Center-vortex ensembles and the asymptotic Casimir law

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The asymptotic properties of pure YM

- ***N*-ality**

  - Center-vortex condensates: 't Hooft (1978), Mack & Petkova (1979), Nielsen & Olesen (1979), Cornwall (1979)...

  - Center dominance: Del Debbio, Faber, Greensite & Olejnik (1997), Langfeld, Reinhardt & Tennert (1998), de Forcrand & D'Elia (1999) . . .

  - Center-vortex models with stiffness: Engelhardt & Reinhardt (2000), Engelhardt, Quandt & Reinhardt (2004)

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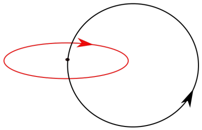
- **Abelian Nielsen-Olesen profiles in 4d**: Cea, Cosmai, Cuteria & Papa (2017), Yanagihara, R.; Iritani, T.; Kitazawa, M.; Asakawa, M.; Hatsuda, 2019, Yanagihara, R.; Kitazawa, 2019

Simplest center-vortex ensembles in 3d

$$\mathcal{W}_{\rm D}[\mathcal{A}] = \frac{1}{\mathfrak{D}} \, {\rm Tr}\, {\rm D} \left(P \left\{ e^{i \oint_{\mathcal{C}_{\rm e}} dx_\mu \, A_\mu(x)} \right\} \right)$$



$$\mathcal{W}_{\rm D}(\mathcal{C}_{\rm e}) = \mathcal{Z}_{\rm D}(\mathcal{C}_{\rm e}) = \frac{1}{\mathfrak{D}} \, {\rm Tr} \left[{\rm D} \left(e^{i \frac{2\pi}{N}} I \right) \right]^{{\rm L}(\omega, \mathcal{C}_{\rm e})}$$

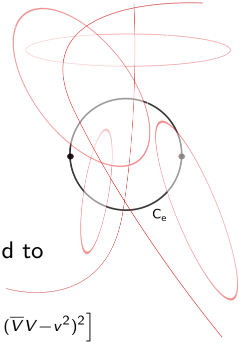
Models for the Wilson loop average at asymptotic distances:

$$\langle \mathcal{Z}_{\rm D}(\mathcal{C}_e) \rangle = \mathcal{N} \sum_{\omega} e^{-S(\omega)} \frac{1}{\mathfrak{D}} \, {\rm Tr} \left[{\rm D} \left(e^{i \frac{2\pi}{N}} I \right) \right]^{{\rm L}(\omega, \mathcal{C}_{\rm e})}$$

- Diluted gas of closed worldlines: LEO & H. Reinhardt, 2018
  $S(\omega)$ contains center-vortex tension (μ) and stiffness ($1/\kappa$) terms

For fundamental quarks,

$$\mathcal{Z}_{\rm F}(\mathcal{C}_{\rm e}) = \left[e^{i\frac{2\pi}{N}} \right]^{{\rm L}(\omega,\mathcal{C}_{\rm e})}$$



Small positive stiffness and repulsive contact interactions lead to

$$\langle \mathcal{Z}_{\rm F}(\mathcal{C}_{\rm e}) \rangle = \mathcal{N} \int [\mathcal{D}V][\mathcal{D}\bar{V}]\, e^{-\int d^3x \left[\frac{1}{3\kappa} \overline{D_\mu V} D_\mu V + \frac{1}{2\zeta} (\bar{V}V - v^2)^2 \right]}$$

$$v^2 \propto -\mu\kappa > 0 \quad , \quad D_\mu = \partial_\mu - i\frac{2\pi}{N} s_\mu \quad , \quad s_\mu \text{ is localized on } S(\mathcal{C}_{\rm e})$$

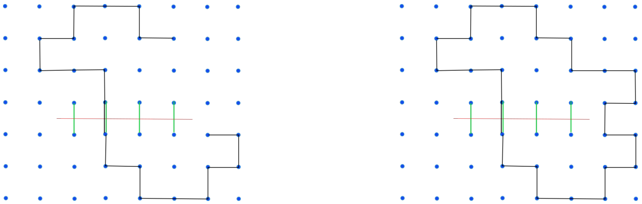
- Percolation ($\mu < 0$) implies $v^2 > 0$ $\rightarrow$ the dynamics can be approximated by soft degrees of freedom (Goldstone modes γ, $V = v\, e^{i\gamma}$)

