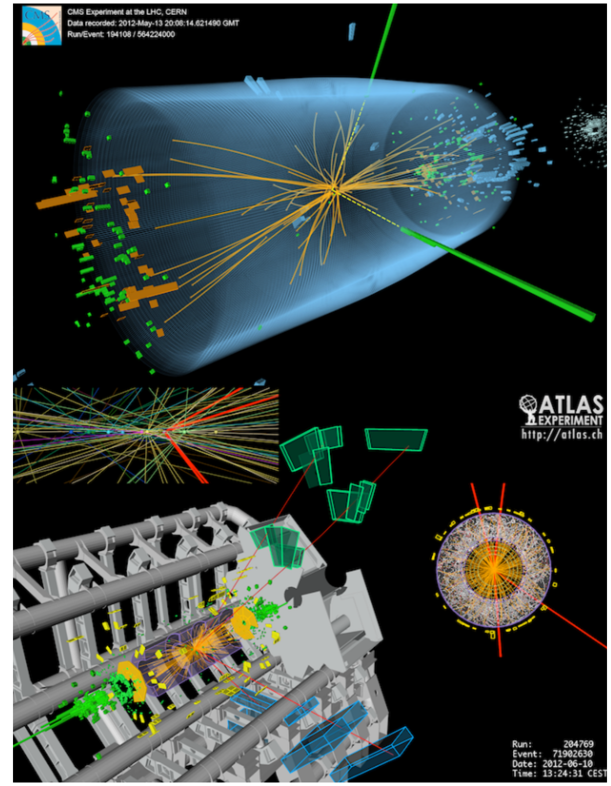
Hadron Cosmological Constant and Origin of Proton Mass

arXiv:2103.15768

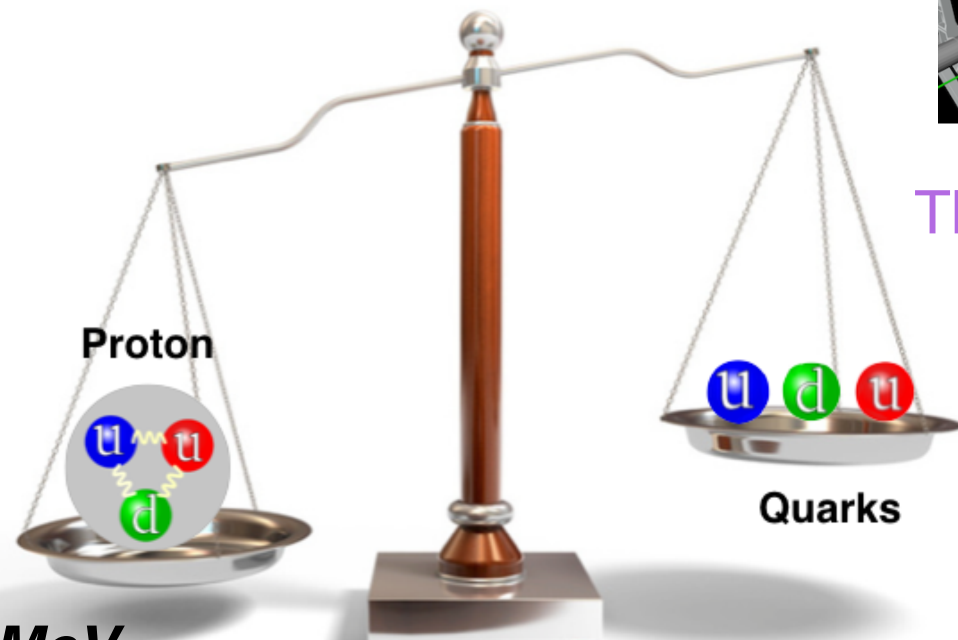
- Mass and Rest Energy
- Rest Energy (Mass) Decomposition from Hamiltonian
- Gravitational Form Factors
- Trace anomaly and cosmological constant

vConf21
Aug. 2, 2021

Motivation



Where does the proton mass come from, and how ?



The Higgs boson make the u/d quark having masses (2GeV MS-bar):

$m_u = 2.08(9)$ *MeV*
$m_d = 4.73(12)$ *MeV*

Laiho, Lunghi, & Van de Water, Phys.Rev.D81:034503,2010

But the mass of the proton is

938.272046(21) MeV.

~**100 times of the sum of the quark masses!**

Mass and Rest Energy

- $E = m c^2$ ⟹ m increases with E? converting mass to energy?
- $E_0 = m c^2$ (Einstein 1905, $m^2 = E^2 - p^2$)
- $e^+ e^- \to \gamma\gamma$ ($m_{\gamma\gamma} = 2m_e$)
- E and p are additive, not mass
- In general relativity, the gravitational field is coupled to the EMT.
- In non-relativistic limit, Newton's law of force and universal gravitational involves E_0 or mass.
- Inertial mass and gravitational mass are the same mass.
- Relativistic mass in a misnomer, rest mass is redundant.
- -- L.B. Okun doi:10.1134/1.1358478

Rest Energy from Hamiltonian

- Separate the EMT into traceless part and trace part

$$T_R^{\mu\nu} = \overline{T}_R^{\mu\nu} + \frac{1}{4}\eta^{\mu\nu}(T_\rho^\rho)_R$$

- Hamiltonian -- $H = \int d^3\vec{x}\, T^{00}(x)$

$$H_q(\mu) = \int d^3\vec{x}\, (\frac{i}{4}\sum_f \bar{\psi}_f \gamma^{\{0} \overleftrightarrow{D}^{0\}} \psi_f - \frac{1}{4} T^{\mu}_{q\,\mu})_R,$$

