The A^2 Asymmetry and Gluon Propagators in Lattice SU(3) Gluodynamics at $T \simeq T_c$

V.Bornyakov, V.Goy, V.Mitrjushkin,
R. Rogalyov[1]

NRC “Kurchatov Institute” - IHEP

03.08.2021

[1] speaker

Our goal: to find critical behavior of the gluon propagators.

Outline

1. Simulation settings
2. Definitions and notation
3. Lessons of the SU(2) case
4. Correlation between the Polyakov loop and the asymmetry
5. Correlation between the Polyakov loop and the longitudinal propagator
6. Screening masses in different Plyakov-loop sectors
7. conclusions

$N_s^3 \times N_t$ lattice with pure glue Wilson action.
$SU(2)$: $N_t = 8$, $N_s = 32, 48, 72,$ scale setting [Karsch et al. 1992]:

$$\ln(a\sqrt{\sigma}) = -\frac{3\pi^2}{11}\beta + \frac{51}{121}\ \ln\beta + 0.296 + \frac{4.25}{\beta}\ ,$$

$\beta_c = 2.5104$, $\sqrt{\sigma} = 0.44$ GeV, $T_c = 297$ MeV.
$SU(3)$: $N_t = 8$, $N_s = 24$, scale setting [Sommer, Necco 2004]:

$$\ln\frac{a}{r_0} = -1.6804 - 1.7331(\beta - 6) + 0.7849(\beta - 6)^2 - 0.4428(\beta - 6)^3\ ,$$

the Sommer parameter $r_0 = 0.5$ fm, $\sqrt{\sigma} = 0.47$ GeV.
$\beta_c = 6.06$ and $\dfrac{T_c}{\sqrt{\sigma}} = 0.63 \quad \Longrightarrow \quad T_c = 294$ MeV

- $SU(2)$ $a_c = 0.084$ fm, $L = 2.6 \div 6.0$ fm

β	2.508	2.512	2.515	2.518
τ	-0.008	0.005	0.015	0.025

- $SU(3)$ $a_c = 0.083$ fm, $L = 2.0$ fm

β	6.000	6.044	6.075	6.122
τ	-0.096	-0.026	0.025	0.104

We use variable $\tau = \dfrac{T - T_c}{T_c}$

Gauge fixing

Gauge transformations:

$$U_{x\mu} \overset{g}{\mapsto} U^g_{x\mu} = g^\dagger_x U_{x\mu} g_{x+\mu}, \qquad g_x \in SU(N_c) \ .$$

Vector potentials:

$$\mathbf{A}_{x\mu} = \frac{1}{2i}\Big(U_{x\mu} - U^\dagger_{x\mu}\Big)_{\text{traceless}} \equiv A^a_{x,\mu} T^a \ , \tag{1}$$

The lattice Landau gauge condition

$$(\partial \mathbf{A})_x = \sum_{\mu=1}^{4} (\mathbf{A}_{x\mu} - \mathbf{A}_{x-\hat{\mu};\mu}) = 0 \tag{2}$$

represents a stationarity condition for the gauge-fixing functional

$$F_U(g) = \frac{1}{4V} \sum_{x\mu} \frac{1}{N_c} \,\Re\, \mathrm{Tr}\ U^g_{x\mu} \ , \tag{3}$$

with respect to gauge transformations g_x .

Definition of the longitudinal (L) and transverse (T) propagators:

$$D^{ab}_{\mu\nu}(p) = \delta_{ab}\left(P^T_{\mu\nu}(p)D_T(p) + P^L_{\mu\nu}(p)D_L(p)\right) ,$$

where $P^{T;L}_{\mu\nu}(p)$ - orthogonal transverse (longitudinal) projectors are defined at $p = (\vec{p} \neq 0;\ p_4 = 0)$ as follows

$$P^T_{ij}(p) = \left(\delta_{ij} - \frac{p_i p_j}{\vec{p}^2}\right), \quad P^T_{\mu 4}(p) = 0 \; ; \tag{4}$$

