Confinement and Chiral Symmetry in the $SU(3)$ Instanton-dyon Ensemble

Quark Confinement and the Hadron Spectrum 2021 (Stavanger)

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Pure $SU(3)$: 2102.11321 [hep-ph]; Two-flavor QCD: in progress
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Outline

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Instantons

- Instantons are 4D topological solitons related to tunneling between vacua of different N_{CS}
- Instantons generate chiral symmetry breaking, but not confinement; monopoles needed
- Instantons do not interact directly with the holonomy and are charge neutral

Instanton-dyons

- At $\langle P \rangle \neq 0$, the instanton is seen to be made of N_c constituents $\rightarrow$ the dyons
- Dyons have non-zero magnetic charge and non-integer topological charge
- Their action $S_i = \frac{8\pi^2}{g^2}\nu_i$ is ≈ 4 near T_c, making them suitable for using semiclassical methods
- Dyon action sets temperature scale $S_0 = \frac{8\pi^2}{g^2} = \frac{11}{3} N_c \ln(T/\Lambda)$

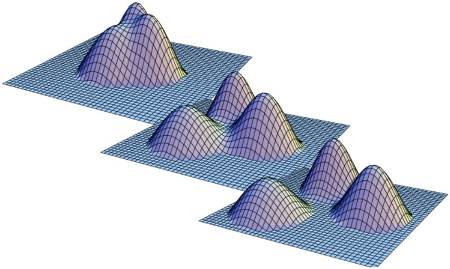
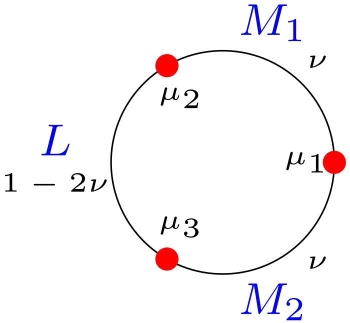


Figure: [Kraan, van Baal (1998)]

Polyakov Loop and Holonomy

- $\langle P \rangle = \frac{1}{3} + \frac{2}{3}\cos(2\pi\nu)$
- $\langle P \rangle = 0$, $\nu = \frac{1}{3} \rightarrow$ confined phase
- Dyons have individual actions $S_{M1} = S_{M2} = S_0\nu$, $S_L = S_0(1-2\nu)$
- Antidyons have the same properties with opposite magnetic and topological charges



Free Energy Density

$$f = \frac{4\pi^2}{3}(2(\nu(1-\nu))^2 + (2\nu(1-2\nu))^2) - 4n_M \ln\left[\frac{d_\nu e}{n_M}\right] - 2n_L \ln\left[\frac{d_{1-2\nu} e}{n_L}\right] + \frac{\ln(8\pi^3 N_M^2 N_L)}{\tilde{V}_3} + \Delta f \tag{1}$$

Compute Δf by Monte-Carlo integration

Minimize $f(T, \nu, n_M, n_L)$ to get $f(T)$, $\nu(T)$, $n_M(T)$, $n_L(T)$

Dyon Interactions

- $\Delta S^{d\bar{d}}_{class} = -\frac{S_0 C_{d\bar{d}}}{2\pi}(\frac{1}{rT} - 2.75\pi\sqrt{\nu_i\nu_j}e^{-1.408\pi\sqrt{\nu_i\nu_j}rT})$
- $C_{d\bar{d}} = 2$ for same kind, -1 for different kinds of dyons
- $\Delta S^{core}_{class} = \frac{\nu V_0}{1+e^{2\pi\nu T(r-r_0)}}$
- Core is used at distances smaller than $x_0 = 2\pi\nu rT$, core radius goes as $1/\nu$
- Diakonov Determinant:

$$G_{im,jn} = \delta_{ij}\delta_{mn}(4\pi\nu_m - \sum_{k\neq i}\frac{2}{T|r_{i,m} - r_{k,m}|} + \sum_{k}\frac{1}{T|r_{i,m} - r_{k,p\neq m}|}) + \frac{2\delta_{mn}}{T|r_{i,m} - r_{j,n}|} - \frac{\delta_{m\neq n}}{T|r_{i,m} - r_{j,n}|}, \quad (2)$$