Quark momentum fraction

$$H_g(\mu) = \int d^3\vec{x}\, \frac{1}{2}(B^2 + E^2)_R,$$

Glue momentum fraction

$$H_{tr} = \int d^3\vec{x}\, \frac{1}{4}(T^\mu_\mu)_R.$$

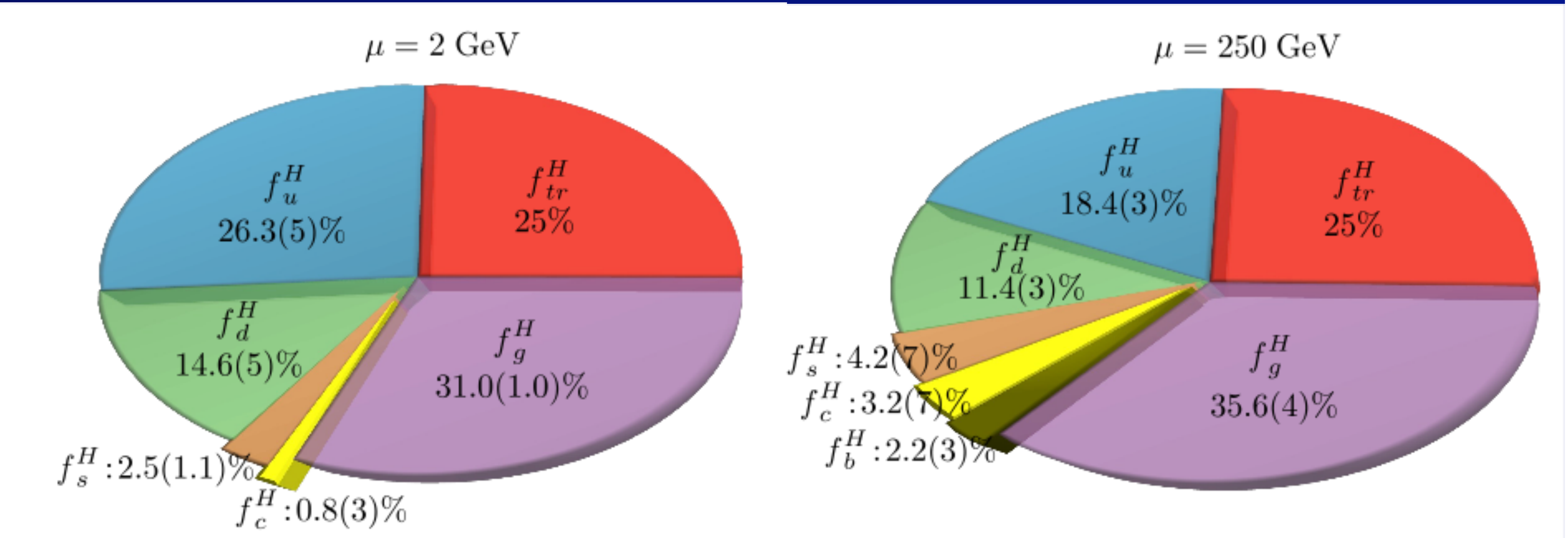
- Rest energy -- $E_0 = M = \langle H_{q_f}(\mu)\rangle + \langle H_g(\mu)\rangle + \langle H_{tr}\rangle,$

$$\langle H_{q_f}(\mu)\rangle = \frac{3}{4}\sum_f \langle x\rangle_f(\mu) M, \qquad \langle H_g(\mu)\rangle = \frac{3}{4}\langle x\rangle_g(\mu) M,$$

$$\langle H_{tr}\rangle = \frac{1}{4} M.$$

<x> - momentum fraction

Rest Energy Decomposition from Hamiltonian



$$f_f^H = \langle H_q \rangle / M = \frac{3}{4} \langle x \rangle_f(\mu), \qquad f_g^H = \langle H_g \rangle / M = \frac{3}{4} \langle x \rangle_g(\mu),$$

$$f_{tr}^H = \langle H_{tr} \rangle / M = \frac{1}{4}$$

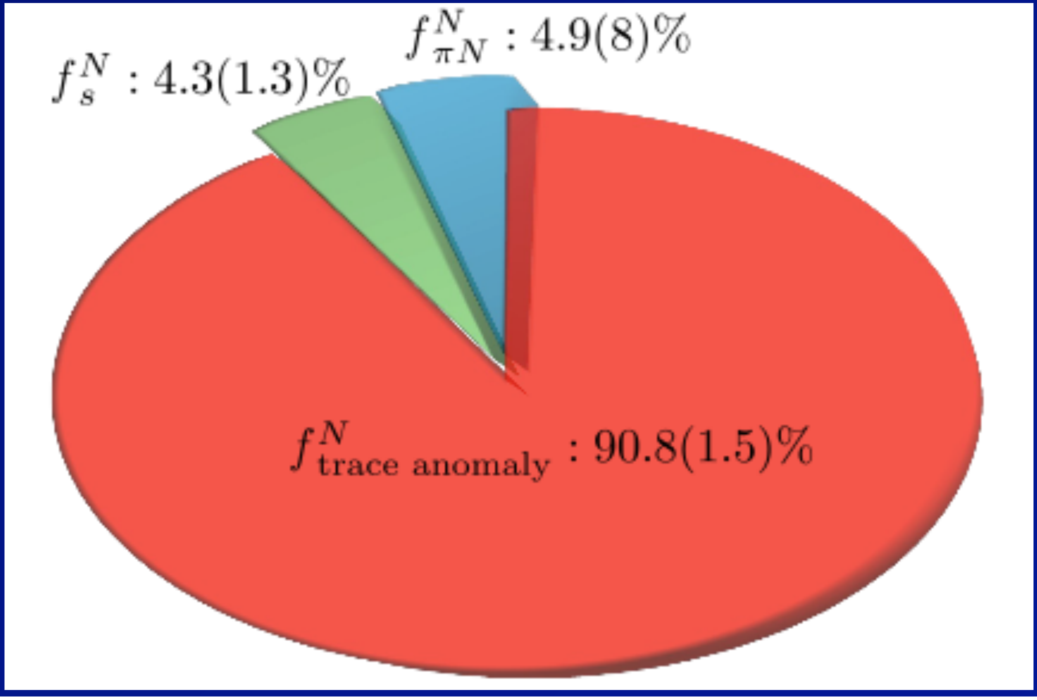
Momentum fractions from CT18 (T.J. Hou et al, PRD, arXiv:1912.10053) at $\mu = 2$ GeV and 250 GeV.