$$P^L_{44}(p) = 1 \; ; \; P^L_{\mu i}(p) = 0 \, . \tag{5}$$

Two scalar propagators - longitudinal $D_L(p)$ and transverse $D_T(p)$ - are given by

$$D_L(p) = \frac{1}{3} \sum_{a=1}^{N_c^2-1} \langle A_0^a(p) A_0^a(-p) \rangle$$

$$D_T(p) = \left[\begin{array}{ll} \frac{1}{2(N_c^2-1)} \sum_{a=1}^{N_c^2-1} \sum_{i=1}^{3} \langle A_i^a(p) A_i^a(-p) \rangle & p \neq 0 \\ \frac{1}{3(N_c^2-1)} \sum_{a=1}^{N_c^2-1} \sum_{i=1}^{3} \langle A_i^a(p) A_i^a(-p) \rangle & p = 0 \end{array} \right.$$

The Chromo-Electric-Magnetic Asymmetry

$$\begin{aligned} \langle A_E^2 \rangle &= g^2 \langle A_4^a(x) A_4^a(x) \rangle, \\ \langle A_M^2 \rangle &= g^2 \langle A_i^a(x) A_i^a(x) \rangle. \end{aligned} \tag{6}$$

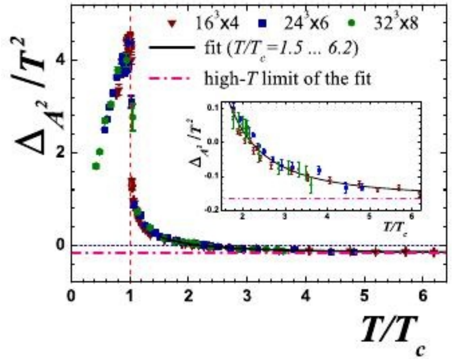
The quantity of particular interest is the chromoelectric-chromomagnetic asymmetry

$$\mathcal{A} = \frac{\langle A_E^2 \rangle - \frac{1}{3} \langle A_M^2 \rangle}{T^2} \ . \tag{7}$$

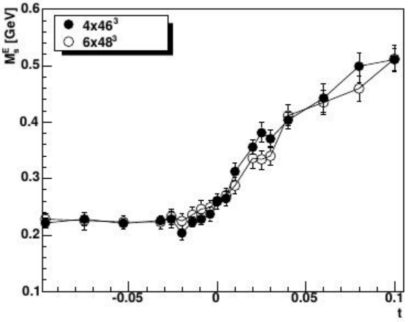
which can be expressed in terms of the propagators,

$$\mathcal{A} = \frac{2 N_c N_t (N_c^2 - 1)}{\beta a^2 N_s^3} \left[D_L(0) - D_T(0) + \sum_{p \neq 0} \left(\frac{3|\vec{p}|^2 - p_4^2}{3p^2} D_L(p) - \frac{2}{3} D_T(p) \right) \right]$$

We work in the Landau gauge $\partial_\mu A_\mu^a = 0$



Chernodub,
Ilgenfritz
2008



Maas, Pawlowski
von Smekal, Spielmann
2011

In 2018 we argued that from

- correlation between the asymmetry and the Polyakov loop
- regression analysis
- univerasality hypothesis

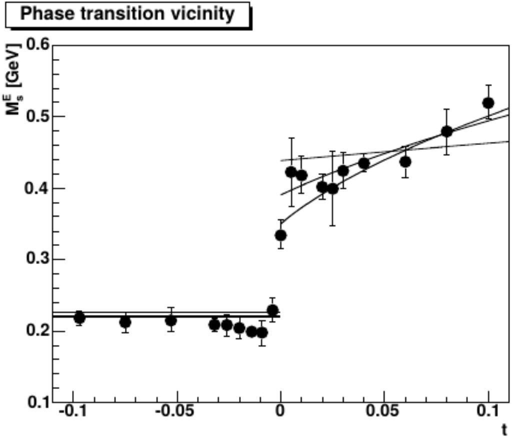
it follows coincidence of the critical exponents
of magnetization in the 3D Ising model
and of $\mathcal{A}$ and $D_L(0)$ in SU(2) gauge theory

$$\mathcal{A} = \mathcal{A}_0 + B_{\mathcal{A}}\tau^{\beta_{\mathcal{A}}} + \overline{o}(\tau^{\beta_{\mathcal{A}}}). \quad (8)$$

