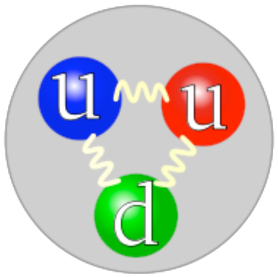
Hadrons as QCD Bound States

Online talk at QCHS, Stavanger August 2021

Paul Hoyer, University of Helsinki

QCD



QED

"Hadrons are nonperturbative bound-states of QCD"

"Precision bound-state calculations are essentially nonperturbative"

Bodwin et al, *Rev. Mod. Phys.* **57** (1985) 723

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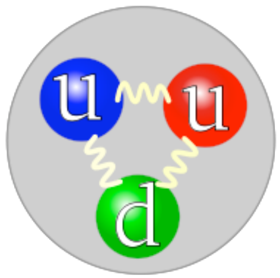

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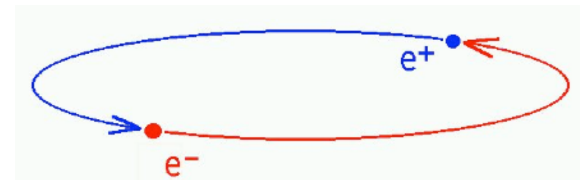

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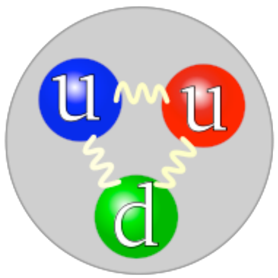

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QED bound state perturbation theory allows precision calculations of atoms:

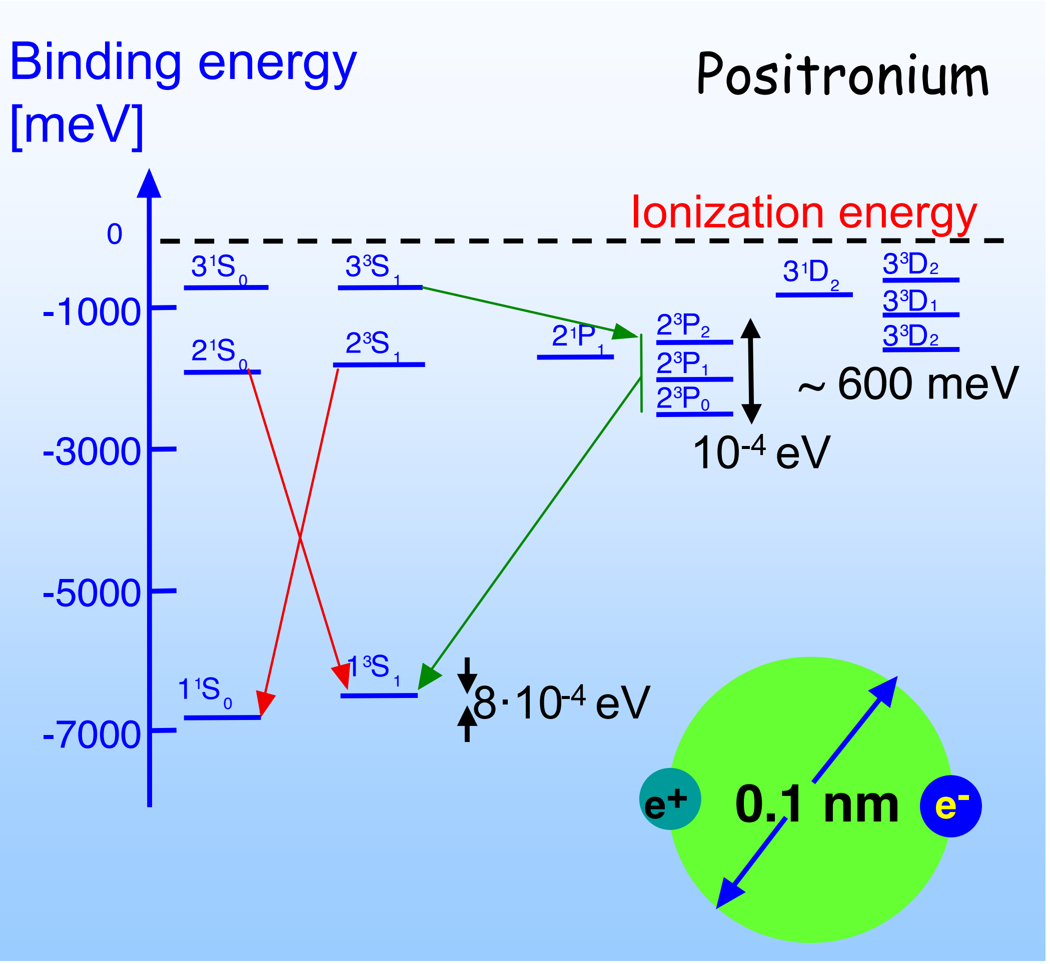
E.g., Hyperfine splitting in Positronium

