Fermion & gluon spectral functions far from equilibrium

TECHNISCHE UNIVERSITÄT WIEN

Kirill Boguslavski

FWF Der Wissenschaftsfonds.

A Virtual Tribute to Quark Confinement
and the Hadron Spectrum 2021,
August 02, 2021

Talk mainly based on:

KB, Lappi, Mace, Schlichting, 2106.11319

Further literature (gluon spectral functions):
KB, Kurkela, Lappi, Peuron, 1804.01966; 2101.02715

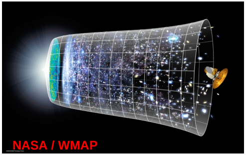
Table of Contents

Highly occupied systems

Heavy-ion collisions

Longitudinally expanding quark-gluon plasmas

Inflationary cosmology



Relativistic scalar systems

- Nonperturbatively strong bosonic fields can be encountered in
  - ⋆ Heavy-ion collisions at early times
  - ⋆ From instabilities during cosmological reheating
  - ⋆ IR sector of gluons and scalars
- Understanding interactions with fermions important
  Some applications in heavy-ion collisions:
  - ⋆ jets (energy loss, jet quenching)
  - ⋆ electromagnetic observables

Spectral functions

For microscopic description of observables and dynamics

Spectral function $\rho(\omega, p)$ of fermions encodes interactions

- Our approach: *far from equilibrium*
  - ⋆ Highly occupied gluon plasma ($A \sim 1/g$), weak coupling ($g^2 \ll 1$)
  - $\Rightarrow$ Then nonperturbative and perturbative methods available!
- Classical-statistical lattice simulations vs. HTL, kinetic theory
- More generally, gluon & fermion spectral functions needed for
  - ⋆ Heavy-ion collisions at early times
  - ⋆ Comparison to thermal properties (general features?)
  - ⋆ Nonperturbative extensions of kinetic theory, anisotropic HTL

Table of Contents

Theory

- $\mathrm{SU}(N_c)$ gauge theory (simulations: $N_c = 2$, $U_j \approx \exp\left(ig\, a_s A_j\right)$)

$$H_{YM} = \frac{1}{g^2 a_s} \sum_{\vec{x},i} \mathrm{Tr}[E_i(t',\vec{x})^2] + \frac{1}{2} \sum_j \mathrm{ReTr}[1 - U_{ij}(t',\vec{x})]$$

$$\hat{H}_W = \frac{1}{2} \sum_{\vec{x}} \left[\hat{\psi}^\dagger(t',\vec{x}), \gamma^0 \left(-i\not{D}_s[U] + m\right) \hat{\psi}(t',\vec{x})\right]$$

- $-i\not{D}_s[U]\hat{\psi}$ tree-level improved Wilson Dirac operator
- Neglect fermionic backreaction (suppressed), temporal gauge $A_0 = 0$
- Mode exp. $\hat{\psi}(t',\vec{x}) = \frac{1}{\sqrt{V}} \sum_{\lambda,\vec{p}} \hat{b}_{\lambda,\vec{p}}(t)\, \phi^u_{\lambda,\vec{p}}(t',\vec{x}) + \hat{d}^\dagger_{\lambda,\vec{p}}(t)\, \phi^v_{\lambda,\vec{p}}(t',\vec{x})$
- At reference time t:
  - ⋆ Creation/annih. operators $\hat{b}$, $\hat{d}$ satisfy anti-commut. relations
  - ⋆ Plane waves $\phi^u_{\lambda,\vec{p}}(t',\vec{x})\Big|_{t'=t} = u_\lambda(\vec{p})e^{+i\vec{p}\cdot\vec{x}}$, $\phi^v_{\lambda,\vec{p}}(t',\vec{x})\Big|_{t'=t} = v_\lambda(\vec{p})e^{-i\vec{p}\cdot\vec{x}}$

See also: Aarts, Smit (1999); Kasper, Hebenstreit, Berges (2014); Mace, Mueller, Schlichting, Sharma (2016); ...

