Inhomogeneous confining-deconfining phases in rotating plasma

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1. **Motivation: vorticity in quark-gluon plasma**

   Finite-temperature phase diagram of QCD; Noncentral collisions and vorticity

2. **Overview: interacting quarks in rotation and chiral phase transition**

   Nambu—Jona-Lasinio model

3. **Rotation and confinement of color (a puzzle)**

   Lattice; holography; hadron resonance model; compact electrodynamics;
   Tolman-Ehrenfest law and inhomogeneity of plasmas

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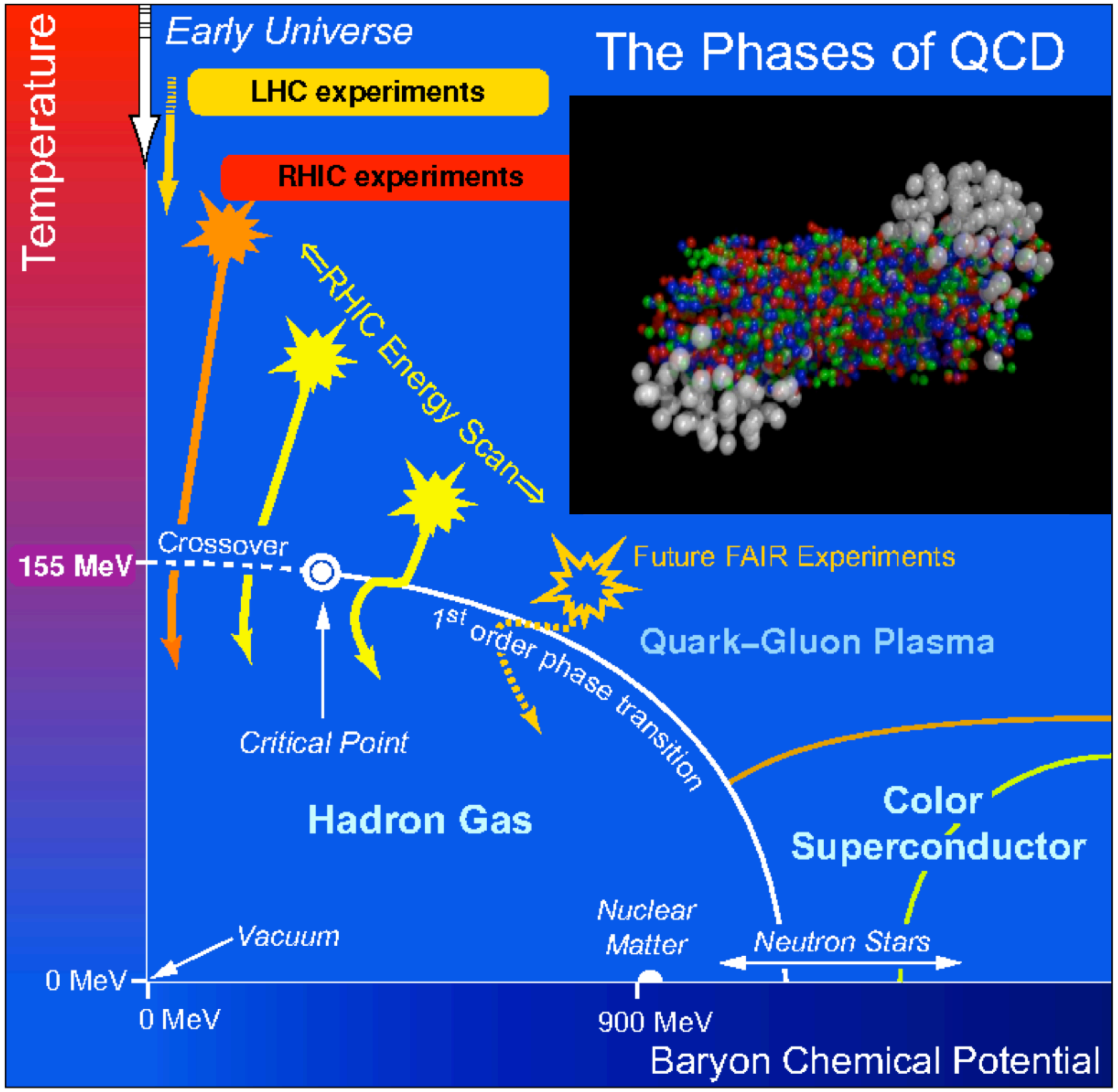
Phase diagram of QCD

1) Hot quark-gluon plasma phase and cold hadron phase constitute, basically, one single phase because they are separated by a nonsingular transition ("crossover").

2) The color superconducting phases at high baryonic chemical potential μ were extensively studied theoretically [they are out of reach of both lattice simulations and Earth-based experiments]



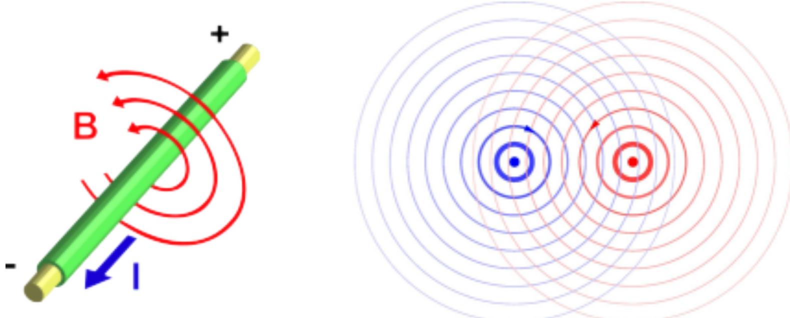
From a BNL webpage

3) The LHC and RHIC experiments probe low baryon density physics. One can safely take $\mu = 0$ in further discussions.

Noncentral collisions

generate magnetic field and angular momentum

Electromagnetism at work:

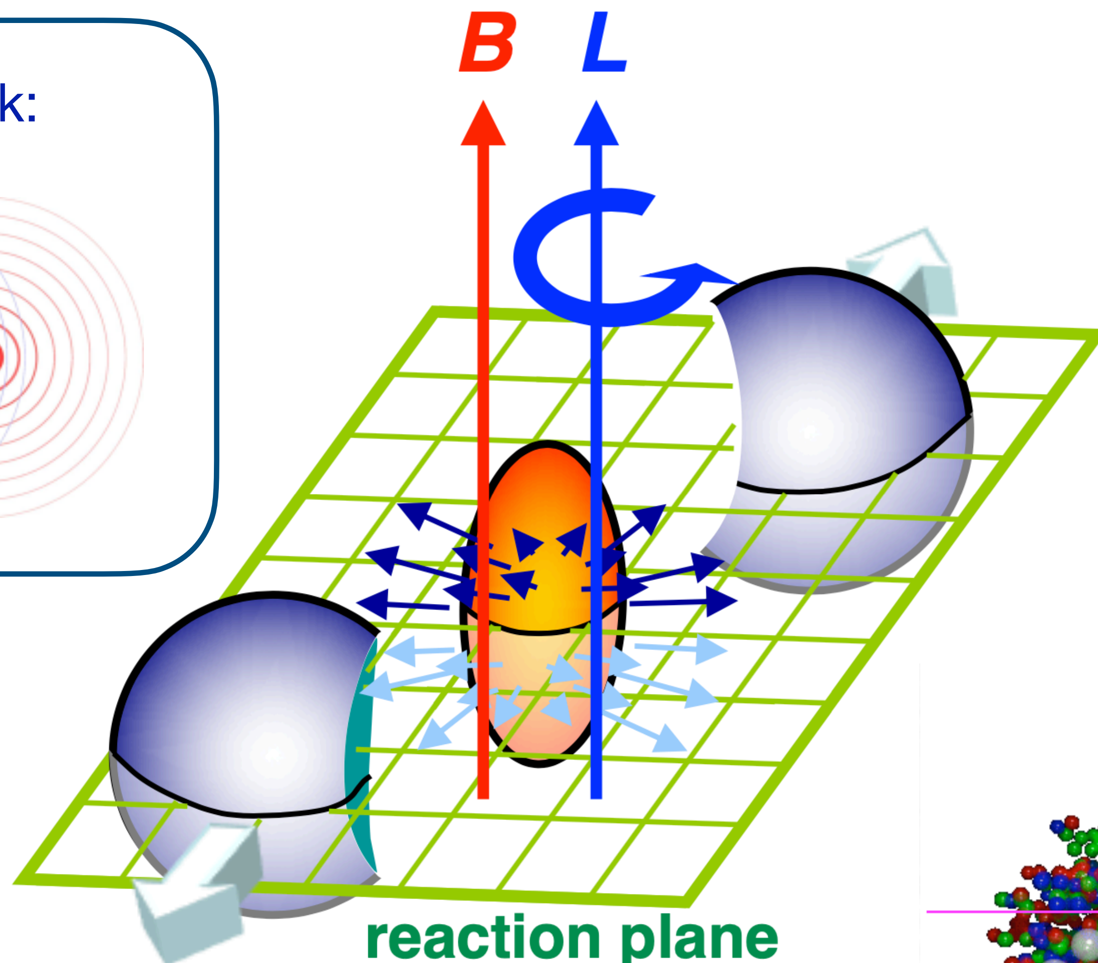


Strong magnetic field

$$B \sim 10^{13}\ \mathrm{T}$$

$$eB \sim m_\pi^2 \quad (\tau \sim 0.2\ \mathrm{fm})$$

et early times of the collision

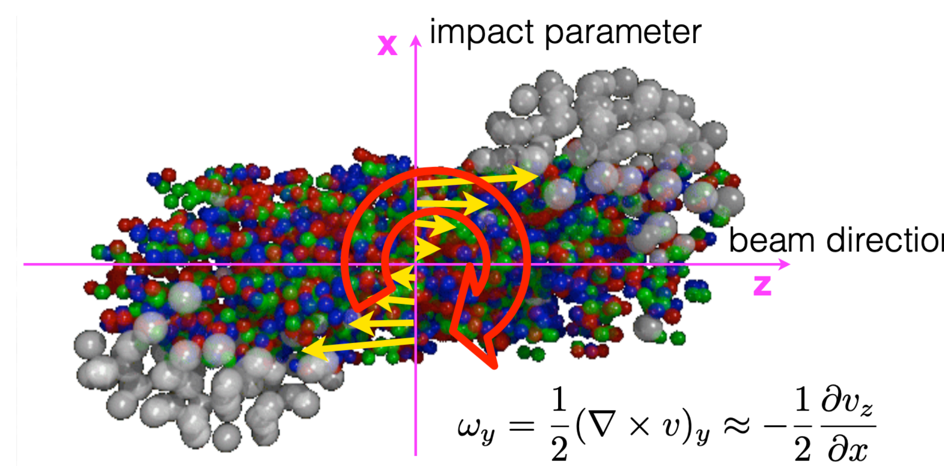


Classical mechanics at work

$$\mathbf{L} = \mathbf{r} \times \mathbf{p}$$

$$\sim 10^6 \hbar$$

Large orbital angular momentum



strong vorticity

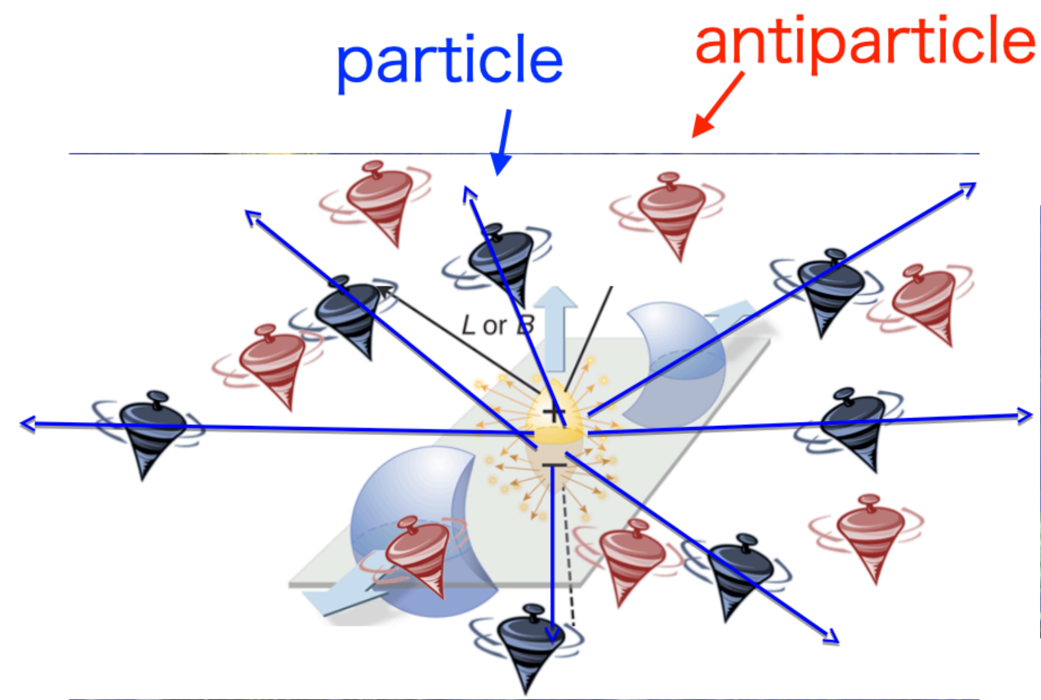
the effects of magnetic fields may be small (under discussion)