Trace of EMT (¼ of Hadron Mass)

- Trace of EMT – scalar, frame independent, RG invariant

$$T^\mu_\mu = \sum_f m_f \bar{\psi}_f \psi_f + \left[\sum_f m_f \gamma_m(g) \bar{\psi}_f \psi_f + \frac{\beta(g)}{2g} F^{\alpha\beta} F_{\alpha\beta} \right]$$

- Lattice calculation of of quark condensate
  - Y.B. Yang et al (χ QCD) [arXiv: 1511.15089]
  - Overlap fermion (Z_m Z_s =1)
  - 3 lattices (one at physical m_π), systematics (volume, continuum)



πN sigma term

$$\sigma_{\pi N} = \frac{m_u + m_d}{2} \langle P | \bar{u}u + \bar{d}d | P \rangle$$

Strangeness sigma term

$$\sigma_s = m_s \langle P | \bar{s}s | P \rangle$$

$$f^N_{\pi N} = \frac{\sigma_{\pi N}}{M_N}, \qquad f^N_s = \frac{\sigma_s}{M_N}$$

Gravitational FF

- Gravitational Form factors from the EMT matrix elements

$$\langle P'|(T^{\mu\nu}_{q,g})_R(\mu)|P\rangle/2M_N = \bar{u}(P')[T_{1_{q,g}}(q^2,\mu)\gamma^{(\mu}\bar{P}^{\nu)} + T_{2_{q,g}}(q^2,\mu)\frac{\bar{P}^{(\mu}i\sigma^{\nu)\alpha}q_\alpha}{2M_N}$$
$$+ D_{q,g}(q^2,\mu)\frac{q^\mu q^\nu - g^{\mu\nu}q^2}{M_N} + \bar{C}_{q,g}(q^2,\mu)M_N\eta^{\mu\nu}]u(P)$$

– T_1 and T_2

$$T_{1_{q,g}}(0) = \langle x\rangle_{q,g}(\mu); \qquad \langle x\rangle_q(\mu) + \langle x\rangle_g(\mu) = 1$$ [Ji]

$$T_{1_{q,g}}(0) + T_{2_{q,g}}(0) = 2J_{g,g}(\mu); \qquad 2J_q(\mu) + 2J_g(\mu) = 1$$

– D term: deformation of space = elastic property - [Polyakov]

– C term: pressure - [Lorce]

$$\bar{C}_q + \bar{C}_g = 0, \qquad \partial_\nu T^{\mu\nu} = 0$$

Rest Energy from Gravitational FF

- What are $\bar{C}_q$ and $\bar{C}_g$?

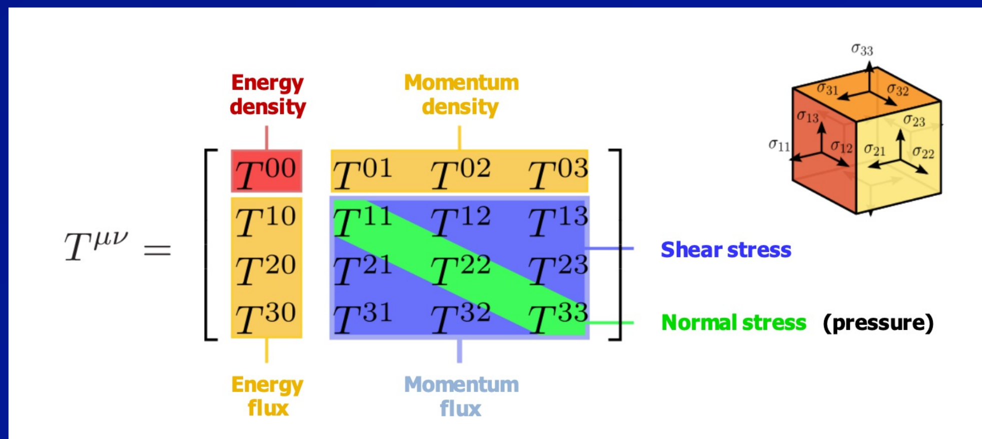
$$\langle P|(T^{00}_{q,g})_{\rm R}(\mu)|P\rangle|_{\vec{P}=0}/2M_N = \langle x\rangle_{q,g}(\mu)M_N + \bar{C}_{q,g}(0,\mu)M_N,$$
$$\langle P|(T^{ii}_{q,g})_{\rm R}(\mu)|P\rangle|_{\vec{P}=0}/2M_N = 3\bar{C}_{q,g}(0,\mu)M_N$$

Note: Being scale dependent, separate quark and glue T^{00} are renormalized and mixed.

$$3\bar{C}_{q,g}(0,\mu)M_N = [\langle P|\eta_{\mu\nu}(T^{\mu\nu}_{q,g})_{RM}|P\rangle - \langle P|(T^{00}_{q,g})_{RM}(\mu)|P\rangle]/2M_N$$

$$\bar{C}_q(0,\mu) = \frac{1}{4}\sum_f (f^N_f - \langle x\rangle_f(\mu)),$$
$$\bar{C}_g(0,\mu) = \frac{1}{4}(f^N_a - \langle x\rangle_g(\mu))$$

$\langle P|(T^{00}_{q,g})_{\rm R}(\mu)|P\rangle|_{\vec{P}=0}/2M_N$ is the same as from the Hamiltonian



Trace Anomaly and Cosmological Constant

- What is trace anomaly? What dynamical role does it play, if any?
  - Nucleon as a statistical system
  - Canonical partition function in Euclidean space