Classical-statistical lattice simulations, algorithm

1. Set initial conditions for gluons at $t' = 0$, generating a configuration with $\langle E_T^*(t{=}0,\vec{p}) E_T(t{=}0,\vec{q})\rangle \propto p\, f(t{=}0,p)\,\delta_{jk}(2\pi)^3\delta(\vec{p}-\vec{q})$
2. Solve classical EOMs of gluonic part for $t' \leq t$, set Coulomb-type gauge $\partial^j A_j\big|_t = 0$ at $t' = t$
3. For each momentum mode $\vec{p}$ initialize $\phi^{u/v}_{\lambda,\vec{p}}$, evolve for $t' > t$
4. Use leap-frog scheme to solve classical EOMs

$$
\begin{aligned}
U_j(t',\vec{x}) &= e^{i a_t/a_s E^j(t'-a_t/2,\vec{x})} U_j(t'-a_t,\vec{x}) \\
E^j(t'+a_t/2,\vec{x}) - E^j(t'-a_t/2,\vec{x}) &= -\frac{a_t}{a_s}\sum_{j\neq i}\left[U_{ij}(t',\vec{x}) + U_{i(-j)}(t',\vec{x})\right]_{\mathrm{ah}} \\
\phi^{u/v}_{\lambda\vec{p}}(t'+a_t,\vec{x}) - \phi^{u/v}_{\lambda\vec{p}}(t'-a_t,\vec{x}) &= -2i a_t\gamma^0\left(-i\gamma^j D^s_{W,j}[U] + m\right)\phi^{u/v}_{\lambda\vec{p}}(t',\vec{x})
\end{aligned}
$$

5. Calculate fermionic spectral function $\rho(t',t,\vec{p})$ for each $\vec{p}$ mode

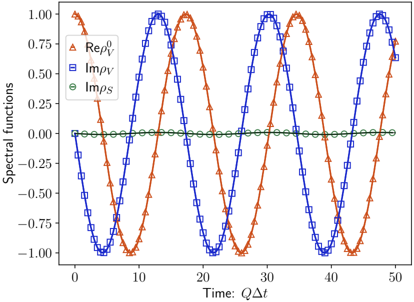
Calculating fermion spectral function nonperturbatively

- Definition $\rho^{\alpha\beta}(x,y) = \left\langle \left\{ \hat{\psi}^{\alpha}(t',\vec{x}), \hat{\bar{\psi}}^{\beta}(t,\vec{y}) \right\} \right\rangle$
- Plug in mode exp., Fourier transform, use plane waves at $t' = t$

$$\begin{aligned}
&\rho^{\alpha\beta}(t',t,\vec{p}) \\
&= \frac{1}{V}\sum_{\lambda,\vec{q}} \left\langle \tilde{\phi}^{u,\alpha}_{\lambda,\vec{q}}(t',\vec{p}) \left(\tilde{\phi}^{u,\gamma}_{\lambda,\vec{q}}(t,\vec{p})\right)^{*} + \tilde{\phi}^{v,\alpha}_{\lambda,\vec{q}}(t',\vec{p}) \left(\tilde{\phi}^{v,\gamma}_{\lambda,\vec{q}}(t,\vec{p})\right)^{*} \right\rangle \gamma_0^{\gamma\beta} \\
&= \frac{1}{V}\sum_{\lambda} \left\langle \tilde{\phi}^{u,\alpha}_{\lambda,\vec{p}}(t',\vec{p}) u^{\dagger,\gamma}_{\lambda}(\vec{p}) + \tilde{\phi}^{v,\alpha}_{\lambda,-\vec{p}}(t',\vec{p}) v^{\dagger,\gamma}_{\lambda}(-\vec{p}) \right\rangle \gamma_0^{\gamma\beta}
\end{aligned}$$

- Huge simplification due to $\tilde{\phi}_{\lambda,\vec{q}}(t,\vec{p}) \propto \delta^{(3)}(\vec{p}-\vec{q})$
- Classical-statistical average over gluonic configurations