D. Kharzeev, L. McLerran, and H. Warringa, Nucl.Phys.A803, 227 (2008);
McLerran and Skokov, Nucl. Phys. A929, 184 (2014)

Z.-T. Liang and X.-N. Wang, PRL94, 102301 (2005);
S. Voloshin, nucl-th/0410089 (2004)

How to measure the vorticity?

the vorticity could be probed via quark's spin polarization



The mechanism:

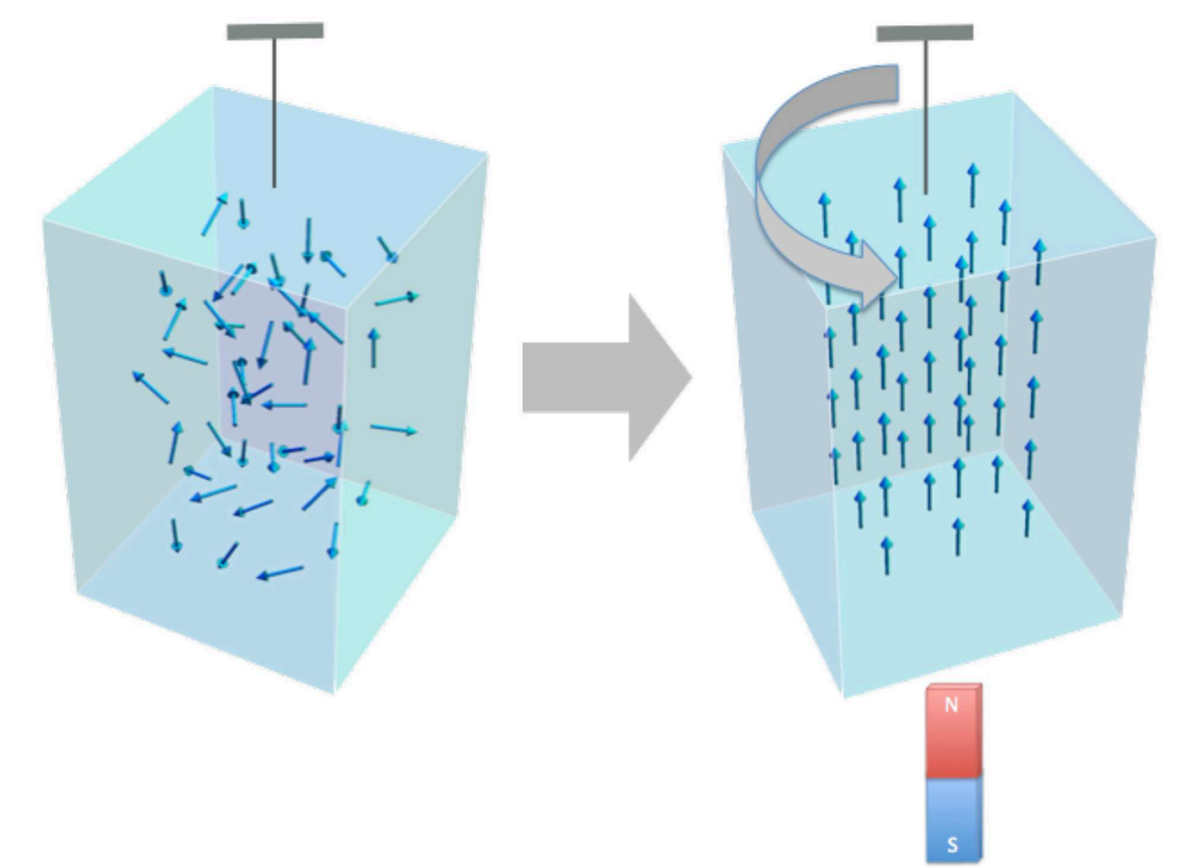
1) orbital angular momentum
   of the rotating quark-gluon plasma
   is transferred to the particle spin

The mechanism is similar to
the Barnett effect (found in 1915)

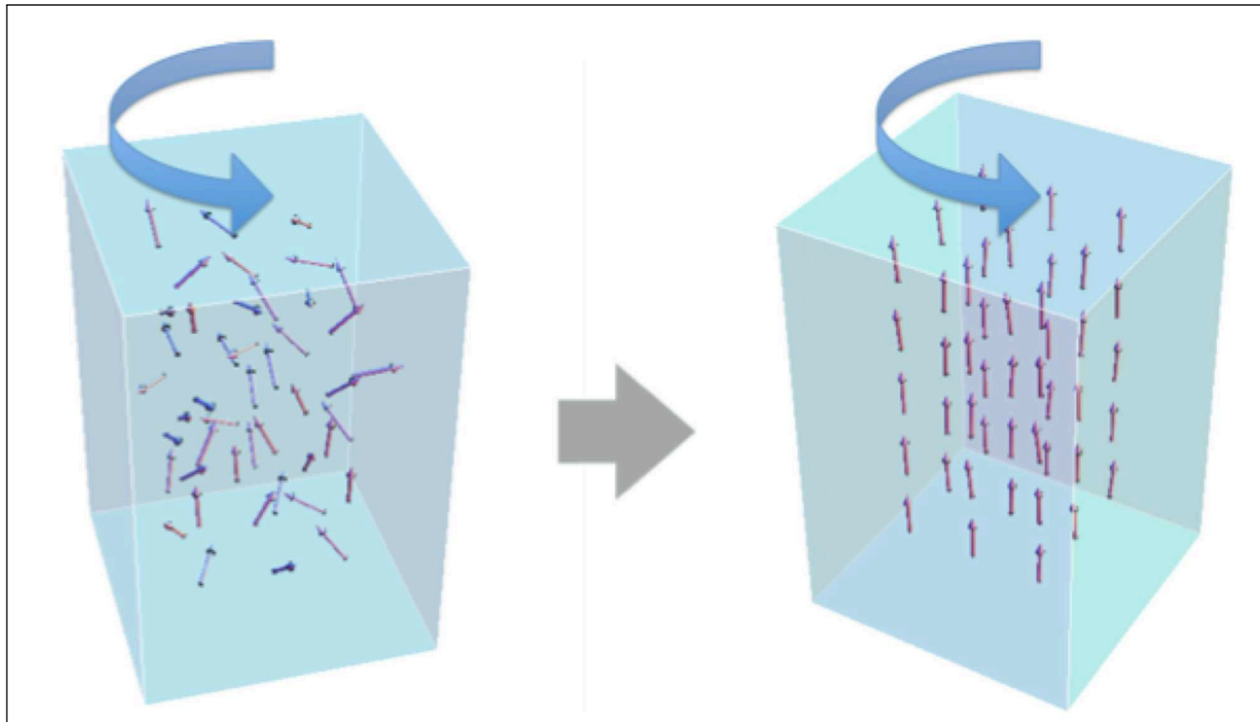
Spin, magnetic field and rotation

The Barnett effect

Coupling between mechanical rotation and spin orientation



Einstein-de Haas effect. Modulation of magnetization of iron by applying the external magnetic filed, magnetic angular momentum is changed. As a result, the mechanical angular momentum is induced for compensating the modulation of the annular momentum.



Barnett effect. Magnetization is induced by applying mechanical rotation since an effective magnetic field, emerges in a rotating body.

Magnetization due to rotation: $M = \chi \Omega / \gamma$

Effective magnetic field: $B_\Omega = \Omega / \gamma$

χ is the magnetization susceptibility of the medium

Figures and subsequent discussion from Matsuo, Ieda, Maekawa, Frontiers in Physics 3, 54 (2015)

Spin and rotation

Relativistic Lagrangian for an electron

$$\mathcal{L} = \bar{\Psi}\left[i\gamma^a c(p_a - qA_a) - mc^2\right]\Psi$$

spin operator

$$\Sigma = \frac{1}{2}\begin{pmatrix} \sigma & O \\ O & \sigma \end{pmatrix}$$

coordinate transformation

$$d\mathbf{r}' = d\mathbf{r} + (\Omega \times \mathbf{r})\, dt, \; dt' = dt$$

The Hamiltonian in rotating frame

$$\bar{H}_D = \beta mc^2 + (c\alpha - \boldsymbol{\Omega} \times \mathbf{r}) \cdot \pi + qA_0 - \hbar\boldsymbol{\Omega} \cdot \boldsymbol{\Sigma}$$

rotation velocity

Spin-rotation coupling

Non-relativistic limit (the Foldy—Wouthuysen—Tani transformation):

$$\bar{H}_e^{(1/m)} = \frac{\pi^2}{2m} - eA_0 - \mathbf{r} \times \pi \cdot \boldsymbol{\Omega} - \frac{e\hbar}{2m}\sigma \cdot (\mathbf{B} + \mathbf{B}_\Omega)$$

