Two Schrodinger-like equations for hadrons

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A virtual tribute to Quark Confinement and the Hadron Spectrum

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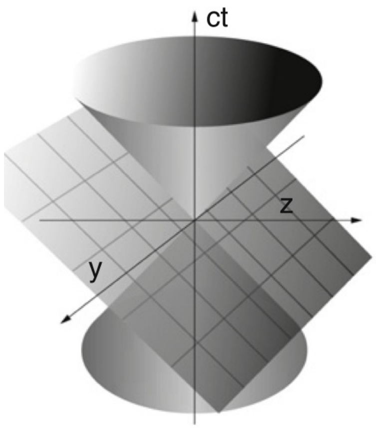
Based on work in progress with M. Ahmady, S. Kaur, S. Lee Mackay and C. Mondal

Light-front QCD



Light-front time

$x^+ = x^0 + x^3$

LF Heisenberg Equation

$$H_{\mathrm{LF}}^{\mathrm{QCD}} |\Psi\rangle = M^2 |\Psi\rangle$$

Fock expansion of hadronic state

$$|\Psi\rangle = \sum_n |n\rangle\langle n|\Psi\rangle$$

"Eliminate" higher Fock states

$$H_{\mathrm{eff}} |q\bar{q}\rangle = M^2 |q\bar{q}\rangle$$

- Allows for fully relativistic Schrodinger-like equations
- Makes contact with quark model

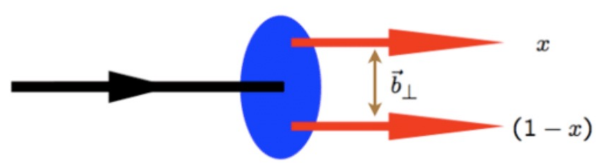
Brodsky, Pinsky, Pauli, Phys. Rep. (1997)

Meson masses in light-front QCD

$$M^2 = \int dx\, d^2\mathbf{b}_\perp \Psi^*(x,\mathbf{b}_\perp)\left[-\frac{\nabla^2_{b_\perp}}{x(1-x)} + \frac{m_q^2}{x} + \frac{m_{\bar{q}}^2}{1-x}\right]\Psi(x,\mathbf{b}_\perp) + \text{interactions}$$

Complicated QCD bound state dynamics

$\psi(x, \vec{b}_\perp)$



Valence meson light-front wavefunction

$$x = \frac{k^+}{P^+}$$

$$\zeta = \sqrt{x(1-x)}\mathbf{b}_\perp$$

Light-front momentum fraction carried by quark

Transverse quark-antiquark separation at equal light-front time

Brodsky, de Teramond, Dosch, Erlich, Phys. Rep. 584, 1 (2015) (review with original references)

Light-front factorization

$$\Psi(x,\zeta,\varphi) = \frac{\phi(\zeta)}{\sqrt{2\pi\zeta}} e^{iL\varphi} X(x)$$

$$X(x) = \sqrt{x(1-x)}\chi(x)$$

$L = |L_z^{max}|$

LF orbital angular momentum

$$M_\perp^2 = \int d^2\zeta \phi^*(\zeta) \left[-\frac{d^2}{d\zeta^2} + \frac{4L^2-1}{4\zeta^2} + \boxed{U_\perp(\zeta)} \right] \phi(\zeta) \, .$$

$$M_\parallel^2 = \int dx \chi^*(x) \left[\frac{m_q^2}{x} + \frac{m_{\bar{q}}^2}{1-x} + \boxed{U_\parallel(x)} \right] \chi(x)$$

$$M^2 = M_\perp^2 + M_\parallel^2$$

Exact derivation of confining potentials in QCD: open question

Holographic Schrodinger Equation

Neglecting quark masses and longitudinal confinement

$$\left(-\frac{d^2}{d\zeta^2} + \frac{4L^2-1}{4\zeta^2} + U_\perp^{\mathrm{LFH}}(\zeta)\right)\phi(\zeta) = M_\perp^2\phi(\zeta)$$

Holographic mapping to AdS_5

Governs transverse dynamics

$$U_\perp^{\mathrm{LFH}}(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(J-1) .$$