Benchmark for free fermions



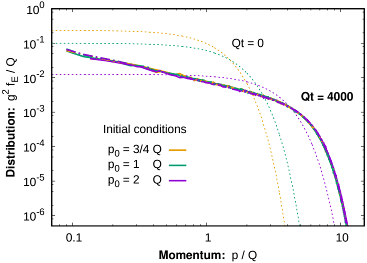
- Analytically: $\rho^{\text{free}}(\Delta t, \vec{p}) = \gamma^0 \cos(E_{\vec{p}}\Delta t) + i\left(\gamma^j \frac{\hat{p}^j}{E_{\vec{p}}} - \frac{m_{\vec{p}}}{E_{\vec{p}}}\right) \sin(E_{\vec{p}}\Delta t)$
- Our method: set gauge fields to $U^i(t', \vec{x}) = 1$, $E^i(t', \vec{y}) = 0$
- Extract $\rho_V^0 = \frac{1}{4}\mathrm{Tr}(\rho\gamma^0)$, $\rho_V = -\frac{E_{\vec{p}}\, p^j}{4\, p^2}\mathrm{Tr}(\rho\gamma^j)$, $\rho_S = \frac{1}{4}\mathrm{Tr}(\rho)$
- Nice agreement!
- 64^3, $a_s\vec{p} = (0.098, 0.195, 0.29)$, mass $m\, a_s = 0.003125$;

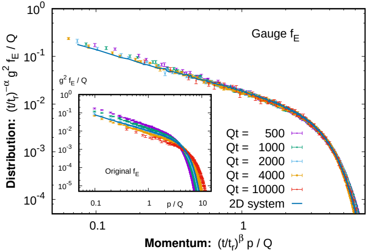
  all other components of ρ at machine prec. $\sim 10^{-16}$

Table of Contents

Non-equilibrium state: self-similar turbulent attractor

Figures: attractor in 2+1D; KB, Kurkela, Lappi, Peuron, PRD 100, 094022 (2019)





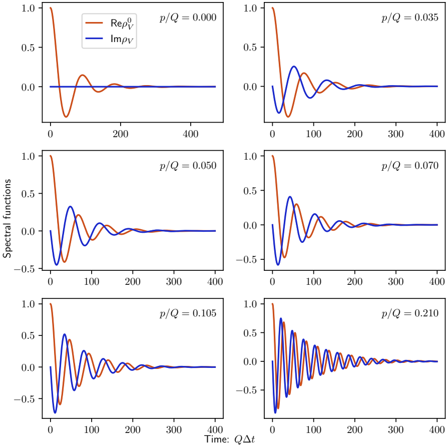
- Gluonic $f_g(t{=}0, p \lesssim Q) \sim \frac{1}{g^2} \gg 1$ approach self-similar attractor

$$f(t,p) = (Qt)^{\alpha} f_s\left((Qt)^{\beta} p\right)$$

- Universal exponents insensitive to details of initial conditions
  - ✓ 2+1D: $\beta = -1/5,\ \alpha = 3\beta,$ KB, Kurkela, Lappi, Peuron (2019)
  - ✓ 3+1D: $\beta = -1/7,\ \alpha = 4\beta,$ Berges, Scheffler, Sexty (2009); Kurkela, Moore (2011, 2012); Berges, Schlichting, Sexty (2012); Schlichting (2012); Berges, KB, Schlichting, Venugopalan (2014); ...
- We extract $\rho^{\alpha\beta}(t, \omega, p)$ at such a typical state in 3+1D

Fermion ρ in 3+1D: non-trivial evolution

Spectral function $\rho^{\alpha\beta}(t+\Delta t, t, p)$



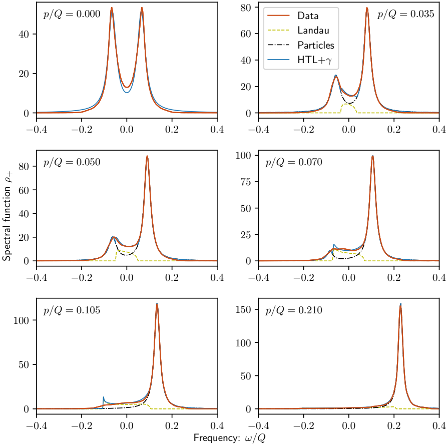
- Lattice 256^3, $Qa_s = 0.75$, $m = 0.003125\, Q$, $Qt = 1500$
- Non-trivial $\rho_V^0 = \frac{1}{4}\mathrm{Tr}(\rho\gamma^0)$, $\rho_V = -\frac{E_{\vec{p}}\, p^j}{4\, p^2}\,\mathrm{Tr}(\rho\gamma^j)$

