Inhomogeneous confining-deconfining phases in rotating plasma

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1. Motivation: vorticity in quark-gluon plasma
   Finite-temperature phase diagram of QCD; Noncentral collisions and vorticity

2. Overview: interacting quarks in rotation and chiral phase transition
   Nambu—Jona-Lasinio model

3. Rotation and confinement of color (a puzzle)
   Lattice; holography; hadron resonance model; compact electrodynamics;
   Tolman-Ehrenfest law and inhomogeneity of plasmas
1) Hot quark-gluon plasma phase and cold hadron phase constitute, basically, one single phase because they are separated by a nonsingular transition ("crossover").

2) The color superconducting phases at high baryonic chemical potential $\mu$ were extensively studied theoretically [they are out of reach of both lattice simulations and Earth-based experiments]

3) The LHC and RHIC experiments probe low baryon density physics. One can safely take $\mu = 0$ in further discussions.
Noncentral collisions

generate magnetic field and angular momentum

Electromagnetism at work:

Strong magnetic field

\[ B \sim 10^{13} \, \text{T} \]

\[ eB \sim m_\pi^2 \left( \tau \sim 0.2 \, \text{fm} \right) \]

et early times of the collision

the effects of magnetic fields may be small (under discussion)

D. Kharzeev, L. McLerran, and H. Warringa, Nucl.Phys.A803, 227 (2008);

Classical mechanics at work

\[ \mathbf{L} = \mathbf{r} \times \mathbf{p} \sim 10^6 \hbar \]

Large orbital angular momentum

Z.-T. Liang and X.-N. Wang, PRL94, 102301 (2005);
How to measure the vorticity?

the vorticity could be probed via quark’s spin polarization

The mechanism:

1) orbital angular momentum of the rotating quark-gluon plasma is transferred to the particle spin

The mechanism is similar to the Barnett effect (found in 1915)
Spin, magnetic field and rotation

The Barnett effect

Coupling between mechanical rotation and spin orientation

**Einstein-de Haas effect.** Modulation of magnetization of iron by applying the external magnetic field, magnetic angular momentum is changed. As a result, the mechanical angular momentum is induced for compensating the modulation of the annular momentum.

**Barnett effect.** Magnetization is induced by applying mechanical rotation since an effective magnetic field, emerges in a rotating body.

**Magnetization due to rotation:** \( M = \chi \Omega / \gamma \)

**Effective magnetic field:** \( B_\Omega = \Omega / \gamma \)

\( \chi \) is the magnetization susceptibility of the medium

Spin and rotation

Relativistic Lagrangian for an electron

\[ \mathcal{L} = \bar{\Psi} \left[ i \gamma^a (p_a - qA_a) - mc^2 \right] \Psi \]

The Hamiltonian in rotating frame

\[ \bar{H}_D = \beta mc^2 + (c\alpha - \Omega \times \mathbf{r}) \cdot \pi + qA_0 - \hbar \Omega \cdot \Sigma \]

Non-relativistic limit (the Foldy—Wouthuysen—Tani transformation):

\[ \bar{H}_e^{(1/m)} = \frac{\pi^2}{2m} - eA_0 - \mathbf{r} \times \pi \cdot \Omega - \frac{e\hbar}{2m} \sigma \cdot (\mathbf{B} + \mathbf{B}_\Omega) \]

The effective Barnett field

\[ \mathbf{B}_\Omega = \frac{m\Omega}{e} \]

aligns spins along rotation axis
Nuclear Barnett Effect found in water

Measured the nuclear Barnett effect by rotating a sample of water at rotational speeds up to 13.5 kHz in a weak magnetic field and observed a change in the polarization of the protons in the sample that is proportional to the frequency of rotation.

Arbogol and Sleator, PRL 122, 177202 (2019)
How to measure the vorticity?

the vorticity could be probed via quark’s spin polarization

The mechanism:

1) orbital angular momentum of the rotating quark-gluon plasma is transferred to the particle spin

The mechanism is similar to the Barnett effect (found in 1915)

2) both particles and anti-particles are polarized in the same way (spin polarization is not sensitive to the particle charge)

3) The vorticity may be measured via the polarization of the produced particles

Which particles? Hyperons! (and other particles with a nonzero spin like vector mesons)
“Self-analysis” of hyperons

Daughter baryon is predominantly emitted in the direction of hyperon’s spin (opposite for anti-particle)

\[
\frac{dN}{d \cos \theta^*} \propto 1 + \alpha_H P_H \cos \theta^*
\]

- $P_H$: hyperon polarization
- $\theta^*$: polar angle of daughter relative to the polarization direction in hyperon rest frame
- $\alpha_H$: hyperon decay parameter

Note: $\alpha_H$ for $\Lambda$ recently updated (BESIII and CLAS)
\[
\alpha_\Lambda = 0.732 \pm 0.014, \quad \alpha_\bar{\Lambda} = -0.758 \pm 0.012
\]

P.A. Zyla et al. (PDG), Prog.Theor.Exp.Phys.2020.083C01

adapted from the talk of T. Niida, ECT* Spin/hydro in heavy-ion collisions 2020
How to measure the polarization?