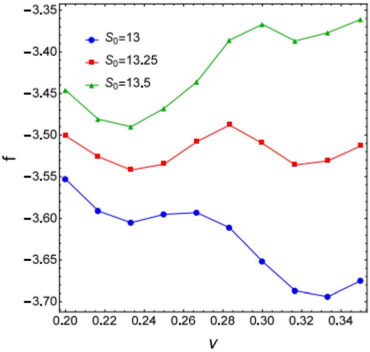
The Simulation Settings

- Compute free energy by standard integration over a dummy parameter in 10 steps $\lambda = 0.1, ...1$
- Perform $\mathcal{O}(50000)$ simulations with different inputs
- For each value of $S_0(T)$, fit to find minimizing input parameters
- Simulations run with $N_D = 120$ dyons in a 3D periodic box setup

	min.	max.	step size	no. of steps
S_0	8	21	1	14
ν	0.1	0.35	$0.01\bar{6}$	16
n_M	0.15	0.6	0.03	16
N_M/N_L	1.0	30.0	varied	16

The First-Order Transition

- Phase transition at $S_0 = 13.18$
- Jumps from $\langle P \rangle = 0$ to $\langle P \rangle \simeq 0.4$ ($\nu = 1/3$ to $\nu \simeq 0.23$)



The First-Order Transition

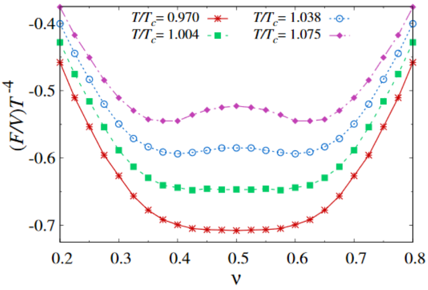
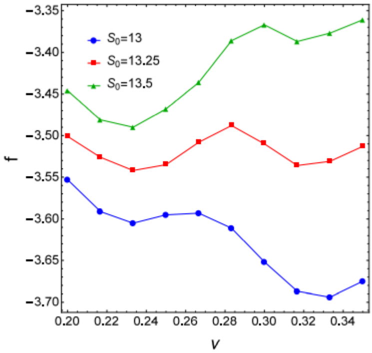


Figure: $SU(2)$ [Lopez-Ruiz, Jiang, Liao (2016)]



Polyakov Loop and T_c

- We can now plot all properties in terms of T/T_c
- Jump in $\langle P \rangle$ nearly identical to lattice
- $\langle P \rangle$ does not continue to grow as quickly as lattice data does

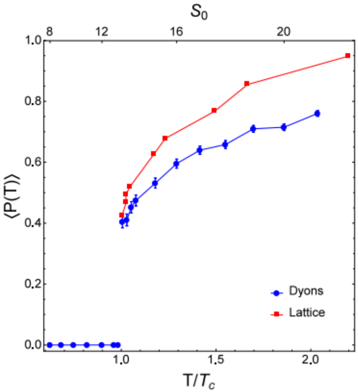
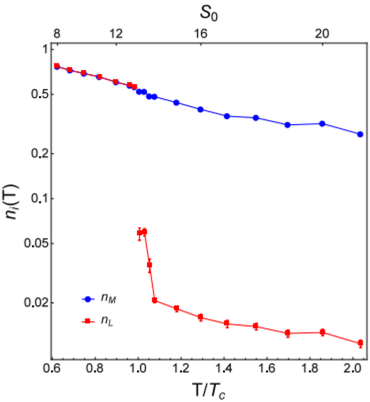


Figure: Lattice data from Kaczmarek et. al. (2002)

Dyon Densities

- M_i dyons: smaller ν and larger core size have opposite effects, resulting in a nearly continuous density
- L dyons: larger value of $1 - 2\nu$ suppresses density

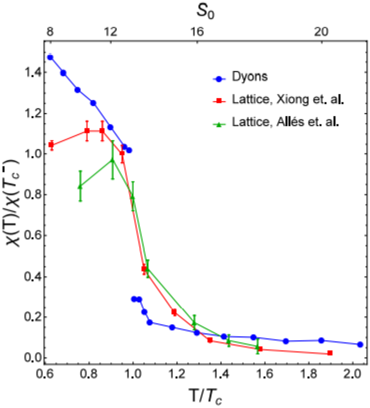


Topological Susceptibility

- Instantons are topologically nontrivial with $Q = \pm 1$
- Dyons carry non-integer charge, (anti)dyons: $Q_i = (-)\nu_i$
- Topological susceptibility $\xi = \langle Q^2 \rangle / V_4$
- Cut the box in half and measure the distribution of charge in that half

Topological Susceptibility

- Dyon model shows a first-order transition, while lattice data show a smooth crossover
- Our ensemble has ~ 40 instantons, much larger than lattice

