Renormalizability of the center-vortex free sector of Yang-Mills theory

A Virtual Tribute to Quark Confinement and the Hadron Spectrum

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CNPq Conselho Nacional de Desenvolvimento Científico e Tecnológico

DAAD Deutscher Akademischer Austauschdienst German Academic Exchange Service

Outline

- Description of the gauge fixing procedure.
- Motivations: could be free from Gribov copies and provide a path from pure Yang-Mills to center vortex ensembles.
- Renormalizability.
- Conclusions and future perspectives.

Preliminary remarks about the gauge fixing procedure

- This gauge was proposed by L. Oxman and G. Rosa (2015), for continuum pure SU(N) Yang-Mills (YM) theory in $3+1$ dimensions.
- The gauge condition is imposed on auxiliary fields.
- This is in contrast with most continuum procedures[1], but resembles the lattice Laplacian Center Gauge (Ph. de Forcrand and M. Pepe (2001)).

[1] See H. Reinhardt and T. Tok (2001) for an exception.

The gauge fixing procedure

- Step 1: correlate A with the auxiliary fields, by means of the minimization of the $SU(N)$ invariant auxiliary action

$$S_{\rm aux}(A,\psi) = \int d^4x \, (\langle D_\mu(A)\psi_I, D_\mu(A)\psi_I\rangle + V_{aux}(\psi)) \ .$$

- ψ_I are a set of N^2-1 adjoint scalar fields. V_{aux} is chosen such that S_{aux} has a $SU(N) \to Z(N)$ SSB.

- This provides a map $A \to \psi_I(A)$.

- Step 2: extract a phase $S \in SU(N)$ from the tuple $\psi_I(A)$. This is accomplished by a generalized polar decomposition, i.e. $\psi_I(A) = S(A)q_I(A)S^{-1}(A)$.

  In general, the phase $S \in SU(N)$ will contain defects associated to center vortices, e.g. $S_{1v}(\varphi) = e^{i\varphi\beta_q T_q}$. β is proportional to a fundamental weight, T_q are the Cartan generators of $SU(N)$.

- In general, the phase $S \in SU(N)$ will contain defects associated to center vortices, e.g. $S_{1v}(\varphi) = e^{i\varphi\beta_q T_q}$.

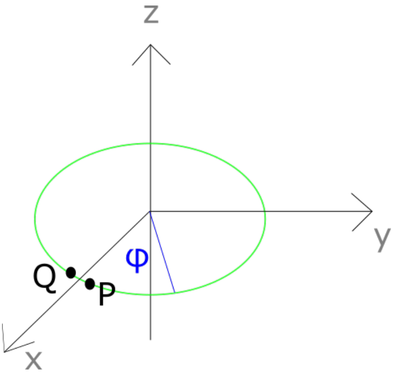


Figure: The phase $S_{1v}(\varphi)$ is close to the identity at point P, and to a center element at point Q, satisfying $S_{1v}(2\pi) = e^{-i2\pi/N} S_{1v}(0)$. This phase can't be eliminated by a regular gauge transformation.

- An equivalence relation between phases is naturally introduced:

$$[S] = [S'] \iff S = US', \; U \text{ regular}.$$

- It is natural to split the configuration space $\{A\}$ into sectors $\mathcal{V}(S_0)$, where $A \in \mathcal{V}(S_0) \iff S(A) = US_0, U$ regular.

- The gauge is fixed separately in each sector $\mathcal{V}(S_0)$ by means of the sector-dependent pure modulus condition

$$f_{S_0}(\psi) = [S_0^{-1}\psi_I(A)S_0, T_I] = 0 \; .$$

This is a local gauge fixing procedure.

- I. M. Singer (1978): no continuous global gauge fixing is possible.

Figure: The connection fiber bundle $\{A\}$. Gauge orbits are represented by green vertical lines.