Underlying conformal symmetry

$$M_{\perp,M}^2 = 4\kappa^2\left(n_\perp + L_M + \frac{S_M}{2}\right)$$

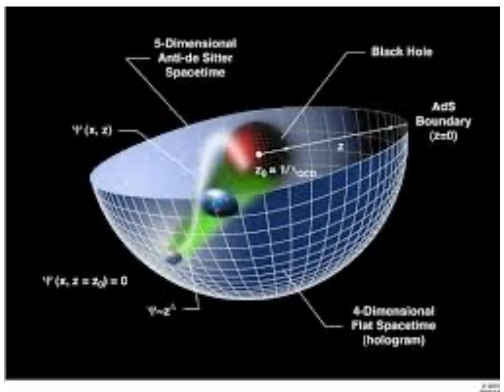
Massless pion as expected in chiral QCD

Light-front holographic dictionary

LF transverse variable

$$\zeta \leftrightarrow z$$

Fifth dimension of AdS



$$\Phi_J(z) = (R/z)^{J-3/2} e^{-\varphi(z)/2} \phi_J(\zeta)$$

$$(m_5 R)^2 = -(2-J)^2 + L^2$$

$$\Phi_J(x,z) = \Phi_J(z) e^{iP\cdot x}$$

Dilaton field in AdS

$$U(\zeta, J) = \frac{1}{2}\varphi''(z) + \frac{1}{4}\varphi'(z)^2 + \left(\frac{2J-3}{2z}\right)\varphi'(z)$$

Confinement potential

Confinement in physical spacetime⟺ dilaton field in AdS

Quadratic dilaton/potential is required by underlying conformal symmetry

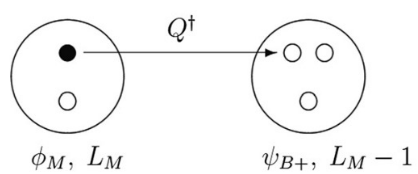
Supersymmetric light-front holography

Brodsky, de Teramond, Dosch, Lorce, Phys. Lett. B 759 (2016)
Dosch, de Teramond, Brodsky, Phys. Rev. D 95 034016 (2017)
Neilson, Brodsky, Phys. Rev. D 97 114001 (2018)

$$H\,|\phi\rangle = M_\perp^2\,|\phi\rangle$$

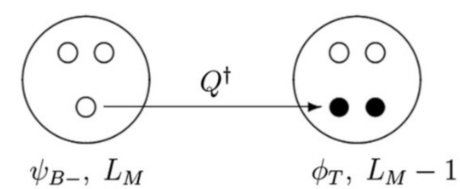
$$H = \begin{pmatrix} -\frac{d^2}{d\zeta^2} + \frac{4L_M^2-1}{4\zeta^2} + U_M(\zeta) & 0 \\ 0 & -\frac{d^2}{d\zeta^2} + \frac{4L_B^2-1}{4\zeta^2} + U_B(\zeta) \end{pmatrix}$$

$$|\phi\rangle = \begin{pmatrix} \phi_M(L_M = L_B+1) & \psi^-(L_B+1) \\ \psi^+(L_B) & \phi_T(L_T = L_B) \end{pmatrix}$$



$$U_M(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L_M + S_M - 1)$$

$$U_B(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L_B + S_D)$$



quark-antiquark quark-diquark quark-diquark diquark-antidiquark

$$M_{\perp,M}^2 = 4\kappa^2\left(n_\perp + L_M + \frac{S_M}{2}\right)$$

$$M_{\perp,B}^2 = 4\kappa^2\left(n_\perp + L_B + \frac{S_D}{2} + 1\right)$$

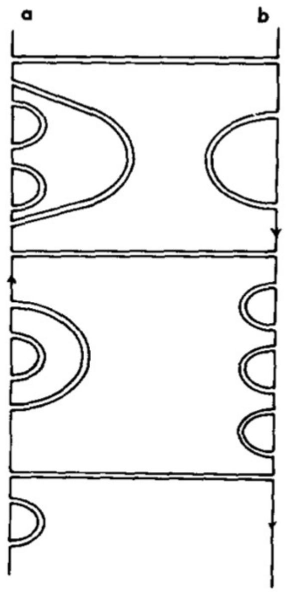
$$M_{\perp,T}^2 = 4\kappa^2\left(n_\perp + L_T + \frac{S_T}{2} + 1\right)$$

Each baryon has two supersymmetric partners: a meson and a tetraquark

The `t Hooft Equation

`t Hooft, Nucl. Phys. B 75 461 (1974)

$$\left(\frac{m_q^2}{x}+\frac{m_{\bar{q}}^2}{1-x}\right)\chi(x)+\boxed{\frac{g^2}{\pi}\mathcal{P}\int \mathrm{d}y\frac{|\chi(x)-\chi(y)|}{(x-y)^2}}=M_\parallel^2\chi(x)$$



- (1+1)-dim QCD Lagrangian
- Large N_c limit: only planar diagrams contribution
- g is the `t Hooft coupling with mass dimensions

