Hadronic contribution to $(g-2)_\mu$ in the Standard Model: data-driven approach

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FOR FUNDAMENTAL PHYSICS

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Outline

Introduction: $(g-2)_\mu$ in the Standard Model

Hadronic Vacuum Polarization contribution to $(g-2)_\mu$

Hadronic light-by-light contribution to $(g-2)_\mu$

Conclusions and Outlook

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White Paper (2020): $(g-2)_\mu$, experiment vs SM

Contribution	Value $\times 10^{11}$
HVP LO (e^+e^-)	6931(40)
HVP NLO (e^+e^-)	$-98.3(7)$
HVP NNLO (e^+e^-)	12.4(1)
HVP LO (lattice , *udsc*)	7116(184)
HLbL (phenomenology)	92(19)
HLbL NLO (phenomenology)	2(1)
HLbL (lattice, *uds*)	79(35)
HLbL (phenomenology + lattice)	90(17)
QED	116 584 718.931(104)
Electroweak	153.6(1.0)
HVP (e^+e^-, LO + NLO + NNLO)	6845(40)
HLbL (phenomenology + lattice + NLO)	92(18)
Total SM Value	116 591 810(43)
Experiment	116 592 061(41)
Difference: $\Delta a_\mu := a_\mu^{\text{exp}} - a_\mu^{\text{SM}}$	252(59)

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White Paper:
T. Aoyama et al. Phys. Rep. 887 (2020) = WP(20)

Muon $g-2$ **Theory Initiative**
Steering Committee:
GC
Michel Davier
Simon Eidelman
Aida El-Khadra (co-chair)
Martin Hoferichter
Christoph Lehner (co-chair)
Tsutomu Mibe (J-PARC E34 experiment)
(*Andreas Nyffeler* until summer 2020)
Lee Roberts (Fermilab E989 experiment)
Thomas Teubner
Hartmut Wittig

White Paper (2020): $(g-2)_\mu$, experiment vs SM

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Muon $g-2$ Theory Initiative
Workshops:

- First plenary meeting, Q-Center (Fermilab), 3-6 June 2017
- HVP WG workshop, KEK (Japan), 12-14 February 2018
- HLbL WG workshop, U. of Connecticut, 12-14 March 2018
- Second plenary meeting, Mainz, 18-22 June 2018
- Third plenary meeting, Seattle, 9-13 September 2019
- Lattice HVP workshop, virtual, 16-20 November 2020
- Fourth plenary meeting, KEK (virtual), 28 June-02 July 2021

White Paper executive summary (my own)

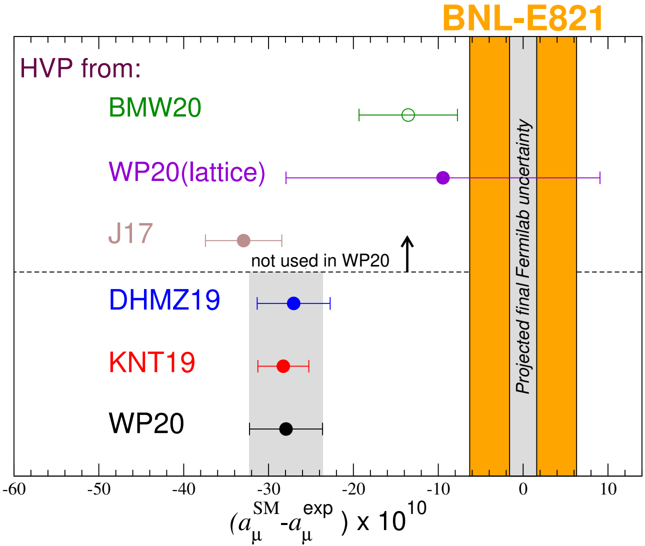
- QED and EW known and stable, negligible uncertainties
- HVP dispersive: consensus number, conservative uncertainty (KNT19, DHMZ19, CHS19, HHK19)
- HVP lattice: consensus number, $\Delta a_\mu^{\rm HVP,latt} \sim 5\,\Delta a_\mu^{\rm HVP,disp}$ (Fermilab-HPQCD-MILC18,20, BMW18, RBC/UKQCD18, ETM19,SK19, Mainz19, ABTGJP20)
- HVP BMW20: central value $\to$ discrepancy $< 2\sigma$; $\Delta a_\mu^{\rm HVP,BMW} \sim \Delta a_\mu^{\rm HVP,disp}$ just published $\to$ not in WP
- HLbL dispersive: consensus number, w/ recent improvements $\Rightarrow$ $\Delta a_\mu^{\rm HLbL} \sim 0.5\,\Delta a_\mu^{\rm HVP}$
- HLbL lattice: single calculation, agrees with dispersive ($\Delta a_\mu^{\rm HLbL,latt} \sim 2\,\Delta a_\mu^{\rm HLbL,disp}$) $\to$ final average (RBC/UKQCD20)

