Quantum Anomalies in Matter

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Upcoming review with M. Chernodub, Y. Ferreiros, A. Grushin, M. Vozmediano,
and
K.L., Y. Ferreiros, Phys. Lett. B816, 136419

Virtual Tribute to Quark Confinement and the Hadron Spectrum

06 August, 2021

Outline

- Chiral Anomalies
- Anomaly induced transport
  - QGP
  - Weyl semimetals
- Torsion
- Conclusions

Chiral Anomalies

$$\langle D_\mu J^\mu_a \rangle = \epsilon^{\mu\nu\rho\lambda} \left(\frac{d_{abc}}{32\pi^2} F^b_{\mu\nu} F^c_{\rho\lambda} + \frac{b_a}{768\pi^2} R^{\alpha}{}_{\beta\mu\nu} R^{\beta}{}_{\alpha\rho\lambda} \right)$$

$$\langle D_\mu T^{\mu\alpha} \rangle = F^{\alpha\mu}_a J^a_\mu + \epsilon^{\mu\nu\rho\lambda} \frac{b_a}{384\pi^2} D_\beta \left(F^a_{\mu\nu} R^{\alpha\beta}{}_{\rho\lambda} \right)$$

$$J^\mu_a = \bar{\psi} Q_a \gamma^\mu \psi$$

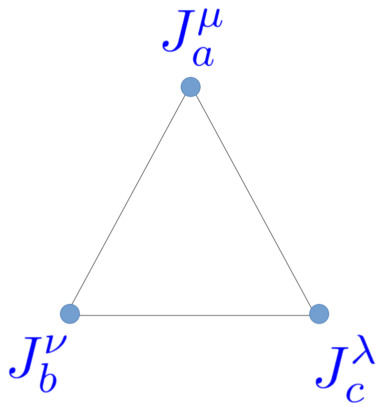
e.g.

$$J^\mu_5 = \bar{\psi} \gamma_5 \gamma^\mu \psi$$

- **Operators** in quantum theory of fermions
- **State independent**
- **Topological**: independent of coupling constants
- Completely determined by **anomaly coefficients**

$$d_{abc} = \sum_r (q_a q_b q_c) - \sum_l (q_a q_b q_c)$$

$$b_a = \sum_r (q_a) - \sum_l (q_a)$$



[Adler], [Bell, Jackiw], [Kimura] 1969

Anomalous Transport Theory

$$\langle \vec{J}_a \rangle = \frac{d_{abc}}{4\pi^2}\mu_b \vec{B}_c + \left(\frac{d_{abc}}{4\pi^2}\mu_b\mu_c + \frac{b_a}{12}T^2\right)\vec{\omega}$$

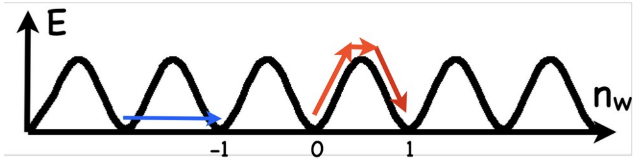
$$\langle \vec{J}_\epsilon \rangle = \left(\frac{d_{abc}}{8\pi^2}\mu_b\mu_c + \frac{b_a}{24}T^2\right)\vec{B}_a + \left(\frac{d_{abc}}{6\pi^2}\mu_a\mu_b\mu_c + \frac{b_a}{6}\mu_a T^2\right)\vec{\omega}$$

- **State dependent:** T, μ
- **Test for anomalies** without external $F_{\mu\nu}$ or $R_{\alpha\beta\mu\nu}$! (*"Matter" vs "Fields"*)
- Formally in **equilibrium**
- **Dissipationless** (no entropy production)

[Vilenkin], [Kharzeev],[Son, Surowka], [K.L., Megías, Pena-Benitez], [Neiman, Oz],[Jensen, Loganayagam, Yarom], [Stone, Kim]...

