Another look at the three-gluon vertex in the minimal Landau gauge

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Outline

Green's functions

- Green's functions summarize the dynamics of the theory
  - QCD: information on confinement and chiral symmetry breaking
- n-point complete Green's functions

$$G^{(n)}(x_1, \ldots, x_n) = \langle 0 | T \left(\phi(x_1) \cdots \phi(x_n) \right) | 0 \rangle,$$

  - decomposition in terms of one particle irreducible (1PI) functions $\Gamma^{(n)}$
  - access to form factors that define $\Gamma^{(n)}$
- lattice approach allows for first principles determination of the complete Green's functions of QCD

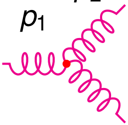
Three gluon vertex

- allow to measure e.g. strong coupling constant
- fundamental role in the structure of Dyson-Schwinger equations
- infrared behaviour predicted by DSE equations
  - infrared suppression — form factors (FF) decrease as $p \to 0$
  - zero crossing — FF change sign in IR region
  - logarithmic divergence at origin — FF $\to -\infty$ for $p \to 0$
- infrared behaviour corroborated by lattice simulations
  - quenched and dynamical simulations

Aguilar, Soto, Ferreira, Papavassiliou, Rodríguez-Quintero, PLB 818(2021)136352
Aguilar, Soto, Ferreira, Papavassiliou, Rodríguez-Quintero, Zafeiropoulos, EPJC 80(2020)154
Aguilar, Ferreira, Papavassiliou, EPJC 80(2020)887
Maas, Vujinović, arXiv:2006.08248 [hep-lat]

Three point complete Green's function

$$\langle A^{a_1}_{\mu_1}(p_1)\, A^{a_2}_{\mu_2}(p_2)\, A^{a_3}_{\mu_3}(p_3)\rangle =$$
$$V\,\delta(p_1+p_2+p_3)\; G^{a_1 a_2 a_3}_{\mu_1\mu_2\mu_3}(p_1,p_2,p_3)$$



$$G^{a_1 a_2 a_3}_{\mu_1\mu_2\mu_3}(p_1,p_2,p_3) =$$
$$D^{a_1 b_1}_{\mu_1\nu_1}(p_1)\; D^{a_2 b_2}_{\mu_2\nu_2}(p_2)\; D^{a_3 b_3}_{\mu_3\nu_3}(p_3)\,\Gamma^{b_1 b_2 b_3}_{\nu_1\nu_2\nu_3}(p_1,p_2,p_3)$$

- color structure:

$$\Gamma^{a_1 a_2 a_3}_{\mu_1\mu_2\mu_3}(p_1,p_2,p_3) = f_{a_1 a_2 a_3}\;\Gamma_{\mu_1\mu_2\mu_3}(p_1,p_2,p_3)$$

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Three gluon vertex

- Bose symmetry requires vertex to be symmetric under interchange of any pair (p_i, a_i, μ_i)
  - $\Gamma_{\mu_1\mu_2\mu_3}(p_1, p_2, p_3)$ must be antisymmetric
- $\Gamma_{\mu_1\mu_2\mu_3}(p_1, p_2, p_3)$ in the continuum: requires six Lorentz invariant form factors
  - two associated to the transverse component $\Gamma^{(t)}$
  - remaining associated to the longitudinal $\Gamma^{(l)}$.

J. S. Ball, T.-W. Chiu, Phys. Rev. **D22**, 2550 (1980)

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Lattice setup

- Wilson gauge action, $\beta = 6.0$
  - 64^4, 2000 configurations

    Duarte, Oliveira, PJS, PRD 94(2016)074502
  - 80^4, 1800 configurations
- rotation to the Landau gauge: FFT-SD method
- gluon field

$$a g_0 A_\mu(x + a\hat{e}_\mu) = \frac{U_\mu(x) - U^\dagger(x)}{2 i g_0} - \frac{\mathrm{Tr}\left[U_\mu(x) - U^\dagger(x)\right]}{6 i g_0}$$

in momentum space:

$$A_\mu(\hat{p}) = \sum_x e^{-i\hat{p}(x+a\hat{e}_\mu)} A_\mu(x + a\hat{e}_\mu) \ , \ \hat{p}_\mu = \frac{2\,\pi\, n_\mu}{a\,L_\mu}$$