  $$\mathrm{Re}\rho_V^0 \approx \quad e^{-\gamma(t,p)\Delta t}\cos(\omega(t,p)\Delta t)$$
  $$\mathrm{Im}\rho_V \approx -e^{-\gamma(t,p)\Delta t}\sin(\omega(t,p)\Delta t)$$

- All other components expected to vanish from symmetries; numerically suppressed by at least $\sim 10^{-2}$

Fermion ρ in 3+1D: Comparison with HTL

$$\rho_+(t,\omega,p) \equiv \rho_V^0 + \rho_V$$



- HTL: Landau cut + q.p. peaks:
  Braaten, Pisarski (1992); Rebhan (1992); Mrowczynski, Thomas (2000); Blaizot, Iancu (2002); ...

$$\rho_+^{\mathrm{HTL}}(\omega,p) = 2\pi\,\beta_+(\omega/p,p) + 2\pi\,[Z_+(p)\delta(\omega-\omega_+(p)) + Z_-(p)\delta(\omega+\omega_-(p))]$$

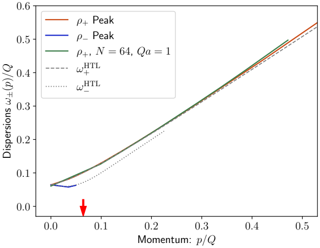
- HTL+γ extension

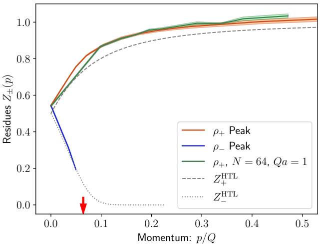
$$\rho_+^{\mathrm{HTL}}(\omega,p) = 2\pi\,\beta_+(\omega/p,p) + \frac{2Z_+(p)\gamma_+(p)}{(\omega-\omega_+(p))^2+\gamma_+^2(p)} + \frac{2Z_-(p)\gamma_-(p)}{(\omega+\omega_-(p))^2+\gamma_-^2(p)}$$

- Very good description; $\omega_\pm(t,p)$, $Z_\pm(t,p)$, $\gamma_\pm(t,p)$ fits

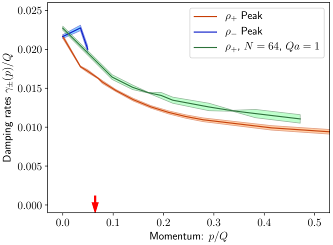
Fermion ρ in 3+1D: $\omega_\pm$, $Z_\pm$, $\gamma_\pm$

(top) $\omega_\pm$, *(bottom)* $Z_\pm$





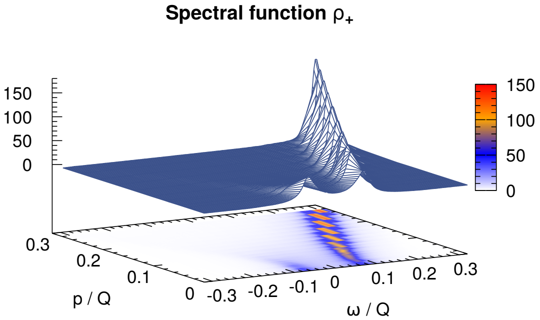
Damping rates (width) $\gamma_\pm$



- HTL dispersions and residues agree well with data
- First-principles insight into p-dependence of damping rates
- Arrows: $m_f = \left[C_F \int \frac{d^3p}{(2\pi)^3} \frac{g^2 f_g(p)}{p} \right]^{1/2}$

