On the mixed 0-form/1-form anomaly in the Hilbert space: pouring new wine into old bottles

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with Erich Poppitz and F. David Wandler in hep-th/2106.11442

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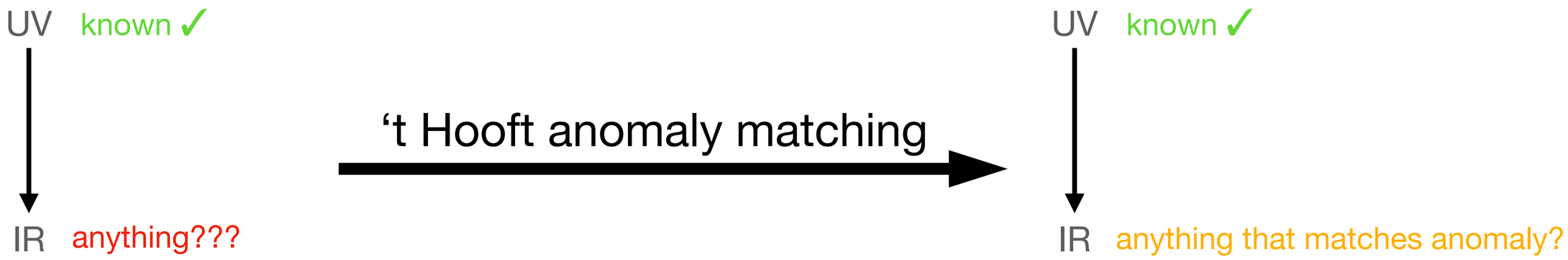
Outline

1. Introduction and Motivation
2. Higher-form Symmetries and Anomalies
3. Pure Yang-Mills
4. QCD (adj)
5. Discussion and Conclusion

Introduction and Motivation

Introduction and Motivation

Anomaly Matching



Anomaly matching allows us to constrain IR dynamics

“Usual” anomaly matching understood since 1980s

New: generalized ’t-Hooft anomaly matching (Gaiotto, Kapustin, Komargodski, Seiberg 2014-… + others)

Introduction and Motivation

This Talk

New mixed anomalies involving higher-form symmetries in Hilbert space

Give clear, down to earth picture of these anomalies

Our work: provide explicit examples of 0-form/1-form anomaly in Hilbert space

Old bottles: van Baal's work on Hamiltonian pure Yang-Mills on torus in 1980s

New wine: new 0-form/1-form anomalies

We study 4d gauge theories: pure Yang-Mills and QCD (adj)

Exact degeneracies between appropriate electric flux states for any torus size

Study all gauge groups (with center): $SU(N)$, $Sp(N)$, $Spin(N)$, E_6, E_7

Higher-form Symmetries and Anomalies

Anomalies in the Hilbert space

A Review

Suppose G and H have a mixed ’t Hooft anomaly $\Longrightarrow \left[\hat{H}, \hat{U}\right] = 0$ and $\left[\hat{H}, \hat{V}\right] = 0$

Introduce backgrounds for G $\longleftrightarrow \hat{U}\hat{V}\left(\hat{V}\hat{U}\right)^{-1}$ changed (group commutator)

Simultaneously diagonalize $\hat{H}$ and $\hat{U} \longrightarrow$ states $|E, u\rangle$

$\left[\hat{H}, \hat{V}\right] = 0 \implies \hat{V}$ cannot change energy

$\hat{U}\hat{V}\left(\hat{V}\hat{U}\right)^{-1} \neq 1 \implies \hat{V}|E, u\rangle \sim |E, u'\rangle,\ u \neq u'$

Takeaway: anomaly informs degeneracy of states

Higher Form Symmetries

A Quick Introduction

Old (0-form) symmetries: act on local objects (fields)

New (higher form) symmetries: act on non-local objects

eg: 1-form center symmetry acting on Wilson loops

Have been known on the lattice for a long time, new insights into continuum description

Our interest: new generalized ‘t Hooft anomalies involving higher form symmetries

Theory	0-form	1-form
Pure Yang-Mills	Parity	Center
QCD (adj)	Chiral	Center

Pure Yang-Mills on the Torus

Pure Yang-Mills

Setup

We will impose twisted boundary conditions on $\mathbb{T}^3$ corresponding to 't Hooft magnetic flux, explicit details not necessary here (see paper for more information)