Status of $(g-2)_\mu$, experiment vs SM

$$a_\mu(BNL) = 116\,592\,089(63) \times 10^{-11}$$
$$a_\mu(FNAL) = 116\,592\,040(54) \times 10^{-11}$$
$$a_\mu(Exp) = 116\,592\,061(41) \times 10^{-11}$$

Status of $(g-2)_\mu$, experiment vs SM

Before the Fermilab result



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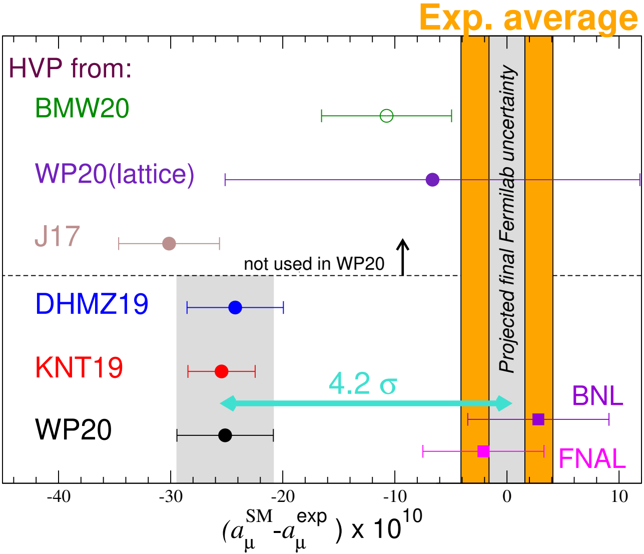
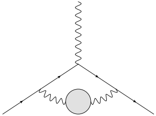
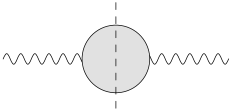


Theory uncertainty comes from hadronic physics

- Hadronic contributions responsible for most of the theory uncertainty
- Hadronic vacuum polarization (HVP) is $\mathcal{O}(\alpha^2)$, dominates the total uncertainty, despite being known to $< 1\%$



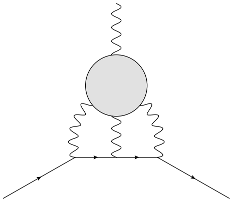


  - unitarity and analyticity $\Rightarrow$ dispersive approach
  - $\Rightarrow$ direct relation to experiment: $\sigma_{\text{tot}}(e^+e^- \to \text{hadrons})$
  - e^+e^- Exps: BaBar, Belle, BESIII, CMD2/3, KLOE2, SND
  - alternative approach: lattice, becoming competitive

(BMW, ETMC, Fermilab, HPQCD, Mainz, MILC, RBC/UKQCD)

Theory uncertainty comes from hadronic physics

- Hadronic contributions responsible for most of the theory uncertainty
- Hadronic vacuum polarization (HVP) is $\mathcal{O}(\alpha^2)$, dominates the total uncertainty, despite being known to $< 1\%$
- Hadronic light-by-light (HLbL) is $\mathcal{O}(\alpha^3)$, known to $\sim 20\%$, second largest uncertainty (now subdominant)



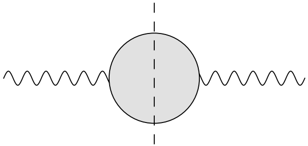
- earlier: "*it cannot be expressed in terms of measurable quantities*"
- recently: dispersive approach $\Rightarrow$ data-driven, systematic treatment
- lattice QCD is becoming competitive

(Mainz, RBC/UKQCD)