Summary: fermion ρ_+ in isotropic 3+1D plasmas

KB, Lappi, Mace, Schlichting, arXiv:2106.11319



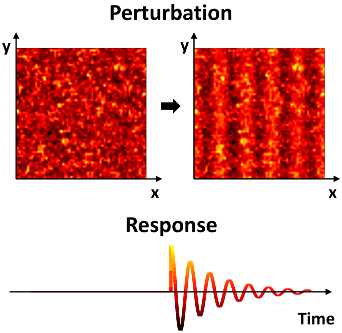
We find:

- Lorentzian quasiparticle peaks + Landau cut ($\omega < p$)
- very good description by HTL
- full p-dependence of $\gamma_+(t,p)$
- (see Backup:) $\gamma_+(t,p{=}0) \sim m_F(t) \sim Q(Qt)^{-1/7}$
  vs. HTL expectation: $\gamma^{\mathrm{HTL}}(t,p{=}0) \propto g^2 T^*(t) \sim Q(Qt)^{-3/7}$

Table of Contents

Classical-statistical $\mathrm{SU}(N_c)$ simulations + linear response theory

KB, Kurkela, Lappi, Peuron, *PRD 98, 014006 (2018),* Editors' suggestion



- Similar algorithm as for fermions
- Split $A(t, \vec{x}) \mapsto A(t, \vec{x}) + \delta A(t, \vec{x})$ at t, perturb with plane wave $j_0(\vec{p})\, \delta\left(t' - t\right)$
- Response $\langle \delta A(t', \vec{p}) \rangle = G_R(t', t, \vec{p})\, j_0(\vec{p})$
- Linearized EOM for $\delta A(t, \vec{x})$ such that Gauss law conserved (also in gauge-cov. formulation)

  Kurkela, Lappi, Peuron, *EUJC 76 (2016) 688*
- $\theta(t' - t)\ \boxed{\rho(t', t, p)} = G_R(t', t, p)$

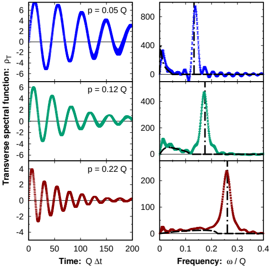
Similar methods for scalars:

Aarts (2001); Piñeiro Orioli, Berges (2019); Schlichting, Smith, von Smekal (2020); KB, Piñeiro Orioli (2020);

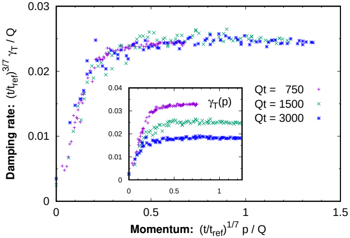
Gluon spectral function in isotropic 3+1D plasmas

KB, Kurkela, Lappi, Peuron, PRD 98, 014006 (2018)

$\rho_T(\omega, p)$

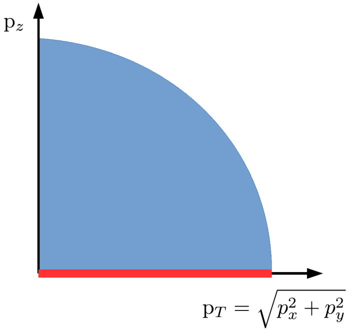


Width $\propto (Qt)^{3/7}\gamma_T(t,p)$



- Narrow Lorentzian q.p. peaks (position $\omega(p)$, width $\gamma(p)$)
- HTL at LO (black dashed) describes main features well
- Landau cut ($\omega < p$) and q.p. peak distinguishable
- Peak width $\gamma(t,p) \ll \omega(t,p)$
- Full p dep., $\gamma_T(t,p) \approx \gamma_L(t,p)$
- $\frac{\gamma_T(t,p)}{\omega(t,p=0)} \sim (Qt)^{-2/7}$ decreases