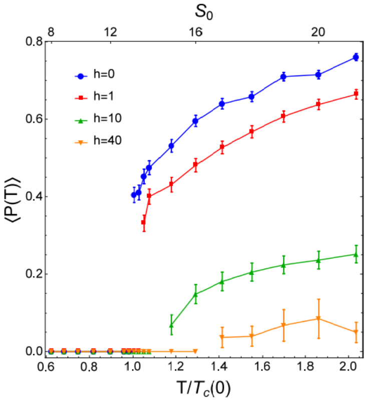


Trace-Deformed Yang-Mills

- $\Delta S_{def} = h \int d^3x |P(\vec{x})|^2$
- Introduced as a way to preserve center symmetry at high temperature
- For a positive h, the deformation term suppresses the value of $\langle P \rangle$ and favors the confined phase
- No need to run any additional MC simulations, the new term $\Delta f_{def} = h \langle P \rangle^2$ can be added and re-fit.

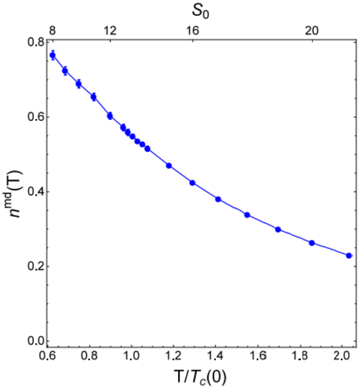
Trace Deformation and the Polyakov Loop

- Increasing h reduces the value of $\langle P \rangle$ and increases T_c
- At $h \sim 80$ the theory is confined at all temperatures we study



The Maximally-deformed Theory

- All dyon densities remain equal and vary smoothly with temperature
- The system remains confining even at low dyon densities

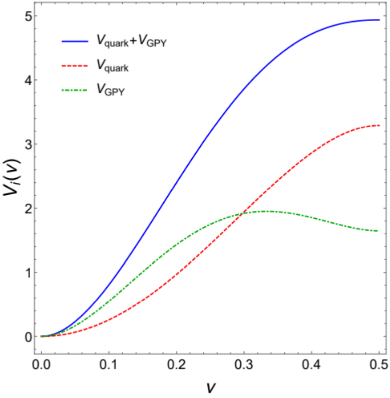


Dyons and Quarks

- We now include $N_f = 2$ flavors of dynamical massless quarks
- Dynamical $\rightarrow$ we include the fermionic determinant in the partition function
- In physical QCD the deconfinement transition is a smooth crossover, unlike the pure $SU(3)$ theory
- We can study the Dirac eigenvalue spectrum and thus the chiral condensate and chiral symmetry

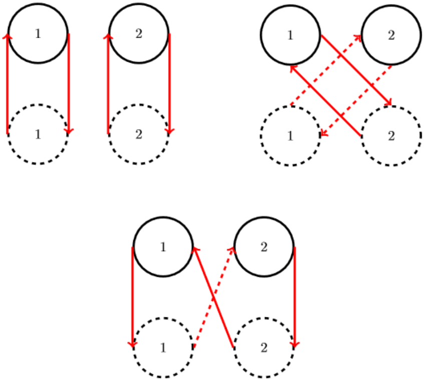
New Partition Function

- Two new terms in our partition function
- Quark perturbative potential $V_{quark} = -N_f \frac{4\pi^2}{3}(2\nu^4 - \nu^2)$
- Fermionic determinant $\det(i\not{D})^{N_f} \simeq \det(\hat{T})^{N_f}$
- Consider only the space spanned by the dyons' zero modes

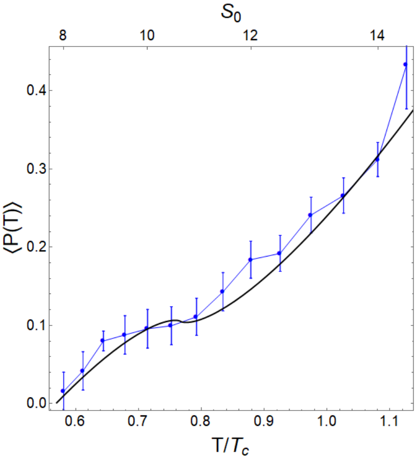


Hopping Matrix

- Quarks can 'hop' from dyon to antidyon with amplitude T_{ij}
- $\hat{T} = \begin{pmatrix} 0 & T_{ij} \\ -T_{ji} & 0 \end{pmatrix}$
- $T_{ij} = \bar{\nu} c' \exp(-\sqrt{11.2 + (\pi \bar{\nu} r T)^2})$
- $\det(\hat{T}) = T_{11}^2 T_{22}^2 + T_{12}^2 T_{21}^2 - 2 T_{11} T_{21} T_{22} T_{12}$