Outline

HVP contribution: Master Formula

Unitarity relation: simple, same for all intermediate states



$$\mathrm{Im}\Pi(q^2) \propto \sigma(e^+e^- \to \mathrm{hadrons}) = \sigma(e^+e^- \to \mu^+\mu^-)R(q^2)$$

Analyticity $\Rightarrow$ Master formula for HVP

Bouchiat, Michel (61)

$$a_\mu^{\mathrm{hvp}} = \frac{\alpha^2}{3\pi^2}\int_{s_{th}}^{\infty}\frac{ds}{s}K(s)R(s)$$

$K(s)$ known, depends on m_μ and $K(s) \sim \frac{1}{s}$ for large s

Comparison between DHMZ19 and KNT19

	DHMZ19	KNT19	Difference
$\pi^+\pi^-$	507.85(0.83)(3.23)(0.55)	504.23(1.90)	3.62
$\pi^+\pi^-\pi^0$	46.21(0.40)(1.10)(0.86)	46.63(94)	-0.42
$\pi^+\pi^-\pi^+\pi^-$	13.68(0.03)(0.27)(0.14)	13.99(19)	-0.31
$\pi^+\pi^-\pi^0\pi^0$	18.03(0.06)(0.48)(0.26)	18.15(74)	-0.12
K^+K^-	23.08(0.20)(0.33)(0.21)	23.00(22)	0.08
K_SK_L	12.82(0.06)(0.18)(0.15)	13.04(19)	-0.22
$\pi^0\gamma$	4.41(0.06)(0.04)(0.07)	4.58(10)	-0.17
Sum of the above	626.08(0.95)(3.48)(1.47)	623.62(2.27)	2.46
$[1.8, 3.7]$ GeV (without $c\bar{c}$)	33.45(71)	34.45(56)	-1.00
J/ψ, $\psi(2S)$	7.76(12)	7.84(19)	-0.08
$[3.7, \infty)$ GeV	17.15(31)	16.95(19)	0.20
Total $a_\mu^{\text{HVP, LO}}$	$694.0(1.0)(3.5)(1.6)(0.1)_\psi(0.7)_{\text{DV+QCD}}$	692.8(2.4)	1.2

2π: comparison with the dispersive approach

The 2π channel can itself be described dispersively $\Rightarrow$ more constrained theoretically Ananthanarayan, Caprini, Das (19), GC, Hoferichter, Stoffer (18)

Energy range	ACD18	CHS18	DHMZ19	KNT19
$\leq 0.6\,\text{GeV}$		110.1(9)	110.4(4)(5)	108.7(9)
$\leq 0.7\,\text{GeV}$		214.8(1.7)	214.7(0.8)(1.1)	213.1(1.2)
$\leq 0.8\,\text{GeV}$		413.2(2.3)	414.4(1.5)(2.3)	412.0(1.7)
$\leq 0.9\,\text{GeV}$		479.8(2.6)	481.9(1.8)(2.9)	478.5(1.8)
$\leq 1.0\,\text{GeV}$		495.0(2.6)	497.4(1.8)(3.1)	493.8(1.9)
$[0.6, 0.7]\,\text{GeV}$		104.7(7)	104.2(5)(5)	104.4(5)
$[0.7, 0.8]\,\text{GeV}$		198.3(9)	199.8(0.9)(1.2)	198.9(7)
$[0.8, 0.9]\,\text{GeV}$		66.6(4)	67.5(4)(6)	66.6(3)
$[0.9, 1.0]\,\text{GeV}$		15.3(1)	15.5(1)(2)	15.3(1)
$\leq 0.63\,\text{GeV}$	132.9(8)	132.8(1.1)	132.9(5)(6)	131.2(1.0)
$[0.6, 0.9]\,\text{GeV}$		369.6(1.7)	371.5(1.5)(2.3)	369.8(1.3)
$\left[\sqrt{0.1}, \sqrt{0.95}\right]\,\text{GeV}$		490.7(2.6)	493.1(1.8)(3.1)	489.5(1.9)

WP(20)

Combination method and final result

Complete analyses DHMZ19 and KNT19, as well as CHS19 (2π) and HHK19 (3π), have been so combined:

- central values are obtained by simple averages (for each channel and mass range)
- the largest experimental and systematic uncertainty of DHMZ and KNT is taken
- $1/2$ difference DHMZ$-$KNT (or BABAR$-$KLOE in the 2π channel, if larger) is added to the uncertainty