3+1D → 2+1D plasmas

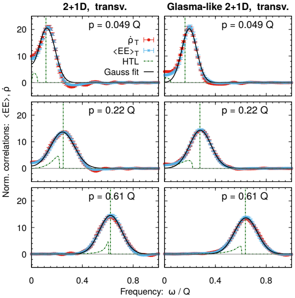


- Isotropic 3+1D: $f(t, p)$
- 2+1D: $f(t, p_T, p_z{=}0)$
- can be regarded as extreme momentum anisotropy

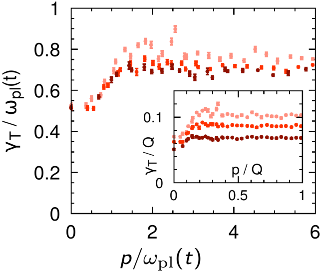
Vs. Gluon spectral function in 2+1D plasmas

KB, Kurkela, Lappi, Peuron, *JHEP 05, 225 (2021)*

$\dot{\rho}_T(\omega, p) \approx \omega \rho_T(\omega, p)$



Width $\gamma_T(t, p)/\omega_{\rm pl}(t)$



- Broad non-Lorentzian peaks
- HTL curves (green) agree poorly
- Landau cut and q.p. peak not distinguishable

- Peak width $\gamma(t, p) \sim \omega_{\rm pl}(t)$

  $(\omega_{\rm pl} \equiv \omega_T(p{=}0))$

$\Rightarrow$ no quasiparticles for $p \lesssim \omega_{\rm pl}$!

Table of Contents

Conclusion

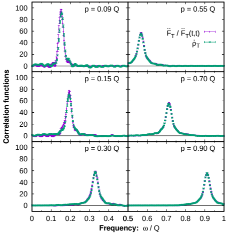
- We have developed tools to extract spectral functions in highly occupied plasmas nonperturbatively
- Applied to classical self-similar attractors:
  fermionic 3+1D, gluonic 3+1D, gluonic 2+1D
- ρ's in 3+1D generally well described by HTL,
  first principles determination of damping rates

Outlook: applications to heavy-ion collisions

- ρ in Bjorken expanding systems or of heavy-flavor quarks
- Anisotropic / expanding kinetic theory, HTL
- Effects on transport coefficients

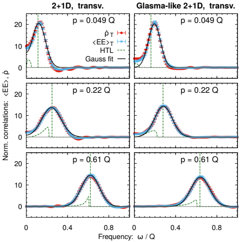
Some talks with classical-statistical simulations:

- B. S. Kasmaei, We, 17:20 (track C)
- J. Peuron, Th, 10:10 (track C)
- D. Schuh, Th, 10:55 (track D)

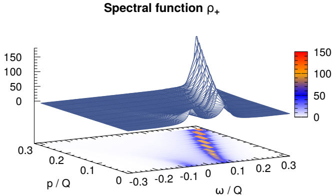


Gluonic 3+1D

Thank you for your attention!



Gluonic 2+1D

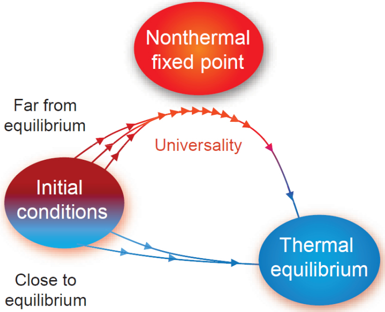


Fermionic 3+1D

Backup slides

Universal (classical) attractors

- Rich nonequilibrium dynamics in gauge and scalar systems
- Share similar universal features:



Nonthermal fixed point (NTFP)

★ Large initial occupancy
$\Rightarrow$ may approach attractor

★ System 'forgets' initial conditions

★ Self-similar dynamics

$$f(t,p) = t^{\alpha} f_s(t^{\beta} p)$$

★ *Universal* α, β, $f_s(p)$

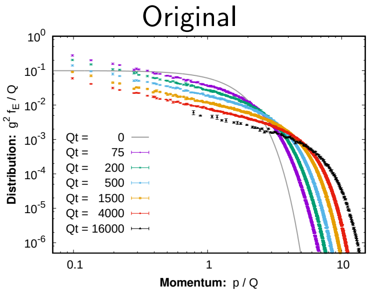
NTFP: Micha, Tkachev (2004); Berges, Rothkopf, Schmid (2008)

