
	8.834668	0.92	2.5×10^5
	9.122755	0.95	2.5×10^5
	9.410842	0.98	2.5×10^5
	9.6029	1.00	1.0×10^5

SU(2) LGT in (2+1) dimensions: Scaling test



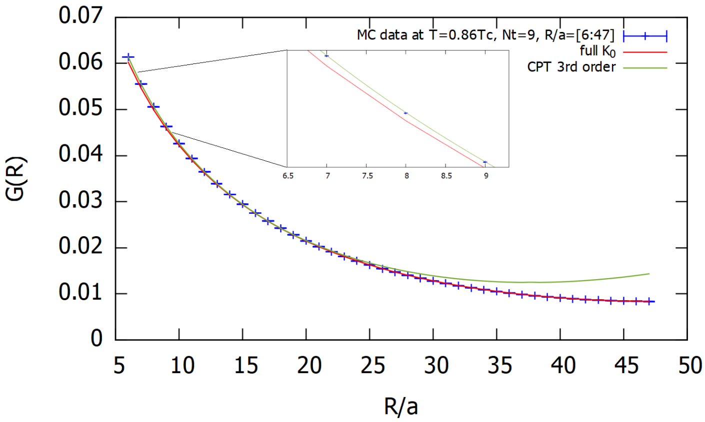
Testing CPT Predictions

- $f(x) = \frac{C_P}{x^{\frac{1}{4}}}\left[1 + \frac{x}{2\xi}\ln\left(\frac{e^{\gamma_E}x}{8\xi}\right) + \frac{1}{16}\left(\frac{x}{\xi}\right)^2 + \frac{1}{32}\left(\frac{x}{\xi}\right)^3\ln\left(\frac{e^{\gamma_E}x}{8\xi}\right)\right]$



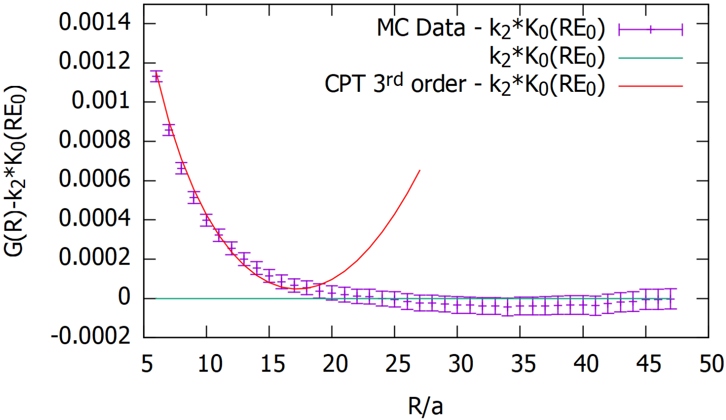
CPT vs EST

- $f(x) = k_2 \cdot K_0(b \cdot x) \Longrightarrow \frac{\xi}{a}\Big|_{EST} = 23.3(8)$
  $\chi^2_{red} = 1.14$ in the range $R/a = [15:47]$

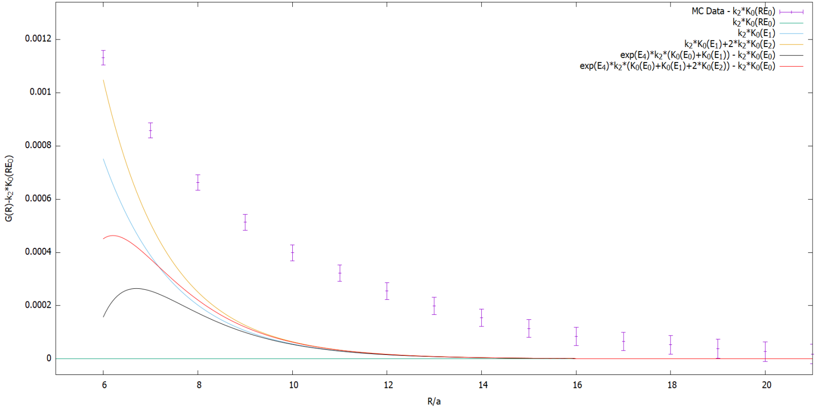


CPT vs EST: Zoom

To see the deviations from the Nambu-Goto EST prediction at short distances we subtract the NG EST from the Montecarlo data:



Adding further masses of the Nambu-Goto spectrum does not fill the gap



Deviations from Nambu-Goto

- Deviations from Nambu-Goto can be better appreciated looking at the T dependence of E_0 at fixed β. We chose two values of β: $\beta = 9$ and $\beta = 12.15266$ and simulated several values of N_t.