orbital momentum

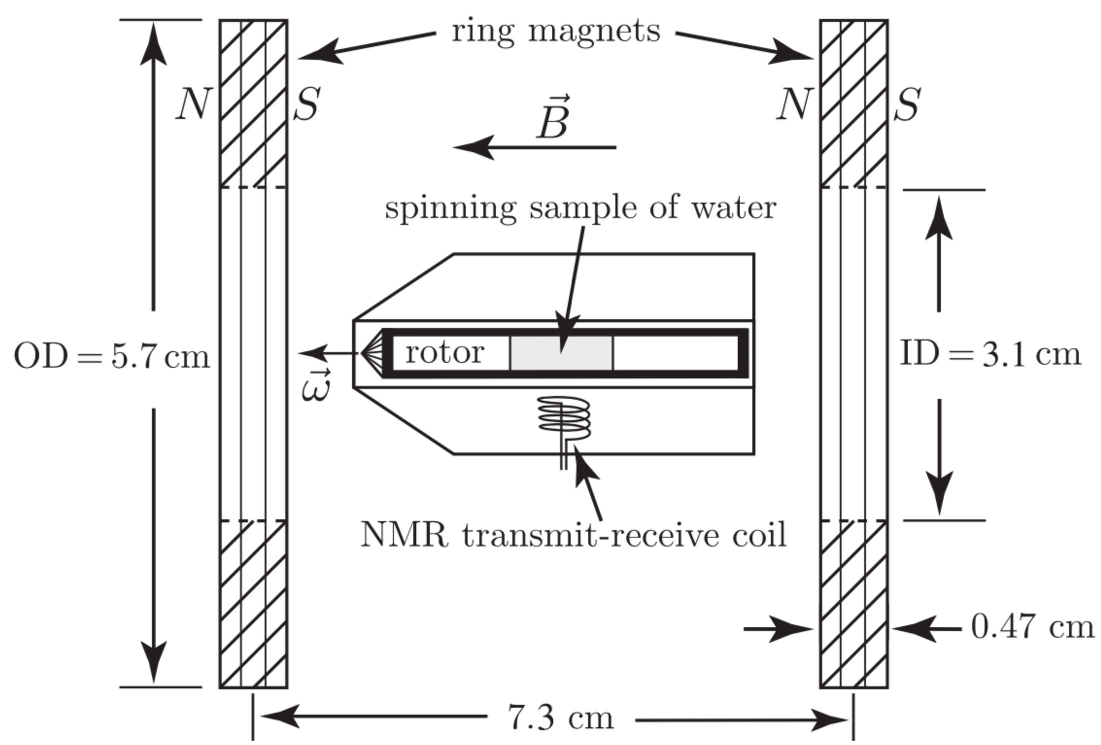
Zeeman coupling

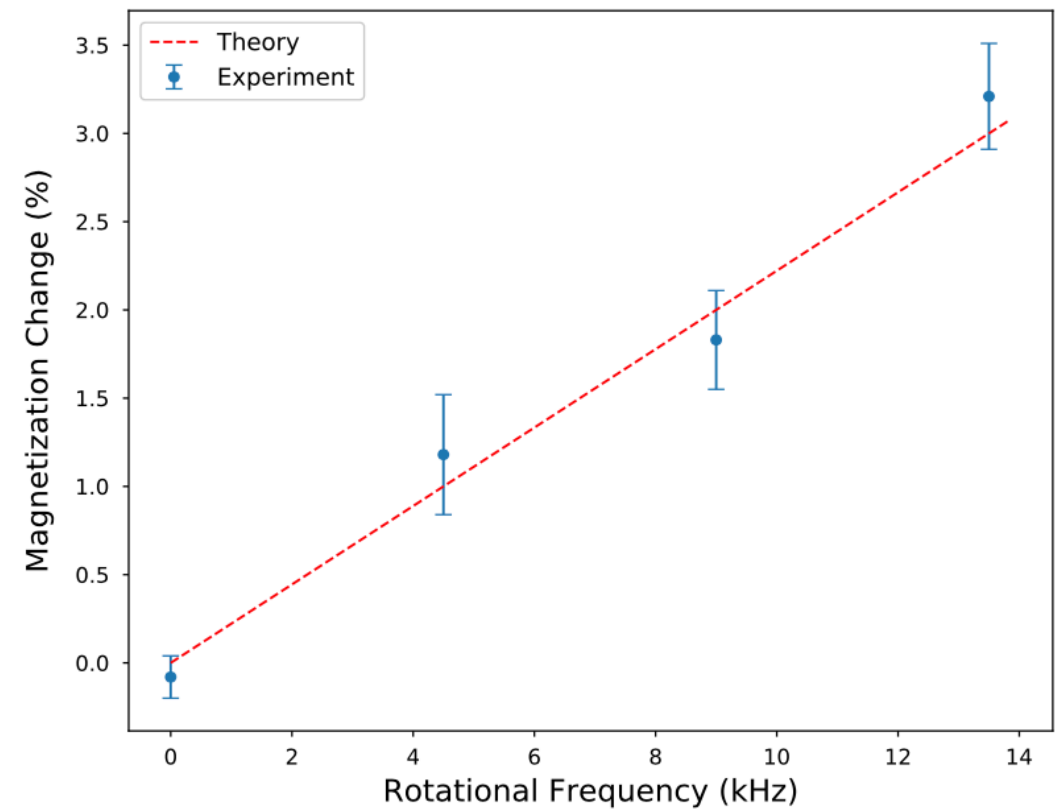
Barnett effect

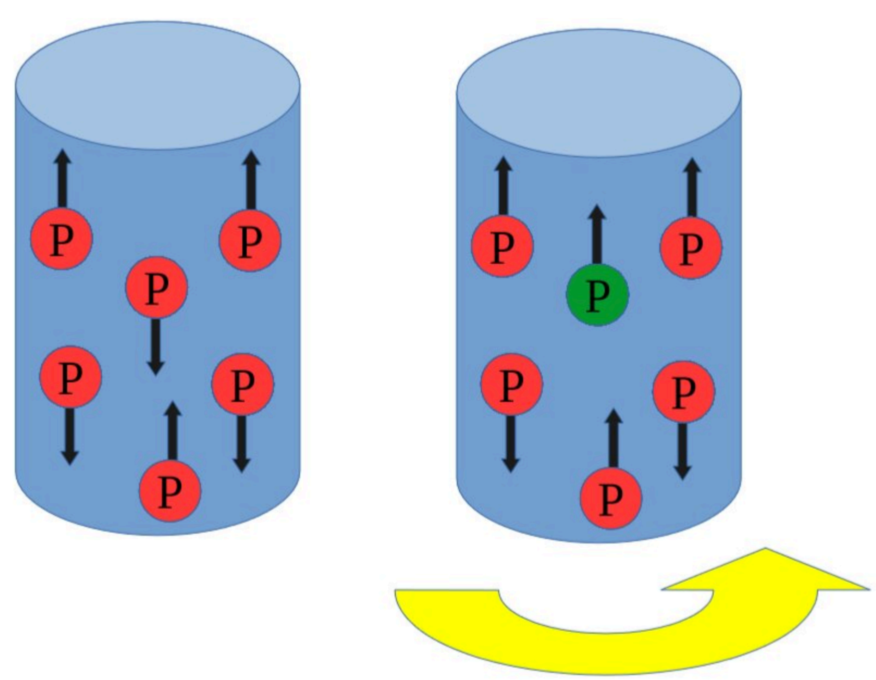
The effective Barnett field $\mathbf{B}_\Omega = m\boldsymbol{\Omega}/e$

aligns spins along rotation axis

Nuclear Barnett Effect found in water





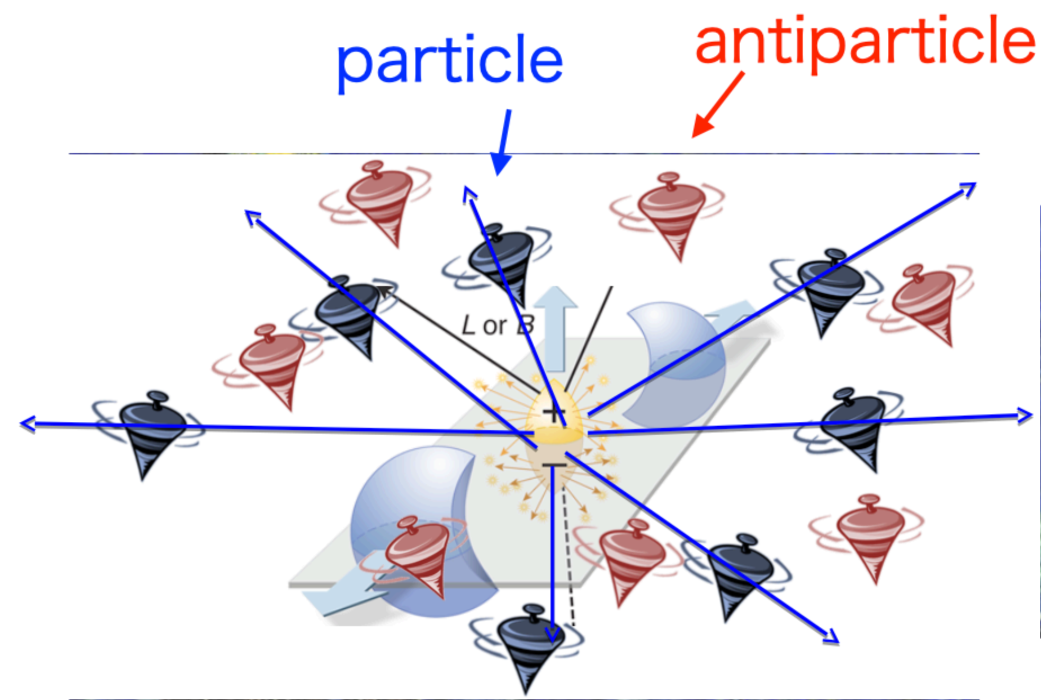


Measured the nuclear Barnett effect by rotating a sample of water at rotational speeds up to 13.5 kHz in a weak magnetic field and observed a change in the polarization of the protons in the sample that is proportional to the frequency of rotation.

Arabgol and Sleator, PRL 122, 177202 (2019)

How to measure the vorticity?

the vorticity could be probed via quark's spin polarization



The mechanism:

1) orbital angular momentum
of the rotating quark-gluon plasma
is transferred to the particle spin

The mechanism is similar to
the Barnett effect (found in 1915)

2) both particles and anti-particles
are polarized in the same way
(spin polarization is not sensitive to the particle charge)

3) The vorticity may be measured via the polarization of the produced particles

Which particles? Hyperons! (and other particles with a nonzero spin like vector mesons)

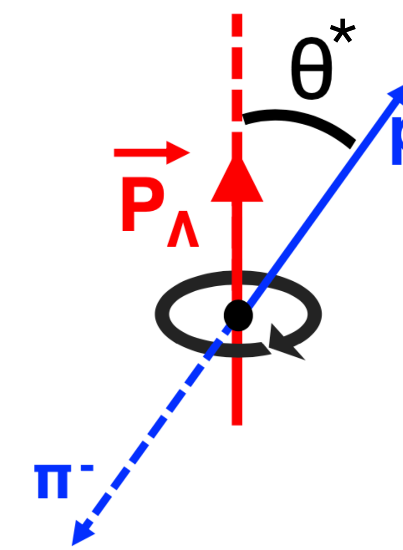
“Self-analysis” of hyperons

Daughter baryon is predominantly emitted in the direction of hyperon’s spin (opposite for anti-particle)

$$\Lambda \rightarrow p + \pi^-$$

(BR: 63.9%, cτ~7.9 cm)

$$\frac{dN}{d\cos\theta^*} \propto 1 + \alpha_H \mathrm{P_H} \cos\theta^*$$



P_H: hyperon polarization

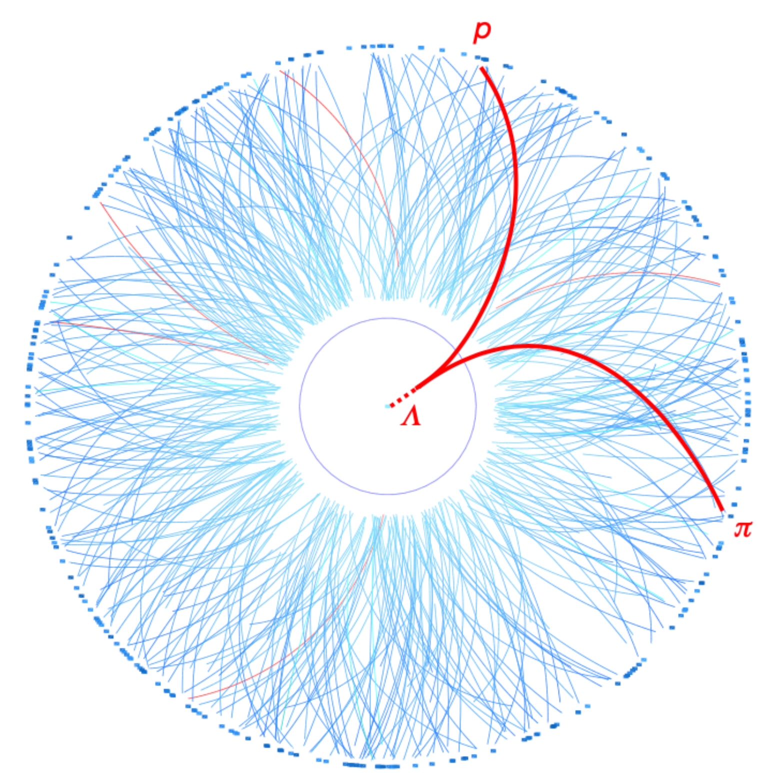
θ^*: polar angle of daughter relative to the polarization directior in hyperon rest frame

α_H: hyperon decay parameter

Note: α_H for Λ recently updated (BESIII and CLAS)

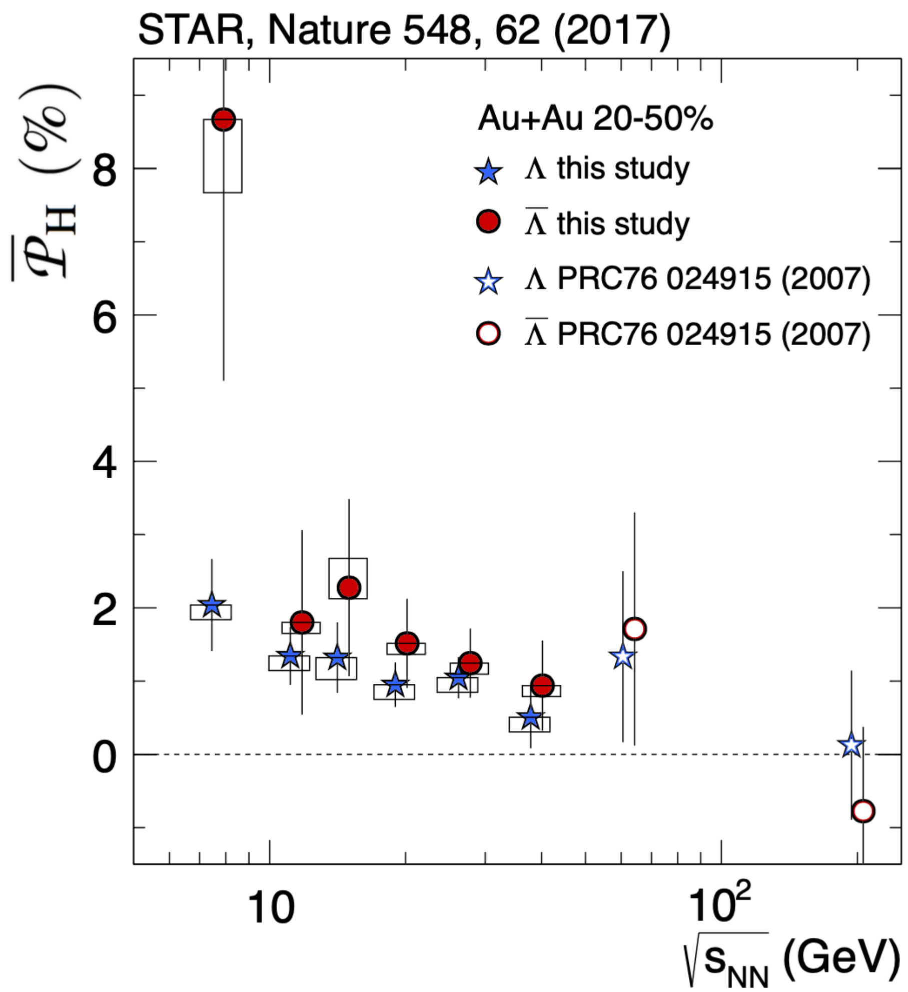
α_Λ=0.732±0.014, $\alpha_{\bar{\Lambda}}$=-0.758±0.012