3d percolating center vortices

Frustrated 3d XY

$$S^{(3)}_{\rm latt} = \beta \sum_{x,\mu} {\rm Re}\left[1 - e^{-i\alpha_\mu(x)} e^{i\gamma(x+\hat{\mu})} e^{-i\gamma(x)}\right]$$

- $e^{i\alpha_\mu(\mathbf{x})} = e^{i\frac{2\pi}{N}}$, if $S(\mathcal{C}_{\rm e})$ is crossed by the link and is trivial otherwise
- $\prod_x \int_{-\pi}^{\pi} d\gamma(x)$ keeps the contribution of $e^{i\gamma(x+\hat{\mu})} e^{-i\gamma(x)}$ on links that form closed loops $\Rightarrow \mathcal{Z}_{\rm F}(\mathcal{C}_{\rm e})$



- $\beta_{\rm c} \approx 0.454 \Rightarrow$

  - large loops are favored (negative tension)
  - in detriment of multiple small (positive stiffness)
  - multiple links are disfavored (excluded volume effects)

Extended ensembles

(David R. Junior, LEO & Gustavo M. Simões (JHEP, 2020))

Additional labels: For oriented center vortices, A_μ can be characterized by

$$S_0 = e^{-i\chi\beta\cdot T} \quad , \quad \beta\cdot T \equiv \beta|_q T_q$$

- For elementary vortices, β is one of the magnetic fundamental weights β_i
- χ changes by 2π when going around ω
- T_q, $q = 1, \ldots, N-1$ are the Cartan generators

$$\mathcal{Z}_{\rm D}(\mathcal{C}_{\rm e}) = \frac{1}{\mathcal{D}}\,{\rm Tr}\,\left[{\rm D}\left(e^{i\frac{2\pi}{N}I}\right)\right]^{{\rm L}(\omega,\mathcal{C}_{\rm e})} = \frac{1}{N}\,{\rm Tr}\,\left[e^{i\int_\omega dx_\mu b_\mu}\right]$$

- $b_\mu = 2\pi\beta_e\cdot T\, s_\mu$
- β_e is a magnetic weight of ${\rm D}$

Extended ensembles: effective description

$$\langle \mathcal{Z}_{\rm D}(\mathcal{C}_{\rm e})\rangle = \left(e^{\int_0^\infty \frac{dL}{L}\int dx \int du \, {\rm Tr}\, Q(x,u,x,u,L)}\right)^N$$

- $Q(x, u; x_0, u_0)$ satisfies a difusion equation in (x, u)-space
- small stiffness limit

$$Q(x,u,x_0,u_0,L) \approx \langle x|e^{-LO}|x_0\rangle \quad , \quad O = -\frac{1}{3\kappa}D_\mu D_\mu + \mu I_N \quad , \quad D_\mu = \partial_\mu - ib_\mu$$

$$\int [D\Phi^\dagger][D\Phi]\, e^{-\int d^3x \left[\frac{1}{3\kappa}{\rm Tr}\left((D_\mu\Phi)^\dagger D_\mu\Phi\right) + \mu {\rm Tr}(\Phi^\dagger\Phi)\right]}$$

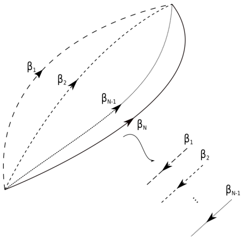
- Φ is a complex $N \times N$ matrix, $\Phi_{ij} = \phi_j|_i$
- ϕ_i is a set of N emergent complex scalar fields in the fundamental irrep

Matching rules

Center-vortex matching:

$$S_0 = e^{-i\chi_1 \beta_1 \cdot T} \dots e^{-i\chi_{N-1}\beta_{N-1}\cdot T}$$

$\sum_i \beta_i = 0 \to N$ center-vortex guiding centers with different weights β_i can be matched



$$\mathcal{Z}_{\rm D}(\mathcal{C}_{\rm e}) = \frac{1}{N!}\epsilon_{i_1\dots i_N}\,\epsilon_{i'_1\dots i'_N}\Gamma_{\gamma_1}[b_\mu]|_{i_1 i'_1}\dots\Gamma_{\gamma_N}[b_\mu]|_{i_N i'_N} \quad , \quad \Gamma_\gamma[b_\mu] = e^{i\int_\omega dx_\mu\, b_\mu}$$

