Another look at the three-gluon vertex in the minimal Landau gauge

Guilherme Catumba$^{1,2}$, Orlando Oliveira $^2$, Paulo Silva $^2$

$^1$ IFIC, University of Valencia, Spain
$^2$ CFisUC, Department of Physics, University of Coimbra, Portugal

August 2, 2021
Outline

1 Introduction and Motivation

2 Results
   • Three gluon vertex

3 Conclusions and outlook
Green’s functions

- Green’s functions summarize the dynamics of the theory
  - QCD: information on confinement and chiral symmetry breaking
- $n$-point complete Green’s functions
  $$G^{(n)}(x_1, \ldots, x_n) = \langle 0 | T (\phi(x_1) \cdots \phi(x_n)) | 0 \rangle,$$
  - decomposition in terms of one particle irreducible (1PI) functions $\Gamma^{(n)}$
  - access to form factors that define $\Gamma^{(n)}$
- lattice approach allows for first principles determination of the complete Green’s functions of QCD
Three gluon vertex

- allow to measure e.g. strong coupling constant
- fundamental role in the structure of Dyson-Schwinger equations
- infrared behaviour predicted by DSE equations
  - infrared suppression — form factors (FF) decrease as $p \to 0$
  - zero crossing — FF change sign in IR region
  - logarithmic divergence at origin — FF $\to -\infty$ for $p \to 0$
- infrared behaviour corroborated by lattice simulations
  - quenched and dynamical simulations

Aguilar, Soto, Ferreira, Papavassiliou, Rodríguez-Quintero, PLB 818(2021)136352
Aguilar, Soto, Ferreira, Papavassiliou, Rodríguez-Quintero, Zafeiropoulos, EPJC 80(2020)154
Aguilar, Ferreira, Papavassiliou, EPJC 80(2020)887
Three point complete Green’s function

\[
\left\langle A_{\mu_1}^{a_1}(p_1) A_{\mu_2}^{a_2}(p_2) A_{\mu_3}^{a_3}(p_3) \rightangle = V \delta(p_1 + p_2 + p_3) \ G_{\mu_1\mu_2\mu_3}^{a_1a_2a_3}(p_1, p_2, p_3)
\]

\[
G_{\mu_1\mu_2\mu_3}^{a_1a_2a_3}(p_1, p_2, p_3) = D_{\mu_1\nu_1}^{a_1b_1}(p_1) \ D_{\mu_2\nu_2}^{a_2b_2}(p_2) \ D_{\mu_3\nu_3}^{a_3b_3}(p_3) \ \Gamma_{\nu_1\nu_2\nu_3}^{b_1b_2b_3}(p_1, p_2, p_3)
\]

- **color structure:**

\[
\Gamma_{\mu_1\mu_2\mu_3}^{a_1a_2a_3}(p_1, p_2, p_3) = f_{a_1a_2a_3} \ \Gamma_{\mu_1\mu_2\mu_3}(p_1, p_2, p_3)
\]
Three gluon vertex

- Bose symmetry requires vertex to be symmetric under interchange of any pair \((p_i, a_i, \mu_i)\)
  \(\Gamma_{\mu_1 \mu_2 \mu_3}(p_1, p_2, p_3)\) must be antisymmetric
- \(\Gamma_{\mu_1 \mu_2 \mu_3}(p_1, p_2, p_3)\) in the continuum: requires six Lorentz invariant form factors
  - two associated to the transverse component \(\Gamma^{(t)}\)
  - remaining associated to the longitudinal \(\Gamma^{(l)}\).

Lattice setup

- Wilson gauge action, $\beta = 6.0$
  - $64^4$, 2000 configurations
- $80^4$, 1800 configurations
- Rotation to the Landau gauge: FFT-SD method
- Gluon field

\[
ag_0 A_\mu(x + a\hat{e}_\mu) = \frac{U_\mu(x) - U_\mu^\dagger(x)}{2ig_0} - \frac{\text{Tr} \left[ U_\mu(x) - U_\mu^\dagger(x) \right]}{6ig_0}
\]

In momentum space:

\[
A_\mu(\hat{p}) = \sum_x e^{-i\hat{p}(x + a\hat{e}_\mu)} A_\mu(x + a\hat{e}_\mu), \quad \hat{p}_\mu = \frac{2\pi n_\mu}{aL_\mu}
\]
Lattice setup

- tree level improved momentum

\[ p_\mu = \frac{2}{a} \sin \left( \frac{a \hat{p}_\mu}{2} \right) \]

- accessing the 1PI three gluon vertex from the lattice

\[
\begin{align*}
G_{\mu_1 \mu_2 \mu_3}(p_1, p_2, p_3) &= \text{Tr} \left\langle A_{\mu_1}(p_1) A_{\mu_2}(p_2) A_{\mu_3}(p_3) \right\rangle = \\
&= V \delta(p_1 + p_2 + p_3) \frac{N_c(N_c^2 - 1)}{4} D(p_1^2) D(p_2^2) D(p_3^2) \\
P_{\mu_1 \nu_1}(p_1) P_{\mu_2 \nu_2}(p_2) P_{\mu_3 \nu_3}(p_3) \Gamma_{\nu_1 \nu_2 \nu_3}(p_1, p_2, p_3)
\end{align*}
\]
Momentum configuration

