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A novel application of Spherical Elementary Currents with Ground Magnetometers – Analysis of By effects on the auroral electrojets

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 This presentation is based on work within my recent masters thesis: <u>https://bora.uib.no/bora-</u> <u>xmlui/handle/11250/2716829</u>

 Code written for modelling ionospheric currents using SECS is available in a public Github repository: <u>https://github.com/08walkersj/SECpy</u>



Spherical Elementary Currents (SECS)

- This study utilises **divergence-free** (df) SECS to model the divergence-free currents that produce the magnetic perturbations seen on the ground.
- Due to the longivity of data coverage and density of magnetometers, Fennoscandia was the chosen region for this study



Abisko(ABK) Andenes(AND) Bear Island(BJN) Brorfelde(BFE) Hankasalmi(HAN) Hel(HLP) Hornsund(HRN) Kevo(KEV) Kilpisiarvi(KIL) Longyearbyen(LYR) Muonio(MUO) New Aalesund(NAL) Nurmajarvi(NUR) Oulujarvi(OUJ) Pello(PEL) Roervik(RVK) Sodankvla(SOD) Soeroeya(SOR) Tromso(TRO) Uppsala(UPS)

 $\theta'=0$ (pole)



DF SECS

- A divergence-free spherical elementary current system (SECS) consists of a pole with a series of closed circulating currents (J_{df}) that decrease in strength with increasing co-latitude from the pole
- This system can then be scaled by an amplitude (I₀), adjusting the amplitude of the surrounding currents.

$$\mathbf{J}_{df}(\mathbf{r}') = \frac{I_{0,df}}{4\pi R_I} cot(\frac{\theta'}{2}) \hat{\mathbf{e}}_{\phi'}$$

 R_{I} : Radial position of the SECS pole (Usually the ionospheric current layer) Θ' : Co-latitude of the point of evaluation of the current from the SECS pole



Divergence-free elementary system

Vanhamäki, H., & Juusola, L. (2020). Introduction to Spherical Elementary Current Systems. In *Ionospheric Multi-Spacecraft Analysis Tools* (pp. 5–33). Springer International Publishing. https://doi.org/10.1007/978-3-030-26732-2_2



DF SECS

The **magnetic field** from the **divergence-free currents** surrounding a **SECS pole** can be found, again in terms of the pole:

$$\mathbf{B}_{r}^{DF}(r,\theta',\phi') = \frac{\mu_{0}I_{0,df}}{4\pi r_{m}} \begin{cases} \frac{1}{\sqrt{1 + (\frac{r_{m}}{R_{I}})^{2} - 2(\frac{r_{m}}{R_{I}})cos(\theta')}} - 1, & r_{m} < R_{I} \\ \frac{\frac{R_{I}}{r_{m}}}{\sqrt{1 + (\frac{R_{I}}{r_{m}})^{2} - 2(\frac{R_{I}}{r_{m}})cos(\theta')}} - \frac{R_{I}}{r_{m}}, & r_{m} > R_{I} \end{cases}$$

$$\mathbf{B}_{\theta}^{DF}(r,\theta',\phi') = \frac{-\mu_0 I_{0,df}}{4\pi r_m sin(\theta')} \begin{cases} \frac{\frac{r_m}{R_I} - cos(\theta')}{\sqrt{1 + (\frac{r_m}{R_I})^2 - 2(\frac{r_m}{R_I})cos(\theta')}} + cos(\theta'), & r_m < R_I \\ \frac{1 - \frac{R_I}{r_m}cos(\theta')}{\sqrt{1 + (\frac{R_I}{r_m})^2 - 2(\frac{R_I}{r_m})cos(\theta')}} - 1, & r_m > R_I \end{cases}$$

The magnetic field is scaled by the amplitude (I_0) the same as the currents.

r_m: radial position of the evaluation of the magnetic field



Divergence-free elementary system

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Superposition of SECS Poles



 $\theta'=0$ (pole)

Divergence-free elementary system

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- Using **multiple SECS poles** and the **superposition** of their currents, a complex current structure can be created by **varying the amplitudes** of each pole.
- The superposition can also be applied to the magnetic field of the SECS poles.
- This allows a current system created by the superposition of SECS poles to be constrained by magnetic field measurements
 - 1. The currents and magnetic field of each pole must be converted into a global system (e.g. Geodetic)
 - 2. The SEC pole amplitudes must be **related** to the **magnetic field** perturbations **measured** on the ground
 - 3. The inversion must be **encouraged** towards a solution that creates **currents** that are **justified** by expectations from **theory**



Conversion from Local to Global System



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$$\cos(\theta') = \cos(\theta_k)\cos(\theta^{el}) + \sin(\theta_k)\sin(\theta^{el})\cos(\phi^{el} - \phi_k)$$

$$\cos(C) = \frac{\cos(\theta^{el}) - \cos(\theta)\cos(\theta')}{\sin(\theta)\sin(\theta')}$$

$$\sin(C) = \frac{\sin(\theta^{el})\sin(\phi^{el} - \phi)}{\sin(\theta')}$$

$$\hat{\mathbf{e}}_{\theta'} = \hat{\mathbf{e}}_{\theta}\cos(C) - \hat{\mathbf{e}}_{\phi}\sin(C)$$

$$\hat{\mathbf{e}}_{\phi'} = \hat{\mathbf{e}}_{\theta}\sin(C) + \hat{\mathbf{e}}_{\phi}\cos(C)$$

 θ^{el} and φ^{el} are the global co-ordinates of the SEC pole, θ_k and φ_k are the global co-ordinates of the evaluated point and θ' is the colatitude of the evaluated point in terms of the SECS system co-ordinates