Matching rules: effective description

Weighting with $e^{-E(\gamma_i)}$, integrating over γ_i with fixed endpoints, and then over L_i, x_0 and x, we obtained

$$\mathcal{Z}_{\rm D}(\mathcal{C}_{\rm e}) \to \int d^3x\, d^3x_0\, \epsilon_{i_1\dots i_N}\, \epsilon_{j_1\dots j_N}\, G(x,x_0)_{i_1 j_1} \dots G(x,x_0)_{i_N j_N}$$

$$O\, G(x,x_0) = \delta(x-x_0)\, I_N$$

- a vertex $\det \Phi + {\rm c.c.}$

$$\langle \mathcal{Z}_{\rm D}(\mathcal{C}_{\rm e})\rangle \approx \mathcal{N} \int [D\Phi^\dagger][D\Phi] \times$$
$$\times\, e^{-\int d^3x \left[\frac{1}{3\kappa} {\rm Tr}\left((D_\mu\Phi)^\dagger D_\mu \Phi \right) + \mu {\rm Tr}(\Phi^\dagger\Phi) + \frac{\lambda_0}{2} {\rm Tr}(\Phi^\dagger\Phi)^2 - \xi_0 (\det\Phi + \det\Phi^\dagger) \right]}$$

$$\Phi \to S_c(x)\,\Phi \quad , \quad b_\mu \to S_c(x) b_\mu S_c^{-1}(x) + i S_c(x) \partial_\mu S_c^{-1}(x)\,,$$
$$\Phi \to \Phi\, S_f \quad , \quad S_f\,, S_c(x) \in SU(N)$$

Matching rules in the lattice

- $\mu < 0$, $\xi_0 = 0 \to \frac{\lambda}{2} \mathrm{Tr}(\Phi^\dagger \Phi - a^2 I_N)^2 \to \mathcal{M} = U(N)$

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$$S^{(3)}_{\mathrm{latt}}(b_\mu) = \beta \sum_{x,\mu} \mathrm{Re}\,\left[\mathbb{I} - U^\dagger_\mu V(x+\hat{\mu}) V^\dagger(x)\right)\right]$$

$U_\mu(x) = e^{i2\pi\beta_{\mathrm{e}}\cdot T} \in Z(N)$, , if $S(\mathcal{C}_{\mathrm{e}})$ is crossed by the link and is trivial otherwise $\Rightarrow \mathcal{Z}_{\mathrm{F}}(\mathcal{C}_{\mathrm{e}})$

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- Up to this point, there is a continuum set of classical vacua $\mathcal{M} = SU(N)$ which precludes the formation of the stable domain wall

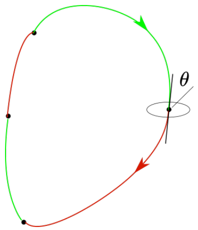
Including nonoriented center vortices

- In 4d, chains $\to$ 97% of the cases: Ambjorn, Giedt & Greensite (2000)

Nonoriented center vortices

- In the continuum, the Lie algebra flux orientation of center vortices in 4d changes at the monopole worldlines: Reinhardt, 2002
- In 3d the change is at the instantons

$$S_0 = e^{-i\varphi\beta\cdot T} W(\theta) \quad , \quad W(\theta) = e^{i\theta\,\sqrt{N}T_\alpha}$$



Center-vortex components with different weights β, β' are interpolated by instantons that carry adjoint weight $\beta' - \beta$

Nonoriented component: effective description

$$S_{\rm eff}(\Phi, b_\mu) = \int d^3x \left(\mathrm{Tr}\, (D_\mu \Phi)^\dagger D_\mu \Phi + V(\Phi, \Phi^\dagger) \right)$$

$$V(\Phi, \Phi^\dagger) = \frac{\lambda}{2}\, \mathrm{Tr}(\Phi^\dagger \Phi - a^2 I_N)^2 - \xi\, (\det \Phi + \det \Phi^\dagger) - \vartheta\, \mathrm{Tr} \left(\Phi^\dagger T_A \Phi T_A \right)$$