KFL, hep-lat/0202026

$$Z_{GC}(V,T,\mu) = \int \mathcal{D}U\mathcal{D}\bar{\psi}\mathcal{D}\psi\, e^{-S_G(U)-S_F(U,\bar{\psi},\psi,\mu)}$$

$$Z_C(V,T,n_B) = 1/2\pi \int_0^{2\pi} d\phi e^{-in_B\phi} Z_{GC}(V,T,\mu)|_{\mu=i\phi T}$$

  - Nucleon Mass at T → 0 (t → ∞ in Z_{GC})

$$\mu(n_B) = -\frac{1}{\beta}\ln\frac{Z_C(n_B+1)}{Z_C(n_B)} = \frac{F_{n_B+1}-F_{n_B}}{(n_B+1)-n_B}$$

- Nucleon is a bubble in the sea of gluon condensate

$$\langle H_a \rangle = \epsilon_{vac} V,$$

where,

$$\epsilon_{vac} = -\frac{\beta(g)}{2g}\langle 0|F^{\alpha\beta}F_{\alpha\beta}|0\rangle > 0$$

Trace Anomaly and Cosmological Constant

- Pressure of anomaly:

$$d\langle H_a\rangle = -P_{vac}\,dV\,(dQ = T\,dS = 0), \qquad P_{vac} = -\epsilon_{vac} < 0$$

- Quark and glue energy

$$\langle H_E(\mu)\rangle + \langle H_g(\mu)\rangle \propto V^P$$

- Volume dependence of total rest energy

$$E_0 = \epsilon_{vac}V + \epsilon_{mat}V^p$$

$$-\frac{dE_0}{dV} = P_{vac} + P_k = -\epsilon_{vac} + p\,\epsilon_{mat}V^{p-1} = 0$$

- E_0 = d E_S (d=4) ⇒ p = -1/3 (MIT bag model, E_0 = BV+ $\Sigma_{q,g}$/R)
- Rest energy as the sum of scalar trace and tensor traceless parts

$$E_0 = E_T + E_S,$$

$$E_T = \langle H_{q_f}(\mu)\rangle + \langle H_g(\mu)\rangle = \frac{3}{4}[\sum_f \langle x\rangle_f(\mu) + \langle x\rangle_g(\mu)]M,$$

$$E_S = \frac{1}{4}[\langle H_m\rangle + \langle H_a\rangle]$$

Trace Anomaly and Cosmological Constant

- Stress-pressure equation

$$\bar{C}_q + \bar{C}_g = 0 \Rightarrow P_{\text{total}} = -\frac{d\,E_0}{d\,V} = -\frac{E_S}{V} + \frac{1}{3}\frac{E_T}{V} = 0$$

- $E_S \propto V$, $E_T \propto V^{1/3}$

$$-\frac{E_S}{V} \to -\epsilon_{\text{vac}}$$

- Vacuum energy density is indeed a constant which is like the cosmological constant in the $g^{\mu\nu}$ term as Einstein introduced for a static universe.

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} + \Lambda\, g_{\mu\nu} = 8\pi G\, T_{\mu\nu} \qquad \Lambda = 4\pi G \rho$$

- Freidman equation for the accelerating expansion of the universe

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P)$$

String tension in charmonium

- Heavy quarkonium is confined by a linear potential.
- Constant vacuum energy density and flux tube

$$V(r) = |\epsilon_{vac}|\, A\, r = \sigma\, r$$

- Infinitely heavy quark with Wilson loop

$$V(r) + r\frac{\partial V(r)}{\partial r} = \frac{\langle \frac{\beta}{2g}(\int d^3\vec{x}\, F^2)\, W_L(r,T)\rangle}{\langle W_L(r,T)\rangle}.$$

- For charmonium

$$2\sigma\langle r\rangle = \langle H_\beta\rangle_{\bar{c}c} = \frac{\langle \bar{c}c|\frac{\beta}{2g}\int d^3\vec{x}\, F^2|\bar{c}c\rangle}{\langle \bar{c}c|\bar{c}c\rangle}$$

$$\langle H_\beta\rangle_{\bar{c}c} = M_{\bar{c}c} - (1+\gamma_m)\langle H_m\rangle_{\bar{c}c}.$$

- Lattice calculation of charmonium (W. Sun et al.,2012.06228)
- $\langle H_\beta\rangle_{\bar{c}c} = 199\,\mathrm{MeV}$ → σ = 0.153 GeV^2
- Cornell potential fit of charmonium → σ = 0.164(11) GeV^2

Mateu:2018zym

Summary and Challenges

- Proton Rest energy (mass) components (scale dependent):
  - Gravitational form factors (related to momentum fraction)
  - Hamiltonian (quark KE+PE, glue field energy, quark mass, anomaly)
- $m_q \leftarrow$ Higgs mechanism
- Quark condensate ← chiral symmetry breaking
- Trace anomaly ← conformal symmetry breaking
- Conformal window with multi-flavors or different gauge group
- Conformal phase at finite temperature and density
  - A. Alexandru and I. Horvath [arXiv:1906.08047]
- Nuclei
- String theory invented in hadron physic finds its home in quantum gravity.
- Cosmological constant introduced in general relativity applies naturally to hadron physics.