Final result:

$$\begin{aligned} a_\mu^{\text{HVP, LO}} &= 693.1(2.8)_{\text{exp}}(2.8)_{\text{sys}}(0.7)_{\text{DV+QCD}} \times 10^{-10} \\ &= 693.1(4.0) \times 10^{-10} \end{aligned}$$

WP(20)

Consequences of the BMW result

A shift in the value of $a_\mu^{\text{HVP, LO}}$ would have consequences:

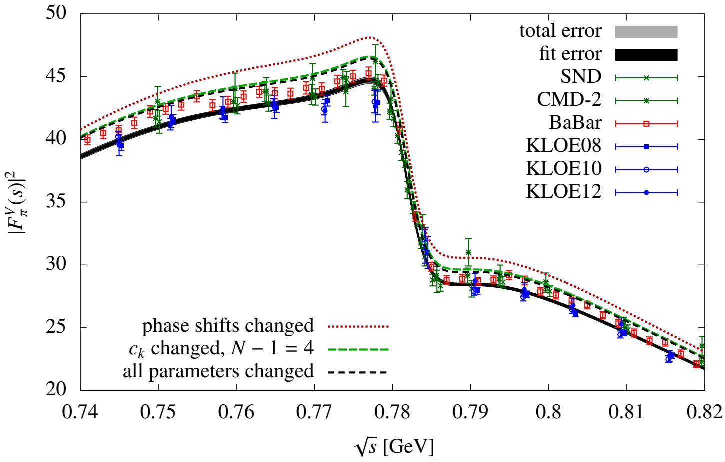
- $\Delta a_\mu^{\text{HVP, LO}} \quad \Leftrightarrow \quad \Delta\sigma(e^+e^- \to \text{hadrons})$
- $\Delta\alpha_{\text{had}}(M_Z^2)$ is determined by an integral of the same $\sigma(e^+e^- \to \text{hadrons})$ (more weight at high energy)
- changing $a_\mu^{\text{HVP, LO}}$ necessarily implies a shift in $\Delta\alpha_{\text{had}}(M_Z^2)$: size depends on the energy range of $\Delta\sigma(e^+e^- \to \text{hadrons})$
- a shift in $\Delta\alpha_{\text{had}}(M_Z^2)$ has an impact on the EW-fit
- to save the EW-fit $\Delta\sigma(e^+e^- \to \text{hadrons})$ must occur below ~ 1 (max 2) GeV

  Crivellin, Hoferichter, Manzari, Montull (20)/Keshavarzi, Marciano, Passera, Sirlin (20)/Malaescu, Schott (20)
- or the need for BSM physics would be moved elsewhere

Changes in $\sigma(e^+e^- \to \text{hadrons})$ below 1 GeV?

- Below $1-2$ GeV only one significant channel: $\pi^+\pi^-$
- Strongly constrained by analyticity and unitarity ($F_\pi^V(s)$)
- $F_\pi^V(s)$ parametrization which satisfies these
  $\Rightarrow$ small number of parameters GC, Hoferichter, Stoffer (18)
- $\Delta a_\mu^{\text{HVP, LO}} \Leftrightarrow$ shifts in these parameters
  analysis of the corresponding scenarios GC, Hoferichter, Stoffer (21)

Changes in $\sigma(e^+e^- \to \text{hadrons})$ below 1 GeV?



GC, Hoferichter, Stoffer (21)

Tension [BMW20 vs e^+e^- data] stronger for KLOE than for BABAR

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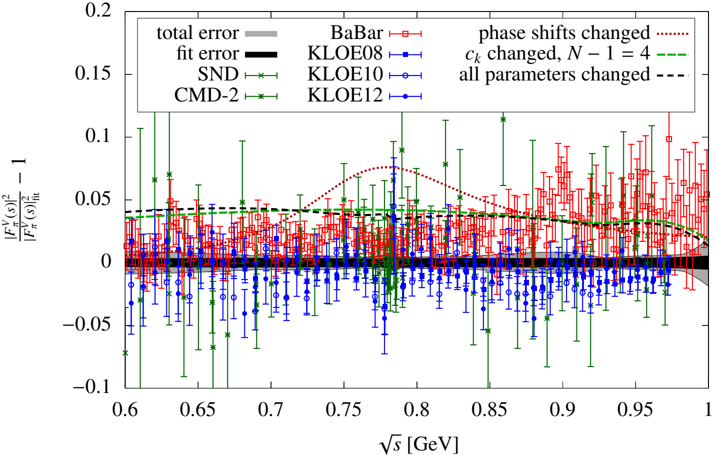