- For small ϑ $\rightarrow$ soft modes in the lattice ($V(x) \in$SU(N)):

$$S^{(3)}_{\rm latt}(b_\mu) \rightarrow \beta \sum_{x,\mu} \mathrm{Re} \left[\mathbb{I} - U^\dagger_\mu V(x+\hat{\mu}) V^\dagger(x)) \right] + \mathrm{const.} \sum_x \mathrm{Tr}\, (\mathrm{Ad}(V(x))$$

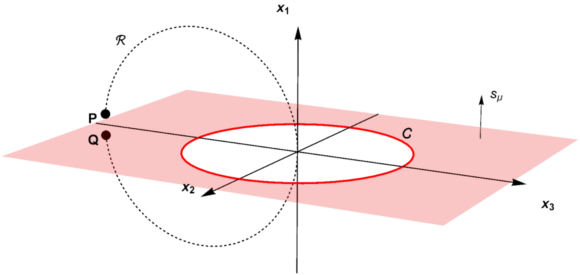
New contribution from the singlet formed with $V(x)$, $V^\dagger(x)$, and $\mathrm{Tr}\, (\mathrm{Ad}(V(x))$ $\rightarrow$ nonoriented center vortices

- The new term is also generated from a weighted nonoriented component

Domain wall

- For λ, ξ, $\vartheta > 0$

$$\mathcal{M} = \{\Phi_n = v\, e^{i\,\frac{2\pi n}{N}}\; I_N \;,\; ; n = 0, 1, \ldots, N-1\}$$



$$\lim_{x_1 \to -\infty} \Phi(x_1, x_2, x_3) = vI_N \quad , \quad \lim_{x_1 \to \infty} \Phi(x_1, x_2, x_3) = ve^{i2\pi \beta_e \cdot T} \quad , \quad (0, x_2, x_3) \in S(\mathcal{C}_{\rm e})$$

- For large Wilson loops, and for any weight of an antisymmetric irrep, we computed the soliton with the ansatz

$$\Phi = (\eta\, I_N + \eta_0\, \beta_e \cdot T)e^{i\theta \beta_e \cdot T}\, e^{i\alpha}$$

  - This closes the eqs. for any N-ality k

- Due to the relation $e^{i2\pi\beta_e \cdot T} = e^{-i\frac{2k\pi}{N}}$

  - α varies with $\theta \approx \mathrm{const.}$ (closely related to the 't Hooft model (1978))
  $\Phi = V I_N$, $V = \eta\, e^{i\alpha}$

$$\mathcal{L} = \partial^\mu \bar{V}\, \partial_\mu V + m^2\, \bar{V} V + \frac{\lambda}{2}\, (\bar{V} V)^2 + \xi\, (V^N + \bar{V}^N)$$

  However, there is no dynamical reason for the path-integral to favor this type of restricted configuration

  - θ varies with $\alpha \approx \mathrm{const.}$

- For $\lambda a^2, \xi v^{N-2} >> \vartheta \rightarrow$ second case, only θ is not frozen

$$\partial^2_{x_1}\theta = \frac{3\vartheta}{2}\sin(\theta)$$

- We obtained the asymptotic Casimir Law

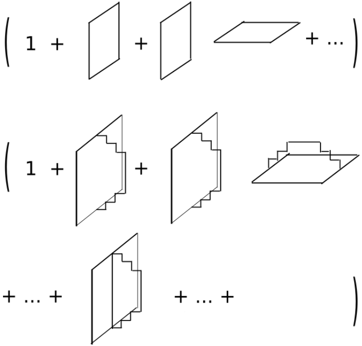
$$\epsilon_k = \frac{k(N-k)}{N-1}\epsilon_1$$

- Collective transverse fluctuations of the domain wall $\rightarrow$ Lüscher term

4d Ensemble measure in the lattice

LEO (2018)

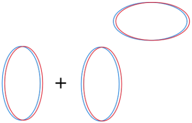
$$Z^{\rm latt}_{\rm mix}[s_{\mu\nu}] = \int [\mathcal{D}V_\mu]\, e^{-S^{(4)}_{\rm latt}} \times$$