Rest Energy from Hamiltonian

- Energy momentum tensor (EMT)

$$T_{\mu\nu} = \frac{1}{4}\bar{\psi}\gamma_{(\mu}\overleftrightarrow{D}_{\nu)}\psi + G_{\mu\alpha}G_{\nu\alpha} - \frac{1}{4}\delta_{\mu\nu}G^2$$

- Separate the EMT into traceless part and trace part

$$T_R^{\mu\nu} = \overline{T}_R^{\mu\nu} + \frac{1}{4}\eta^{\mu\nu}(T_\rho^\rho)_R$$

X. Ji (1995)

- Hamiltonian -- $H = \int d^3\vec{x}\, T^{00}(x)$

- With equation of motion (scale dependent)

$$H_m = \int d^3\vec{x} \sum_f m_f \bar{\psi}_f \psi_f,$$

$$H_E(\mu) = \int d^3\vec{x} \sum_f (\psi_f^\dagger i\vec{\alpha}\cdot\vec{D}\psi_f)_M,$$

Quark kinetic and potential energy

$$H_g(\mu) = \int d^3\vec{x}\, \frac{1}{2}(B^2+E^2)_M,$$

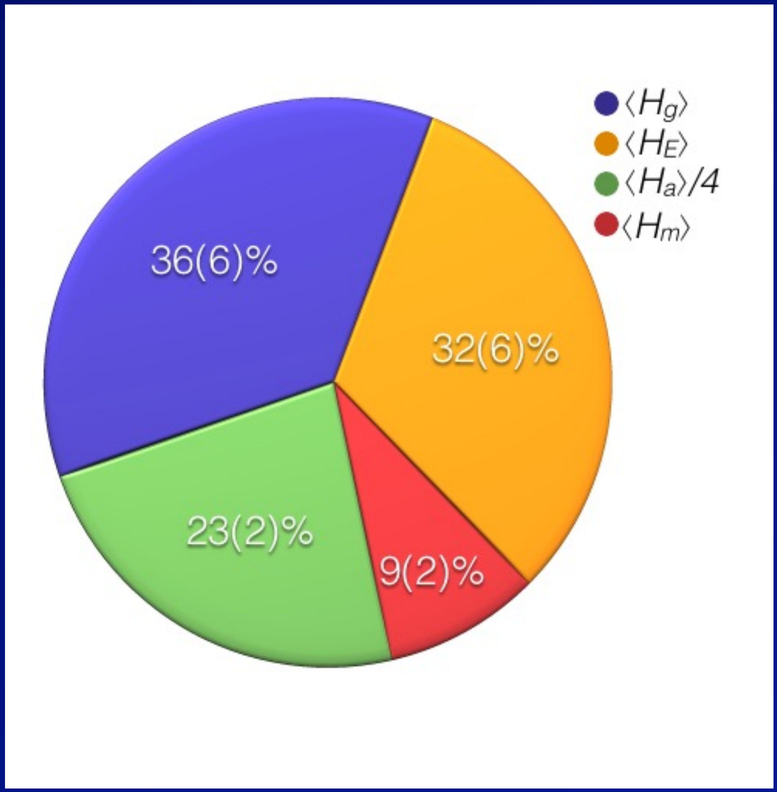
Glue field energy

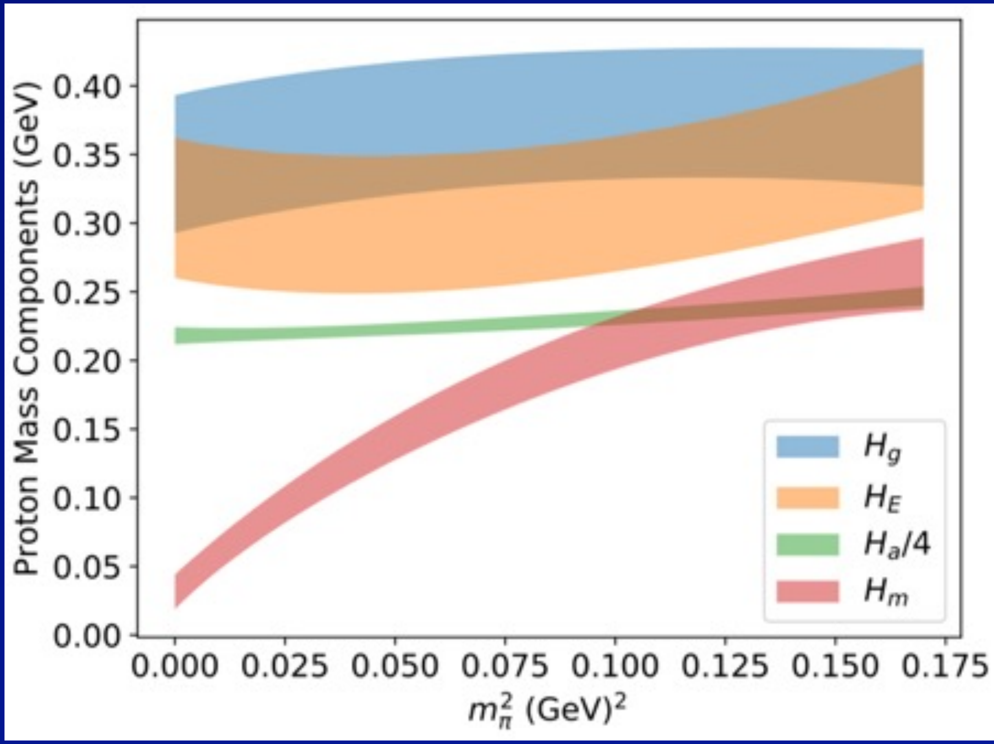
$$H_{tr} = \int d^3\vec{x}\, \frac{1}{4}(T_\mu^\mu)_R.$$

$$E_0 = M = \langle H_M\rangle + \langle H_E(\mu)\rangle + \langle H_g(\mu)\rangle + \langle H_{tr}\rangle,$$

Proton Mass Decomposition

Lattice calculation with systematics (physical pion mass, continuum, infinite volume extrapolations, renormalization)





Y.B. Yang et al (χQCD), PRL 121, 212001 (2018)
Physic 11, 118 (2018); ScienceNews, Nov. 16 (2018)

EIC Insights

- $$\partial_\nu T^{\mu\nu} = 0 \longrightarrow \bar{C}_q(t) + \bar{C}_g(t) = 0$$
- This leads to