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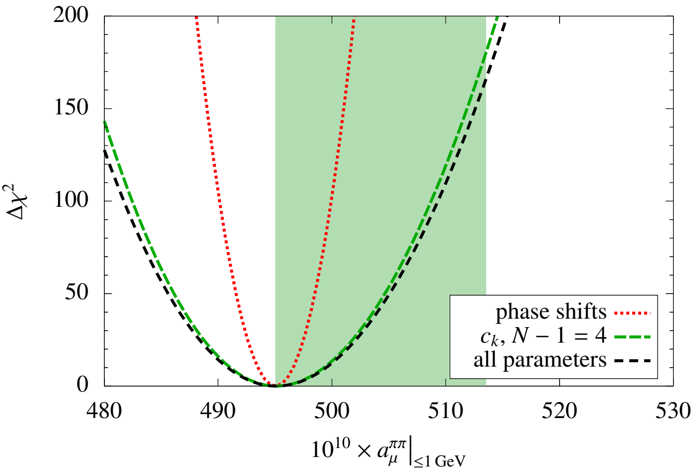

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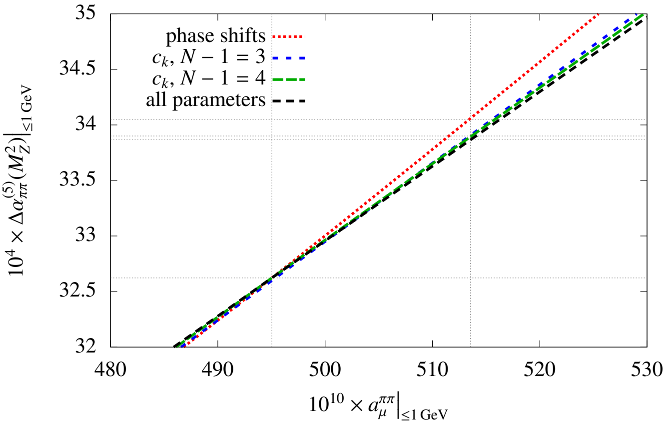

GC, Hoferichter, Stoffer (21)

$$10^4 \Delta\alpha^{(5)}_{\text{had}}(M_Z^2) = \begin{cases} 272.2(4.1) & \text{EW fit} \\ 276.1(1.1) & \sigma_{\text{had}}(s) \end{cases}$$

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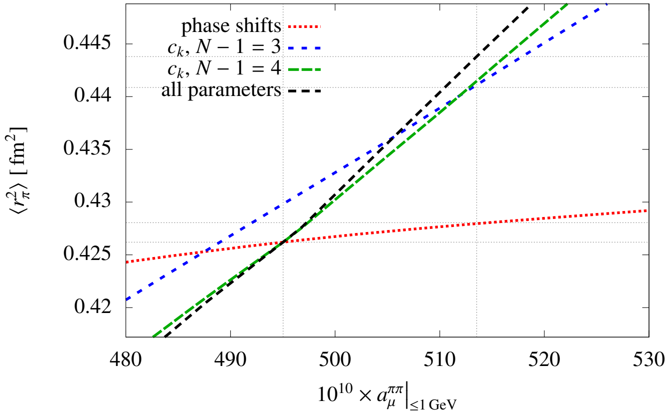
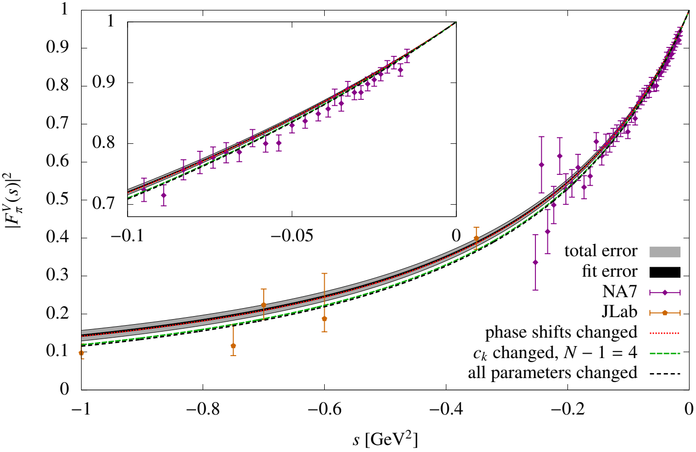