Inversion

- The SECS pole amplitudes can be factorised out of the equations
- The remaining variables describe the geometry of the system
- The remaining variables are evaluated at the **location of the magnetometers** and **converted to a global system** (converting from being in a system relative to each SECS pole into a geographic system)
- This allows the problem to be set up to be described in terms of **matrices**

$G\mathbf{m} = \mathbf{d}$

 Ge_{11} Ge_{12} Ge_{1p} Ge_{22} Ge_{21} Ge_{2p} $Ge_{m2} \implies$ Ge_{m1} Ge_{mp} Gn_{12} Gn_{1p} Gn_{11} \implies Gn_{2p} Gn_{21} Gr_{22} G = Gn_{m2} Gn_{mp} \implies Gr_{11} Gr_{12} Gr_{1p} $Gr_{22} \implies$ Gr_{21} Gr_{2p}

Gx_{mp} contains the geometric information of magnetometer **m** in terms of pole **p** that will scale the effect of the magnetic perurtabation measured in the direction **x** on the amplitude of **p**

- Matrix G contains the geometry of the system (converted to produce geographic vectors)
- Vector m contains the amplitudes of each SECS pole (I₀ for each pole)
- d contains the measurements at each magnetometer station

By finding the inverse of G, the SECS pole amplitudes can be found. Therefore, the corresponding divergencefree ionospheric current that can produce the perturbations seen can be found



Helping the Inversion

- The nature of this type of modelling is that we have **more unknowns than knowns**
- This means that there are many current systems that could create the magnetic perturbations we see on the ground and, therefore, many combinations of SECS pole amplitudes
- The goal is to pick the combination of amplitudes that create a current that is closest to the **expected behaviour**
- Most studies use **truncated singular value decomposition**, this method will use an adjustable condition which controls how structured the currents will be
- Ground magnetometers are long distance from the ionospheric currents (typically 110km in altitude) and have a large seperation. This means that they are **unlikely to be able see small scale current structures**.
- Encouraging currents to be less structured is, therefore, justifiable

This study uses **regularisation** that both encourages the solution to be **less structured** but also **discourages variations in the magnetic east-west direction**



Regularisation

• Inversions rely on a **cost function**, f_0 , that must minimised to find the best solution.

 $f_0 = (\mathbf{d} - G\mathbf{m})^T (\mathbf{d} - G\mathbf{m})$

- This standard cost function will minimise the **difference** between the **data** and the **model** (therefore finding the solution that matches the data most closesly)
- On its own this will create **highly structured currents** in order match the magnetic perturbations as closely as possible

 $f = f_0 + \lambda_1 ||I\mathbf{m}||_2 + \lambda_2 ||L_e\mathbf{m}||_2$

- This new cost function, *f*, contains the original cost function but adds two extra parameters to be minimised:
- $\lambda_1 ||Im||_2$ Has a scaling parameter λ_1 that controls how much it should be minimised. $||Im||_2$ is the euclidean norm of the pole amplitudes.
- By increasing λ₁, variations in the amplitudes over small spatial distances are discourage (reducing the overall structure of the currents)

Has a scaling parameter λ_2 . $||Lem||_2$ is the euclidean norm of the gradient of the pole amplitudes in the magnetic east-west direction.

 By increasing λ₂, variations in the amplitudes in east to west are discouraged (encouraging smooth currents in the magnetic east-west direction)





- This is a **substorm** occuring within the region of analysis.
- The currents in the magnetic east and north direction and radial component of the magnetic field (B_r) are evaluated along a magnetic meridian of 105° mlon for each minute when all chosen magnetometers are available which covers the period 2000-2019.



Using the magnetic meridian **all MLTs are scanned** and can produce a **statistical description** of the **divergence-free ionospheric currents** in the northern hemisphere

> Average Current Structure Winter Bz>0







Example Profiles for Electrojet Detection

- From each minute of meridian currents, information on the auroral electrojets are extracted
- The electrojets are identified using a combination of **profile peak**, the **quantiles** and **gradients** within the profile.



Properties found

The **three strongest** (indentified by peak sheet current value) **westward** and three strongest **eastward** electrojets in each profile have the following **properties** recorded:

• Boundaries:

- The magnetic latitude of the **poleward** and **equatorward** boundaries of each electrojet are found using the electrojet detection algorithm

- Units: ^omlat
- Width:
 - The width of each electrojet in magnetic latitude is recorded by finding the **difference** in the poleward
 - and equatorward boundaries
 - Units: ^omlat
- Peak:
 - The peak of the profile between the boundaries (peak sheet current density) is found for each electrojet
 - Units: Am⁻¹
- Total Current
 - The total current of each electrojet is found by **integrating the profile** between the boundaries
 - Units: ^omlatAm⁻¹ (Is converted to just Amps when plotted)



IMF By Dependence of the Eastward Electrojet Width between 18-21 MLT in the Summer





IMF By Dependence of the Eastward Electrojet Width between 18-21 MLT in the Winter













- There is a clear **By trend** for both electrojets during the **Summer** but **not during the Winter**
- To investigate this further the sheet current density profiles, that contribute to the properties being analysed, are plotted.



















<u>Results</u>

- In the profile plots the **poleward edge** of the electrojet **shifts** due to $\mathbf{B}_{\mathbf{y}}$ during the **Summer**
- This trend is **not** visible during the **Winter**.
- This **B**_v trend is **opposite** for the **eastward** and **westward** electrojets
- This leads to the idea that it may be a result of **convection patterns**
- The **seasonal trend** suggests the influence of **lobe reconection**
- A process that is more **dominant during the Summer** (+ve dipole tilt)