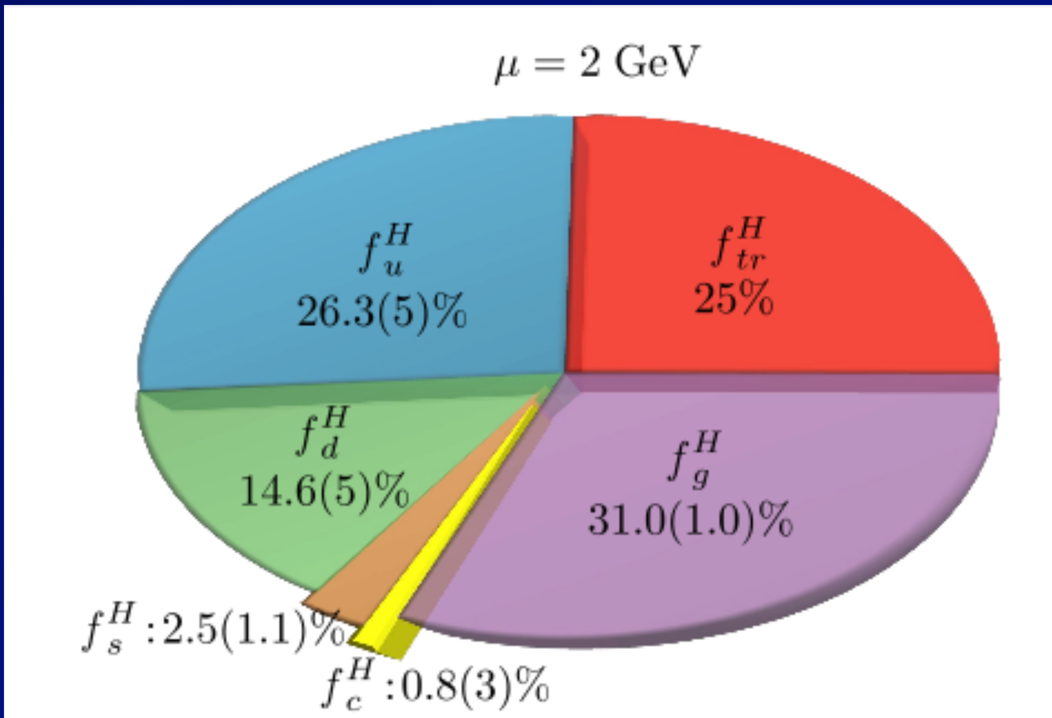
$$T_{1_q}(t) + T_{1_g}(t) - (f^N_{\pi N}(t) + f^N_s(t) + f^N_a(t)) = 0$$

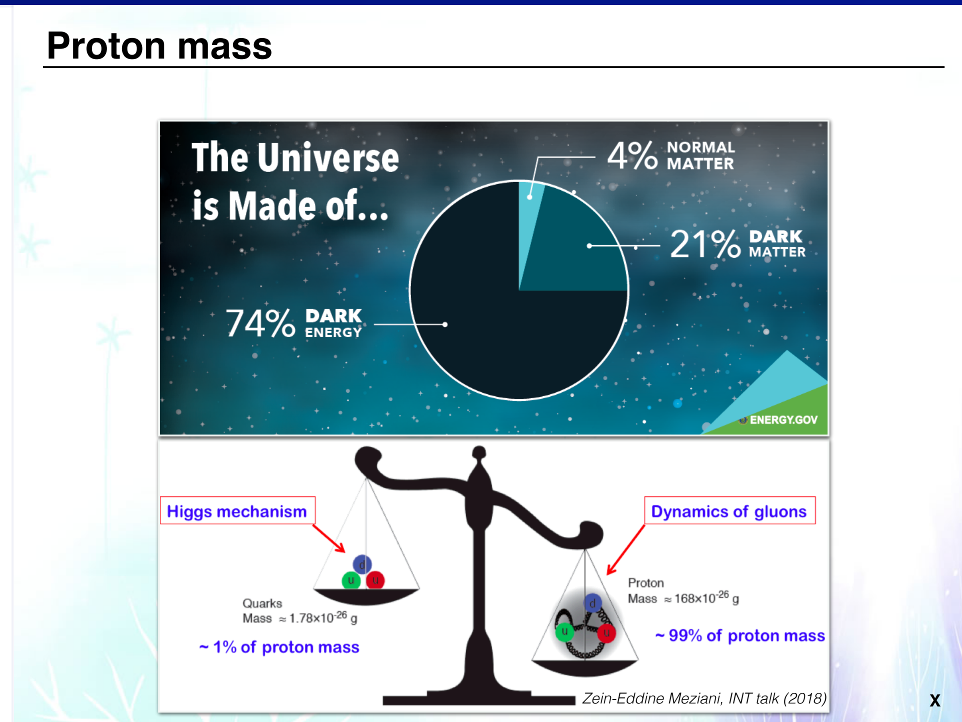
- $T_{1_{q,g}}(t)$ from GPD (X. Ji)

$$T_{1_{q,g}}(t) = \int dx\, x H_{q,g}(x,0,t)$$

- $f^N_a(t)$ from photo production of charmonium and bottomonium at threshold (D. Kharzeev)
- $f^N_{\pi N}(t), f^N_s(t)$ from lattice calculations

Where does the proton mass come from?





3/4 comes from the quark and glue energies (kinetic, potential, field).
1/4 comes from the quark masses and vacuum energy.