Therefore,

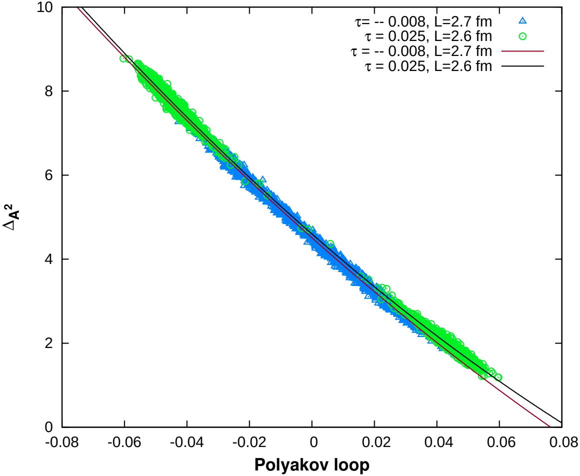
$$\beta_{\mathcal{A}} = \beta = 0.326419(3),$$
$$B_{\mathcal{A}} = A_1 B = -54.02(24)$$

At $\tau < 0$ $\mathcal{A} \approx 0$ is a smooth function



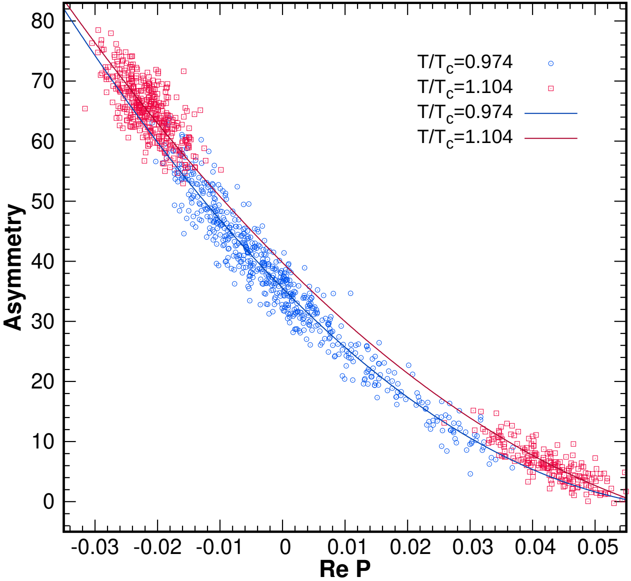
$$m_e = \frac{1}{\sqrt{D_L(0)}}$$

A.Maas et al., 2011



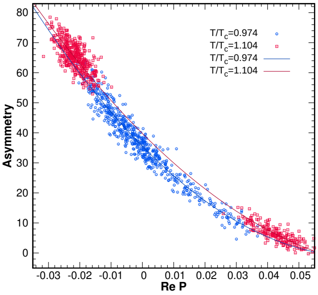
Correlation on the scatter plot

SU(2)



Correlation on the scatter plot

SU(3)



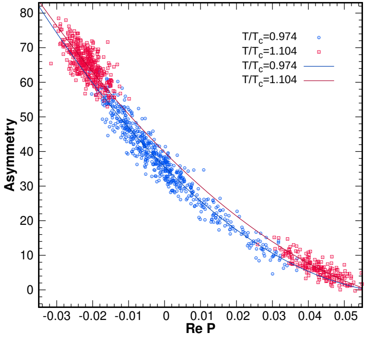
We consider conditional CDF $F(\mathcal{A}|\mathcal{P})$ of the asymmetry at a given value of Polyakov loop and the conditional average