Lattice setup

- tree level improved momentum

$$p_\mu = \frac{2}{a} \sin\left(\frac{a\,\hat{p}_\mu}{2}\right)$$

- accessing the 1PI three gluon vertex from the lattice

$$G_{\mu_1\mu_2\mu_3}(p_1, p_2, p_3) = \mathrm{Tr}\,\langle A_{\mu_1}(p_1)\,A_{\mu_2}(p_2)\,A_{\mu_3}(p_3)\rangle =$$
$$= V\delta(p_1 + p_2 + p_3)\frac{N_c(N_c^2 - 1)}{4}\,D(p_1^2)\,D(p_2^2)\,D(p_3^2)$$
$$P_{\mu_1\nu_1}(p_1)\,P_{\mu_2\nu_2}(p_2)\,P_{\mu_3\nu_3}(p_3)\,\Gamma_{\nu_1\nu_2\nu_3}(p_1, p_2, p_3)$$

Momentum configuration

- focus on the asymmetric momentum configuration: $p_2 = 0$

B. Allés et al, Nucl. Phys. B**502**, 325 (1997)

$$G_{\mu_1\mu_2\mu_3}(p,0,-p) = V\frac{N_c(N_c^2-1)}{4}\left[D(p^2)\right]^2 D(0)\frac{\Gamma(p^2)}{3}\; p_{\mu_2}\; T_{\mu_1\mu_3}(p)$$

$$\Gamma(p^2) = 2\left[A(p^2,p^2;0) + p^2\, C(p^2,p^2;0)\right]$$

$$G_{\mu\,\alpha\,\mu}(p,0,-p)\,p_\alpha = V\frac{N_c(N_c^2-1)}{4}\left[D(p^2)\right]^2 D(0)\;\Gamma(p^2)\;p^2$$

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A word about statistical errors on $\Gamma(p^2)$

- measurement of $\Gamma(p^2)$ requires to compute the ratio

$$G_{\mu\alpha\mu}(p,0,-p)p_\alpha / \left[D(p^2)\right]^2 \, D(0)$$

- large statistical fluctuations at high momenta:

$$\left[\Delta\Gamma(p^2)\right]^2 = \frac{1}{\left[D(p^2)\right]^4}\left\{ \left[\frac{\Delta G_{\mu\alpha\mu}p_\alpha}{D(0)}\right]^2 + \left[2\,\Delta D(p^2)\,\frac{G_{\mu\alpha\mu}p_\alpha}{D(p^2)\,D(0)}\right]^2 + \left[2\,\Delta D(0)\,\frac{G_{\mu\alpha\mu}p_\alpha}{[D(0)]^2}\right]^2 \right\}$$

- for large momenta:
  - $D(p^2) \sim 1/p^2$
  - $\Delta\Gamma(p^2) \sim p^4$

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Handling of noise, lattice artefacts

- binning in momentum
- $H(4)$ extrapolation of the lattice data
  - lattice momentum p_μ
  - invariants of the remnant $H(4)$ symmetry group associated with a hypercubic lattice

$$p^2 = p^{[2]} = \sum_\mu p_\mu^2, \quad p^{[4]} = \sum_\mu p_\mu^4, \quad p^{[6]} = \sum_\mu p_\mu^6, \quad p^{[8]} = \sum_\mu p_\mu^8$$

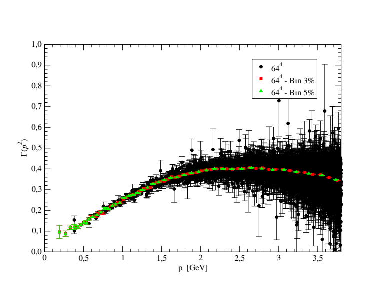
  - on the lattice, a scalar quantity F is a function of all $H(4)$ invariants, i.e. $F_{Lat} = F(p^{[2]}, p^{[4]}, p^{[6]}, p^{[8]})$
  - continuum limit given by $F(p^{[2]}, 0, 0, 0)$

Becirevic et al, PRD 60(1999)094509
Soto, Roiesnel, JHEP 09(2009)007

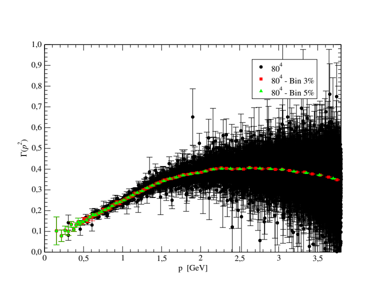
Outline

Original and binned $\Gamma(p^2)$

64^4



80^4



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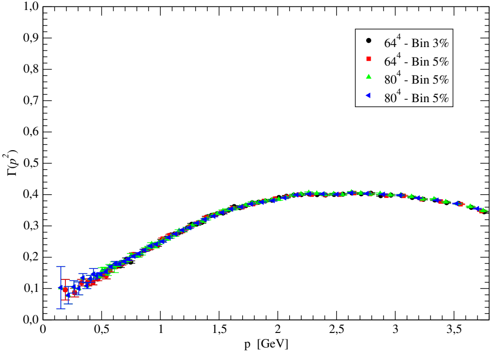
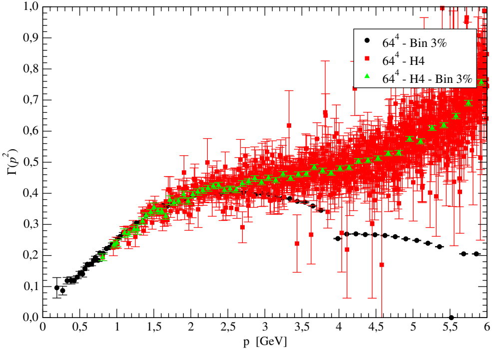
Binned $\Gamma(p^2)$