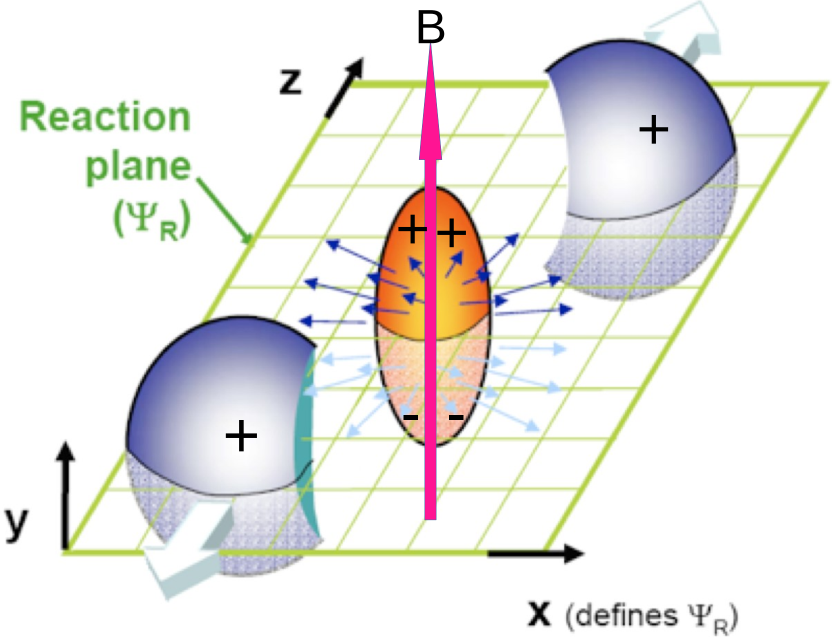
CME @ HIC

Quark Gluon Plasma:



$$\partial_\mu J^\mu_A = \frac{N_f}{32\pi^2}\epsilon^{\mu\nu\rho\lambda}G^a_{\mu\nu}G^a_{\rho\lambda}$$

QCD out of equilibrium topological gluon field configurations



Axial anomaly (QED):

$$\partial_\mu J^\mu_5 = \frac{N_f N_c}{16\pi^2}\epsilon^{\mu\nu\rho\lambda}F_{\mu\nu}F_{\rho\lambda}$$

CME:

$$\vec{J} = \frac{N_f N_c}{2\pi^2}\mu_5 \vec{B}$$

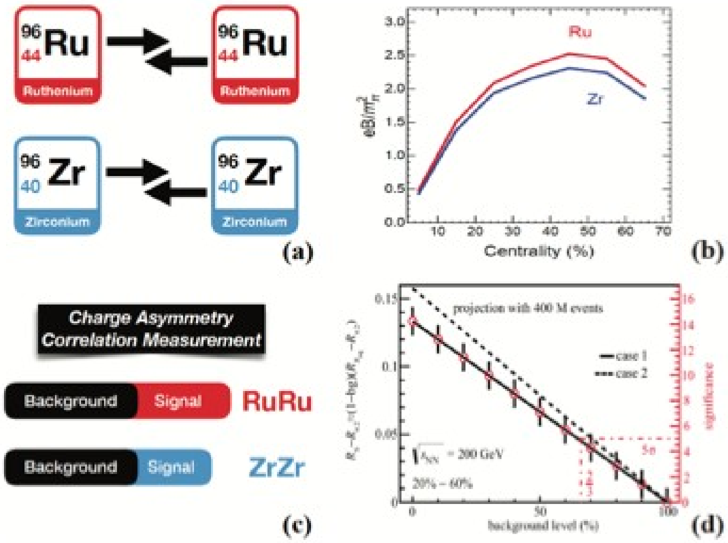
[Kharzeev, McLarren, Warringa]

CME @ HIC

Huge effort to get experimental grip on CME: new methods (“event shape engeneering”), new improved correlators, ...