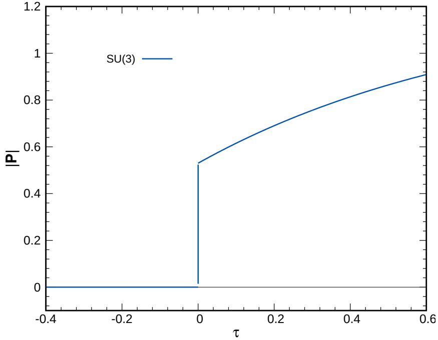
$$E(\mathcal{A}|\mathcal{P}) = \int \frac{dF(\mathcal{A}|\mathcal{P})}{d\mathcal{A}} \mathcal{A} d\mathcal{A} \qquad (9)$$

It can be fitted by the formula

$$E(\mathcal{A}|\mathcal{P}) \simeq \mathcal{A}_0 + \mathcal{A}_1 \,\mathrm{Re}\,\mathcal{P} + \mathcal{A}_2 \big(\mathrm{Re}\,\mathcal{P}\big)^2 \qquad (10)$$

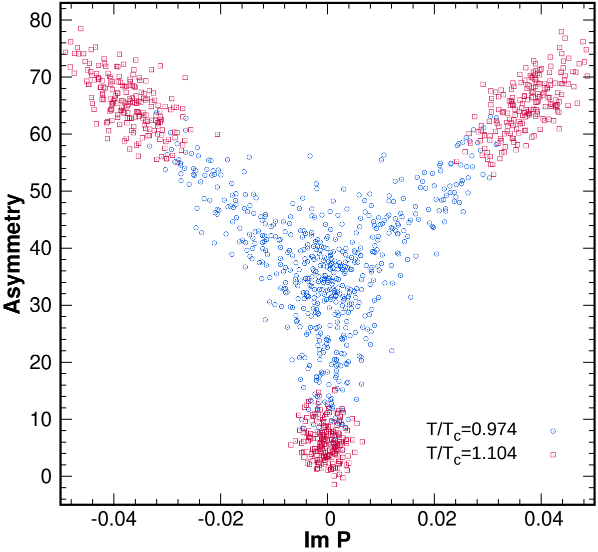
assuming its independence of $\mathrm{Re}\,\mathcal{P}$





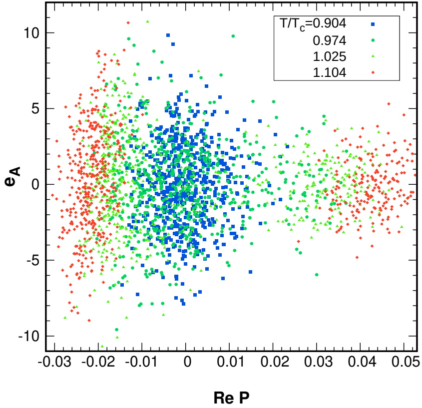
Smooth dependence of $\mathcal{A}$ on $\mathcal{P}$ & jump of $|\mathcal{P}|$ at $\{\tau = 0, V \to \infty\}$

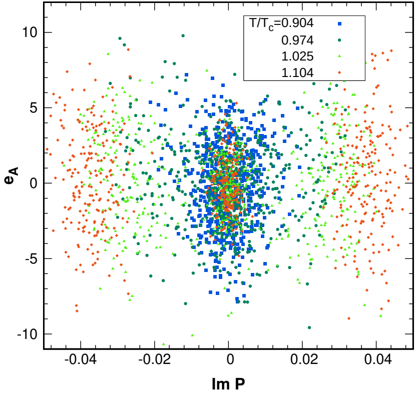
$\Longrightarrow$ a jump of $\mathcal{A}$ at the transition
when $V \to \infty$



