A novel approach to lattice QCD at finite baryon density

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in Collaboration with:

partially based on JHEP 05 (2020) 088; Giordano, Kapas, Katz, Nogradi, Pasztor
The conjectured phase diagram of QCD

- QGP
- Heavy Ion Collisions
-早宇宙
-化学的凍結
-越界
-重粒子衝突
-温度
-核子密度 / 化学ポテンシャル / 掺入
-中性子星
-中性子星の合体
-核崩壊超新星

Temperature
Early universe
Crossover
Hadron gas
Chemical freeze-out

Baryon density / chemical potential / doping
Neutron stars
Neutron star mergers
Core collapse supernovae

Heavy Ion Collisions
$E_{CM}$ increases
In addition to the sign problem, known approaches to finite density QCD suffer from additional serious problems. E.g.

- Taylor and imaginary $\mu$: **analytic continuation problem**
- Reweighting and Taylor: **overlap problem**
- Complex Langevin: **convergence issues**
- ... 

These problems are just as or even more crippling than the sign problem. This talk:

\[ \rightarrow \text{a method where the only problem is the sign problem} \]

If the sign problem is dealt with by sufficient statistics, the results are reliable, and errors (on a fixed lattice) are statistical only.
Reweighting in general

Target theory: $Z_w$  
Simulated theory: $Z_r$

$$Z_w = \int \mathcal{D} U \ w(U) \quad w(U) = \text{det}M[U, \mu]e^{-S_g[U]} \in \mathbb{C}$$

$$Z_r = \int \mathcal{D} U \ r(U) \quad r(U) > 0$$

$$\langle O \rangle_w = \frac{\int \mathcal{D} U \ w(U)O(U)}{\int \mathcal{D} U \ w(U)} = \frac{\int \mathcal{D} U \ r(U)\frac{w(U)}{r(U)}O(U)}{\int \mathcal{D} U \ r(U)\frac{w(U)}{r(U)}} = \frac{\langle \frac{w}{r}O \rangle_r}{\langle \frac{w}{r} \rangle_r}$$

Two problems that are exponentially hard in the volume:

- $\frac{w}{r} \in \mathbb{C} \rightarrow \text{sign problem}$
- Tails of $\rho(\frac{w}{r})$ long $\rightarrow \text{overlap problem}$  
  The first bottleneck when reweighting from $\mu = 0$
Why does reweighting from $\mu = 0$ fail?

The expectation value of any observable:

$$\langle O \rangle_w = \frac{\langle \frac{w}{r} O \rangle_r}{\langle \frac{w}{r} \rangle_r}$$

The weights are the $w/r$, to calculate anything, we need to have control over this observable.

The sign problem is under control, the overlap problem is not:
Giordano, Kapas, Katz, Nogradi, Pasztor; PRD 102, 034503 (2020)
Sign reweighting

\[ Z = \int DUE^{-S_g} \det M = \int DUE^{-S_g} \text{Re} \det M \]

- Beware: the substitution \( \det M \to \text{Re} \det M \) can be done in \( Z \) but not in generic expectation values.
- Can calculate e.g. \( \frac{\partial^n \log Z}{\partial \mu_{ud}^n} \), \( \frac{\partial^n \log Z}{\partial m_{ud}^n} \) and \( \frac{\partial^n \log Z}{\partial \beta^n} \)

A new choice of a theory to reweight to and from:

\[ w = e^{-S_g} \text{Re} \det M \]
\[ r = e^{-S_g} |\text{Re} \det M| \]

\[ \Rightarrow \frac{w}{r} \equiv \epsilon = \pm 1 \]

- The weights are \( \epsilon = \pm 1 \) \to No tail, \textbf{no overlap problem}
- \( \langle \pm \rangle_r \) measures the strength of the \textbf{sign problem}

Early mentions of the idea:

Alexandru, Faber, Horvath, Liu: hep-lat/0507020; PRD72 114513
First test of the new method - unimproved staggered $N_\tau = 4$

Strength of the sign problem at $T_c(\mu)$

Giordano, Kapas, Katz, Nogradi, Pasztor; JHEP 05 (2020) 088

For simplicity we take $\mu_s = 0$ and $\mu_u = \mu_d = \mu_B/3$
First test of the new method - unimproved staggered $N_T = 4$

Finite volume scaling at $\mu_B/T = 2.4$

Consistent with Fodor, Katz; JHEP 04 (2004) 050. BUT: to start being relevant for phenomenology, a much better lattice action has to be used
Second test - 2stout $N_T = 6$ (PRELIMINARY)

\[ \langle \bar{\psi} \psi \rangle_R = (\langle \bar{\psi} \psi \rangle_{0, 0} - \langle \bar{\psi} \psi \rangle_{T, \mu}) \frac{m_{ud}}{f_\pi^4} \]

2stout, $16^3 \times 6$

No sign of the transition getting stronger
\[ \langle \bar{\psi}\psi \rangle_R = (\langle \bar{\psi}\psi \rangle_{0,0} - \langle \bar{\psi}\psi \rangle_{T,\mu}) \frac{m_{ud}}{f_\pi^4} \]

\[ (\mu_B/T)^2 \leq 0 \text{ data} \]
\[ (\mu_B/T)^2 > 0 \text{ data} \]

Polynomial fit to \(-10 < (\mu_B/T)^2 \leq 0\)

2stout, 16^3 \times 6
PRELIMINARY
• Current methods to study finite density QCD are typically not bottlenecked by the sign problem itself
• In particular reweighting from $\mu = 0$ is bottlenecked by the overlap problem
• We proposed a new reweighting method that is free from the overlap problem in the weights and is therefore only bottlenecked by the sign problem itself
• First test: CEP for unimproved staggered at $N_\tau = 4$, expected to be dominated by cut-off effects
• Second test: 2stout at $N_\tau = 6$; preliminary
  • width of transition at $\mu_B/T = 1.5 \approx$ width at 0
  • so far matches analytic continuation from imaginary $\mu$