Rest Energy from Hamiltonian

- Separate out H_m from equation of motion

$$H = H_m + H_E(\mu) + H_g(\mu) + \frac{1}{4}H_a$$

X. Ji

$$\begin{aligned} H_m &= \int d^3x \sum_f m_f \bar{\psi}_f \psi_f - \text{quark mass} \\ H_E(\mu) &= \sum_f (\psi_f^\dagger i\vec{\alpha}\cdot\vec{D}\psi_f)_{\rm M} + \left[-Z_{gq}H_m + \frac{4}{3}Z_{qg}H_g(\mu_r)\right] - \text{quark energy} \\ H_g(\mu) &= \int d^3x \,\frac{1}{2}(B^2+E^2)_{\rm M} - - \text{glue field energy.} \end{aligned}$$

$$\begin{aligned} f_f^E &= \langle H_{E_f}(\mu)\rangle / M_N = \frac{3}{4}(\langle x\rangle_{q_f} - f_f^N) \\ f_g^E &= \langle H_g\rangle / M_N = \frac{3}{4}\langle x\rangle_g . \end{aligned}$$

More scheme dependence

Quark and Glue Components of Hadron Mass and Rest Energy

- Energy momentum tensor (EMT)

$$T_{\mu\nu} = \frac{1}{4}\bar{\psi}\gamma_{(\mu}\overleftrightarrow{D}_{\nu)}\psi + G_{\mu\alpha}G_{\nu\alpha} - \frac{1}{4}\delta_{\mu\nu}G^2$$

- Mass from trace of EMT (trace anomaly) – scalar, frame independent, components are RG invariant

$$T^\mu_\mu = \sum_f m_f \bar{\psi}_f \psi_f + \left[\sum_f m_f \gamma_m(g)\bar{\psi}_f\psi_f + \frac{\beta(g)}{2g}F^{\alpha\beta}F_{\alpha\beta}\right]$$

Chanowitz, Ellis, Crewther, Collin, Duncan, Joglekar

$$\langle P|T^\mu_\mu|P\rangle = 2(E^2 - P^2) = 2M_N^2$$

$$(T^{\mu\nu})_R = T^{\mu\nu}, \qquad \partial_\nu T^{\mu\nu} = 0$$

- Rest energy from EMT – quark and glue components are frame and scale dependent

$$\langle P|T^{\mu\nu}|P\rangle = 2P^\mu P^\nu$$

Normalization of EMT Trace

- Mass from trace of EMT – scalar, frame independent, components are RG invariant

$$T^\mu_\mu = \sum_f m_f \bar{\psi}_f \psi_f + \left[\sum_f m_f \gamma_m(g) \bar{\psi}_f \psi_f + \frac{\beta(g)}{2g} F^{\alpha\beta} F_{\alpha\beta} \right]$$

- However $\langle P | T^\mu_\mu | P \rangle = 2(E^2 - P^2) = 2M_N^2$ not M_N.
- Expectation value -- frame dependent, mass not additive

$$\frac{\langle P | \int d^3\vec{x}\, T^\mu_\mu(x) | P \rangle}{\langle P | P \rangle} = \frac{M_N^2}{P^0}, \qquad \langle P | P \rangle = (2\pi)^3 2P^0 \delta^3(0)$$

- Rest frame $\displaystyle \frac{\langle P | \int d^3\vec{x}\, T^\mu_\mu(x) | P \rangle}{\langle P | P \rangle} \Big|_{\vec{P}=0} = M_N$

- Proper volume – same as in rest frame

$$\frac{\langle P | \int d^3\vec{x}\, \gamma\, T^\mu_\mu(x) | P \rangle}{\langle P | P \rangle} = M_N, \qquad \gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

Rest Energy from Gravitational FF

- Interpretation in terms of perfect liquid (Lorce)

$$\langle P|(T^{\mu\nu}_{q,g})_{\rm M}(\mu)|P\rangle/2M_N \;=\; T_{1_{q,g}} P^\mu P^\nu/M_N + \bar{C}_{q,g}(0,\mu)\,\eta^{\mu\nu}\,M_N\,,$$
$$T^{\mu\nu} \;=\; (\epsilon+p)u^\mu u^\nu - p\,\eta^{\mu\nu}$$

- Identify

$$\epsilon_{q,g} \equiv \left[T_{1_{q,g}}(0) + \bar{C}_{q,g}(0)\right]\frac{M}{V}, \qquad p_{q,g} \equiv -\bar{C}_{q,g}(0)\frac{M}{V}$$

- Internal energy and pressure-volume work

$$U_q \;=\; \epsilon_q V = \left[\langle x\rangle_q + \bar{C}_q(0)\right]M = \left[3/4\langle x\rangle_q + 1/4\sum_f f_f^N\right]M,$$
$$U_g \;=\; \epsilon_g V = \left[\langle x\rangle_g + \bar{C}_g(0)\right]M = \left[3/4\langle x\rangle_q + 1/4 f_a^N\right]M$$
$$W_{q,g} \;=\; p_{q,g}V = -\bar{C}_{q,g}(0)M$$