Most important: Isobar run @ RHIC in 2018

Expect ~20% higher CME signal in Ru



Isobar Collisions at RHIC to Test Local Parity Violation in Strong Interactions

D. E. Kharzeev & J. Liao

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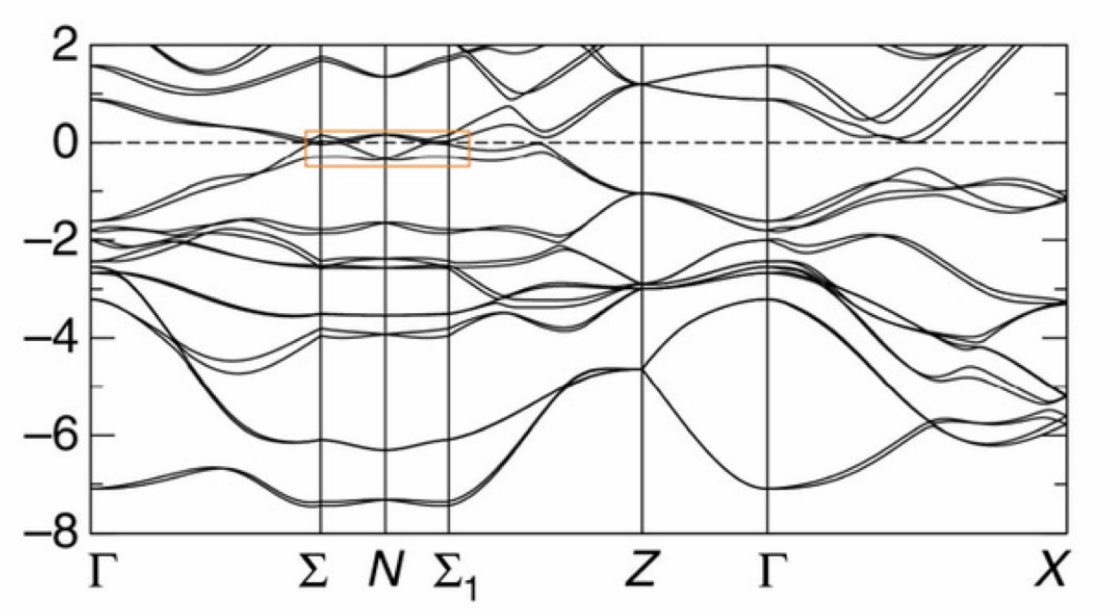
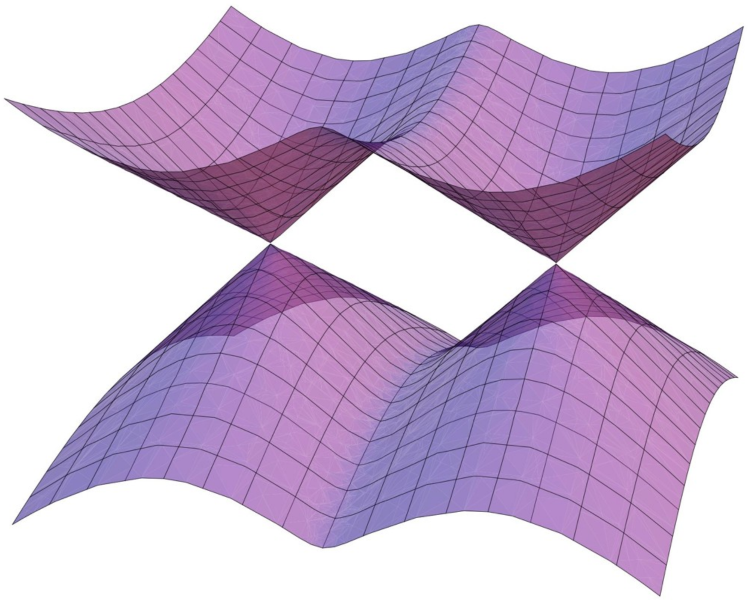
Results expected to be (finally) out this year!

Weyl semimetals



TaAs



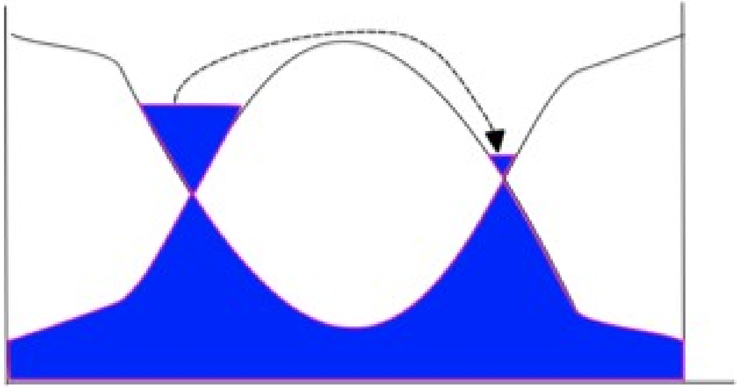
[Nielsen, Ninomiya]