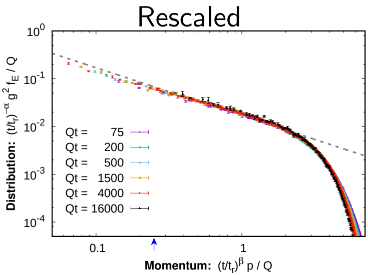
Universality: Berges, KB, Schlichting, Venugopalan (2015); Piñeiro Orioli, KB, Berges (2015)

Experimental observations: Prüfer et al., Nature 563, 217 (2018); Erne et al., Nature 563, 225 (2018)

Self-similarity of 2+1D theory

Figures: attractor in 2+1D; KB, Kurkela, Lappi, Peuron, PRD 100, 094022 (2019)





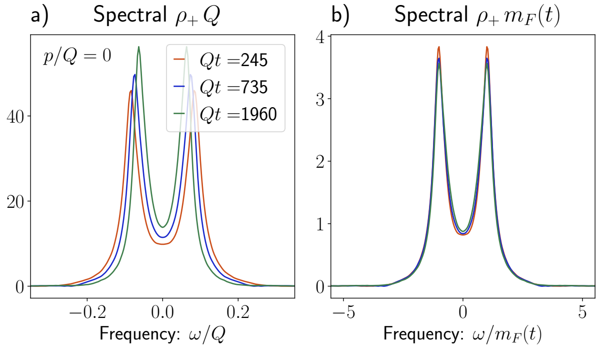
- Self-similar evolution

$$f(t,p) = (Qt)^{\alpha} f_s \left((Qt)^{\beta} p \right)$$

- Via energy conservation: $\alpha = (d+1)\beta$

Fermion ρ in 3+1D: Time evolution

KB, Lappi, Mace, Schlichting, arXiv:2106.11319



- $\rho_+(t, \omega, p{=}0)$ scales with fermion mass $m_f(t) \equiv \omega_\pm(t, p{=}0)$
- Expected from HTL: $\gamma^{\mathrm{HTL}}(t, p{=}0) \propto g^2 T^*(t) \sim Q(Qt)^{-3/7}$
  $\Rightarrow$ observed $\gamma(t, p{=}0) \sim m_f(t)$ is surprising

Gluon HTL results: perturbative computation

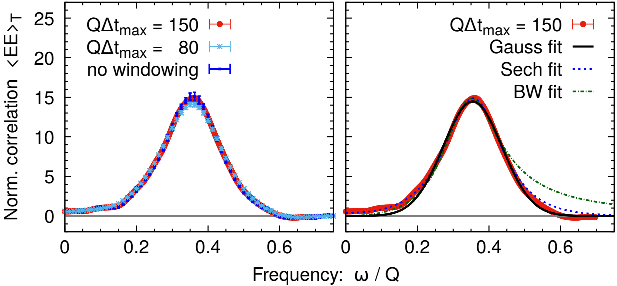
- Hard loop (HTL) framework requires $m_D/\Lambda \ll 1$;
  in thermal equ. for $g \sim m_D/T \ll 1$, Braaten, Pisarski (1990); Blaizot, Iancu (2002); ...
- In 3+1D $m_D^2 = 4N_c \int \frac{\mathrm{d}^3p}{(2\pi)^3} \frac{g^2 f(t,p)}{p} \sim g^2 f\,\Lambda^2 \Rightarrow$ HTL applicable
- In 2+1D soft-soft interactions important

$$m_D^2 \approx d_{\mathrm{pol}} N_{\mathrm{c}} \int \frac{\mathrm{d}^2p}{(2\pi)^2} \frac{g^2 f(t,p)}{\sqrt{m^2+p^2}} \sim g^2 f\,\Lambda \ln\left(\frac{\Lambda}{m_D}\right)$$