- focus on the asymmetric momentum configuration: \( p_2 = 0 \)
  

\[
G_{\mu_1 \mu_2 \mu_3}(p, 0, -p) = V \frac{N_c(N_c^2 - 1)}{4} \left[ D(p^2) \right]^2 D(0) \frac{\Gamma(p^2)}{3} p_{\mu_2} T_{\mu_1 \mu_3}(p)
\]

\[
\Gamma(p^2) = 2 \left[ A(p^2, p^2; 0) + p^2 C(p^2, p^2; 0) \right]
\]

\[
G_{\mu \alpha \mu}(p, 0, -p) p_{\alpha} = V \frac{N_c(N_c^2 - 1)}{4} \left[ D(p^2) \right]^2 D(0) \Gamma(p^2) p^2
\]
A word about statistical errors on $\Gamma(p^2)$

- Measurement of $\Gamma(p^2)$ requires to compute the ratio
  \[ G_{\mu\alpha\mu}(p, 0, -p)\rho_\alpha / [D(p^2)]^2 D(0) \]

- Large statistical fluctuations at high momenta:
  \[
  \left[ \Delta \Gamma(p^2) \right]^2 = \frac{1}{[D(p^2)]^4} \left\{ \left[ \frac{\Delta G_{\mu\alpha\mu} p_\alpha}{D(0)} \right]^2 + \left[ 2 \Delta D(p^2) \frac{G_{\mu\alpha\mu} p_\alpha}{D(p^2) D(0)} \right]^2 + \left[ 2 \Delta D(0) \frac{G_{\mu\alpha\mu} p_\alpha}{[D(0)]^2} \right]^2 \right\}
  \]

  For large momenta:
  - $D(p^2) \sim 1/p^2$
  - $\Delta \Gamma(p^2) \sim p^4$
Handling of noise, lattice artefacts

- binning in momentum
- $H(4)$ extrapolation of the lattice data
  - lattice momentum $p_\mu$
  - invariants of the remnant $H(4)$ symmetry group associated with a hypercubic lattice

\[
p^2 = p^{[2]} = \sum_\mu p^2_\mu, \quad p^{[4]} = \sum_\mu p^4_\mu, \quad p^{[6]} = \sum_\mu p^6_\mu, \quad p^{[8]} = \sum_\mu p^8_\mu
\]

- on the lattice, a scalar quantity $F$ is a function of all $H(4)$ invariants, i.e. $F_{\text{Lat}} = F(p^{[2]}, p^{[4]}, p^{[6]}, p^{[8]})$
- continuum limit given by $F(p^{[2]}, 0, 0, 0)$

Becirevic et al, PRD 60(1999)094509
Soto, Roiesnel, JHEP 09(2009)007
Outline

1 Introduction and Motivation

2 Results
   • Three gluon vertex

3 Conclusions and outlook
Original and binned $\Gamma(p^2)$

64$^4$ and 80$^4$
$H(4)$ extrapolation of $\Gamma(p^2)$

A Virtual Tribute to QCHS 2021
Introduction and Motivation

Results

Conclusions and outlook

H(4) extrapolation — infrared region

\[ p^2 \Gamma(p^2) \]

\[ \Gamma(p^2) \]

A Virtual Tribute to QCHS 2021
\( \Gamma(p^2) \) — Infrared behaviour

\[ \Gamma(p^2) = A + Z \ln(p^2) \]
\[ 80^4 \text{ lattice, } p < 1 \text{GeV} \]
\[ A = 0.2395(16) \]
\[ Z = 0.0646(21) \]
\[ \chi^2/\text{dof} = 1.23 \]
\[ p_o = 157 \text{MeV} \]
\[ \Gamma(p^2) = A + Z \ln(p^2 + m^2) \]

- \( 80^4 \) lattice, \( p < 1 \text{GeV} \)
- \( A = 0.208(24) \)
- \( Z = 0.124(27) \)
- \( m = 0.61(15) \)
- \( \chi^2/dof = 0.95 \)
Infrared behaviour

\[ \Gamma(p^2) = 1 + c p^{-d} \]

- \(80^4\) lattice, \( p < 1 \text{GeV}\)
- \(c = -0.7621(15)\)
- \(d = 0.1558(49)\)
- \(\chi^2/dof = 1.35\)
- \(p_o = 175\text{MeV}\)
\( \Gamma(p^2) \) — Infrared behaviour

\[
\Gamma(p^2) = a + bp^2 + cp^4
\]

- 80\(^4\) lattice, \( p < 1\text{GeV} \)
- \( a = 0.0978(60) \)
- \( b = 0.218(22) \)
- \( c = -0.070(18) \)
- \( \chi^2/dof = 0.98 \)
Conclusions

- Improved calculation of the three gluon vertex on the lattice
  - particular kinematical configuration $p_2 = 0$
  - two different lattice volumes: $(6.5 \text{ fm})^4$ and $(8.2 \text{ fm})^4$
  - lattice spacing $a = 0.102 \text{ fm}$
- $H(4)$ extrapolation pushes the vertex to higher values in UV regime
- functional study in the infrared region
  - some functional forms compatible with zero crossing and IR divergence
Outlook

- Three gluon vertex
  - explore other momentum configurations
- Four gluon vertex

Work supported by national funds from FCT – Fundação para a Ciência e a Tecnologia, I.P., Portugal, within projects UID/04564/2020 and CERN/FIS-COM/0029/2017. P. J. S. acknowledges financial support from FCT (Portugal) under contract CEECIND/00488/2017. Simulations performed in supercomputers Navigator, managed by LCA – University of Coimbra [url: www.uc.pt/lca], Lindgren, Sisu (through PRACE projects COIMBRALATT [DECI-9] and COIMBRALATT2 [DECI-12]) and Bob through FCT project CPCA/A2/6816/2020.