Other models for longitudinal confinement:
de Teramond, Brodsky, arXiv: 2103.10930 [hep-ph]
Yang Li, Vary. arXiv: 2103.09993 [hep-ph]

`t Hooft and light-front holographic potentials

Consistent and complementary

$$U_{\parallel}^{\mathrm{tHooft}}(x) = \frac{g^2}{\pi}\mathcal{P}\int \mathrm{d}y \frac{|\chi(x) - \chi(y)|}{(x-y)^2}$$

$$U_{\perp}^{\mathrm{LFH}}(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(J-1)\ .$$

- Chiral limit: together they predict the Gell-Mann-Oakes-Renner (GMOR) relation

$$M_\pi^2 = M_\parallel^2 + \cancelto{0}{M_\perp^2}$$

$$M_\pi^2 = g\sqrt{\frac{\pi}{3}}(m_u + m_d) + \mathcal{O}(m_u + m_d)^2$$

- Heavy quark limit: together, with $g = \kappa$, they imply a rotationally symmetric linear instant form potential

Ahmady, Dahiya, Kaur, Mondal, Sharma, Sandapen: arXiv: 2105.01018 [hep-ph]

Computing the hadron spectrum

- Light hadrons: $M_\perp^2 \gg M_\parallel^2$

expect for ground state pseudoscalars

$M_\perp^2 = 0$

- Heavy hadrons: $M_\parallel^2 \gg M_\perp^2$

Holographic SE `t Hooft

$$M_M^2 = M_{\perp,M}^2(n_\perp, L_M, S_M; \kappa) + M_{\parallel,M}^2(n_\parallel; m_q, m_{\bar{q}}, g) \, ,$$

$$M_B^2 = M_{\perp,B}^2(n_\perp, L_B, S_D; \kappa) + M_{\parallel,B}^2(n_\parallel; m_q, m_{[qq]}, g) \, ,$$

$$M_T^2 = M_{\perp,B}^2(n_\perp, L_T, S_T; \kappa) + M_{\parallel,T}^2(n_\parallel; m_{[\bar{q}\bar{q}]}, m_{[qq]}, g)$$

Diquark mass=2 x quark mass

$$P = (-1)^{L_M+1} = (-1)^{L_B} = (-1)^{L_T} \, .$$

$$C = (-1)^{n_\parallel + L_M + S_M} = (-1)^{n_\parallel + L_T + S_T - 1}$$

$$\boxed{n_\parallel \geq n_\perp + L}$$

A radial and/or orbital transverse excitation $\Longleftrightarrow$ a longitudinal excitation

Parameters

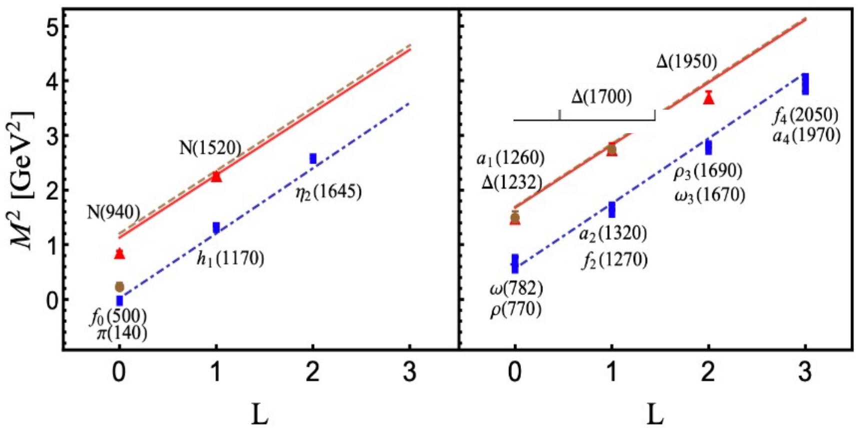
Hadron	g	$m_{u/d}$	m_s	m_c	m_b
Light	0.128	0.046	0.357	-	-
Heavy-light	0.410	0.330	0.500	1.370	4.640
Heavy-heavy	0.523	-	-	1.370	4.640