$\Rightarrow$ HTL breaks down already at soft scale $p \sim m_D$

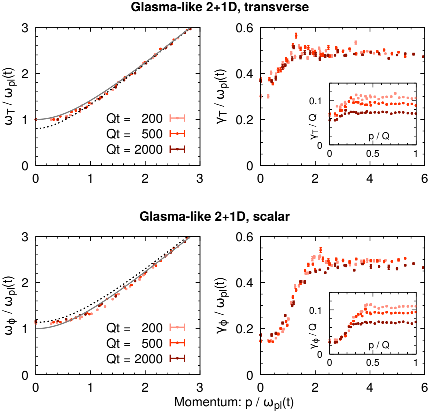
- Comparison to HTL still useful to extract nonperturbative features
- Quasiparticles in $\rho^{\mathrm{HTL}}(\omega, p)$ as $\sim \delta\big(\omega - \omega_\alpha^{\mathrm{HTL}}(p)\big)$
- All expressions depend only on m_D, computed consistently in HTL

Gluon ρ in 2+1D: Shape of the excitation peak



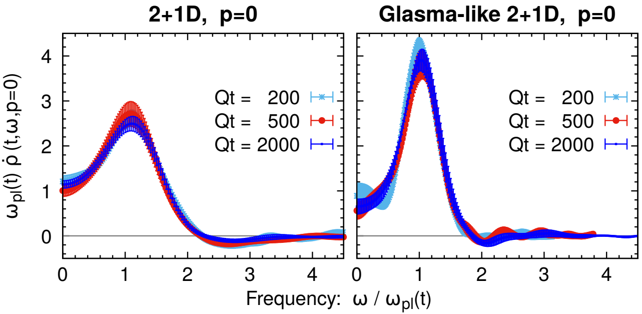
- *Left:* Different ways of computing the Fourier transform are consistent
- *Right:* Peak has non-Lorentzian shape (not Breit-Wigner)

Gluon ρ in 2+1D: Dispersion relations, damping rates



- *Left:* Dispersions $\omega_\alpha(t,p)/\omega_{\rm pl}(t)$
- *Right:* Peak width $\gamma_\alpha(t,p)/\omega_{\rm pl}(t)$
- As functions of $p/\omega_{\rm pl}(t)$ time independent $\Rightarrow \gamma(t,p) \sim \omega_{\rm pl}(t)$
- Scalar excitation narrow for $p \lesssim m_D$, but same t dependence

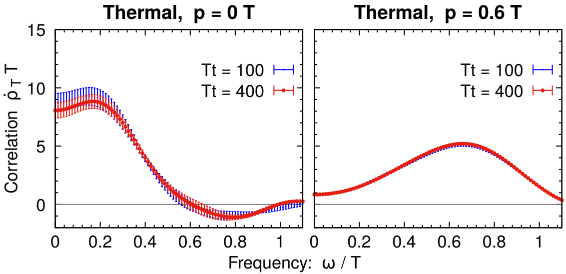
Gluon ρ in 2+1D: Time dependence



- $\omega_{\rm pl}\,\dot\rho(t, \omega/\omega_{\rm pl}, p/\omega_{\rm pl})$ is time independent
- This implies $\boxed{\gamma_\alpha(t,p) \sim \omega_{\rm pl}(t)} \sim m_D \sim Q(Qt)^{-1/5}$
- No quasiparticles at low p seems to be quite general in 2+1D

Gluon spectral function in 2+1D class. thermal equilibrium

KB, Kurkela, Lappi, Peuron, JHEP 05, 225 (2021)



- Classical thermal equilibrium $f(p) \approx \frac{T}{\omega(p)}$
- $\rho(\omega, p)$ qualitatively similar as far from equilibrium
  - ✓ Broad gluonic excitations with $\gamma(p) \sim \omega_{\rm pl}$
  - ✓ HTL provides poor description
  - ✓ For $\omega \to 0$, $\dot{\rho}_T = \omega \rho_T$ finite at low p
- Interpretion: Features seem generic in 2+1D gauge theories