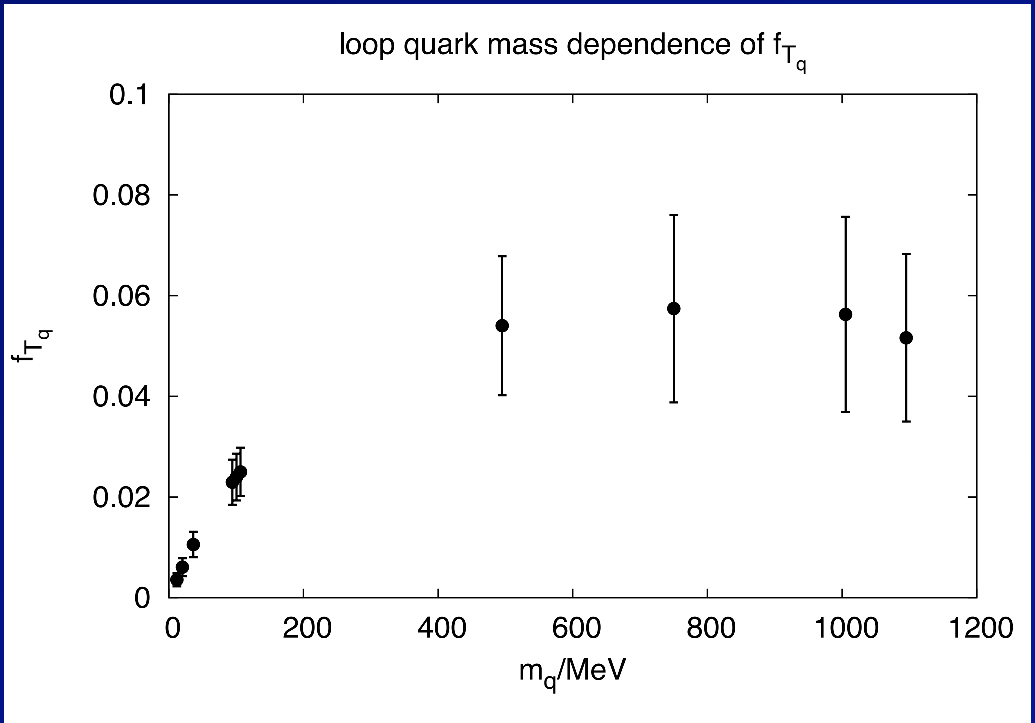
- Total energy and pressure-volume work

$$E_0 \;=\; U_q + U_g,$$
$$W \;=\; W_q + W_g = 0$$

→ Same as from Hamiltonian

Heavy Quarks

- At electroweak scale, the standard model includes Higgs, t, b, c quarks in external states
  - M. Gong et al (χ QCD) [arXiv: 1304.1191]



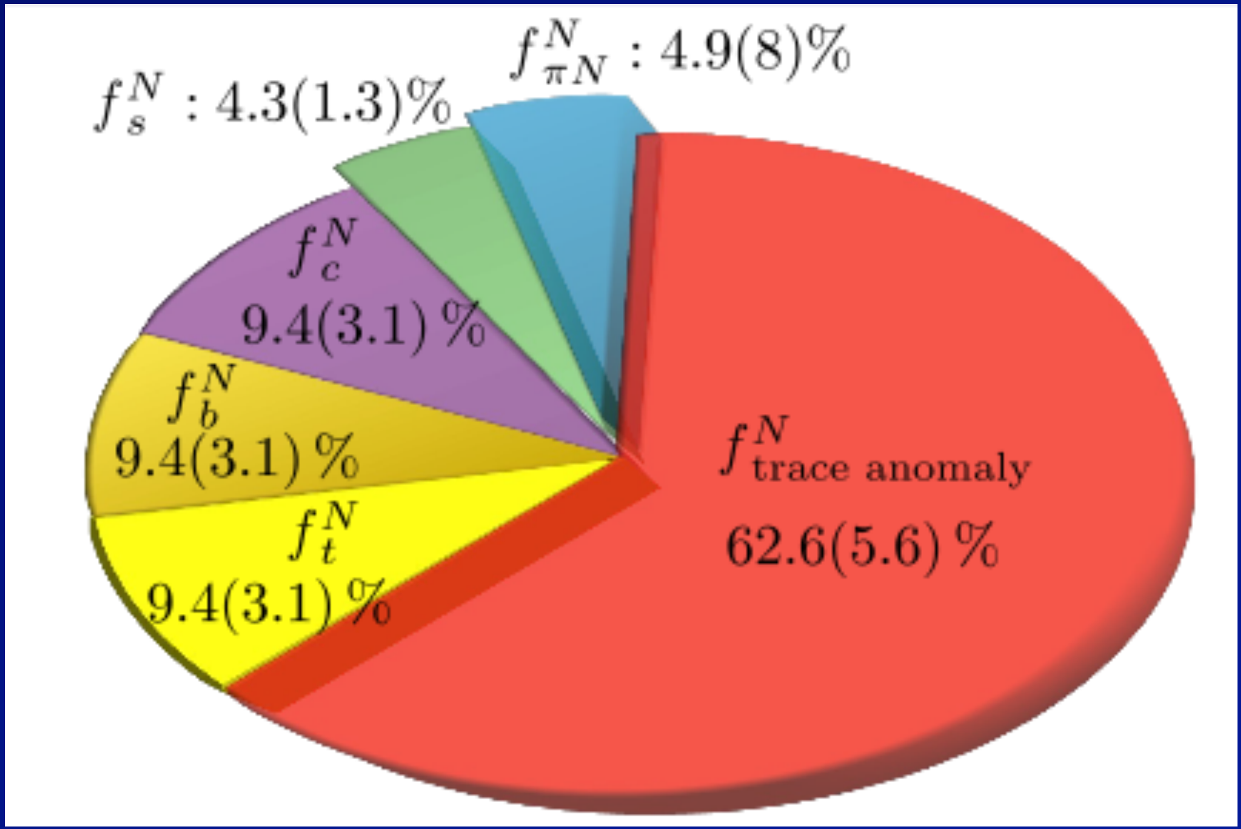
For m_q > ~ 500 MeV, $m_f \langle N|\bar{\psi}_f \psi_f|N\rangle \sim \text{constant}$

Heavy Quark Sigma Terms

- M. A. Shifman, A. Vainshtein, and V. I. Zakharov, Phys.Lett. B 78, 443 (1978) -- heavy quark expansion

$$m_h\langle N|\bar{\psi}_h\psi_h|N\rangle \sim -\frac{n_f}{3}\frac{\alpha_s}{4\pi}\langle N|G^2|N\rangle + \mathcal{O}(1/m_h)$$

$$\frac{\beta(g)}{2g} = -\frac{\beta_0}{2}\left(\frac{\alpha_s}{4\pi}\right) - \frac{\beta_1}{2}\left(\frac{\alpha_s}{4\pi}\right)^2 - \frac{\beta_2}{2}\left(\frac{\alpha_s}{4\pi}\right)^3 + \ldots \qquad \beta_0 = 11 - \frac{2}{3}n_f$$



Higgs coupling in dark matter search

$$f_{f=c,b,t}^N = \frac{m_f\langle P|\bar{\psi}_f\psi_f|P\rangle}{M_N}$$

Decoupling theorem:

$$f_c^N + f_b^N + f_t^N + f_a^N \sim \sum_H \mathcal{O}_H(1/m_H)$$