P.A. Zyla et al. (PDG), Prog.Theor.Exp.Phys.2020.083C01

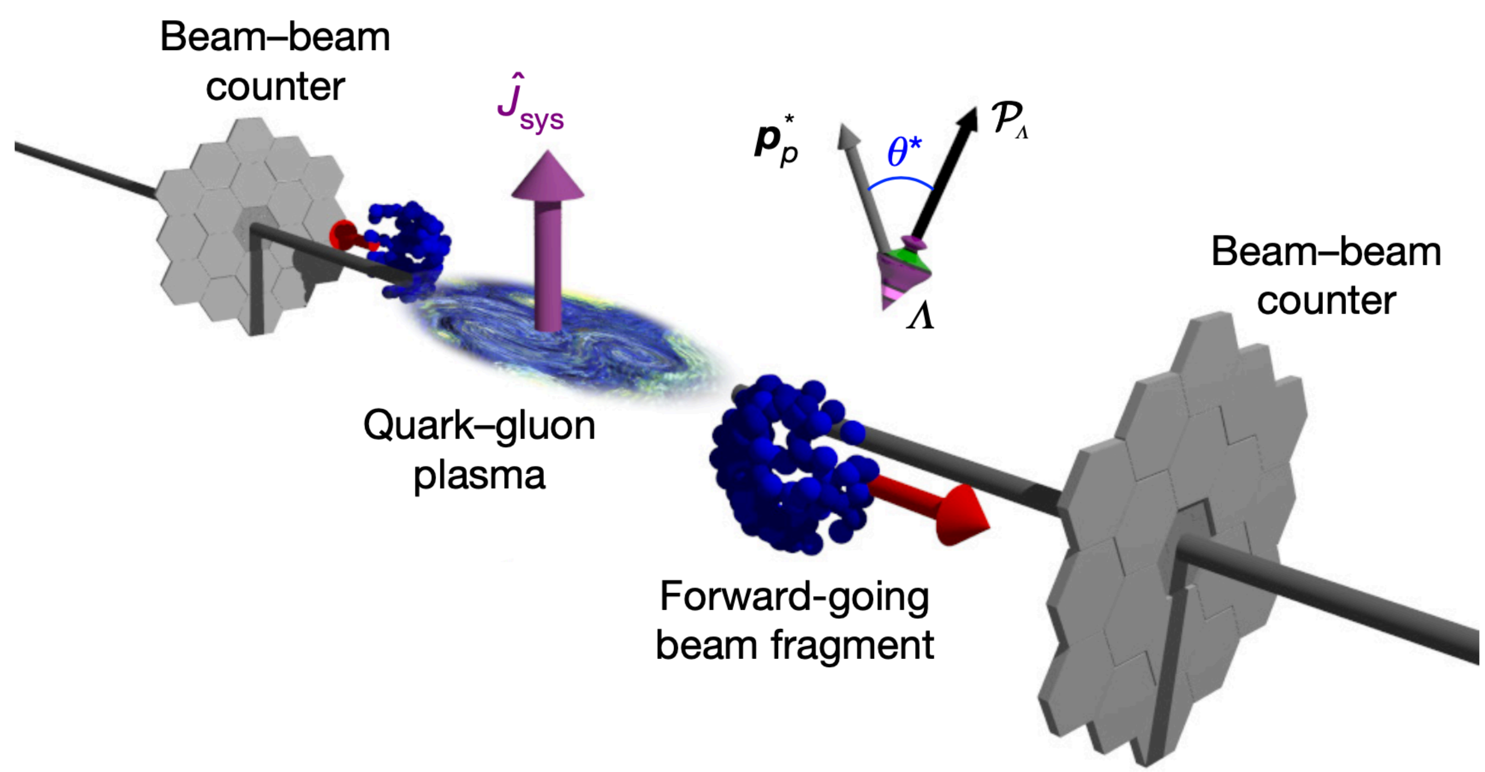


adapted from the talk of T. Niida, ECT* Spin/hydro in heavy-ion collisions 2020

How to measure the polarization?



The observed asymmetry in the hyperon spin polarization ignited much interest.



Overview of the experimental situation: T. Niida, talk at the workshop "Spin and hydrodynamics in relativistic nuclear collisions" ECT*, Trento, Italy, Oct. 05-16, 2020.

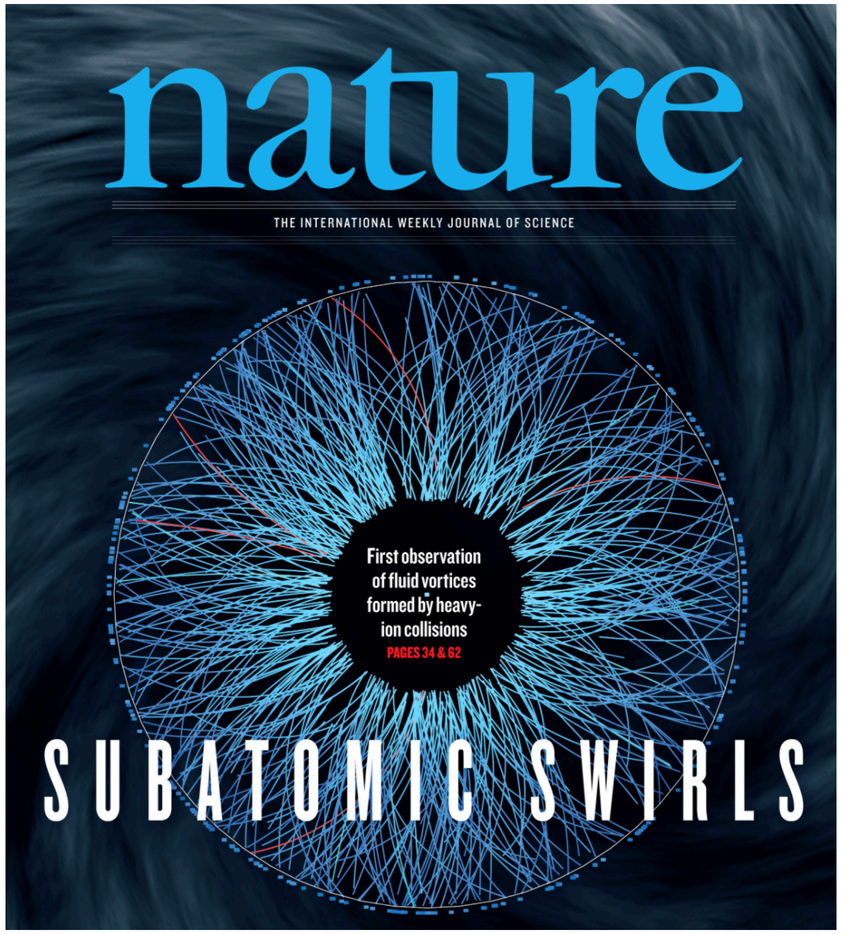
Overview of the theoretical situation: "Vorticity and Spin Polarization in Heavy Ion Collisions: Transport Models", X.-G. Huang, J. Liao, Q. Wang, X.-L. Xia, arXiv:2010.08937

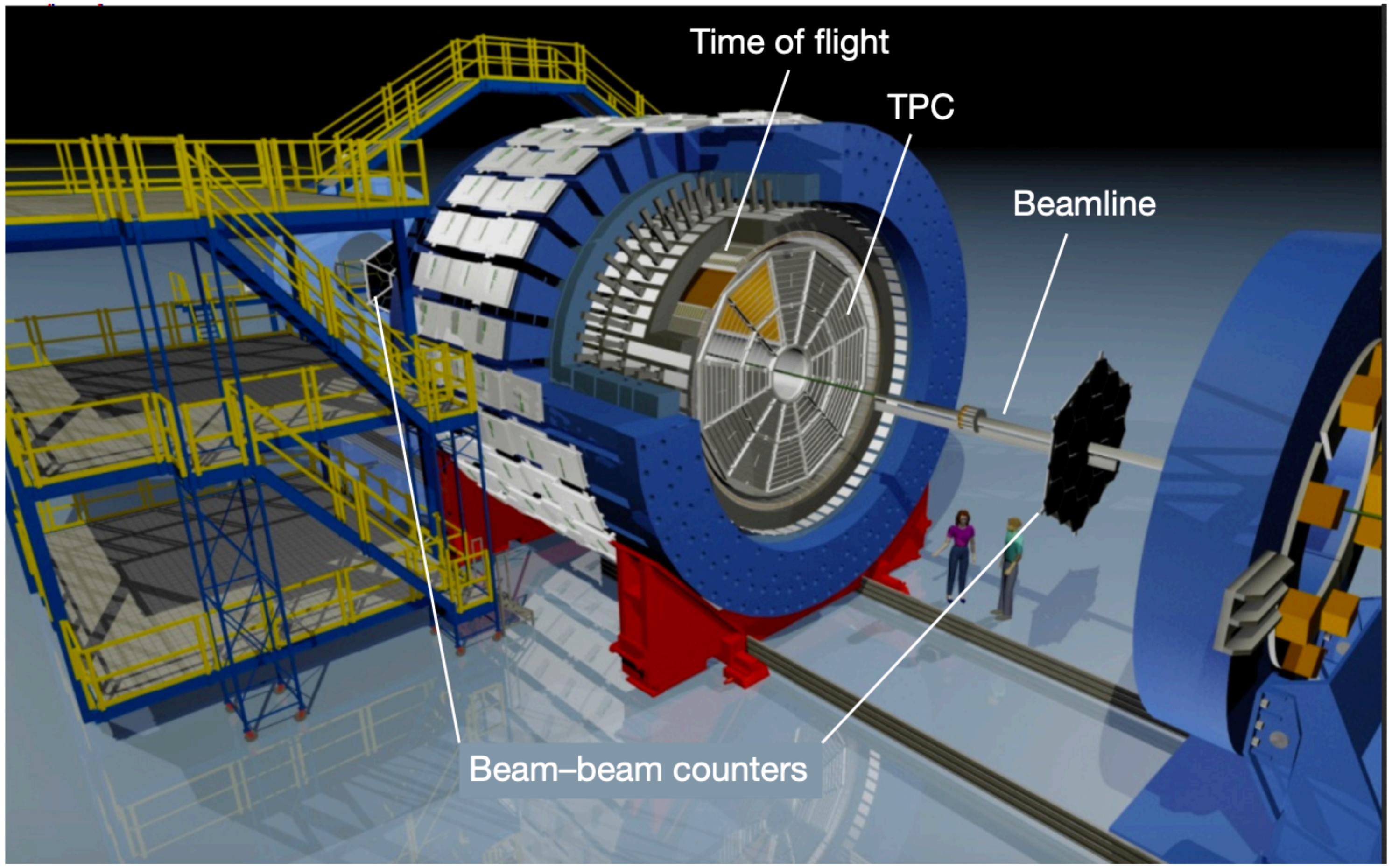
Baznat, Gudima, Sorin, Teryaev, Phys. Rev. C 88, 061901(R) (2013); Sorin and Teryaev, Phys. Rev. C 95, no.1, 011902(R) (2017).
Becattini, Karpenko, Lisa, Upsal, Voloshin, Phys.Rev.C 95 (2017) 5, 054902; Teryaev, Zakharov, Phys.Rev.D 96 (2017) 9, 096023.
Baznat, Gudima, Sorin and Teryaev, Phys. Rev. C 97, no.4, 041902(R) (2018); Csernai, Kapusta, and Welle, Phys. Rev. C 99, no.2, 021901(R) (2019);
D-Xian Wei, Wei-Tian Deng, and Xu-Guang Huang, Phys. Rev. C 99, 014905 (2019); Vitiuk, Bravina and Zabrodin, Phys. Lett. B 803, 135298 (2020)
B. Fu, K. Xu, X.-G. Huang, H.Song, ArXiv:2011.03740; V. E. Ambrus, M.N. Chernodub ArXiv:2010.05831, and others