SU(3)

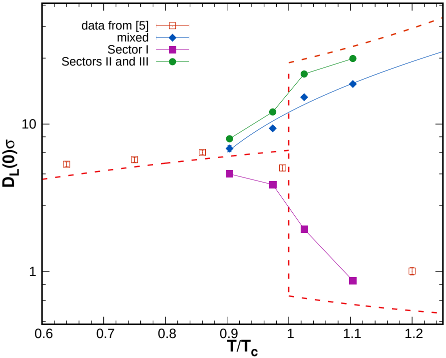
This scatter plot is readily explained by correlation between $\mathcal{A}$ and Re $\mathcal{P}$ only



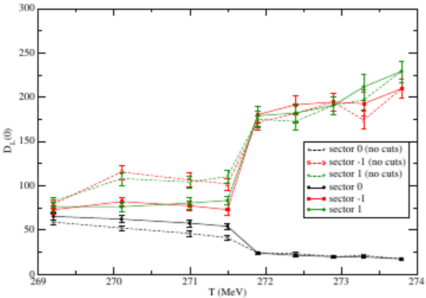


Residuals $e_A(n) = \mathcal{A}_n - \mathcal{A}_0 - \mathcal{A}_1 \,\mathrm{Re}\,\mathcal{P}_n - \mathcal{A}_2(\mathrm{Re}\,\mathcal{P}_n)^2$

show correlation with neither $\mathrm{Re}\,\mathcal{P}$ nor $\mathrm{Im}\,\mathcal{P}$

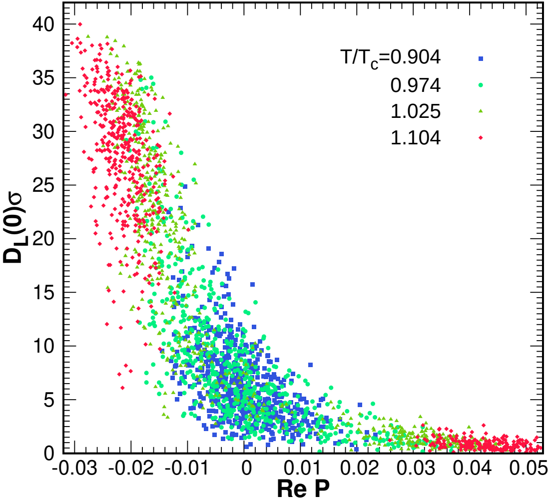


Our results;
dashed line - predicted behavior
at the phase transition



(b) $72^3 \times 8$ lattices.

O.Oliveira, P.Silva 2016



40
35
30
25
20
15
10
5
0
-0.03 -0.02 -0.01 0 0.01 0.02 0.03 0.04 0.05
Re P
D_L(0)σ
T/T_c=0.904
0.974
1.025
1.104

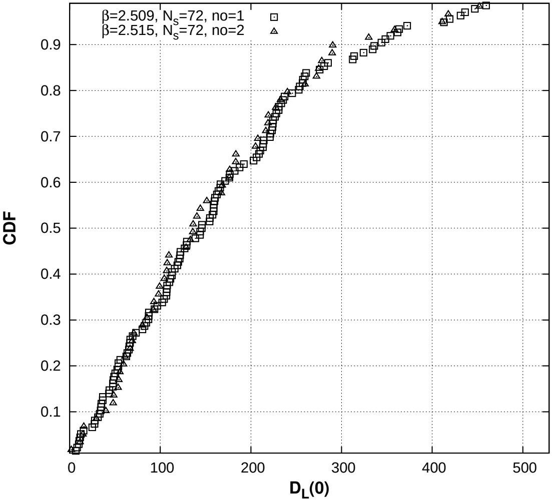
Homoscedasticity

- Independence of the variance of the conditional distribution on the predictor (independence of the variance of $(D_L(0)|\mathcal{P})$) on $\mathrm{Re}\,\mathcal{P}$).