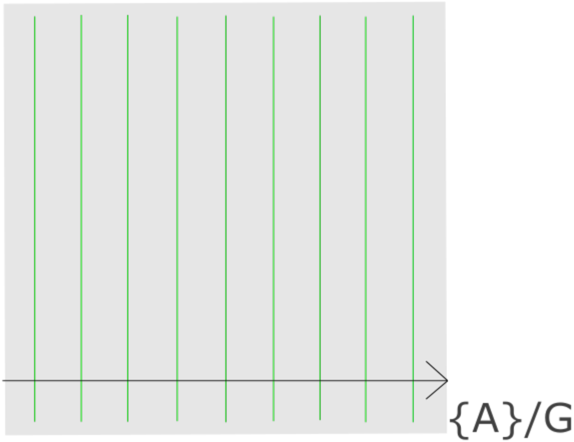


Figure: The connection fiber bundle $\{A\}$. Gauge orbits are represented by green vertical lines. Blue line: attempt at a global gauge fixing condition (global cross-section). Singer's theorem implies that this type of gauge fixing is impossible.

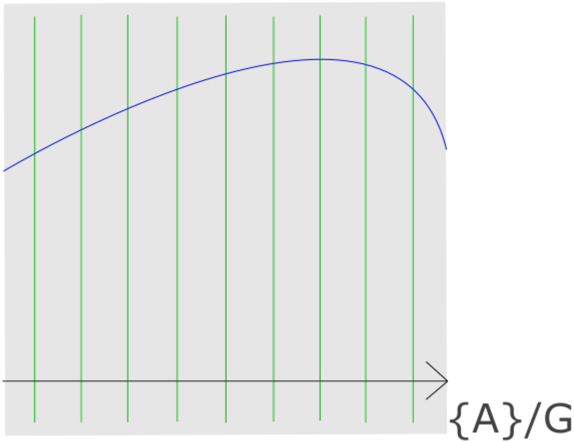
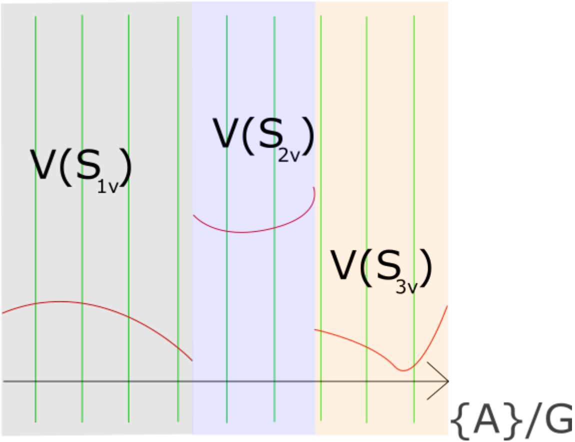


Figure: The connection fiber bundle $\{A\}$. Gauge orbits are represented by vertical green lines. The different shaded regions represent the sectors V(S_{1v}), V(S_{2v}), V(S_{3v}). The red lines correspond to a local gauge fixing procedure (local cross-sections).



- After gauge fixing, the YM partition function is written as

$$Z_{\rm YM} = \sum_{S_0} Z_{(S_0)} \quad , \quad Z_{(S_0)} \propto \int_{\mathcal{V}(S_0)} [DA_\mu][D\phi] e^{-S_{YM}(A)} e^{-S_{gf}^{(S_0)}(A,\phi)} ,$$

$$\phi \equiv \{b, c, \bar{c}, b_I, \bar{c}_I, c_I, \psi_I, \lambda_I, \xi_I\}.$$

- The action $S_{gf}^{(S_0)}$ implements the sector-dependent gauge condition.

- The full action $S_{YM} + S_{gf}^{(S_0)}$ is sector dependent.

- In each sector, the action is invariant under a BRST symmetry s.

- $S_{gf}^{(S_0)}$ may be written as a trivial variation, i.e., all the terms associated to the gauge fixing procedure belong to the trivial part of the cohomology of s.