The observed asymmetry in the hyperon spin polarization ignited much interest.


The most vortical fluid ever observed

The experimental result for the vorticity:

\[ \omega \approx (9 \pm 1) \times 10^{21} \text{ s}^{-1} \]
Phase diagram at finite temperature

Rotation decreases the critical temperature of the chiral phase transition.

The critical temperature of the chiral symmetry breaking transition.

Uniform rotation restores the chiral symmetry.


What is the mechanism?
The “Barnett coupling” in QCD

Uniform rotation restores the chiral symmetry

What is the mechanism?

The chiral condensate is a spin-0 object

\[ \langle \bar{\psi} \psi \rangle = -\frac{\sigma}{2G} \]

The Barnett effect polarized both the spin of a quark and the spin of an anti-quark along the axis of rotation

The chiral condensate is destroyed by rotation due to an analogue of the Barnett effect
Finite-size effects are expected to be strong.

\[ R_{\text{max}} \Omega = 1 \quad \Rightarrow \quad R_{\text{max}} \approx 0.3 \text{ fm} \]

Small transverse size close to the perturbative regime

Chirally symmetry and rotation in QCD

Juang and Liao, PRL 117, 192302 (2016)
What is the effect of rotation on confinement?

Disclaimer: we don’t know/understand for sure. But let’s talk about it anyway.

Papers on the subject (exhaustive list, in order of appearance):

1. V. Braguta, A. Kotov, D. Kuznedelev, and A. Roenko, JETP Lett. 112, 6 (2020); more details in Phys. Rev. D 103 (2021) 9; first-principles lattice calculation: temperature increases with rotation

2. X. Chen, L. Zhang, D. Li, D. Hou, and M. Huang, JHEP 07,132 (2021); holographic approach: temperature decreases with rotation

3. M. Chernodub, Phys. Rev. D 103, 054027 (2021); toy model analysis: temperature decreases with rotation


5. V. Braguta, A. Kotov, D. Kuznedelev, and A. Roenko, “Lattice 21” Symposium; first-principles lattice calculation with fermions included (last week, July 28, 2021) gluons/quarks force the critical temperature to increase/decrease with $\Omega$. 

The confusion is a solid signature that the situation is far from trivial: three independent theoretical papers [2,3,4] based on three different approaches agree with each other and they together contradict qualitatively (!) the first-principles simulations [1], but some hope arises from [5].
Rotation effect from holography

Phase diagram for pure gluodynamics (no quarks)

Dense rotating gluon matter at high-temperature

→ rotation decreases deconfinement temperature

Hadron resonance gas (HRG)

Partition function

\[
Z = \int dm \, \rho(m) \, e^{-m/T}
\]

Hadrons mass spectrum

\[
\rho(m) = e^{m/T_H}
\]

Boltzmann factor

(we omit an integration measure which gives a polynomial factor)

Partition function diverges at \( T > T_H \)

→ hadrons melt and the deconfinement occurs

Pressure of hadrons is

\[
p(T, \mu, \omega; \Lambda) = \sum_{m; M_i \leq \Lambda} p_m + \sum_{b; M_b \leq \Lambda} p_b
\]

cutoff mesons baryons
Hadron resonance gas (HRG)

Taking into account the rotation:

\[ \hat{H} \rightarrow \hat{H} - J \cdot \omega \]

Total angular momentum:

\[ J = L + S \]

Shift of energies in the rotating frame:

\[ \varepsilon \rightarrow \varepsilon - (l + s)\omega \]
Deconfinement due to rotation in HGR

The phase diagram of rotating hadron resonance gas

Deconfinement due to rotation: General arguments

Gluons and quarks are living in the corotating frame, which rotates together with the plasma.

→ The laboratory system is the flat Minkowski spacetime
→ The corotating system corresponds to the curvilinear reference system with the following metric tensor

\[
g_{\mu\nu} = \begin{pmatrix}
1 - (x^2 + y^2)\Omega^2 & y\Omega & -x\Omega & 0 \\
y\Omega & -1 & 0 & 0 \\
-x\Omega & 0 & -1 & 0 \\
0 & 0 & 0 & -1
\end{pmatrix}
\]

corresponding to the line element of the curved space-time:

\[
ds^2 \equiv g_{\mu\nu}dx^\mu dx^\nu = (1 - \rho^2\Omega^2)\,dt^2 - 2\rho^2\Omega\,dt\,d\varphi - d\rho^2 - \rho^2\,d\varphi^2 - dz^2
\]
Tolman-Ehrenfest law

In a static background gravitational field, the temperature of a system in a thermal equilibrium is not constant:

\[ T(x) \sqrt{g_{00}(x)} = T_0 \]