The most vortical fluid ever observed

The experimental result for the vorticity:

$$\omega \approx (9 \pm 1) \times 10^{21}\,\mathrm{s}^{-1}$$



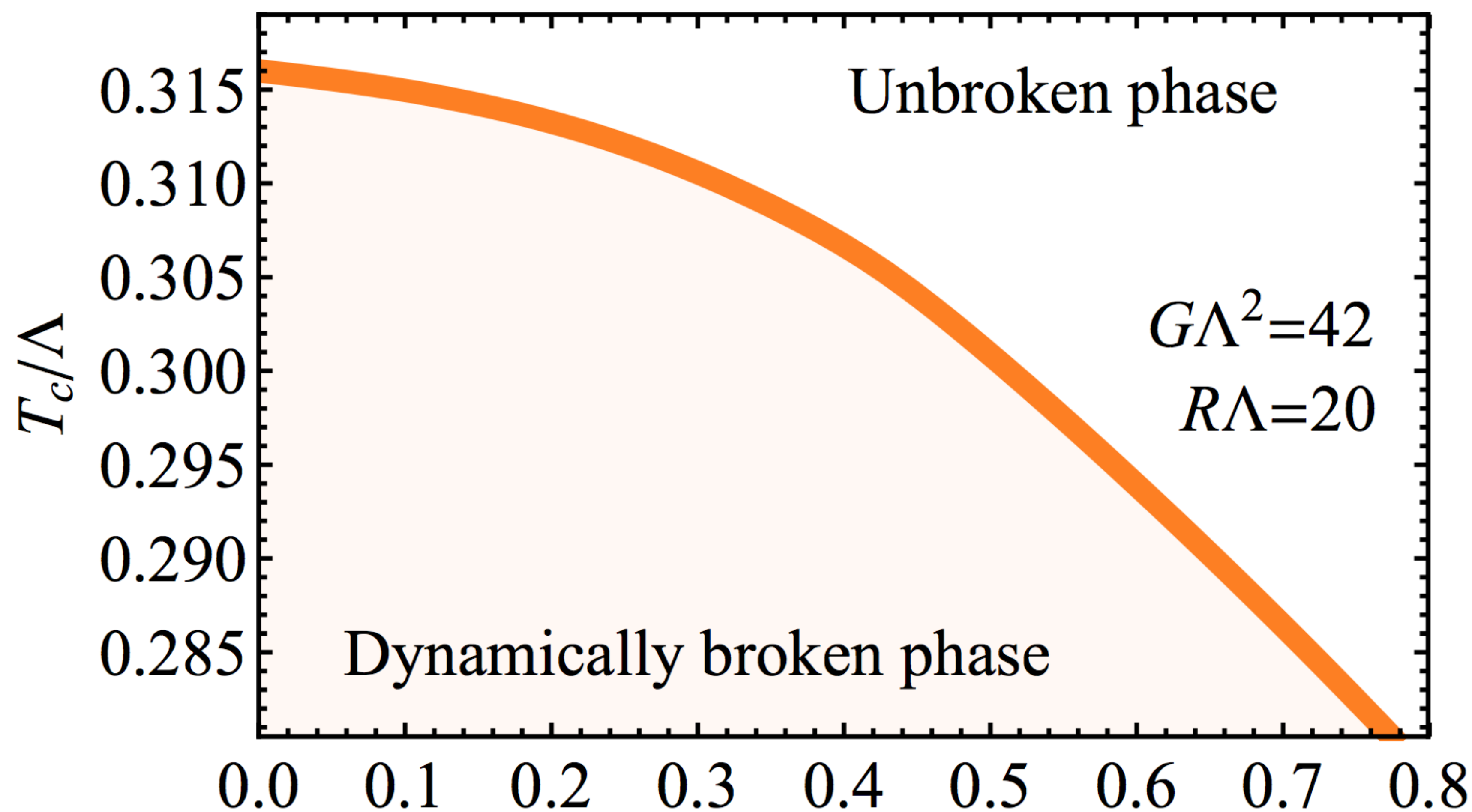


The STAR Collaboration, Nature 62, 548 (2017)

Phase diagram at finite temperature

Rotation decreases the critical temperature of the chiral phase transition

The critical temperature of the chiral symmetry breaking transition



nonrotating system ΩR ultrarelativistic rotation

Uniform rotation restores the chiral symmetry

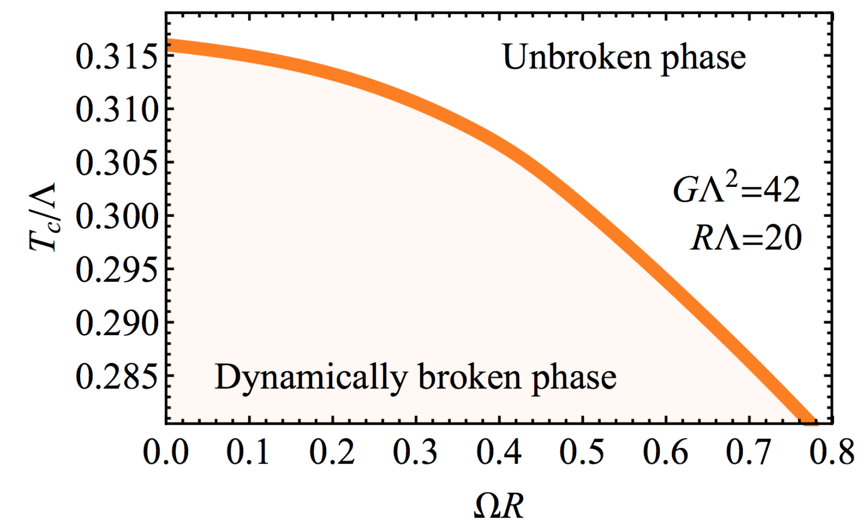
Holographic approaches [B. McInnes, Nucl.Phys. B911 (2016) 173],
Nambu—Jona-Lasinio models [H.-L. Chen, K. Fukushima, X.-G. Huang, K. Mameda, Phys.Rev. D93 (2016) 104052],
[Y. Jiang, J. Liao, Phys.Rev.Lett. 117 (2016), 192302]; M.Ch. and Shinya Gongyo, JHEP 01, 136 (2017)

What is the mechanism?

The "Barnett coupling" in QCD

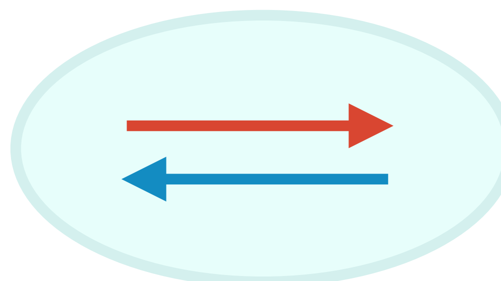
Uniform rotation restores the chiral symmetry

What is the mechanism?

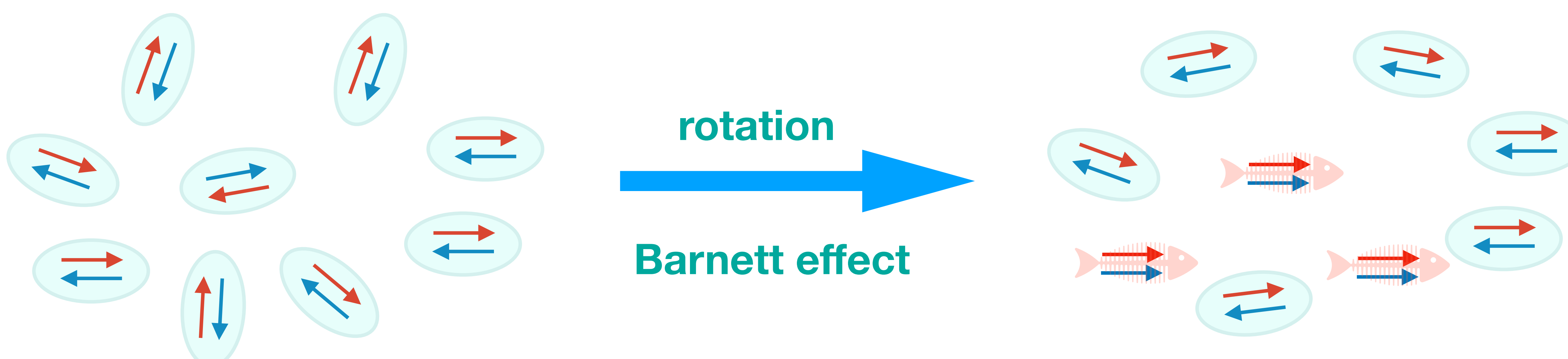


The chiral condensate is a spin-0 object

$$\langle \bar{\psi}\psi \rangle = -\frac{\sigma}{2G}$$



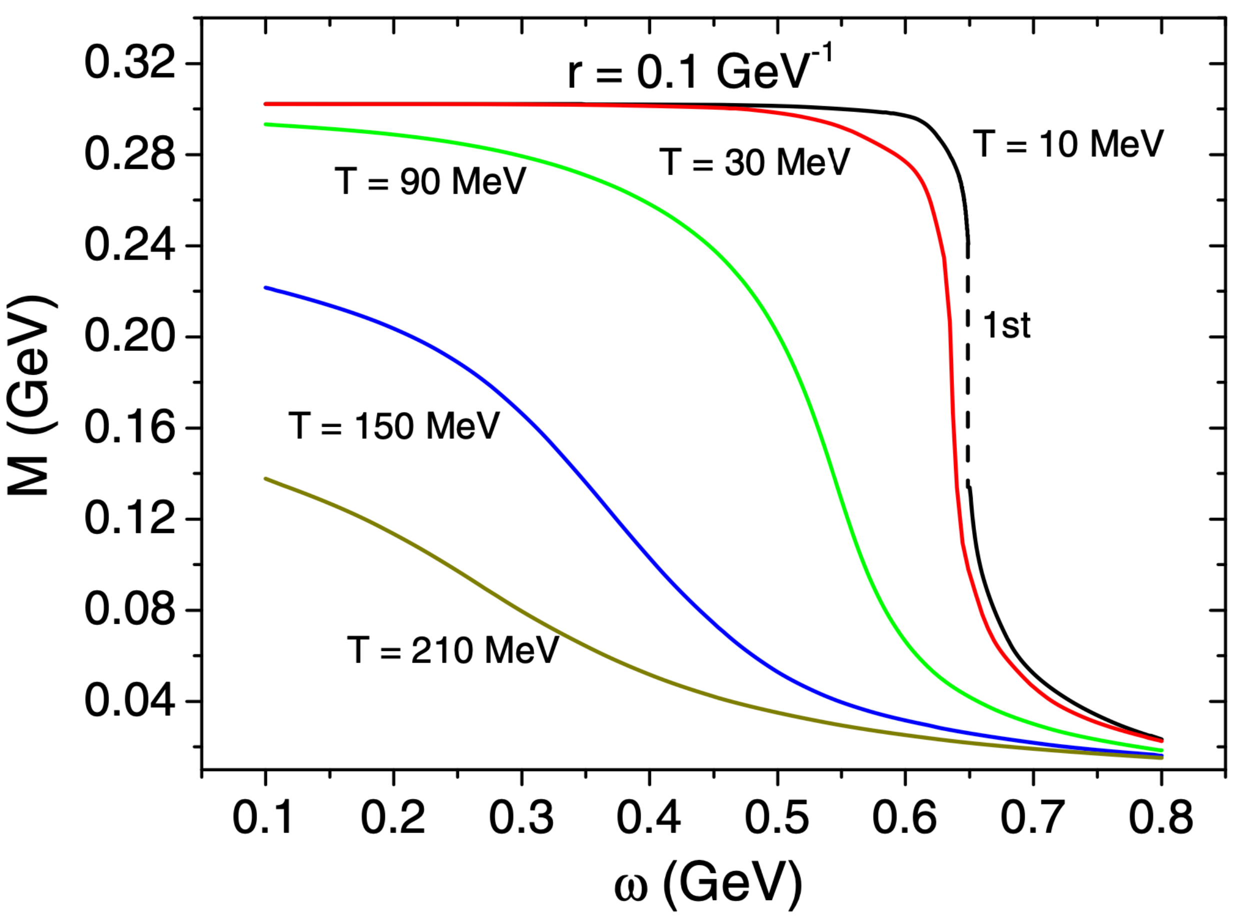
The Barnett effect polarized both the spin of a quark and the spin of an anti-quark along the axis of rotation