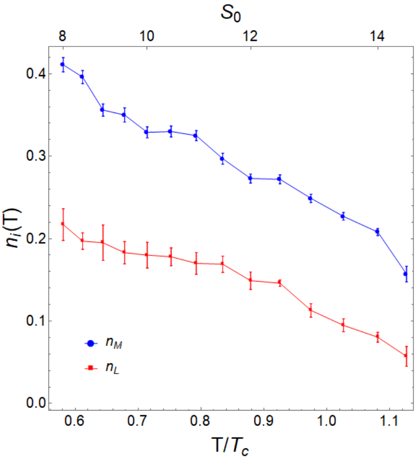
Confinement with Quarks



- No more first-order transition
- Fit to $O(4)$ form gives $S_0(T_{deconf}) = 10.44$

Dyon Densities

- Linear zero-mode-induced interaction suppresses L-dyon density
- Broken $\mathbb{Z}_3$ symmetry $\rightarrow$ dyon densities aren't equal even in the confined phase



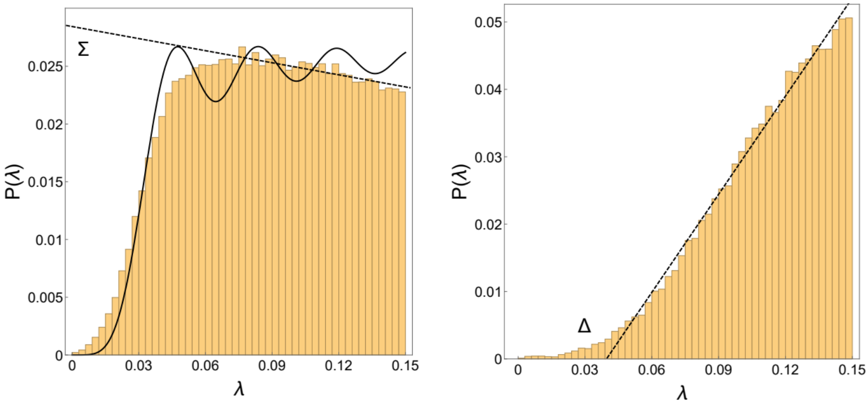
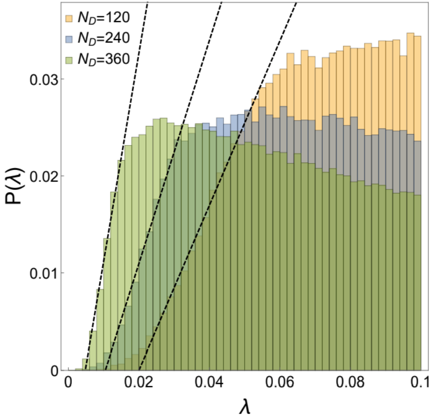


Figure: Left: $S_0 = 8$, Right: $S_0 = 14$

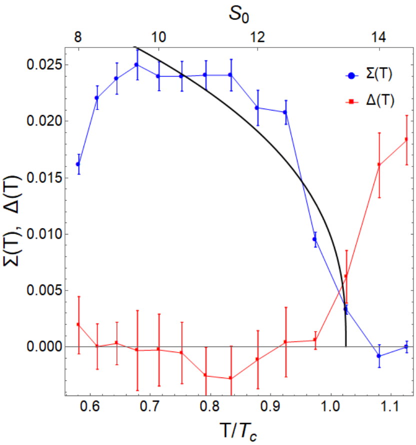
Low T: there is a finite density $\rho(0) \to$ broken chiral symmetry
High T: an eigenvalue gap Δ appears, restored chiral symmetry

Determining the Gap



- Three ensemble sizes
- Fit gaps to a function linear in $1/V$ to determine the infinite volume limit

Chiral Phase Transition



- The condensate quickly goes to zero at the same time a non-zero eigenvalue gap forms
- From this, we estimate $S_0(T_c) = 13.25$
- Should also belong to $O(4)$ universality class

Conclusions

- The dyon ensemble does possess a first-order deconfinement phase transition to the same value of $\langle P \rangle$ as the lattice for the pure $SU(3)$ theory and is compatible with a second-order transition when $N_f = 2$
- Trace deformation allows for confinement at low dyon density and has interesting phases.
- High dyon density leads to collectivization of zero modes and generates a nonzero chiral condensate at below T_c and finite eigenvalue gap above T_c