In GeV

$\kappa = 0.523$ GeV   Universal

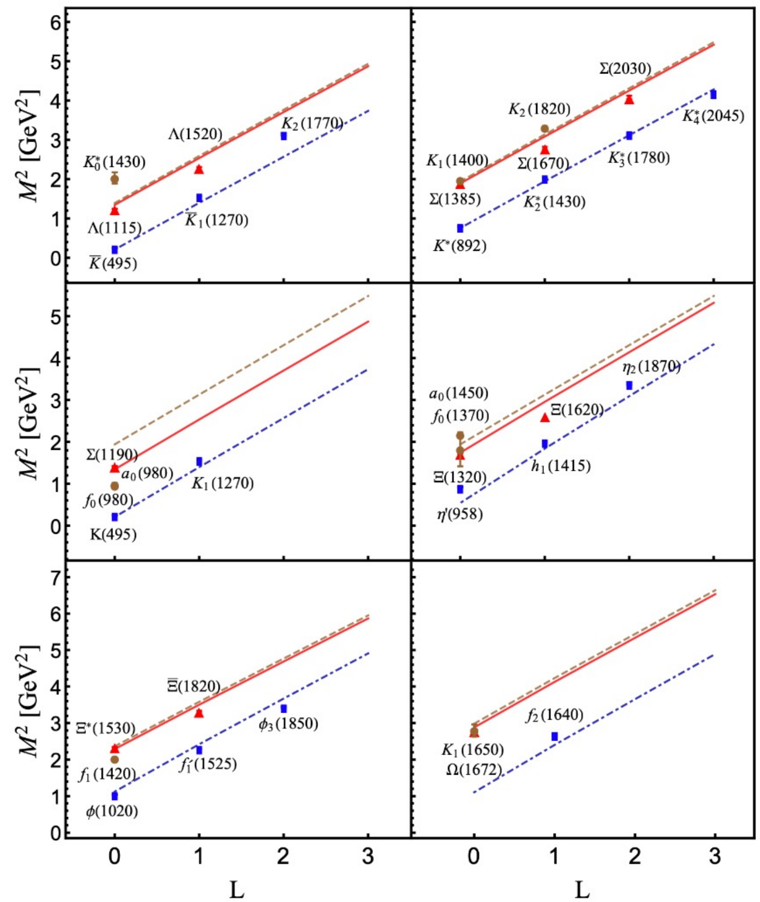
$g = \kappa$ as expected in non-relativistic limit