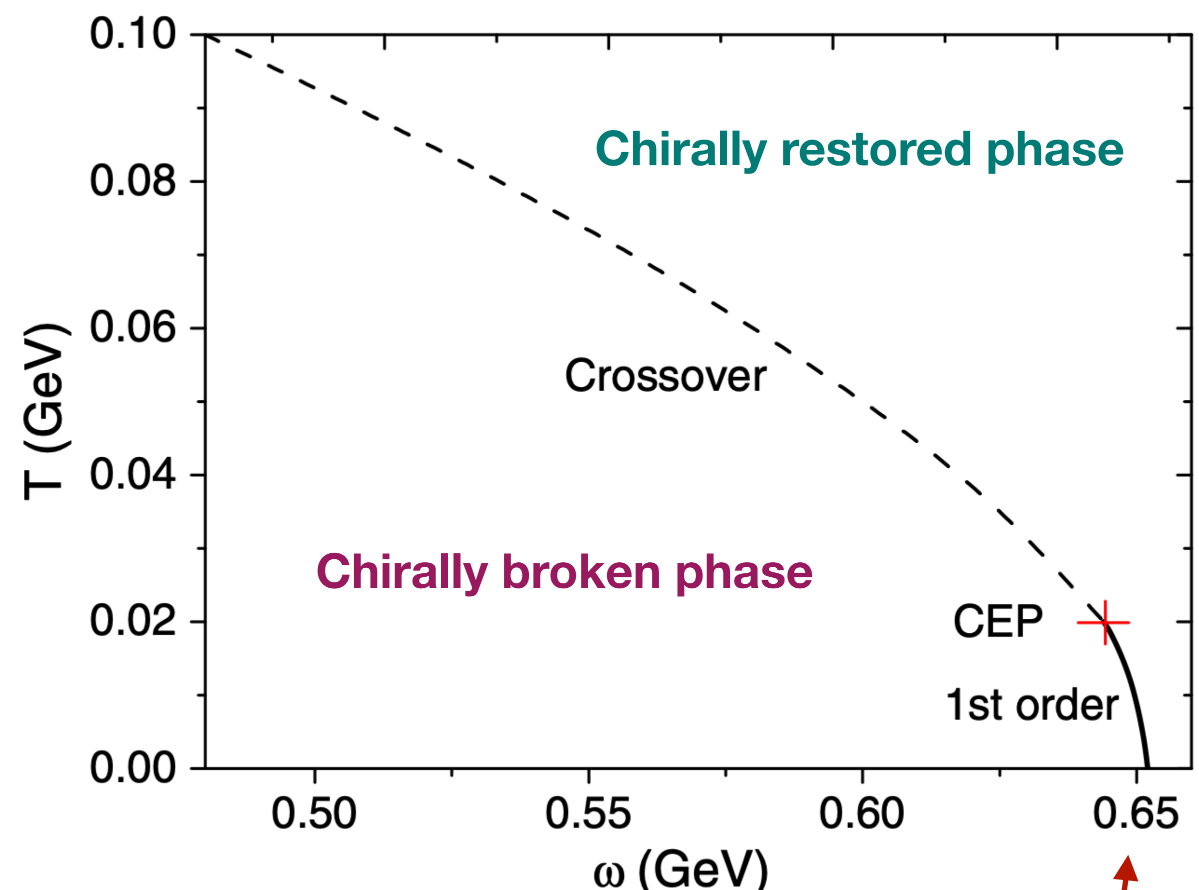
The chiral condensate is destroyed by rotation due to an analogue of the Barnett effect

Chiral symmetry and rotation in QCD

chiral condensate vs. rotation frequency



the phase diagram



Finite-size effects are expected to be strong.

$$R_{\max}\Omega = 1 \quad \longrightarrow \quad R_{\max} \simeq 0.3\,\mathrm{fm}$$

Small transverse size close to the perturbative regime

Juang and Liao, PRL 117, 192302 (2016)

What is the effect of rotation on confinement?

Disclaimer: we don't know/understand for sure. But let's talk about it anyway.

Papers on the subject (exhaustive list, in order of appearance):

1. V. Braguta, A. Kotov, D. Kuznedelev, and A. Roenko, JETP Lett. 112, 6 (2020); more details in Phys. Rev. D 103 (2021) 9;
   first-principles lattice calculation: temperature increases with rotation

2. X. Chen, L. Zhang, D. Li, D. Hou, and M. Huang, JHEP 07,132 (2021);
   holographic approach: temperature decreases with rotation;

3. M. Chernodub, Phys. Rev. D 103, 054027 (2021);
   toy model analysis: temperature decreases with rotation;

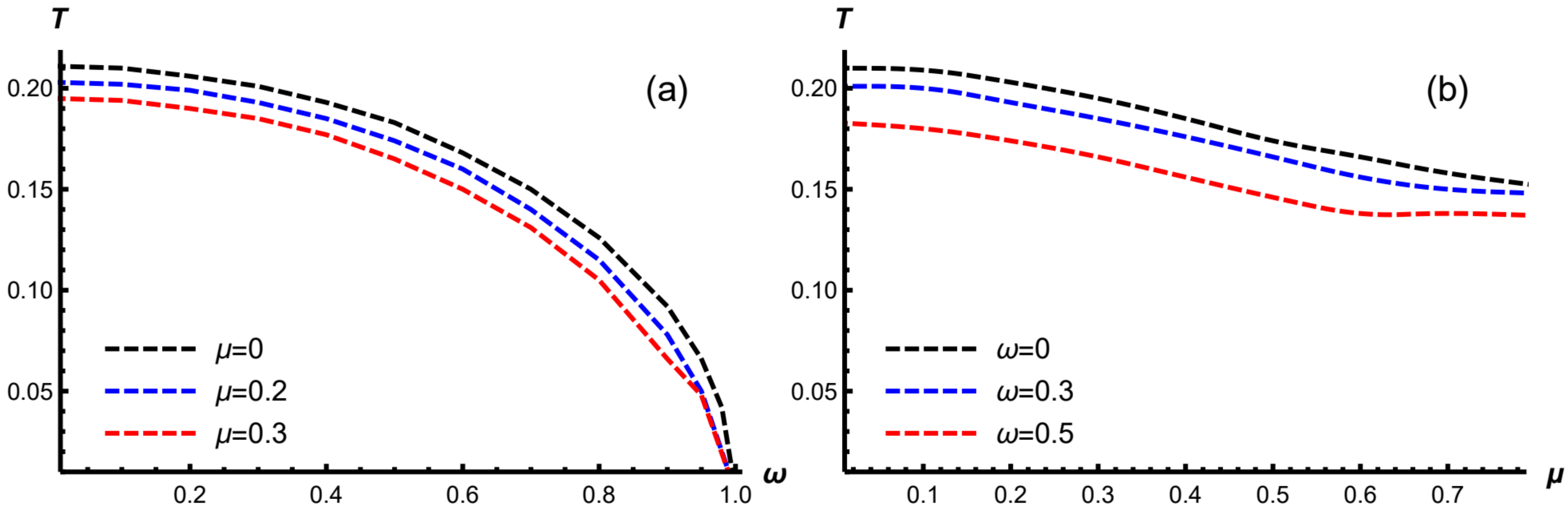
4. Y. Fujimoto, K. Fukushima, and Y. Hidaka, Phys.Lett.B 816 (2021);
   hadron resonance gas model: temperature decreases with rotation;

5. V. Braguta, A. Kotov, D. Kuznedelev, and A. Roenko, "Lattice 21" Symposium;
   first-principles lattice calculation with fermions included (last week, July 28, 2021)
   gluons/quarks force the critical temperature to increase/decrease with Ω.

The confusion is a solid signature that the situation is far from trivial: three independent theoretical papers [2,3,4] based on three different approaches agree with each other and they together **contradict** qualitatively (!) the first-principles simulations [1], but some hope arises from [5].

Rotation effect from holography

phase diagram for pure gluodynamics (no quarks)



dense rotating gluon matter at high-temperature

→ rotation decreases deconfinement temperature

X. Chen, L. Zhang, D. Li, D. Hou, and M. Huang, JHEP 07,132 (2021)

Hadron resonance gas (HRG)

Partition function

$$\rho(m) = e^{m/T_H}$$

Hadrons mass spectrum

$$Z = \int dm\, \rho(m)\, \boxed{e^{-m/T}}$$

Boltzmann factor

(we omit an integration measure which gives a polynomial factor)

Partition function diverges at $T > T_H$

→ **hadrons melt and the deconfinement occurs**

Pressure of hadrons is

$$p(T, \mu, \omega; \underset{\text{cutoff}}{\Lambda}) = \underbrace{\sum_{m;\, M_i \le \Lambda} p_m}_{\text{mesons}} + \underbrace{\sum_{b;\, M_b \le \Lambda} p_b}_{\text{baryons}}$$

Hadron resonance gas (HRG)

Taking into account the rotation:

$$\hat{H} \;\to\; \hat{H} - \boldsymbol{J} \cdot \boldsymbol{\omega}$$

angular frequency of rotation

total angular momentum

$$\boldsymbol{J} = \boldsymbol{L} + \boldsymbol{S}$$

orbital

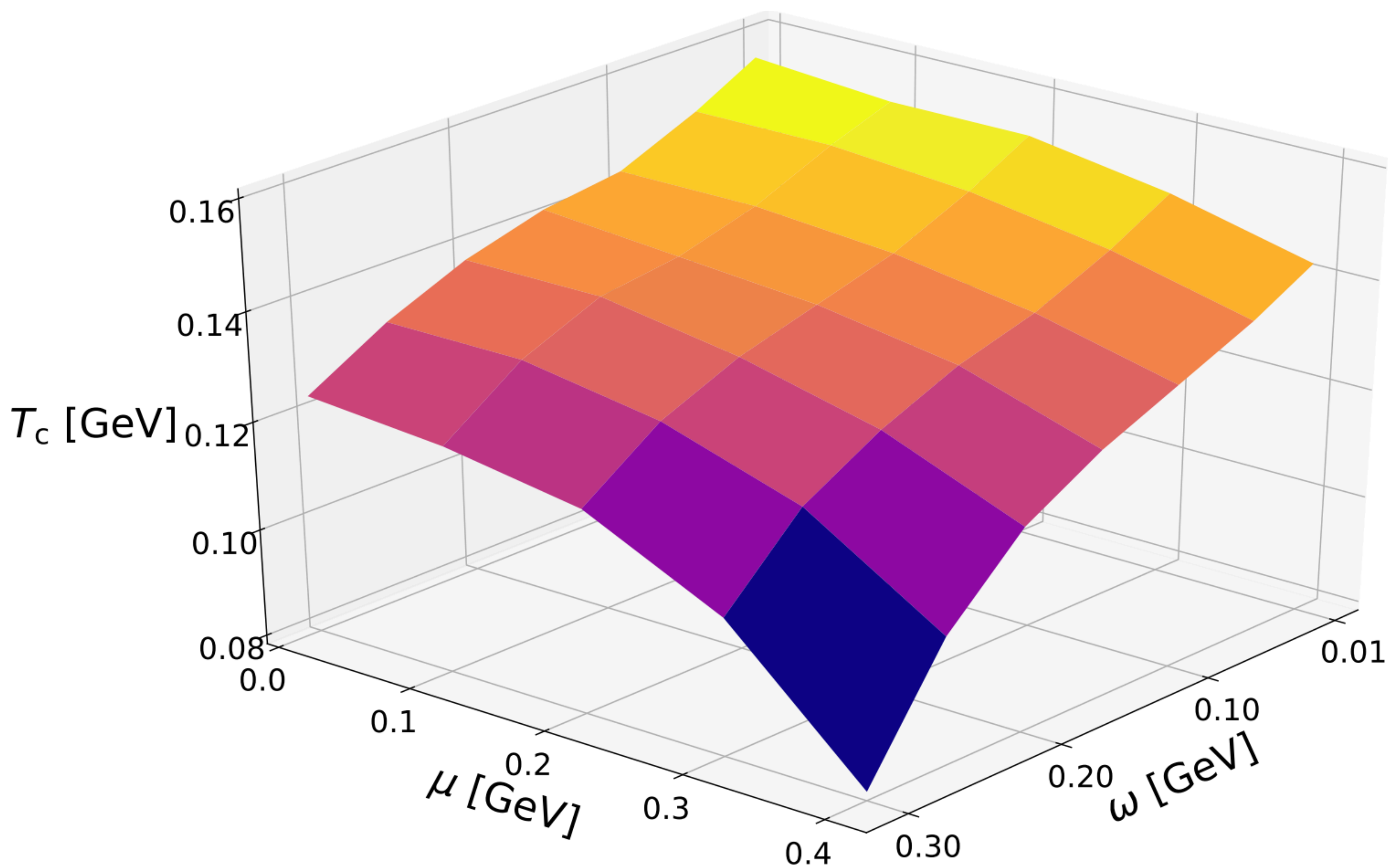
spin

Shift of energies in the rotating frame:

$$\varepsilon \;\to\; \varepsilon - (\ell + s)\omega$$

Deconfinement due to rotation in HGR

The phase diagram of rotating hadron resonance gas



follow Y. Fujimoto, K. Fukushima, and Y. Hidaka, Phys.Lett.B 816 (2021)

Deconfinement due to rotation: General arguments

Gluons and quarks are living in the corotating frame, which rotates together with the plasma.