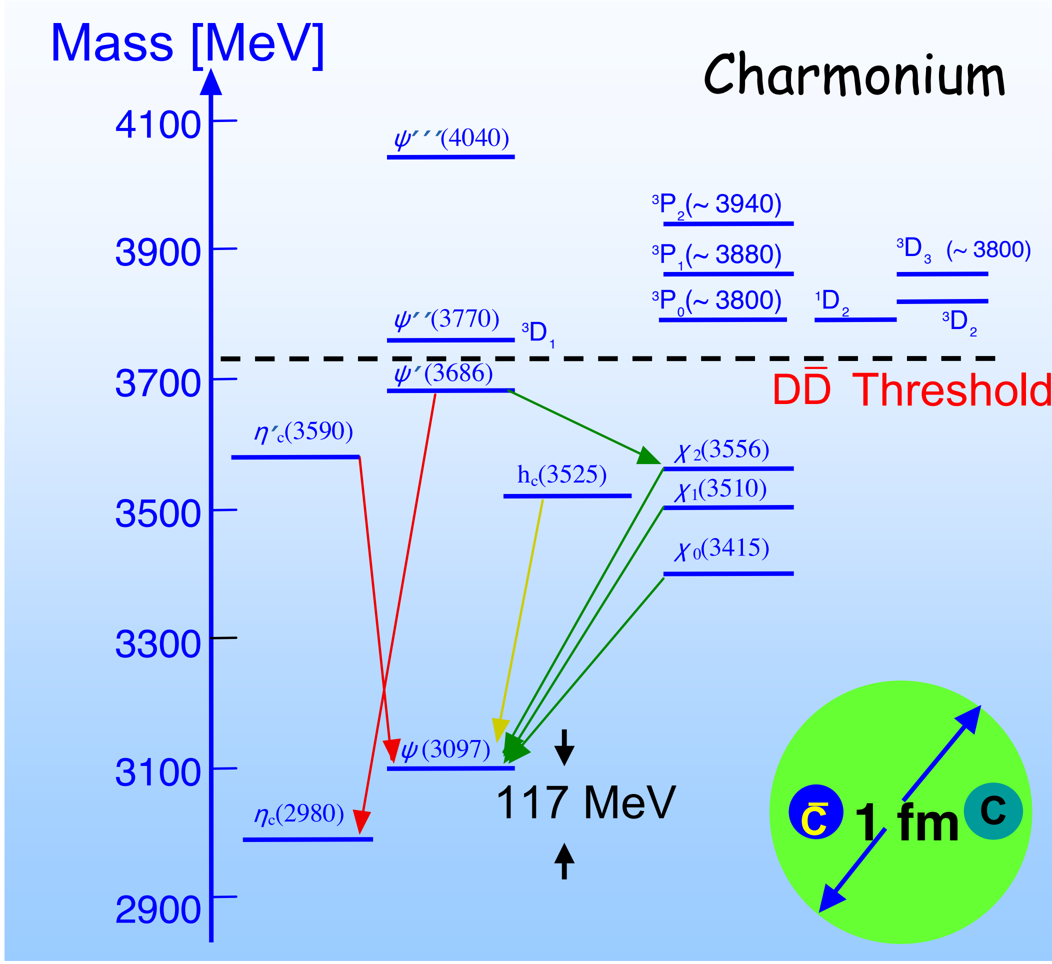
$\Delta\nu_{QED} = 203.39169(41)$ GHz

$\Delta\nu_{EXP} = 203.394 \pm .002$ GHz

Learn from 70 years of QED

Quarkonia are like atoms with confinement





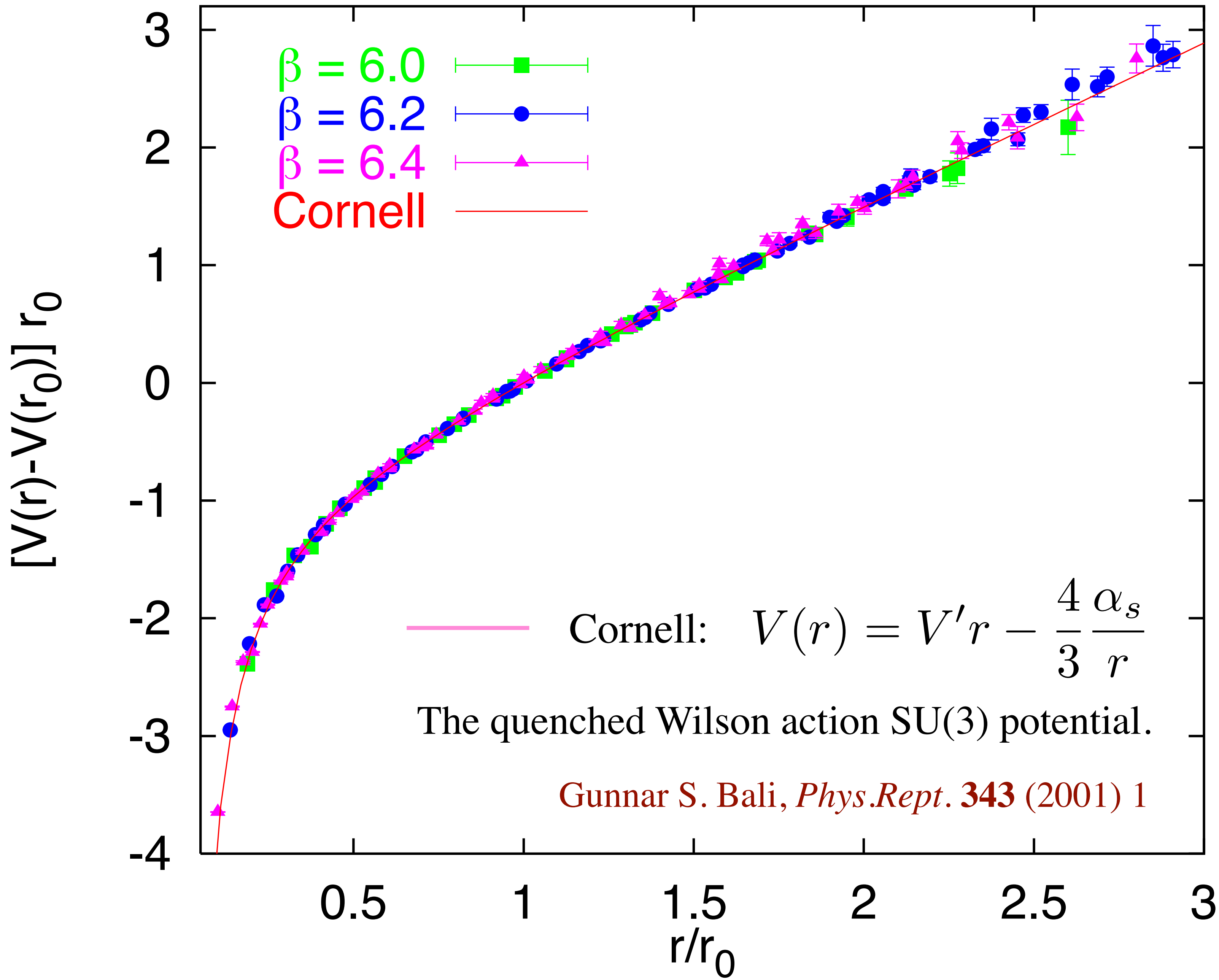
$$V(r) = -\frac{\alpha}{r}$$

$$V(r) = V'r - \frac{4}{3}\frac{\alpha_s}{r} \quad (1980)$$

E. Eichten, S. Godfrey, H. Mahlke and J. L. Rosner, Rev. Mod. Phys. **80** (2008) 1161

“The J/ψ is the Hydrogen atom of QCD”

Lattice QCD agrees with the Cornell potential



Take the Positronium - Quarkonium analogy seriously

Is it possible in QFT?

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Recall how the Schrödinger equation follows from the action S_{QED}

$$V(r) = -\frac{\alpha}{r}$$

Classical potential from Gauss' law

$$V(r) = V'r - \frac{4}{3}\frac{\alpha_s}{r}$$

Involves dimensionful parameter V': Not in S_{QCD}

Can arise from a boundary condition on Gauss' law.