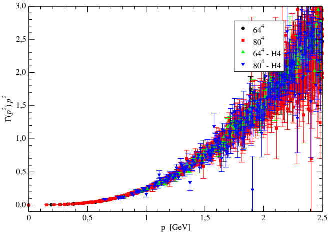


$H(4)$ extrapolation of $\Gamma(p^2)$

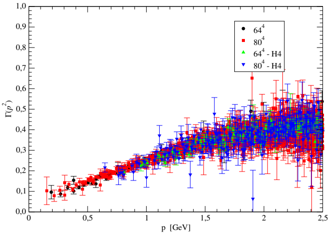


H(4) extrapolation — infrared region

$p^2\Gamma(p^2)$



$\Gamma(p^2)$



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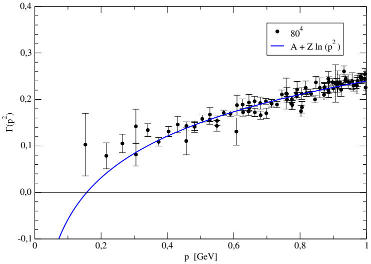
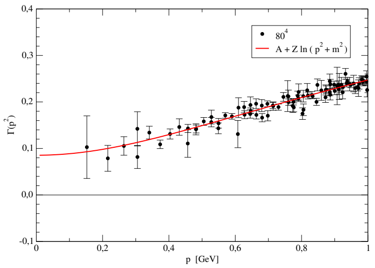
$\Gamma(p^2)$ — Infrared behaviour



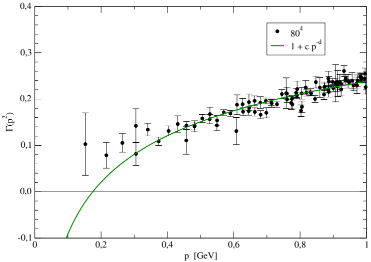
- $\Gamma(p^2) = A + Z\ln(p^2)$
- 80^4 lattice, $p < 1\text{GeV}$
- $A = 0.2395(16)$
- $Z = 0.0646(21)$
- $\chi^2/dof = 1.23$
- $p_o = 157\text{MeV}$

$\Gamma(p^2)$ — Infrared behaviour



- $\Gamma(p^2) = A + Z \ln(p^2 + m^2)$
- 80^4 lattice, $p < 1$GeV
- $A = 0.208(24)$
- $Z = 0.124(27)$
- $m = 0.61(15)$
- $\chi^2/dof = 0.95$

$\Gamma(p^2)$ — Infrared behaviour



- $\Gamma(p^2) = 1 + c\, p^{-d}$
- 80^4 lattice, $p < 1\text{GeV}$
- $c = -0.7621(15)$
- $d = 0.1558(49)$
- $\chi^2/dof = 1.35$
- $p_o = 175\text{MeV}$

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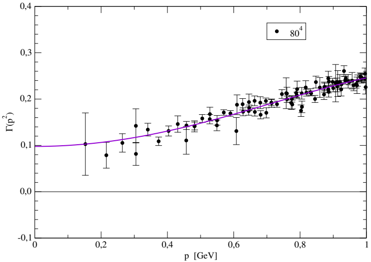
$\Gamma(p^2)$ — Infrared behaviour



- $\Gamma(p^2) = a + bp^2 + cp^4$
- 80^4 lattice, $p < 1\text{GeV}$
- $a = 0.0978(60)$
- $b = 0.218(22)$
- $c = -0.070(18)$
- $\chi^2/dof = 0.98$

Conclusions

- Improved calculation of the three gluon vertex on the lattice
  - particular kinematical configuration $p_2 = 0$
  - two different lattice volumes: $(6.5\text{ fm})^4$ and $(8.2\text{ fm})^4$
  - lattice spacing $a = 0.102$ fm
- H(4) extrapolation pushes the vertex to higher values in UV regime
- functional study in the infrared region
  - some functional forms compatible with zero crossing and IR divergence

Outlook

- Three gluon vertex
  - explore other momentum configurations
- Four gluon vertex

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LCA Laboratory for Advanced Computing

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REPÚBLICA PORTUGUESA

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