→ The laboratory system is the flat Minkowski spacetime
→ The corotating system corresponds to the curvilinear reference system with the following metric tensor

$$g_{\mu\nu} = \begin{pmatrix} 1-(x^2+y^2)\Omega^2 & y\Omega & -x\Omega & 0 \\ y\Omega & -1 & 0 & 0 \\ -x\Omega & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

corresponding to the line element of the curved space-time:

$$ds^2 \equiv g_{\mu\nu} dx^\mu dx^\nu = \left(1-\rho^2\Omega^2\right) dt^2 - 2\rho^2\Omega dt d\varphi - d\rho^2 - \rho^2 d\varphi^2 - dz^2$$

Tolman-Ehrenfest law

In a static background gravitational field, the temperature of a system in a thermal equilibrium is not constant:

$$T(\boldsymbol{x})\sqrt{g_{00}(\boldsymbol{x})} = T_0$$

Metric in rotating frame:

local temperature on the axis of rotation

$$T_0 \equiv T(0)$$

$$g_{\mu\nu} = \begin{pmatrix} \boxed{1-(x^2+y^2)\Omega^2} & y\Omega & -x\Omega & 0 \\ y\Omega & -1 & 0 & 0 \\ -x\Omega & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

in cylindrical coordinates: $$g_{00} = 1 - \rho^2\Omega^2$$ **distance from the axis**

Temperature rises as the distance from the axis of rotation increases:

$$T(\rho) = \frac{T(0)}{\sqrt{1-\rho^2\Omega^2}}$$

R. C. Tolman, "On the Weight of Heat and Thermal Equilibrium in General Relativity," Phys. Rev. 35, 904 (1930);
R. Tolman and P. Ehrenfest, "Temperature Equilibrium in a Static Gravitational Field," Phys. Rev. 36, 1791 (1930).

Thermal equilibrium in rotating QGP

Temperature is colder in the center and higher at the edges of the system:

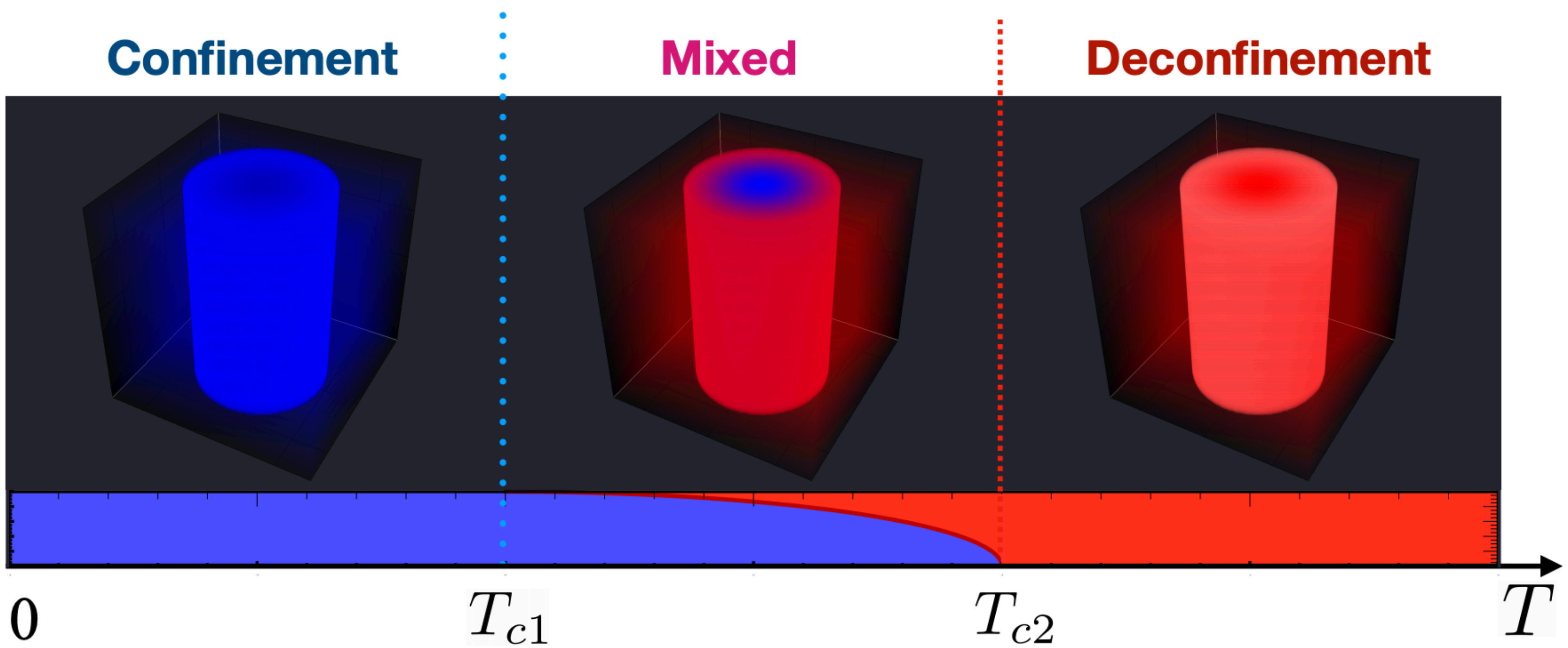
$$T(\rho) = \frac{T(0)}{\sqrt{1-\rho^2\Omega^2}}$$

$$\begin{aligned} T_\Omega(\rho) &< T_{c,\infty} \qquad \text{(confinement)}, \\ T_\Omega(\rho) &> T_{c,\infty} \qquad \text{(deconfinement)} \end{aligned}$$

$T_{c,\infty}$ **the critical temperature in a thermodynamically large, nonrotating system**

The phase structure:

New mixed/inhomogeneous phase!!

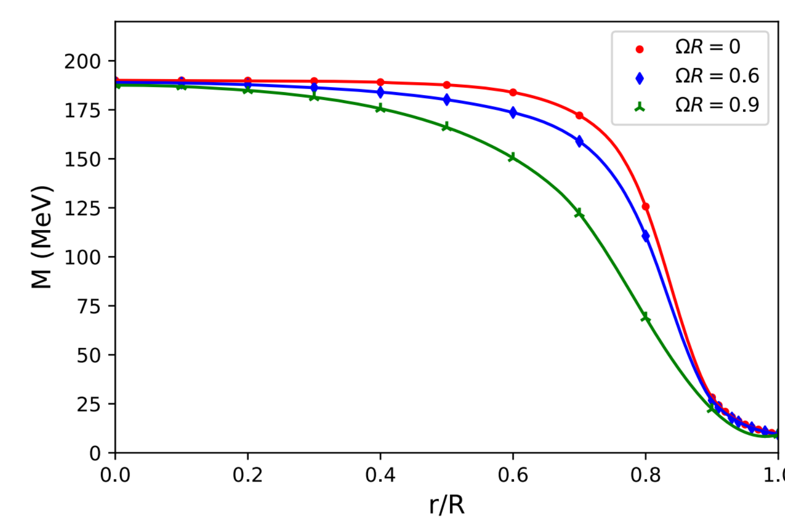


Two critical temperatures:

$$T_{c1} = T_{c,\infty}\sqrt{1-\Omega^2 R^2}, \qquad T_{c2} = T_{c,\infty}$$

theory from a toy model: M. Ch. Phys. Rev. D 103, 054027 (2021)

cf. chiral inhomogeneity (modulo boundary condition)

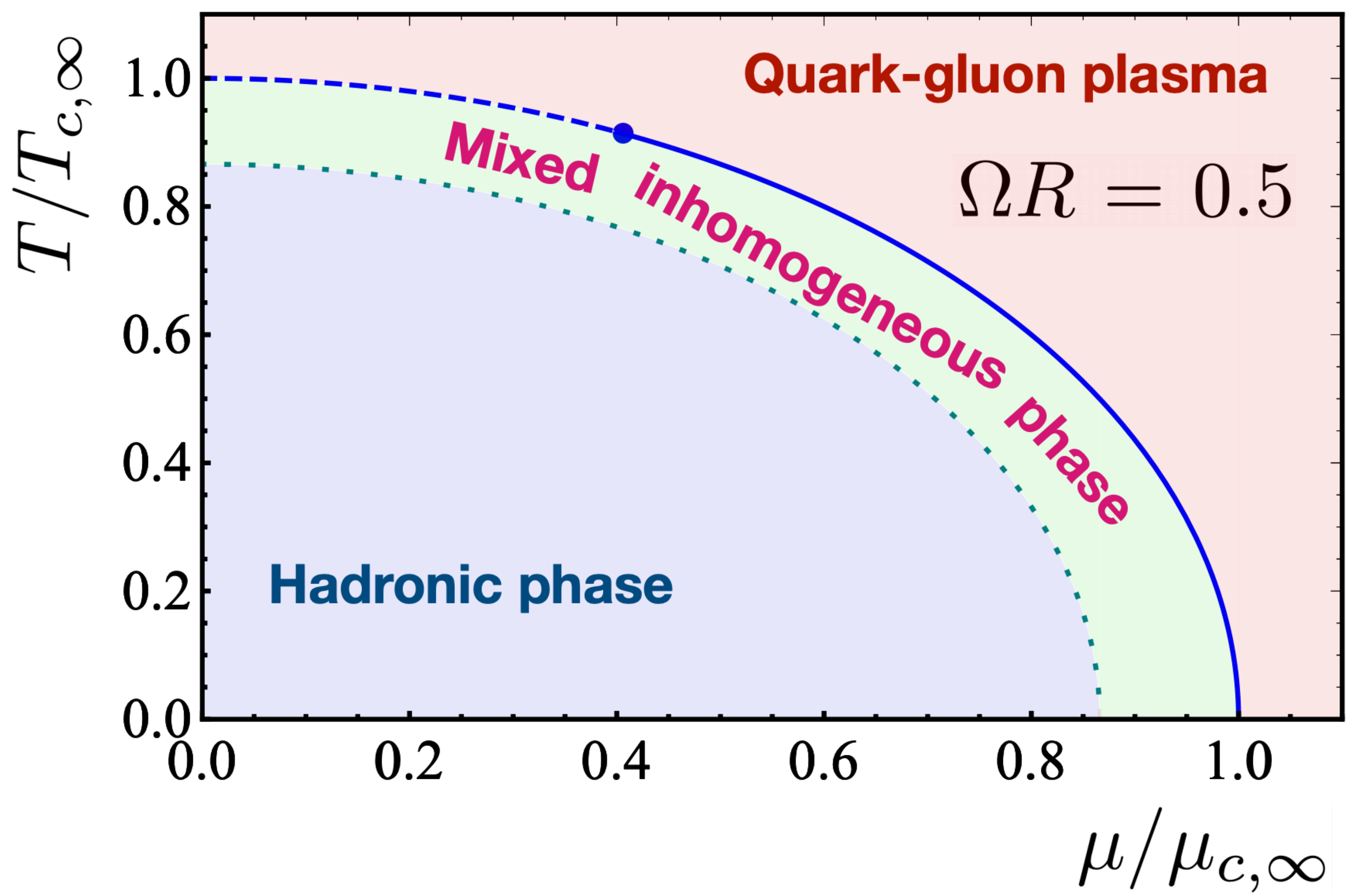


Z.Zhang, C.Shi, X.He, X.Luo, H.Zong, Phys.Rev.D 102 (2020) 11, 114023

Hot dense rotating quark-gluon plasma

The Tolman-Ehrenfest law for temperature and chemical potential

$$T(\boldsymbol{x})\sqrt{g_{00}(\boldsymbol{x})} = T_0, \qquad \mu_B(\boldsymbol{x})\sqrt{g_{00}(\boldsymbol{x})} = \mu_{B0}$$



Check: Compact U(1) gauge theory in (2+1)d

(also known as “compact electrodynamics”, cU(1) or cQED, despite the absence of matter fields)

Confinement picture is well-established!