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Can arise from a boundary condition on Gauss' law.

But:

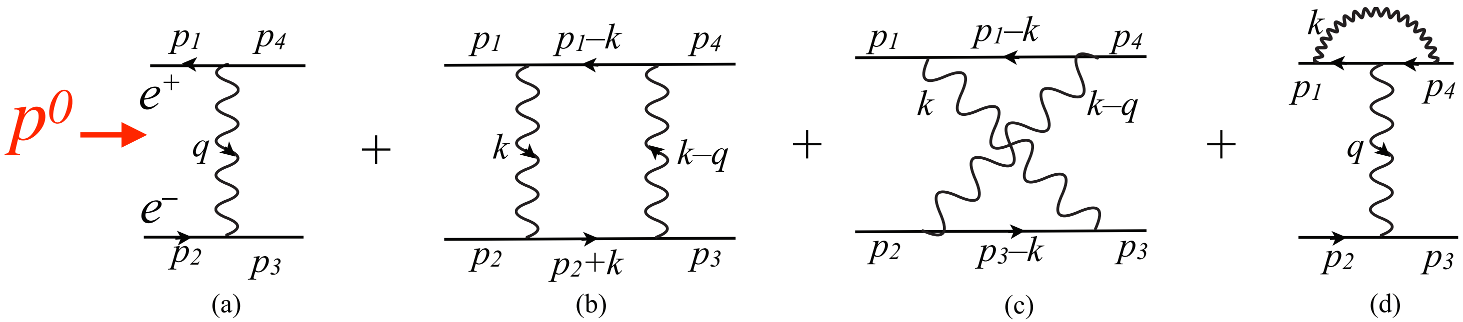
QFT bound states are derived using Feynman diagrams

This assumes free field B.C's: $A^\mu = 0$

Feynman diagrams do not have bound state poles

The Schrödinger equation from Feynman diagrams

$e^+e^- \to e^+e^-$



$$+ \ldots = \frac{|\varphi_{e^+e^-}|^2}{p^0 - E + i\varepsilon} + \ldots$$

> Bound state poles in $e^+e^- \to e^+e^-$ arise only through a
> divergence of the perturbative sum

Bohr scale $|\boldsymbol{p}| \sim \alpha m$:
propagators $\propto 1/\alpha$

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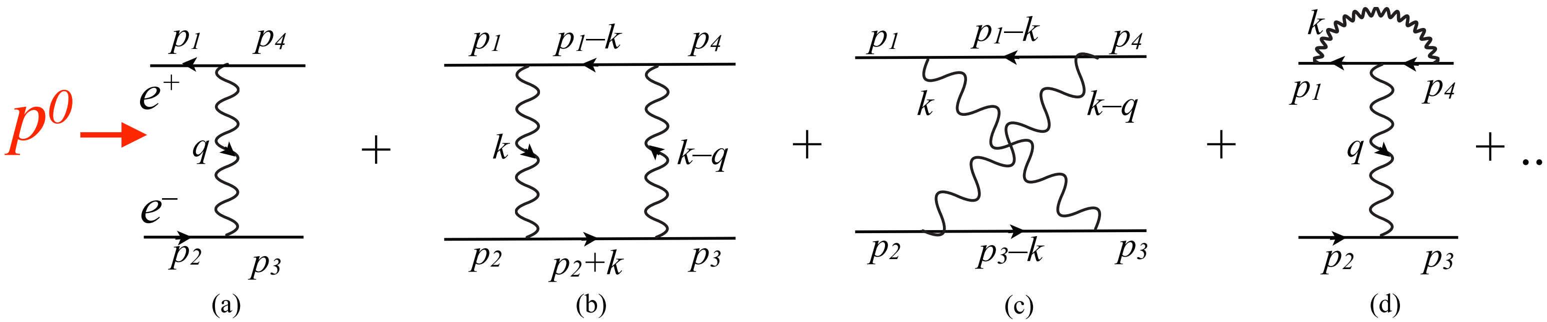

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For $|\boldsymbol{p}| << \alpha m$: classical physics dominates:

Bound states are at the borderline to classical physics

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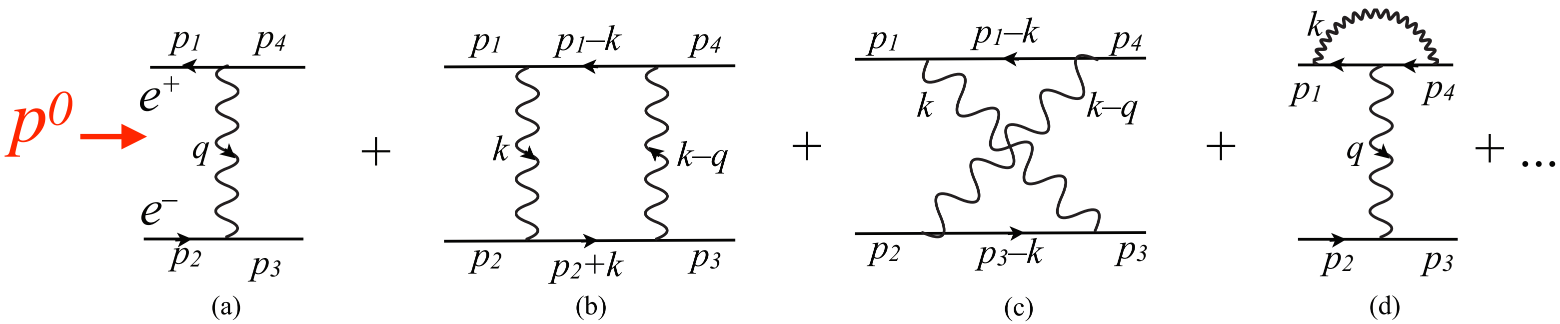

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QED: Sum of “ladder diagrams” generates the classical field $V(r) = -\dfrac{\alpha}{r}$

QCD: $V(r) = V'r$ Confinement probably not generated by Feynman diagrams

⇒ **Need to derive Schrödinger equation with correct boundary conditions**

Temporal gauge: $A^0 = 0$ (crucial)

Gauge theories have instantaneous interactions, despite their local action

A^0 and A_L do not propagate, they are fixed by the choice of gauge.

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Temporal gauge is optimal for bound states defined at an instant of time t.

- Preserves translation and rotation symmetry
- Canonical quantisation straightforward (unlike in $\nabla\cdot \boldsymbol{A} = 0$ gauge)

  $$\left[E^i(t,\boldsymbol{x}), A^j(t,\boldsymbol{y})\right] = i\delta^{ij}\delta(\boldsymbol{x}-\boldsymbol{y})$$

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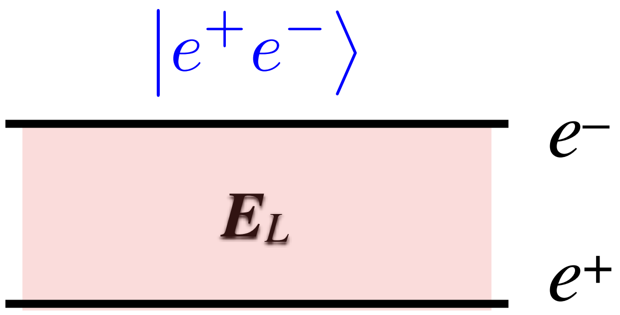
QED: $$\boldsymbol{E}_L(t,\boldsymbol{x})\,|phys\rangle = -\boldsymbol{\nabla}_x \int d\boldsymbol{y}\,\frac{e}{4\pi|\boldsymbol{x}-\boldsymbol{y}|}\psi^\dagger\psi(t,\boldsymbol{y})\,|phys\rangle$$

$\mathcal{H}_V \equiv \frac{1}{2}\int d\boldsymbol{x}\,\boldsymbol{E}_L^2$ determines the potential energy of $|e^-(\boldsymbol{x}_1)\, e^+(\boldsymbol{x}_2)\rangle$ states,

$$\mathcal{H}_V\,\bar\psi_\alpha(\boldsymbol{x}_1)\psi_\beta(\boldsymbol{x}_2)\,|0\rangle = -\frac{\alpha}{|\boldsymbol{x}_1-\boldsymbol{x}_2|}\bar\psi_\alpha(\boldsymbol{x}_1)\psi_\beta(\boldsymbol{x}_2)\,|0\rangle$$

Fock state expansion for Positronium in $A^0=0$ gauge

A perturbative expansion in α can start from the $|e^+e^-\rangle$ Fock state, bound by the classical field $\boldsymbol{E}_L$ of its constituents:



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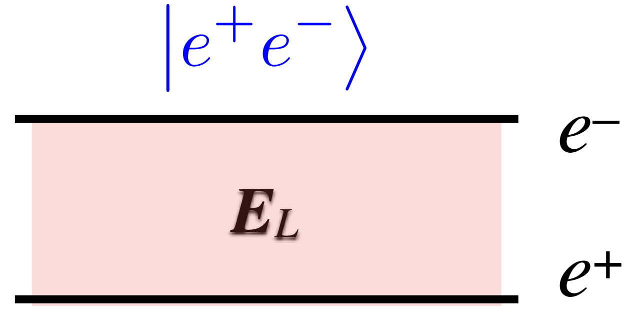
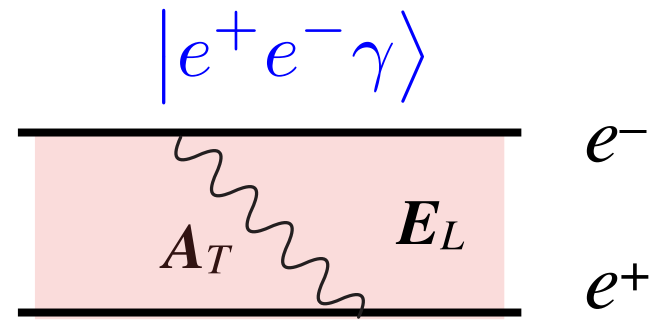


Higher order corrections given by states with transverse photons and e^+e^- pairs



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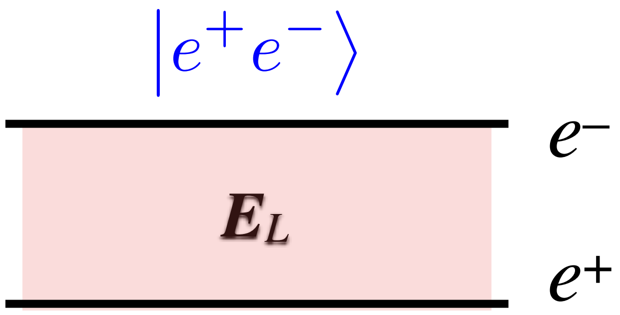

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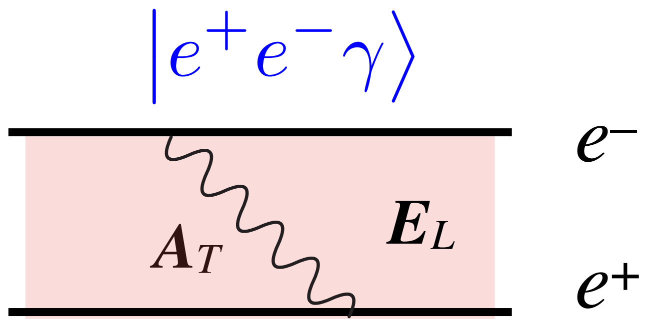

Each Fock component of the bound state includes the instantaneous $\boldsymbol{E}_L$ field.

This Fock expansion is valid in any frame.

Temporal gauge in QCD: $A_a^0 = 0$

Gauss' constraint determines $\boldsymbol{E}_{L,a}$ for all hadron Fock states:

$$\partial_i E^i_{L,a}(\boldsymbol{x})\,|phys\rangle = g\big[-f_{abc}A^i_b E^i_c + \psi^\dagger T^a \psi(\boldsymbol{x})\big]\,|phys\rangle$$

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$$E^i_{L,a}(\boldsymbol{x})\,|phys\rangle = -\partial^x_i \int d\boldsymbol{y}\Big[\kappa\,\boldsymbol{x}\cdot\boldsymbol{y} + \frac{g}{4\pi|\boldsymbol{x}-\boldsymbol{y}|}\Big]\mathcal{E}_a(\boldsymbol{y})\,|phys\rangle$$

where $\mathcal{E}_a(\boldsymbol{y}) = -f_{abc}A^i_b E^i_c(\boldsymbol{y}) + \psi^\dagger T^a\psi(\boldsymbol{y})$

> The homogeneous solution $\propto \varkappa$ is the only one that is compatible with Poincaré invariance