Light hadrons

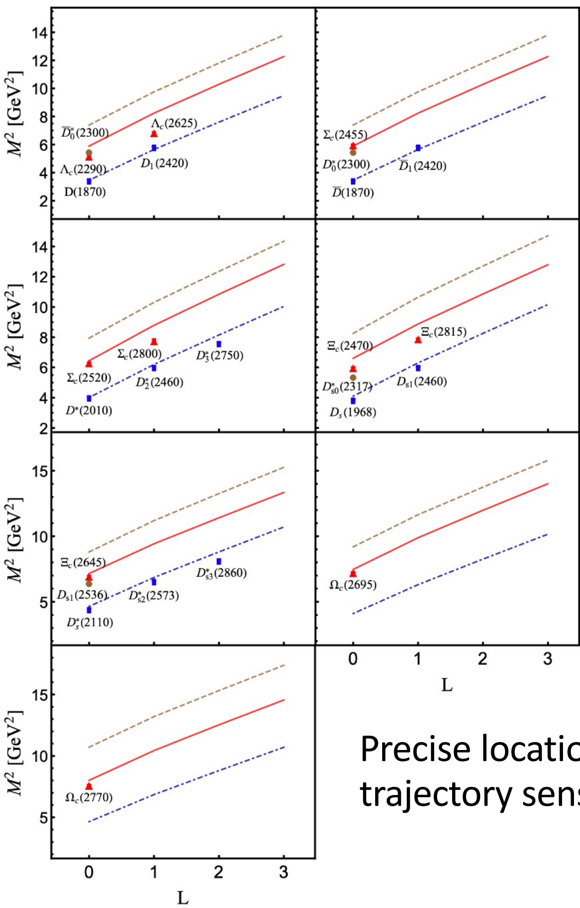


Blue: mesons
Red: baryons
Brown: tetraquarks

Precise location and slope
of Regge trajectory sensitive to both g and κ



Heavy-light hadrons



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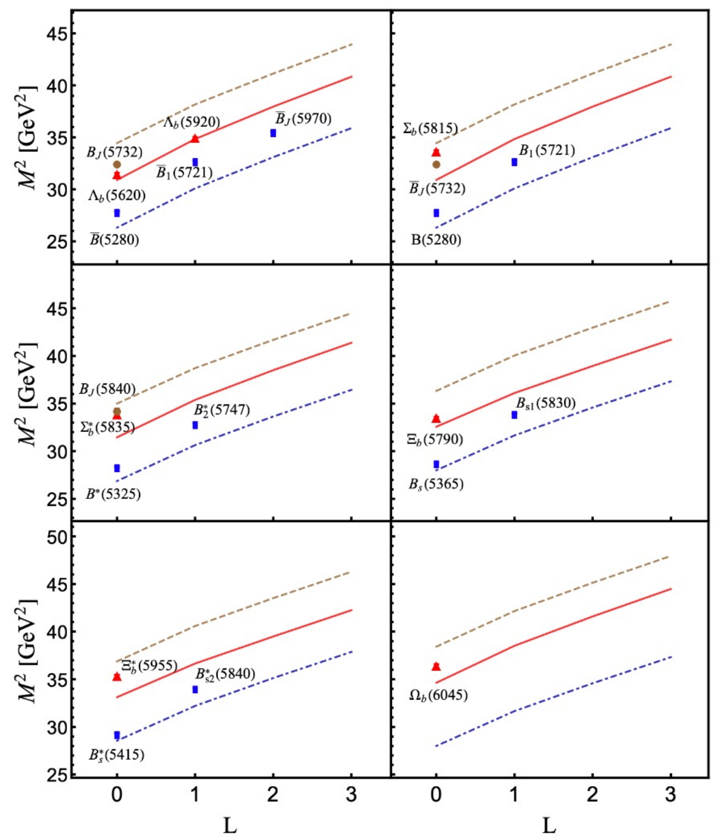
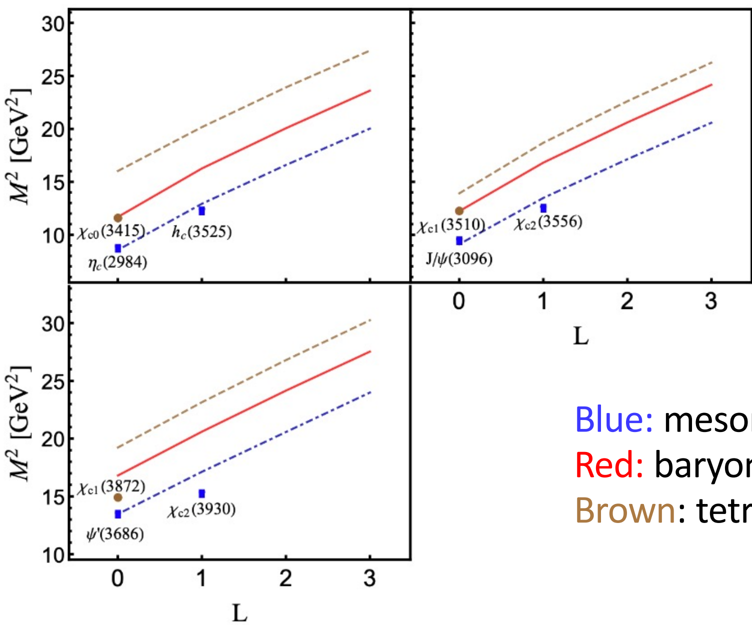


Precise location and slope of Regge trajectory sensitive to both g and κ

Heavy-heavy hadrons



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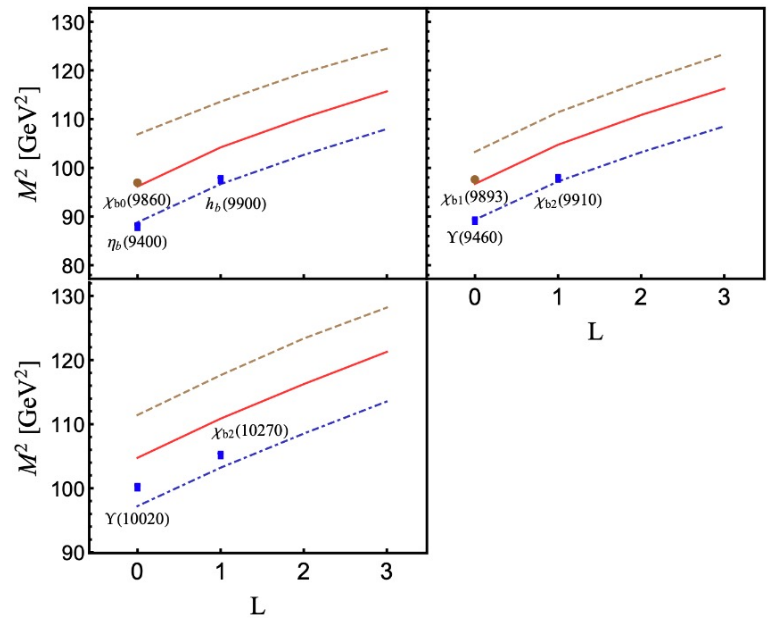

Universality of κ across full spectrum non-trivial

Conclusions and Acknowledgements

- The holographic Schrodinger Equation and the `t Hooft Equation are consistent and complementary

- Together, they provide a good description of hadron spectroscopy, especially for mesons and baryons

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