Lagrangian:

$$\mathcal{L} = \frac{1}{4} F_{\mu\nu}^2$$

$$F^{\mathrm{ph}}_{\mu\nu}[A] = \partial_\mu A_\nu - \partial_\nu A_\mu$$

photon–field strength tensor
“perturbative part”
“regular”

Field strength tensor: $F_{\mu\nu} = F^{\mathrm{ph}}_{\mu\nu} + F^{\mathrm{mon}}_{\mu\nu}$

monopole strength tensor
“non-perturbative part”
“singular”

$$\partial_\mu \widetilde{F}^\mu = \partial_\mu \widetilde{F}^{\mathrm{mon},\mu} = \rho$$

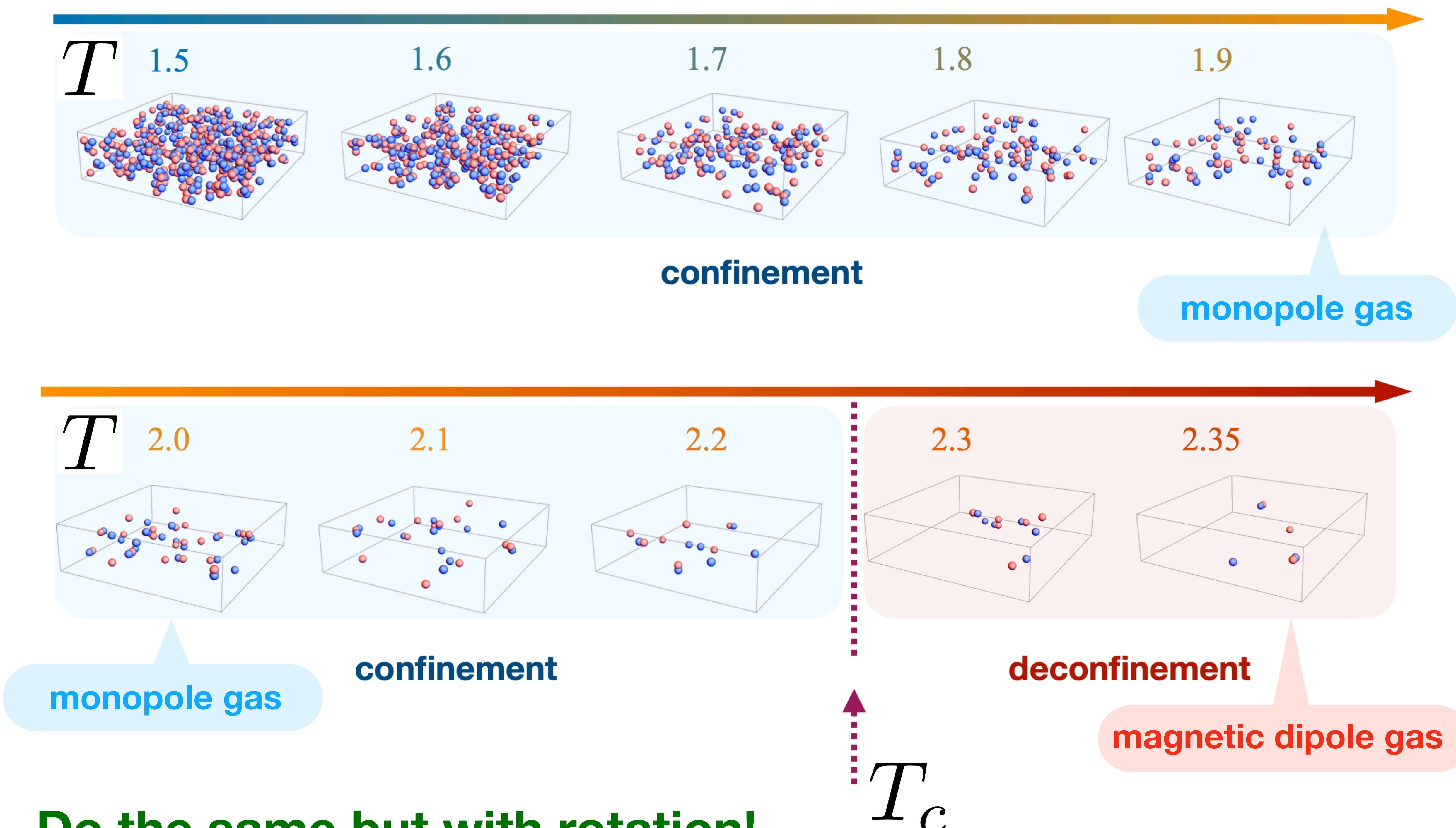
$$\widetilde{F}^\mu = \frac{1}{2} \epsilon^{\mu\alpha\beta} F_{\alpha\beta}$$

dual field-strength vector

density of magnetic monopoles

Compact U(1) gauge theory in (2+1)d

— Confinement due to monopoles (first-principle lattice simulations)



Do the same but with rotation!

Uniform rotation in compact QED

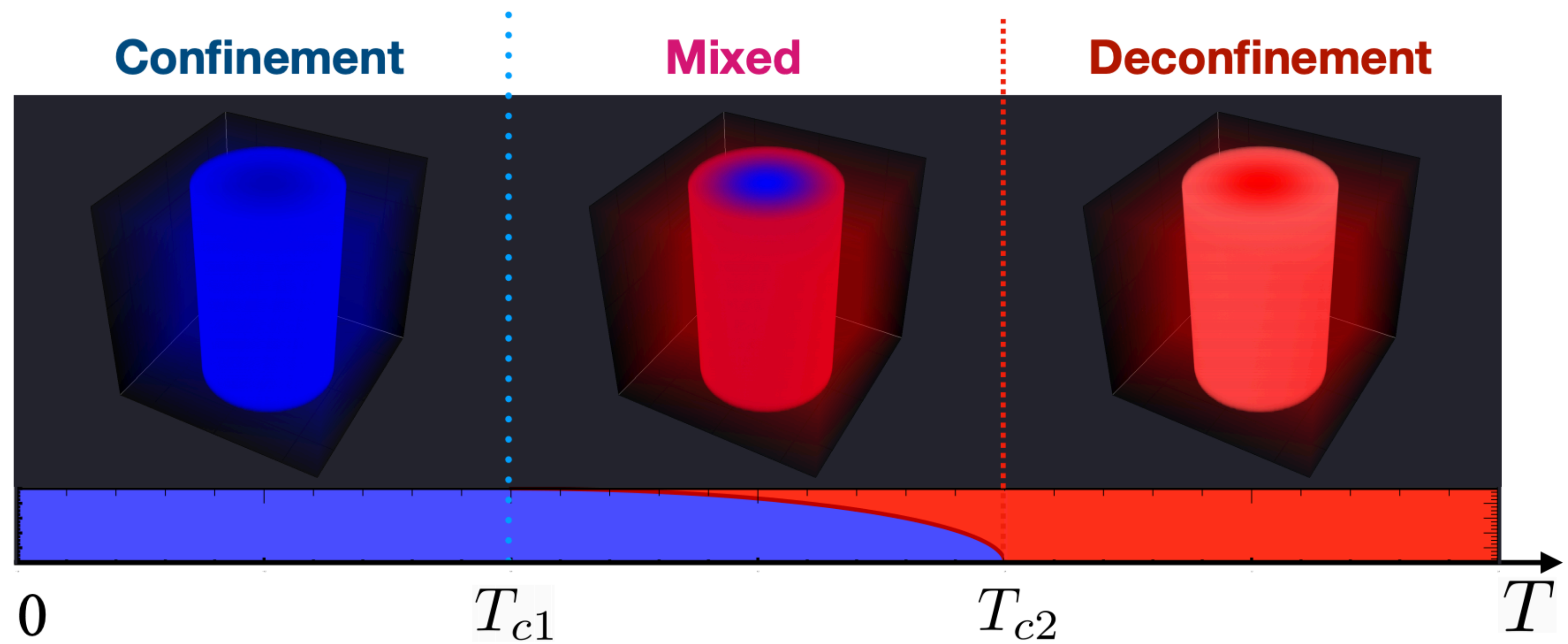
– **Critical deconfinement temperature close to the center of rotation**

$$T_\Omega(\rho) = T(0)\left(1 + \frac{1}{2}\rho^2\Omega^2 + O(\Omega^4)\right)$$

calculated as the temperature at which the monopoles are binding into the magnetically neutral monopole-antimonopole pairs

– **Consistent with Tolman-Ehrenfest law** $$T(\rho) = \frac{T(0)}{\sqrt{1-\rho^2\Omega^2}}$$

– **Leads to inhomogeneous confining plasmas**

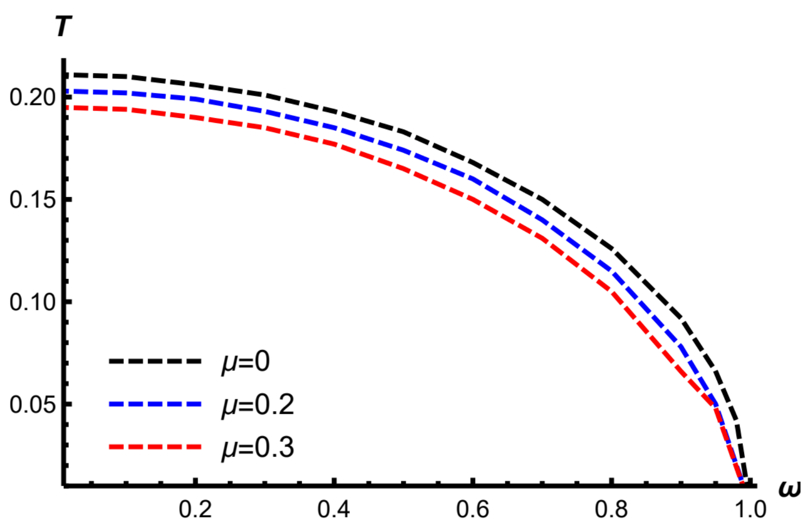


M. Ch., Phys. Rev. D 103, 054027 (2021)

Head-on collision of analytics and numerics

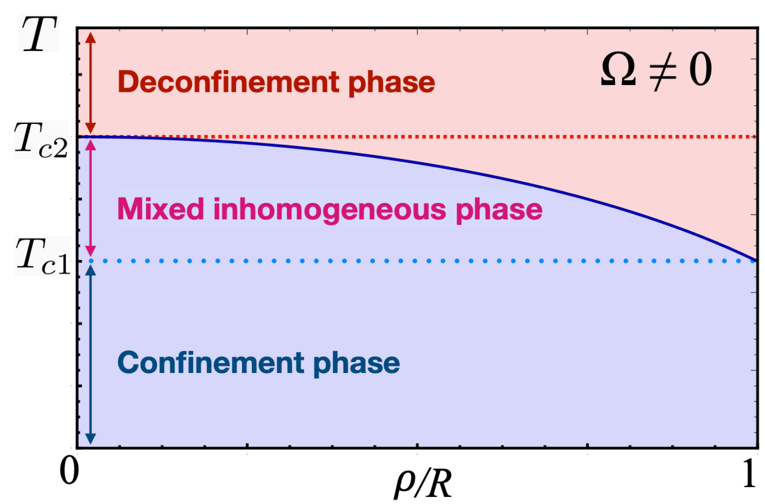
Theoretical results: rotation decreases deconfinement temperature

holography



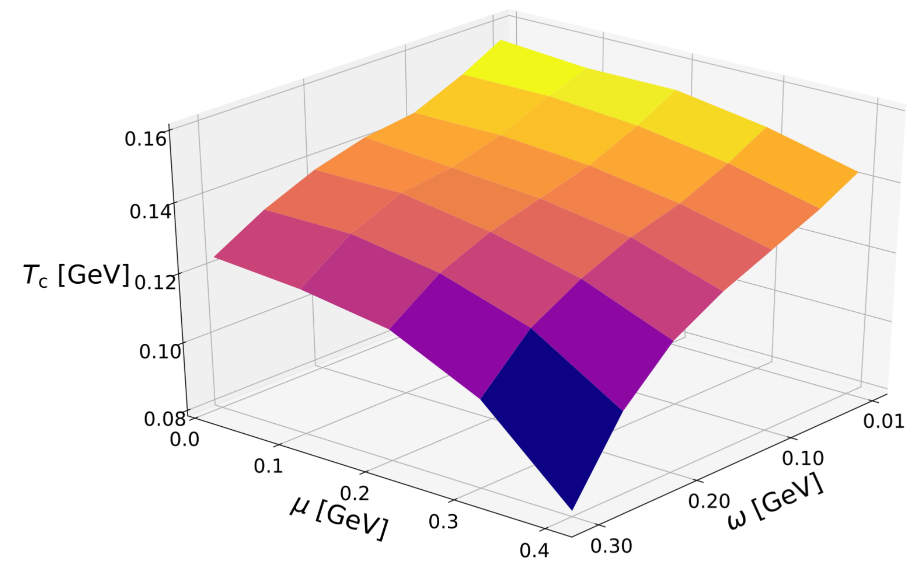
Chen, Zhang, Li, Hou, Huang (arxiv:2010.14478) - **gluons**

confining toy model



M. Ch. (arxiv: 2012.04924) - **gluons**

hadron resonance gas



Fujimoto, Fukushima, Hidaka (arxiv:2101.09173) - **quarks included**

First-principle numerical results in gluodynamics: rotation increases deconfinement temperature

?

$$T_c(\Omega)/T_c(0) = 1 + C_2\Omega^2 \text{ with } C_2 > 0$$

V. Braguta, A. Kotov, D. Kuznedelev, and A. Roenko, JETP Lett. 112, 6 (2020); Phys. Rev. D 103 (2021) 9;
add fermions (while the pion is still heavy ~ 690 MeV): “Lattice 21” Symposium (Wednesday, 2021);
gluons/quarks force the critical temperature to increase/decrease with Ω (a partial resolution?).

Conclusions

— **Quark-Gluon plasma is the most vortical fluid ever observed**

The experimentally measured vorticity $\omega \approx (9 \pm 1) \times 10^{21}\,\mathrm{s}^{-1}$

— **Effect of rotation on the phase structure of QCD?**
A uniform rotation is a simplest tractable approximation to investigation of quark-gluon plasma with large angular momentum created in noncentral heavy-ion collisions

— **Rotation restores chiral symmetry and leads to a decrease of the temperature of the chiral phase transition in QCD**

— **The effect of the rotation on the deconfinement temperature is still controversial. Independent theoretical approaches signal that the deconfinement temperature decreases with temperature while lattice results with pure glue suggest the opposite (quarks try change the slope).**

More efforts are needed! →