- The correlation length diverges at the critical point as $\xi \sim |T - T_c|^{-\nu}$. Two predictions for the critical exponent:
  1. Nambu-Goto: $\nu = 1/2$

$$E_0 = \sigma N_t \sqrt{1 - \frac{\pi}{3\sigma N_t^2}} \to \xi = \xi_0 \sqrt{1 - T^2/T_c^2} = \xi_0 \sqrt{1 - \frac{N_{t,c}}{N_t}}$$

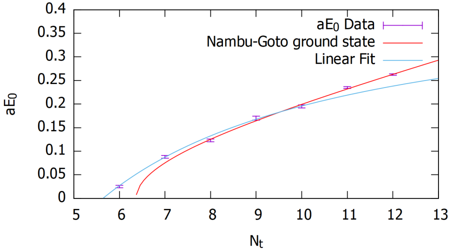
  2. Svetitsky-Yaffe conjecture: $\nu = 1$

- For the linear divergent behaviour we fit the string ground state with the function

$$f(N_t) = k_s \left(1 - \frac{N_{t,c}}{N_t}\right)$$

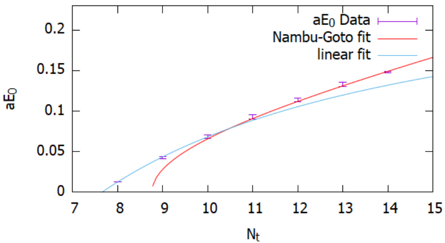
Deviations From Nambu-Goto

At $\beta = 9$:



And $N_{t,c} = 5.64(2)$

At $\beta = 12.15266$:



And $N_{t,c} = 7.65(1)$

Deviations from Nambu-Goto

- Due to the Poincarè invariance and to consistency requirements, the first correction to the Nambu-Goto energy spectrum is expected to be of the order $\sim \frac{1}{\sigma^3 N_t^7}$

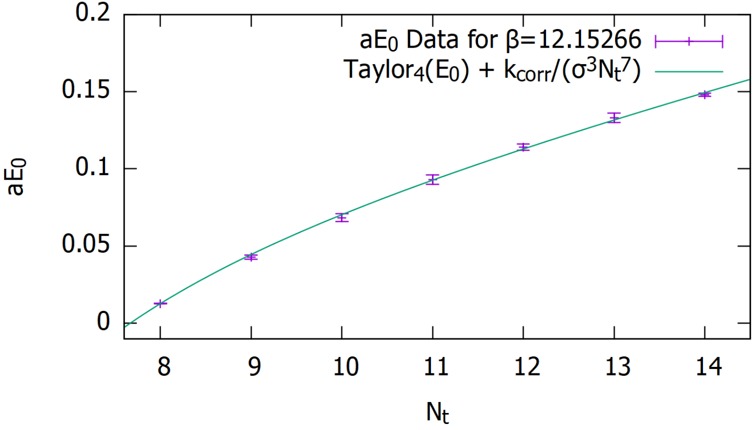
- We fit the temperature dependence of the string ground state E_0 with the function

  $f(N_t) = \text{Taylor}_4(E_{0,NG}) + \frac{k_c}{\sigma^3 N_t^7}$
  where:
  $\text{Taylor}_4(E_{0,NG}) = a^2\sigma N_t - \frac{\pi}{6N_t} - \frac{\pi^2}{72(a^2\sigma)N_t^3} - \frac{\pi^3}{432(a^2\sigma)^2 N_t^5} - \frac{5\pi^4}{10368(a^2\sigma)^3 N_t^7}$

- We get
  1. For $\beta = 9$: $k_c = 0.030(15)$ and $\chi^2_{red} = 1.85$
  2. For $\beta = 12.15266$: $k_c = 0.0490(14)$ and $\chi^2_{red} = 0.72$

- The two values for k_c are almost compatible within the errors: the correction shows the correct scaling behaviour in the continuum limit!

Deviations from Nambu-Goto



Conclusions

- The interquark potential in the SU(2) LGT in (2+1) dimensions near the deconfinement transition is perfectly fitted by the 2d Ising spin-spin correlator in the whole range of distances that we studied, down to $0.8T_c$.
- For $R > \xi$ the 2d Ising correlator coincides with the EST prediction (but not with Nambu-Goto) and both agree with the data
- For $R < \xi$ the 2d Ising correlator coincides with the Conformal Perturbation prediction which can be used even beyond the Ising universality class and perfectly agrees with the data.
- The deviations with respect to Nambu-Goto can be better appreciated looking at the T dependence of the energy levels. A reliable estimate of the first correction to Nambu-Goto can be extracted from the data. As expected from consistency constraints of the EST, this correction is proportional to $1/(\sigma^3 N_t^7)$.
- Thanks to CPT and EST most of these results could be extended to other gauge groups and to the $(3+1)$ dimensions.

Acknowledgements

Collaborators:

Fabrizio Caristo[◇]**,**

Alessandro Nada[◇]**,**

Marco Panero[◇]**,**

Antonio Smecca[◇]**,**

Nicodemo Magnoli[*],

[◇] Dipartimento di Fisica, Università di Torino

[*] Dipartimento di Fisica, Università di Genova