Spacetime is $\mathbb{T}^3 \times \mathbb{R}$ - $\mathbb{T}^3$ size is $L_1 \times L_2 \times L_3$

$SU(N)$ gauge group (will generalize later)

$$\hat{H}_{\theta=0} = \int_{\mathbb{T}^3} \mathrm{d}^3x \, \mathrm{Tr} \left(g^2 \hat{\Pi}_i \hat{\Pi}_i + \frac{1}{g^2} \hat{B}_i \hat{B}_i \right)$$

$A_0 = 0$ gauge

$$\hat{\Pi}_i^a(\vec{x}) = -i \frac{\delta}{\delta A_i^a(\vec{x})} \qquad \hat{B}_i = \frac{1}{2} \varepsilon_{ijk} \hat{F}_{jk}$$

Pure Yang-Mills

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$$\hat{H}_{\theta=\alpha} = \hat{V}_\alpha \hat{H}_{\theta=0} \hat{V}_\alpha^{-1}$$

$\hat{V}_\alpha \left[\hat{A} \right] = e^{i\alpha \int_{\mathbb{T}^3} K_0(\hat{A})}$ shifts the θ-angle by α

$$K_0(\hat{A}) = \frac{1}{8\pi^2} \mathrm{Tr}\left(A \wedge F - \frac{i}{3} A \wedge A \wedge A \right)$$

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Setup: Hilbert Space

Gauge field states $\hat{A}(\overrightarrow{x})|A\rangle = A(\overrightarrow{x})|A\rangle$

Gauge transformations: $\hat{U}|A\rangle = |U \circ A\rangle$

$U \circ A = U(A - i\mathrm{d})U^{-1}$

U obey appropriate $\mathbb{T}^3$ boundary conditions for magnetic flux

Hilbert space: $\mathscr{H} = \left\{ |\psi\rangle \mid \hat{U}|\psi\rangle = |\psi\rangle \right\}$

U are homotopic to identity, preserve $\mathbb{T}^3$ boundary conditions

Under large gauge transformations $\hat{U}[\nu]|\psi\rangle = |\psi\rangle$

We take the $\theta = 0$ Hilbert space, placing the θ dependence in the Hamiltonian

$\nu \neq 0$ is the instanton number of U

Pure Yang-Mills

Setup: Center Symmetry

$\mathbb{Z}_N$ 1-form center symmetry acts on (winding) Wilson loops

Generated by improper* gauge transformations $C[\overrightarrow{k}]$

$$C[\overrightarrow{k}] : W_l \equiv \mathrm{Tr}_F \left(\mathscr{P} e^{-i \oint \mathrm{d}x^l A_l} \right) \to e^{2\pi i k_l / N} W_l$$

Implemented on Hilbert space as $\hat{T}_l |A\rangle = \left| C[\overrightarrow{e}_l] \circ A \right\rangle$

*The transformations are improper because they do not obey the $\mathbb{T}^3$ boundary conditions

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Setup: Parity

$\mathbb{Z}_2$ 0-form parity symmetry at $\theta = 0, \pi$

$$P: \; A(x, y, z) \to A^P(x, y, z) = -\Gamma_P A(L_1 - x, L_2 - y, L_3 - z)\Gamma_P^{-1}$$

$\Gamma_P \in SU(N)$, $\Gamma_P^2 = \pm 1$ is necessary to preserve twisted boundary conditions on $\mathbb{T}^3$

$\theta = 0$: Parity acts directly as above $\hat{P}_0|A\rangle = |A^P\rangle$

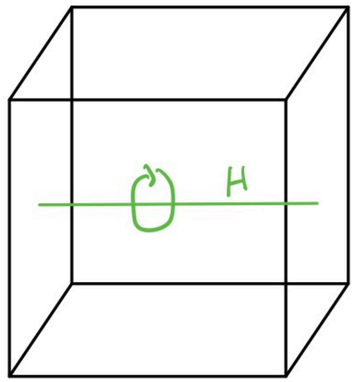
$\theta = \pi$: Parity maps $\theta = \pi \to -\pi$, need to also insert $\hat{V}_{2\pi}$: $\hat{P}_\pi = \hat{V}_{2\pi}\hat{P}_0$