Homoscedasticity is severely broken

Non-Gaussian behavior in the $SU(3)$ case:

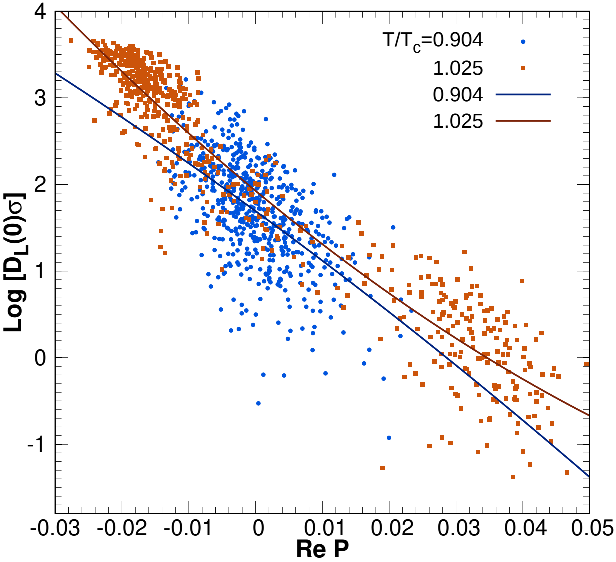
The Kolmogorov-Smirnov test for the $D_L(0)$ distribution at $-0.005 < \mathrm{Re}\,\mathcal{P} < 0.005$ indicates that the probability that it is Gaussian is less than 0.002.



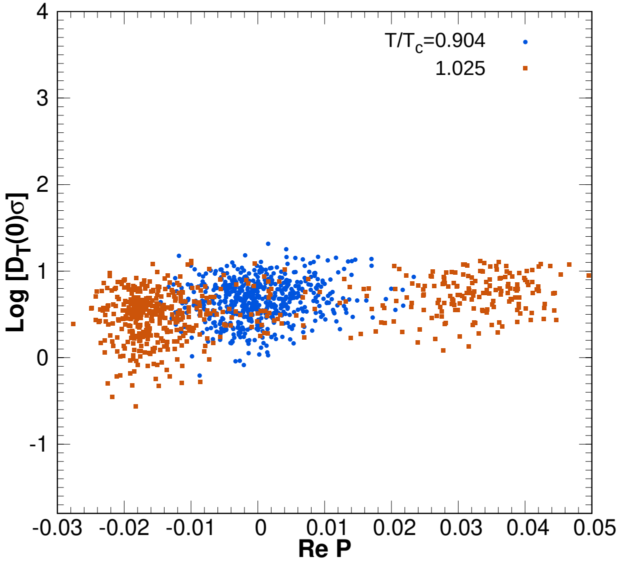
$SU(2)$ theory non-Gaussian distribution:

$-0.030 < \mathcal{P} < -0.025$;
$L = 6$ fm;

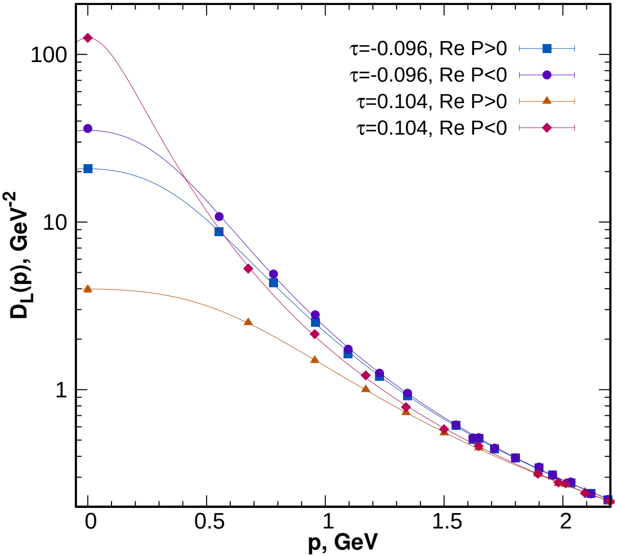
$1 \to \tau = -0.0045$;
$2 \to \tau = 0.0148$