GC, Hoferichter, Stoffer (21)

$$\langle r_\pi^2 \rangle = \begin{cases} 0.429(4)\text{fm}^2 & \text{CHS(18)} \\ 0.436(5)(12)\text{fm}^2 & \chi\text{QCD(20)} \end{cases}$$

Changes in $\sigma(e^+e^- \to \text{hadrons})$ below 1 GeV?



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HLbL contribution: Master Formula

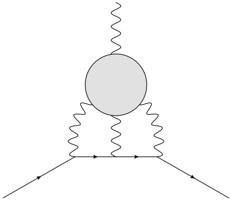
$$a_\mu^{\text{HLbL}} = \frac{2\alpha^3}{48\pi^2}\int_0^\infty dQ_1 \int_0^\infty dQ_2 \int_{-1}^1 d\tau \sqrt{1-\tau^2}\sum_{i=1}^{12} T_i(Q_1,Q_2,\tau)\bar{\Pi}_i(Q_1,Q_2,\tau)$$

Q_i^μ are the Wick-rotated four-momenta and τ the four-dimensional angle between Euclidean momenta:

$$Q_1 \cdot Q_2 = |Q_1||Q_2|\tau$$

The integration variables $Q_1 := |Q_1|$, $Q_2 := |Q_2|$. CHPS (15)

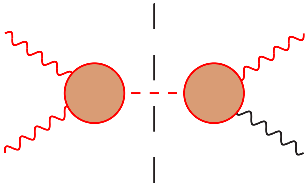
- T_i: known kernel functions
- $\bar{\Pi}_i$ are amenable to a dispersive treatment: their imaginary parts are related to measurable subprocesses



Setting up the dispersive calculation

We split the HLbL tensor as follows:

$$\Pi_{\mu\nu\lambda\sigma} = \Pi^{\pi^0\text{-pole}}_{\mu\nu\lambda\sigma} + \Pi^{\pi\text{-box}}_{\mu\nu\lambda\sigma} + \bar{\Pi}_{\mu\nu\lambda\sigma} + \cdots$$



Pion pole: imaginary parts = δ-functions

Projection on the BTT basis: easy ✓

Our master formula=explicit expressions in the literature ✓

Input: pion transition form factor — Hoferichter et al. (18)

First results of direct lattice calculations — Gerardin, Meyer, Nyffeler (16,19)

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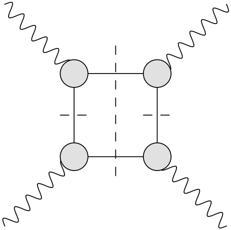
π-box with the BTT set:



– we have constructed a Mandelstam representation for the contribution of the 2-pion cut with LHC due to a pion pole
– we have explicitly checked that this is identical to sQED multiplied by $F_V^\pi(s)$ (FsQED)

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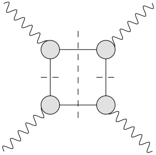
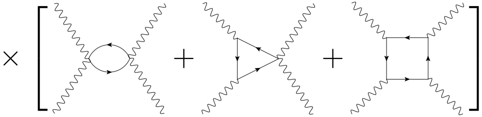


$$\equiv F_\pi^V(q_1^2) F_\pi^V(q_2^2) F_\pi^V(q_3^2)$$



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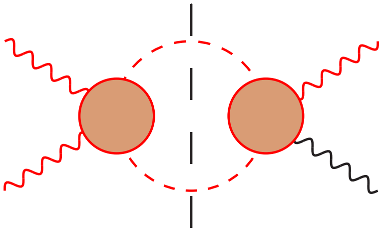

The "rest" with 2π intermediate states has cuts only in one channel and will be
calculated dispersively after partial-wave expansion

Setting up the dispersive calculation

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Contributions of cuts with anything else other than one and two pions in intermediate states are neglected in first approximation

of course, the η, η' and other pseudoscalars pole contribution, or the kaon-box/rescattering contribution can be calculated within the same formalism