**Metric in rotating frame:**

\[
g_{\mu\nu} = \begin{pmatrix}
1 - (x^2 + y^2)\Omega^2 & y\Omega & -x\Omega & 0 \\
y\Omega & -1 & 0 & 0 \\
-x\Omega & 0 & -1 & 0 \\
0 & 0 & 0 & -1
\end{pmatrix}
\]

**Local temperature on the axis of rotation**

\[ T_0 \equiv T(0) \]

**Temperature rises as the distance from the axis of rotation increases:**

\[ T(\rho) = \frac{T(0)}{\sqrt{1 - \rho^2\Omega^2}} \]

Thermal equilibrium in rotating QGP

Temperature is colder in the center and higher at the edges of the system:

$$T(\rho) = \frac{T(0)}{\sqrt{1 - \rho^2 \Omega^2}}$$

$$T_\Omega(\rho) < T_{c,\infty}$$ (confinement),

$$T_\Omega(\rho) > T_{c,\infty}$$ (deconfinement)

The critical temperature in a thermodynamically large, nonrotating system:

The phase structure:

New mixed/inhomogeneous phase!!

Two critical temperatures:

$$T_{c1} = T_{c,\infty} \sqrt{1 - \Omega^2 R^2}, \quad T_{c2} = T_{c,\infty}$$

Theory from a toy model: M. Ch. Phys. Rev. D 103, 054027 (2021)


cf. chiral inhomogeneity (modulo boundary condition)
Hot dense rotating quark-gluon plasma

The Tolman-Ehrenfest law for temperature and chemical potential

\[ T(x) \sqrt{g_{00}(x)} = T_0, \quad \mu_B(x) \sqrt{g_{00}(x)} = \mu_{B0} \]

\[ \frac{T}{T_c, \infty} \quad \frac{\mu}{\mu_c, \infty} \]

- **Quark-gluon plasma**
- **Mixed inhomogeneous phase**
- **Hadronic phase**

\[ \Omega R = 0.5 \]
Check: Compact U(1) gauge theory in (2+1)d
(also known as “compact electrodynamics”, cU(1) or cQED, despite the absence of matter fields)

Confinement picture is well-established!

Lagrangian:

$$\mathcal{L} = \frac{1}{4} F_{\mu\nu}^2$$

Field strength tensor:

$$F_{\mu\nu} = F_{\mu\nu}^{\text{ph}} + F_{\mu\nu}^{\text{mon}}$$

- photon–field strength tensor
  - “perturbative part”
  - “regular”
- monopole strength tensor
  - “non-perturbative part”
  - “singular”

$$\partial_\mu \tilde{F}_\mu = \partial_\mu \tilde{F}_{\text{mon},\mu} = \rho$$

dual field-strength vector

density of magnetic monopoles
Compact $U(1)$ gauge theory in (2+1)d

- Confinement due to monopoles (first-principle lattice simulations)

Do the same but with rotation!
Uniform rotation in compact QED

— Critical deconfinement temperature close to the center of rotation

\[ T_\Omega(\rho) = T(0) \left( 1 + \frac{1}{2} \rho^2 \Omega^2 + O(\Omega^4) \right) \]

calculated as the temperature at which the monopoles are binding into the magnetically neutral monopole-antimonopole pairs

— Consistent with Tolman-Ehrenfest law

\[ T(\rho) = \frac{T(0)}{\sqrt{1 - \rho^2 \Omega^2}} \]

— Leads to inhomogeneous confining plasmas

Head-on collision of analytics and numerics

Theoretical results: rotation decreases deconfinement temperature

First-principle numerical results in gluodynamics: rotation increases deconfinement temperature

\[ T_c(\Omega)/T_c(0) = 1 + C_2\Omega^2 \text{ with } C_2 > 0 \]

V. Braguta, A. Kotov, D. Kuznedelev, and A. Roenko, JETP Lett. 112, 6 (2020); Phys. Rev. D 103 (2021) 9; add fermions (while the pion is still heavy ~ 690 MeV): “Lattice 21” Symposium (Wednesday, 2021);

 gluons/quarks force the critical temperature to increase/decrease with \( \Omega \) (a partial resolution?).
Conclusions

- Quark-Gluon plasma is the most vortical fluid ever observed
  The experimentally measured vorticity \( \omega \approx (9 \pm 1) \times 10^{21} \text{s}^{-1} \)

- Effect of rotation on the phase structure of QCD?
  A uniform rotation is a simplest tractable approximation to investigation of quark-gluon plasma with large angular momentum created in noncentral heavy-ion collisions

- Rotation restores chiral symmetry and leads to a decrease of the temperature of the chiral phase transition in QCD

- The effect of the rotation on the deconfinement temperature is still controversial. Independent theoretical approaches signal that the deconfinement temperature decreases with temperature while lattice results with pure glue suggest the opposite (quarks try change the slope).

  More efforts are needed! →