Weyl semimetals

NMR and Magneto-thermal transport:

Hierarchy $\tau_{\mathrm{relax}} < \tau_{inter-valley} < \tau_{ee}$

„Schmutzphysik"



“Inter-valley scattering”

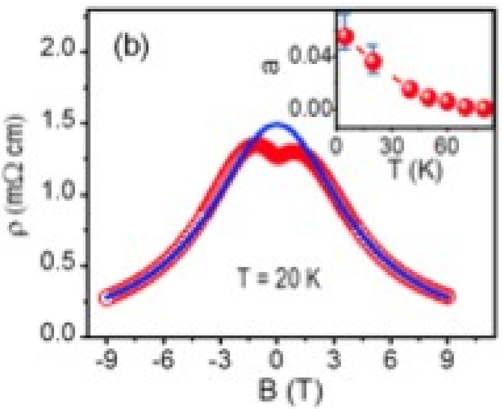
In each single Weyl cone:

$$(-i\omega + \frac{1}{\tau})\delta\rho + \vec{\nabla}\vec{j} = \frac{1}{4\pi^2}\vec{E}.\vec{B}$$
$$(-i\omega + \frac{1}{\tau})\delta\epsilon + \vec{\nabla}\vec{j}_\epsilon = 0$$

Gradients of CME currents for right- and left-
Handed fermions and anomaly

Weyl semimetals

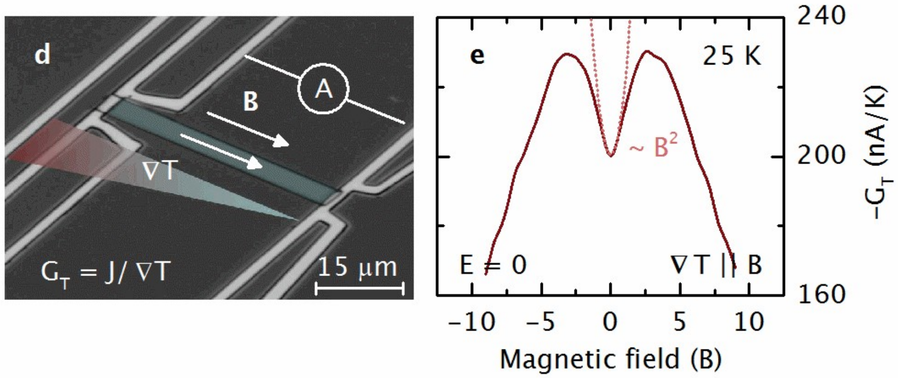
Experimental results



nature physics

[Li, Kharzeev, et al.]

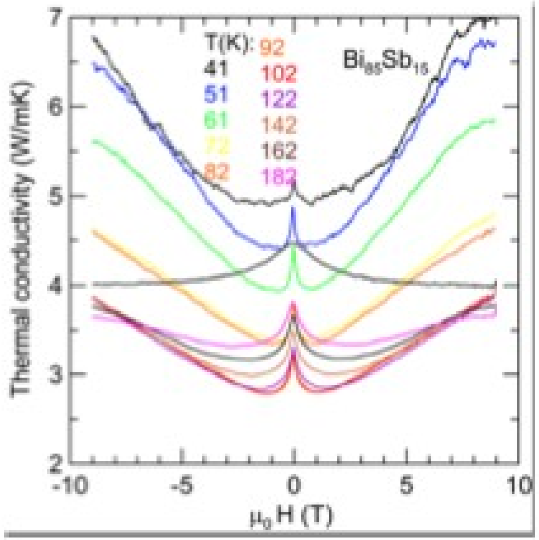
Electric conductivity



nature

[Gooth, et al.]