Short-distance contraints

Two different kinematic configurations for large Q_i^2:

1. All momenta large — Melnikov-Vainshtein (04), Bijnens et al (19)

$$\lim_{Q\to\infty} Q^4\,\bar{\Pi}_1(Q^2,Q^2,Q^2) = -\frac{4}{9\pi^2}.$$

2. $Q^2 \equiv Q_1^2 \sim Q_2^2 \gg Q_3^2$: — Melnikov-Vainshtein (04)

$$\lim_{Q_3\to\infty}\lim_{Q\to\infty} Q^2 Q_3^2\,\bar{\Pi}_1(Q^2,Q^2,Q_3^2) = -\frac{2}{3\pi^2}$$

In fact, in the chiral (and large-N_c) limit

$$\lim_{Q\to\infty} Q^2\,\bar{\Pi}_1(Q^2,Q^2,Q_3^2) = -\frac{2}{3\pi^2 Q_3^2}$$

the Q_3^2 dependence is known exactly

No individual contribution can satisfy these constraints

Approaches to satisfying the SDC

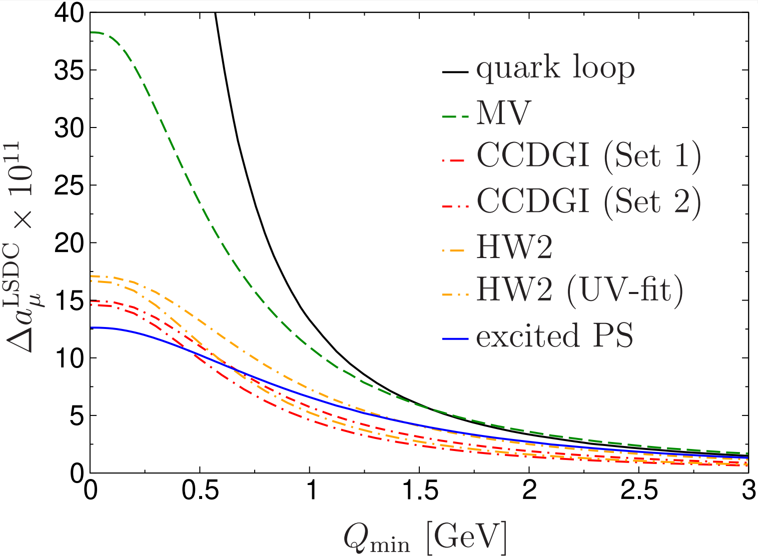
SDCs have been satisfied with three different approaches:

1. pion-pole modification — Melnikhov-Vainshtein (04)
2. Regge model of pseudoscalars — GC, Hagelstein, Hoferichter, Laub, Stoffer (19)
3. Holographic QCD: axial resonances — Leutgeb-Rebhan, Cappiello et al. (20)

	MV model	HW2/CCDGI (set 2)	Regge model
π^0/a_1	17	$(4-5)$	2.7(1.3)
η/f_1	10	$(4-5)$	3.4(1.3)
η'/f_1^*	12	$(6-8)$	6.5(2.0)
Total	38	$(9.6-13)$	12.6(4.1)

- SDCs satisfied with pseudoscalars or axials lead to similar effects on a_μ
- a fourth approach based on interpolants confirms the estimate of HW2/CCDGI and Regge model — Lüdtke, Procura (20)

Approaches to satisfying the SDC



WP(20)

Improvements obtained with the dispersive approach

Contribution	PdRV(09) *Glasgow consensus*	N/JN(09)	J(17)	WP(20)
π^0, η, η'-poles	114(13)	99(16)	95.45(12.40)	93.8(4.0)
π, K-loops/boxes	−19(19)	−19(13)	−20(5)	−16.4(2)
S-wave $\pi\pi$ rescattering	−7(7)	−7(2)	−5.98(1.20)	−8(1)
subtotal	88(24)	73(21)	69.5(13.4)	69.4(4.1)
scalars	–	–	–	− 1(3)
tensors	–	–	1.1(1)	
axial vectors	15(10)	22(5)	7.55(2.71)	6(6)
u, d, s-loops / short-distance	–	21(3)	20(4)	15(10)
c-loop	2.3	–	2.3(2)	3(1)
total	105(26)	116(39)	100.4(28.2)	92(19)