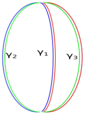


Emergent Yang-Mills-Higgs description

$$V_\mu(x) = e^{ia\Lambda_\mu(x)} \quad , \quad \Lambda_\mu \in \mathfrak{su}(N)$$

$$Z_{\rm mix}[s_{\mu\nu}] = \int [\mathcal{D}\Lambda_\mu]\, e^{-\int d^4x\, \frac{1}{4g^2}\left(F_{\mu\nu}(\Lambda) - 2\pi s_{\mu\nu}\beta_{\rm e}\cdot T\right)^2} \times$$

$\Big(1 +$  $+ \dots \Big)$

$\Big(1 +$ 

$+ \dots + \Big)$

$$Z_{\rm mix}[s_{\mu\nu}] = \int [\mathcal{D}\Lambda_\mu][\mathcal{D}\psi]\, e^{-\int d^4x \left[\frac{1}{4g^2}\left(F_{\mu\nu}(\Lambda) - 2\pi s_{\mu\nu}\beta_{\rm e}\cdot T\right)^2 + \frac{1}{2}(D_\mu\psi_I, D_\mu\psi_I) + V_{\rm H}(\psi)\right]}$$

- The system can easily undergo $SU(N) \to Z(N)$ SSB

Abelian profiles, N-ality and the Casmir law

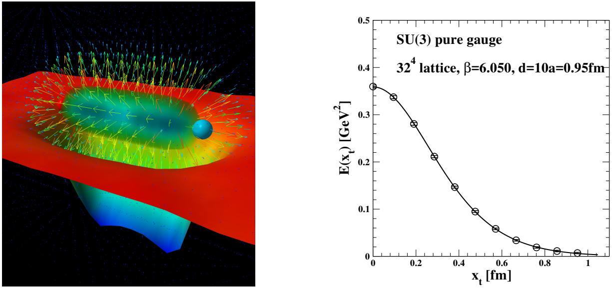


Figure: D. Leinweber simulation, University of Adelaide - Interpolation of the longitudinal chromoelectric field with an Abelian-like model vs. the transverse distance in $SU(3)$, Cea, Cosmai, Cuteria & Papa (2017)

- Abelianization for all N and Casimir law, LEO & G. M. Simões (2019)
- BPS point: Completing the proof of Casimir law: D. R. Junior, LEO & G. M. Simões (2020)

Conclusions

We followed a road that leads from 3d/4d ensembles with

- percolating center-vortex worldlines/worldsurfaces
- attached instantons/monopole worldlines with fusion

to confining asymptotic properties

- domain wall/flux tube with N-ality
- Lüscher term
- Casimir law
- Sine-Gordon/Nielsen-Olesen profiles

- The new term is also generated from a weighted nonoriented component

$$\mathcal{W}_{\rm D}(\mathcal{C}_{\rm e})|_{\rm loop} \propto \int d\mu(g)\, \langle g, \omega | \Gamma_l[b_\mu] | g, \omega\rangle \quad , \quad |g,\omega\rangle = g|\omega\rangle \quad , \quad g \in SU(N)$$

$$\mathcal{W}_{\rm D}(\mathcal{C}_{\rm e})|_{N-{\rm lines}} \propto \int d\mu(g) d\mu(g_0)\, \langle g, \omega_1 | \Gamma_{\gamma_1}[b_\mu] | g_0, \omega_1\rangle \dots \langle g, \omega_N | \Gamma_{\gamma_N}[b_\mu] | g_0, \omega_N\rangle$$

$$C_4 = \int d\mu(g_1) \cdots \int d\mu(g_4)\, {\rm Tr}\, \left(|g_1,\omega'\rangle\langle g_1,\omega|\; |g_2,\omega\rangle\langle g_2,\omega'| \dots |g_4,\omega\rangle\langle g_4,\omega'|\right) \times$$
$$\times \langle g_1,\omega'|\Gamma_{\gamma_n}[b_\mu]|g_4,\omega'\rangle \dots \langle g_3,\omega'|\Gamma_{\gamma_2}[b_\mu]|g_2,\omega'\rangle\langle g_2,\omega|\Gamma_{\gamma_1}[b_\mu]|g_1,\omega\rangle \; .$$