A novel approach to lattice QCD at finite baryon density

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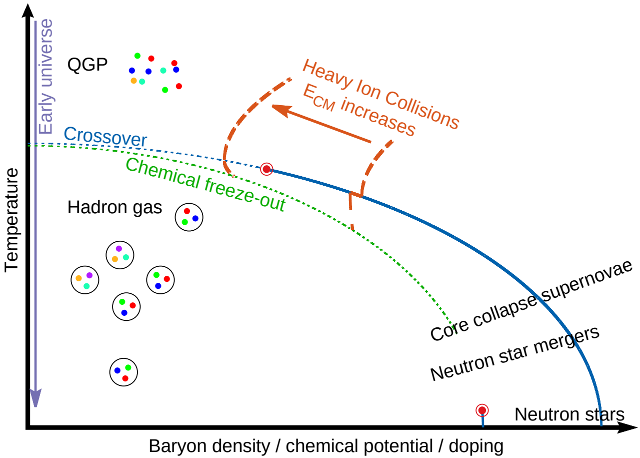
partially based on JHEP 05 (2020) 088; Giordano, Kapas, Katz, Nogradi, Pasztor

The conjectured phase diagram of QCD



Approaches to finite density lattice QCD

In addition to the sign problem, known approaches to finite density QCD suffer from additional serious problems. E.g.

- Taylor and imaginary μ: **analytic continuation problem**
- Reweighting and Taylor: **overlap problem**
- Complex Langevin: **convergence issues**
- ...

These problem are just as or even more crippling than the sign problem. This talk:

$\rightarrow$ a method where the only problem is the sign problem

If the sign problem is dealt with by sufficient statistics, the results are reliable, and errors (on a fixed lattice) are statistical only.

Reweighting in general

Target theory: Z_w Simulated theory: Z_r

$$Z_w = \int \mathcal{D}U \; w(U) \qquad w(U) = detM[U,\mu]e^{-S_g[U]} \in \mathbb{C}$$

$$Z_r = \int \mathcal{D}U \; r(U) \qquad r(U) > 0$$

$$\langle O \rangle_w = \frac{\int \mathcal{D}U \; w(U) O(U)}{\int \mathcal{D}U \; w(U)} = \frac{\int \mathcal{D}U \; r(U) \frac{w(U)}{r(U)} O(U)}{\int \mathcal{D}U \; r(U) \frac{w(U)}{r(U)}} = \frac{\left\langle \frac{w}{r} O \right\rangle_r}{\left\langle \frac{w}{r} \right\rangle_r}$$

Two problems that are exponentially hard in the volume:

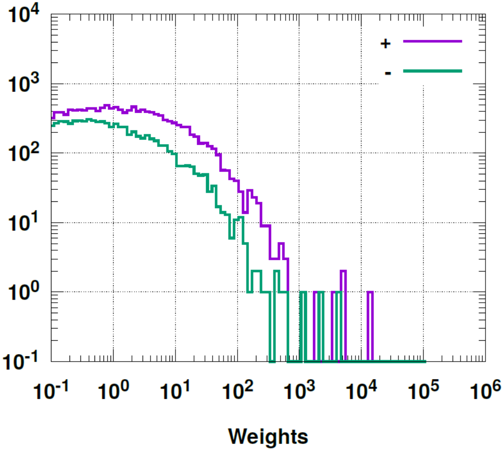
- $\frac{w}{r} \in \mathbb{C} \rightarrow$ **sign problem**
- Tails of $\rho(\frac{w}{r})$ long $\rightarrow$ **overlap problem** $\leftarrow$ The first bottleneck when reweighting from $\mu = 0$

Why does reweighting from $\mu = 0$ fail?

The expectation value of any observable:

$$\langle O \rangle_w = \frac{\left\langle \frac{w}{r} O \right\rangle_r}{\left\langle \frac{w}{r} \right\rangle_r}$$

The weights are the w/r, to calculate anything, we need to have control over this observable



The **sign problem is under control, the overlap problem is not:**
Giordano, Kapas, Katz, Nogradi, Pasztor; PRD 102, 034503 (2020)

Sign reweighting

$$Z = \int \mathcal{D}U e^{-S_g} \det M = \int \mathcal{D}U e^{-S_g} \mathrm{Re}\det \mathrm{M}$$

- Beware: the substitution $\det M \to \mathrm{Re}\det M$ can be done in Z but not in generic expectation values.
- Can calculate e.g. $\frac{\partial^n \log Z}{\partial \mu_{ud}^n}$, $\frac{\partial^n \log Z}{\partial m_{ud}^n}$ and $\frac{\partial^n \log Z}{\partial \beta^n}$

A new choice of a theory to reweight to and from:

$$\begin{aligned} w &= e^{-S_g} \mathrm{Re}\det \mathrm{M} \\ r &= e^{-S_g} \left|\mathrm{Re}\det \mathrm{M}\right| \end{aligned} \quad \Rightarrow \quad \frac{w}{r} \equiv \epsilon = \pm 1$$

- The weights are $\epsilon = \pm 1 \to$ No tail, **no overlap problem**
- $\langle \pm \rangle_r$ measures the strength of the **sign problem**