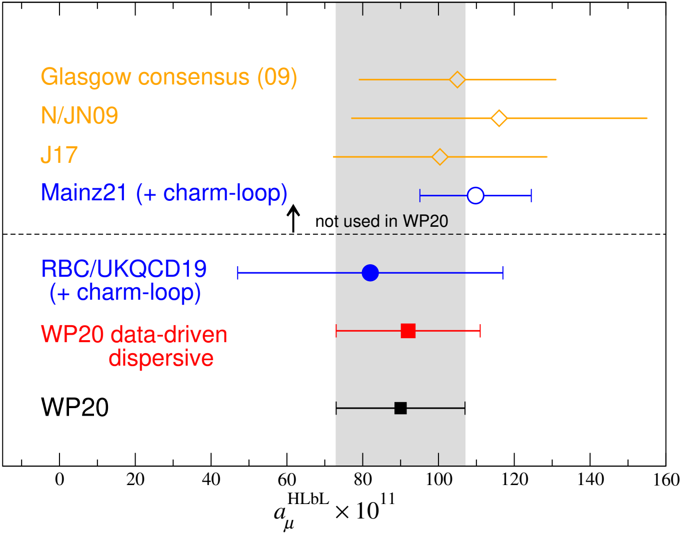
- significant reduction of uncertainties in the first three rows: low-energy region well constrained by a dispersive approach

  CHPS (17), Masjuan, Sánchez-Puertas (17) Hoferichter, Hoid, Kubis, Leupold, Schneider (18)

- $1-2$ GeV and asymptotic region (short distance constraints) have been improved, but still work in progress (see WP(20))

  Melnikov, Vainshtein (04), (.......), Bijnens, Hermansson-Truedsson, Laub, Rodríguez-Sánchez (20,21)

Situation for HLbL



Outline

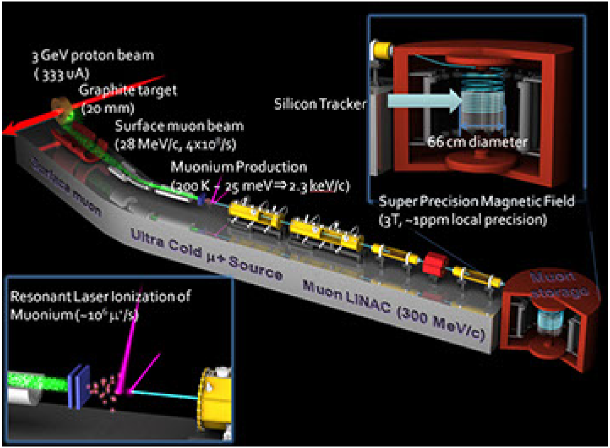
Conclusions

- The WP provides the current status of the SM evaluation of $(g-2)_\mu$: 4.2σ discrepancy with experiment (w/ FNAL)
- Evaluation of the HVP contribution based on the dispersive approach: 0.6% error $\Rightarrow$ dominates the theory uncertainty
- Recent lattice calculation [BMW(20)] has reached a similar precision but differs from the dispersive one (=from e^+e^- data).
  If confirmed $\Rightarrow$ discrepancy with experiment $\searrow$ below 2σ
- Evaluation of the HLbL contribution based on the dispersive approach: 20% accuracy. Two recent lattice calculations [RBC/UKQCD(20), Mainz(21)] agree with it

Outlook

- The Fermilab experiment aims to reduce the BNL uncertainty by a factor four $\Rightarrow$ potential 7σ discrepancy
- Improvements on the theory side:
  - HVP data-driven:
    Other e^+e^- experiments are available or forthcoming:
    SND, BESIII, CMD3, BaBar $\Rightarrow$ Further error reductions
  - HVP lattice:
    BMW result must be confirmed (or refuted) by others.
    Difference to data-driven evaluation must be understood
  - HLbL data-driven: goal of $\sim$ 10% uncertainty within reach
  - HLbL lattice: RBC/UKQCD $\Rightarrow$ similar precision as Mainz.
    Good agreement with data-driven evaluation.

Future: Muon $g-2$/EDM experiment @ J-PARC



Credit: J-PARC