Thermo-electric



nature materials

[Vu, Zhang, Şahin, Flatté, Trivedi, Heremans]

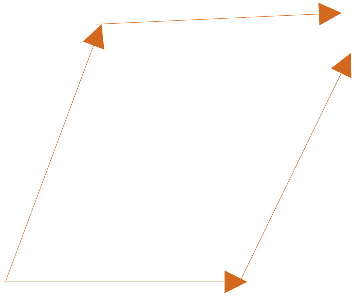
Thermal conductivity

Wiedemann-Franz law

Torsion

- Curvature: Failure of a vector to come back to itself upon parallel transport along a closed curve
- Torsion: Failure of a parallelogram spanned by two vectors to form a closed curve

$$[\nabla_\mu, \nabla_\mu] v_\lambda = R^\sigma_{\lambda\mu\nu} v_\sigma + \theta^\sigma_{\mu\nu} \nabla_\sigma v_\lambda$$



$$\Gamma^\lambda_{\mu\nu} - \Gamma^\lambda_{\nu\mu} = \theta^\lambda_{\mu\nu}$$ Antisymmetric part of connection

Vielbein formalism:

$$De^a = de^a + \omega^a{}_b e^b = \theta^a$$ Torsion

$$d\omega^a{}_b + \omega^a{}_c \wedge \omega^c{}_b = R^a{}_b$$ Curvature

$$e^a = e^a_\mu dx^\mu$$ Tensor

$$\omega^a{}_b = \omega^a_{\mu\ b} dx^\mu$$ Spin Connection

$$\Gamma^\lambda_{\mu\nu} = \frac{1}{2} g^{\lambda\rho} P^{\alpha\beta\gamma}_{\mu\lambda\nu} \left(\partial_\alpha g_{\beta\gamma} - \theta^\sigma_{\alpha\beta} g_{\sigma\gamma} \right)$$ Connection

Torsion

Nieh-Yan term $\theta^a \wedge \theta_a - R_{ab} \wedge e^a \wedge e^b = d(\underbrace{e^a \wedge \theta_a})$

[Nieh, Yan] Ann. Phys. 1982

Well defined tensor!

Torsional contribution to axial anomaly?

$$\delta\Gamma = \Lambda^2 \int \lambda_5 (\theta^a \wedge \theta_b - R_{ab} \wedge e^a \wedge e^b)$$

[Obhukov] 1982
[Bañados, Teitelboim, Zanelli] 1994
[Chandia, Zanelli] 1995
[Kreimer, Mielke] 1999

- Nieh-Yan term has wrong dimension
- Cutoff dependent
- Structure of a mixed anomaly
- Removed by counterterm $\Gamma_{ct} = \Lambda^2 \int A \wedge e^a \wedge \theta_a$

Not a usual anomaly in QFT sense

Still possible in Cond-mat ?

[Hughes, Lee, Parrikar] 2012
[Ferreiros, Kedem, Bergholtz, Bardason] 2018

Compare to $F_V \wedge F_V = d(V \wedge F_V)$

Torsion

Even if its not an anomaly, is there chiral torsional transport?

[Volovik] [Volovik, Nissinen], [Zubkov]
[Huang, Han, Stone] , [Huang, Han]
[Imaki, Yamamoto], [Imaki, Qiu]

Idea: instead of UV scale use an IR scale $T, \mu \longrightarrow J_5^\mu \propto T^2 \epsilon^{\mu\nu\rho\lambda} \theta^a_{\nu\rho} e^a_\lambda$?

Strategy: all possible T-invariant terms + Kubo formulas

[Kharzeev, Yee]

	$A_{(5)}$	e^a	ω^{ab}	d	$J_{(5)}$	S_{ab}	u_a	μ	T	$\star$
$\mathcal{T}$:	-1	$+1$	$+1$	$+1$	$+1$	-1	-1	$+1$	$+1$	-1

TABLE I. Action of time reversal

$$J = \rho * u + c_V u \wedge du + c_T^{\parallel} u_a u_b \theta^a \wedge e^b + c_T^{\perp} \mathcal{P}^{\perp}_{ab} \theta^a \wedge e^b$$

$$\mathcal{P}^{\perp}_{ab} = \eta_{ab} - u_a u_b$$

Unique solution: $c_V = \pm \left(\frac{\mu^2}{4\pi^2} + \frac{T^2}{12} \right)$, $c_T^{\parallel} = 0$, $c_T^{\perp} = 0$

Summary

- Anomalies manifest in Matter (T,μ)
- Measurable transport
- Heavy Ion Collisions
- Topological Metals
- Cosmology

THANKS!