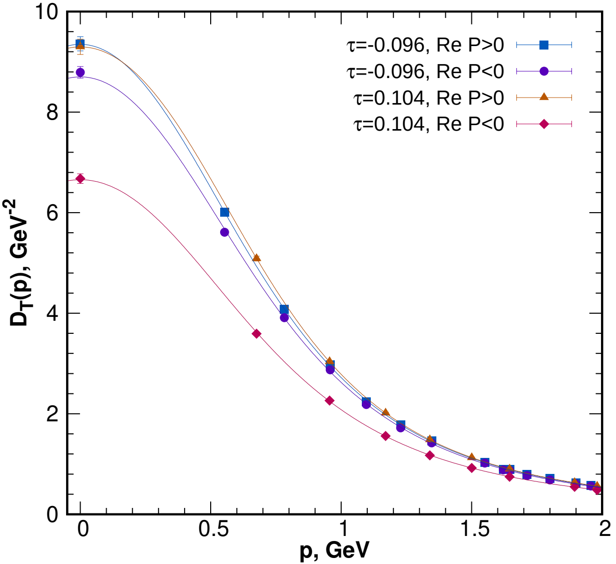
T/T_c=0.904
1.025
0.904
1.025
Log [D_L(0)σ]
Re P
4
3
2
1
0
-1
-0.03
-0.02
-0.01
0
0.01
0.02
0.03
0.04
0.05



T/T_c=0.904
1.025
Log [D_T(0)σ]
Re P
4
3
2
1
0
-1
-0.03
-0.02
-0.01
0
0.01
0.02
0.03
0.04
0.05



τ=-0.096, Re P>0
τ=-0.096, Re P<0
τ=0.104, Re P>0
τ=0.104, Re P<0
100
10
1
$D_L(p)$, GeV^{-2}
0
0.5
1
1.5
2
p, GeV


10
8
6
4
2
0
0
0.5
1
1.5
2
p, GeV
$D_T(p)$, GeV^{-2}
τ=-0.096, Re P>0
τ=-0.096, Re P<0
τ=0.104, Re P>0
τ=0.104, Re P<0

τ	m_E^2 $\mathrm{Re}\,\mathcal{P} > 0$	m_E^2 $\mathrm{Re}\,\mathcal{P} < 0$	m_M^2 $\mathrm{Re}\,\mathcal{P} > 0$	m_M^2 $\mathrm{Re}\,\mathcal{P} < 0$
-0.096	0.373(31)	0.214(31)	0.638(34)	0.642(39)
-0.026	0.445(71)	0.136(11)	0.609(24)	0.586(32)
0.025	0.523(56)	0.0498(38)	0.672(37)	0.565(18)
0.104	0.95(20)	0.0272(11)	0.664(43)	0.611(8)

Table: Values of the chromoelectric and chromomagnetic screening masses (in GeV^2) in different Polyakov-loop sectors. No difference between sectors (*II*) and (*III*) has been found, they are referred to as "$\mathrm{Re}\,\mathcal{P} < 0$".

Conclusions

- Both the asymmetry $\mathcal{A}$ and the zero-momentum longitudinal propagator $D_L(0)$ have a significant correlation with the real part of the Polyakov loop $\mathcal{P}$.
- We determined critical behavior of $\mathcal{A}$ and $D_L(0)$ in the infinite-volume limit. No discontinuities at a finite volume can take place.
- Chromoelectric interactions relative to chromomagnetic are weakly suppressed and short-range in the sector $\mathrm{Re}\mathcal{P} > 0$ and moderately suppressed and long-range in each sector with $\mathrm{Re}\mathcal{P} < 0$.

To be studied: Dependence of the conditional variance of $D_L(0)$ and $D_T(0)$ on lattice volume