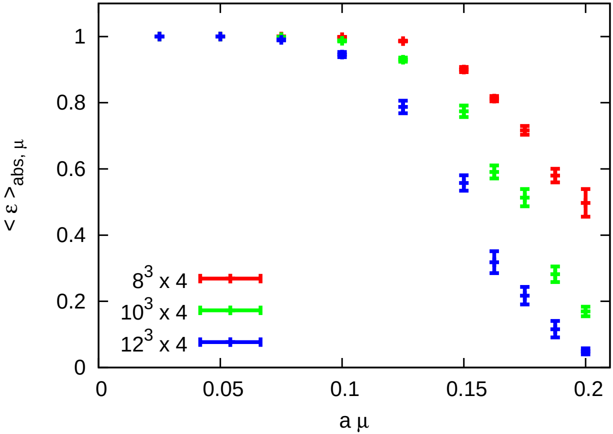
Early mentions of the idea:

de Forcrand, Kim, Takaishi: hep-lat/0209126; Nucl.Ph.B Proc.S. 119 (2003)

Alexandru, Faber, Horvath, Liu: hep-lat/0507020; PRD72 114513

First test of the new method - unimproved staggered $N_\tau = 4$

Strength of the sign problem at $T_c(\mu)$

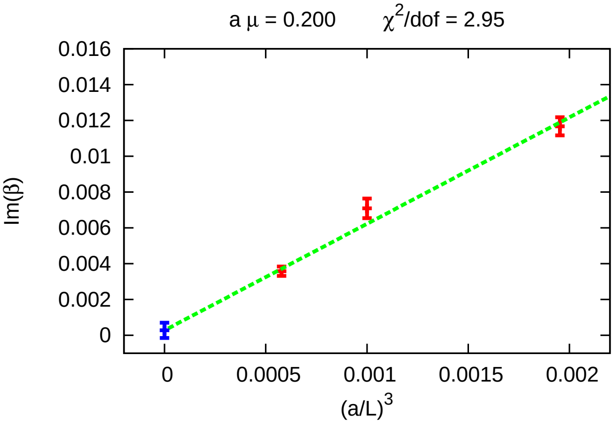


Giordano, Kapas, Katz, Nogradi, Pasztor; JHEP 05 (2020) 088

For simplicity we take $\mu_s = 0$ and $\mu_u = \mu_d = \mu_B/3$

First test of the new method - unimproved staggered $N_\tau = 4$

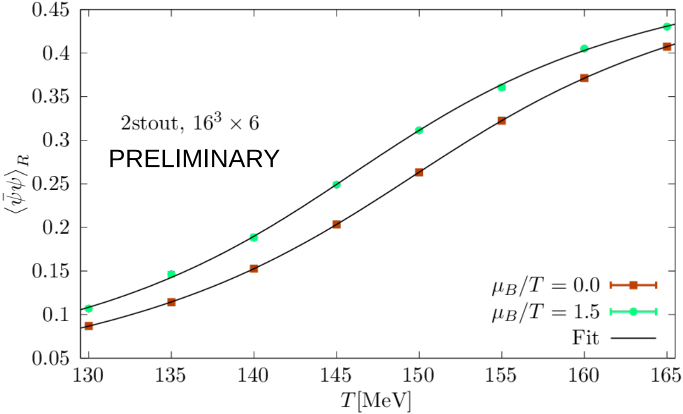
Finite volume scaling at $\mu_B/T = 2.4$



Consistent with Fodor, Katz; JHEP 04 (2004) 050. BUT: to start being relevant for phenomenology, a much better lattice action has to be used

Second test - 2stout $N_\tau = 6$ (PRELIMINARY)

$$\langle\bar{\psi}\psi\rangle_R = (\langle\bar{\psi}\psi\rangle_{0,0} - \langle\bar{\psi}\psi\rangle_{T,\mu})\frac{m_{ud}}{f_\pi^4}$$



No sign of the transition getting stronger

Second test - 2stout $N_\tau = 6$ (PRELIMINARY)

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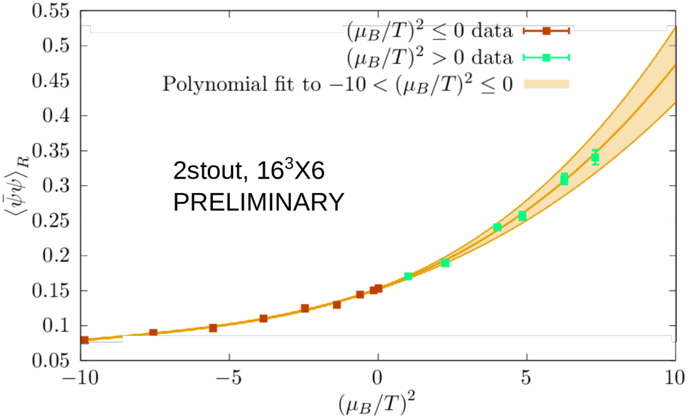

Summary

- Current methods to study finite density QCD are typically not bottlenecked by the sign problem itself
- In particular reweighting from $\mu = 0$ is bottlenecked by the overlap problem
- We proposed a new reweighting method that is free from the overlap problem in the weights and is therefore only bottlenecked by the sign problem itself
- First test: CEP for unimproved staggered at $N_\tau = 4$, expected to be dominated by cut-off effects
- Second test: 2stout at $N_\tau = 6$; preliminary
  - width of transition at $\mu_B/T = 1.5 \approx$ width at 0
  - so far matches analytic continuation from imaginary μ