- The s symmetry of the vortex-free sector ($S_0 = I$):

$$
\begin{aligned}
sA^a_\mu &= \tfrac{i}{g} D^{ab}_\mu c^b \,, & sc^a &= -\frac{i}{2} f^{abc} c^b c^c \,, \\
s\bar{c}^a &= -b^a \,, & sb^a &= 0 \,, \\
s\psi^a_I &= if^{abc}\psi^b_I c^c + c^a_I \,, & s\bar{c}^a_I &= -if^{abc}\bar{c}^b_I c^c - b^a_I \,, \\
sb^a_I &= if^{abc} b^b_I c^c \,, & sc^a_I &= -if^{abc} c^b_I c^c \,, \\
s\mu^2 &= U^2 \,, & sU^2 &= 0 \,, \\
s\kappa &= \mathcal{K} \,, & s\mathcal{K} &= 0 \,, \\
s\lambda &= \Lambda \,, & s\Lambda &= 0 \,, \\
sM^{ab}_I &= N^{ab}_I \,, & sN^{ab}_I &= 0 \,, \\
s\bar{C}^a = sK^a_\mu = sL^a_I &= s\bar{L}^a_I & = sQ^a_I &= sB^a_I = 0 \,.
\end{aligned}
$$

- In this framework, observables are computed as follows:

$$\langle O \rangle_{\mathrm{YM}} = \frac{1}{Z_{\mathrm{YM}}} \sum_{S_0} \frac{Z_{(S_0)}}{Z_{(S_0)}} \int_{\mathcal{V}(S_0)} [DA_\mu][D\phi]\, O\, e^{-S_{\mathrm{YM}}(A)} e^{-S_{gf}(A,\phi)} \ .$$

- What if...

$$\frac{Z_{(S_0)}}{Z_{YM}} = e^{-S_{eff}(S_0)} \ .$$

- Then,

$$\langle O \rangle_{\mathrm{YM}} = \sum_{S_0} e^{-S_{eff}(S_0)} \langle O \rangle_{S_0} \ ,$$

with S_0 ranging over the set of all possible center-vortex configurations.

This is a glimpse of a path from pure YM to center-vortex ensembles in the continuum.

- Phenomenological center-vortex ensembles containing tension and stiffness terms in $S_{eff}(S_0)$ are known to reproduce the properties of the confining flux tube at asymptotic distances. For a recent review on this subject, see D. R. Junior, L. E. Oxman, G. M. Simões, Universe 7(8), 253 (2021). See also Luis Oxman's talk tomorrow!

Renormalizability

- Are these partial contributions $e^{-S_{eff}(S_0)}$ calculable?

- As a first step to answer this question, we analyzed the renormalizability of the vortex-free (perturbative) sector $S_0 = I$.

- We used the algebraic method, where one characterizes $\Sigma^{c.t.}$ by using the Ward Identities of the action Γ (O. Piguet and S. Sorella, 1995).

- Ward identities: Slavnov-Taylor, ghost equation, ghost number, gauge-fixing equation, antighost equation, exact rigid symmetry, color-flavor symmetry.

- Ghost equation:

$$\left(\frac{\delta}{\delta c^a}+(vf^{abc}f^{clm}+if^{abn}M_I^{mn})\frac{\delta}{\delta N_I^{mb}}\right)\Sigma=\Phi^a\ ,$$

Φ^a being a linear polynomial in the fields.

The most general counterterm $\Sigma^{c.t.}$ compatible with the Ward Identities may be absorbed by a redefinition of the parameters and fields.

Conclusions

- The vortex-free sector is perturbatively renormalizable.
- Work in progress: renormalizability of a general sector $\mathcal{V}(S_0)$.
- Future perspectives: compute $S_{eff}(S_0)$. Casimir-like problem with codimension 2. See Ref. C. D. Fosco, D. R. Junior, L. E. Oxman (2020) for a much simpler, but analogous problem.

Thank you!