Including the κ ≠ 0 homogeneous solution for $E^i_{L,a}$

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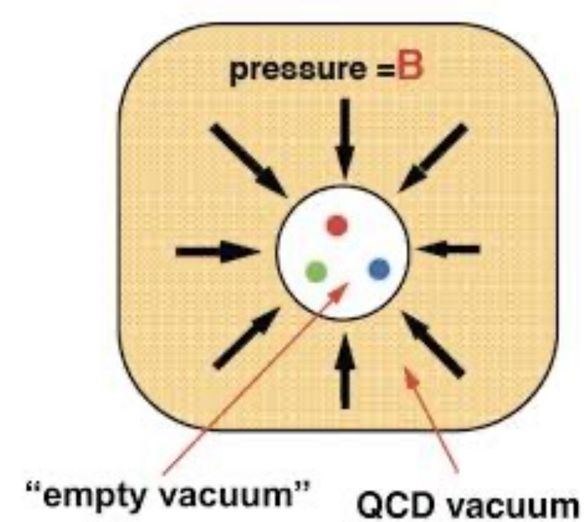
The field energy ∝ volume of space is irrelevant only if it is universal.
This relates the normalisation ϰ of all Fock components, leaving an overall scale Λ as the single parameter.



Compatible with S_{QCD}

"Bag model without a bag"

The potential energy $\mathcal{H}_V = \frac{1}{2}\int d\boldsymbol{x} \sum_a \boldsymbol{E}_L^a \cdot \boldsymbol{E}_L^a$

$$H_V = \int d\boldsymbol{y} d\boldsymbol{z} \Big\{ \boldsymbol{y}\cdot\boldsymbol{z}\Big[\tfrac{1}{2}\kappa^2 \int d\boldsymbol{x} + g\kappa\Big] + \tfrac{1}{2}\frac{\alpha_s}{|\boldsymbol{y}-\boldsymbol{z}|}\Big\} \mathcal{E}_a(\boldsymbol{y})\mathcal{E}_a(\boldsymbol{z})$$

Recall: $\mathcal{E}_a(\boldsymbol{y}) = -f_{abc}A_b^i E_c^i(\boldsymbol{y}) + \psi^\dagger T^a \psi(\boldsymbol{y})$

Gives translation invariant potentials only for (globally) color singlet states

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Meson: $|q(\boldsymbol{x}_1)\bar{q}(\boldsymbol{x}_2)\rangle \equiv \sum_A \bar{\psi}^A(\boldsymbol{x}_1)\,\psi^A(\boldsymbol{x}_2)\,|0\rangle$

$$V_{q\bar{q}}(\boldsymbol{x}_1,\boldsymbol{x}_2) = \Lambda^2|\boldsymbol{x}_1-\boldsymbol{x}_2| - C_F\frac{\alpha_s}{|\boldsymbol{x}_1-\boldsymbol{x}_2|}$$

Cornell potential

This potential is valid also for relativistic $q\bar{q}$ Fock states, in any frame

Baryon Fock state potential

Baryon: $|q(\boldsymbol{x}_1)q(\boldsymbol{x}_2)q(\boldsymbol{x}_3)\rangle \equiv \sum_{A,B,C} \epsilon_{ABC}\psi_A^\dagger(\boldsymbol{x}_1)\,\psi_B^\dagger(\boldsymbol{x}_2)\,\psi_C^\dagger(\boldsymbol{x}_3)\,|0\rangle$

$$V_{qqq}(\boldsymbol{x}_1,\boldsymbol{x}_2,\boldsymbol{x}_3) = \Lambda^2 d_{qqq}(\boldsymbol{x}_1,\boldsymbol{x}_2,\boldsymbol{x}_3) - \frac{2}{3}\,\alpha_s\Big(\frac{1}{|\boldsymbol{x}_1-\boldsymbol{x}_2|}+\frac{1}{|\boldsymbol{x}_2-\boldsymbol{x}_3|}+\frac{1}{|\boldsymbol{x}_3-\boldsymbol{x}_1|}\Big)$$

$$d_{qqq}(\boldsymbol{x}_1,\boldsymbol{x}_2,\boldsymbol{x}_3) \equiv \frac{1}{\sqrt{2}}\sqrt{(\boldsymbol{x}_1-\boldsymbol{x}_2)^2+(\boldsymbol{x}_2-\boldsymbol{x}_3)^2+(\boldsymbol{x}_3-\boldsymbol{x}_1)^2}$$

Analogous potentials obtained for any quark and gluon Fock state, such as $q\bar{q}g$.

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Homogeneous solution of Gauss' constraint gives confinement in QCD

Many features of hadrons thus obtained look promising & intriguing

Further info in 2101.06721v2 and/or contact me !

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