Pure Yang-Mills

Center Symmetry: Diagonalizing and Backgrounds

Introduce center background ('t Hooft flux) magnetic flux $\vec{m}$

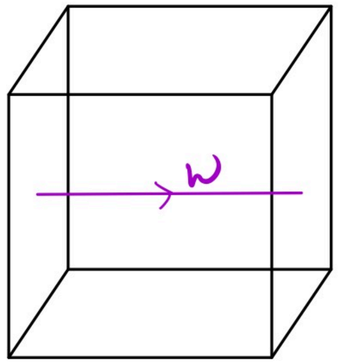
$\vec{m}$ related to $\mathbb{T}^3$ twisted boundary conditions



Introducing a static center vortex in the lth direction increases m_l by one unit

Introduce "electric flux" states $\hat{T}_l|\vec{e}\rangle = e^{2\pi i e_l/N}|\vec{e}\rangle$

Recall $\left[\hat{H}, \hat{T}_l\right] = 0 \longrightarrow$ can label states by energy and $\vec{e}$



Introducing a Wilson loop in the lth direction increases e_l by one unit

Pure Yang-Mills

Setup: Center Symmetry and θ Shifts, an Important Commutation Relation

Very important commutation relation:

$$\boxed{\hat{T}_l \hat{V}_{2\pi}[\hat{A}] \hat{T}_l^{-1} = e^{2\pi i \frac{m_l}{N}} \hat{V}_{2\pi}[\hat{A}] = \left(e^{2\pi i Q[T_l]} \hat{V}_{2\pi}[\hat{A}] \right)}$$

This can be obtained by direct calculation, see the paper for more explicit details

$Q[T_l]$ is the topological charge of a configuration on $\mathbb{T}^4$ with boundary conditions twisted by $C[\overrightarrow{e}_l, 0]$ in the time direction

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Center-Parity at $\theta = 0$

Act with $\hat{P}_0 \hat{T}_i \hat{P}_0$ on $|A\rangle$

Acts as "gauge transformation":

$$\Gamma_P C[\vec{e}_i, 0](L_1 - x, L_2 - y, L_3 - z)\Gamma_P \sim C[-\vec{e}_i, 0](x, y, z)$$

$$\boxed{\text{Result: } \hat{P}_0 \hat{T}_i \hat{P}_0 = \hat{T}_i^{-1}}$$

We see that parity and centre symmetry form a D_N (dihedral) algebra

No mixed anomaly

Pure Yang-Mills

Center-Parity at $\theta = \pi$

Use important commutation relation and $\theta = 0$ result

$$\hat{T}_l \hat{V}_{2\pi}[\hat{A}] \hat{T}_l^{-1} = e^{2\pi i \frac{m_l}{N}} \hat{V}_{2\pi}[\hat{A}] = \left(e^{2\pi i Q[T_l]} \hat{V}_{2\pi}[\hat{A}] \right) \qquad \hat{P}_\pi = \hat{V}_{2\pi} \hat{P}_0$$

$$\boxed{\text{Result: } \hat{T}_l \hat{P}_\pi = e^{2\pi i m_l / N} \hat{P}_\pi \hat{T}_l^{-1} \left(= e^{2\pi i Q[T_l]} \hat{P}_\pi \hat{T}_l^{-1} \right)}$$

We see that parity and center symmetry form a central extension of the D_N (dihedral) algebra

Notice: center symmetry background modifies the group commutator $\hat{T}_l \hat{P}_\pi \left(\hat{P}_\pi \hat{T}_l \right)^{-1}$ according to the anomaly!

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Implications of Center-Parity Algebra

Consider $\overrightarrow{m} = (0,0,1)$ as example

Remember $\left[\hat{H}_{\theta=0,\pi}, \hat{T}_l\right] = \left[\hat{H}_{\theta=0,\pi}, \hat{P}_{0,\pi}\right] = 0$

Label simultaneous $\hat{H}_{\theta=0,\pi}$ and $\hat{T}_l$ eigenstates by e_3 only

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At $\boldsymbol{\theta = 0}$:

$$\hat{T}_l \hat{P}_0 = \hat{P}_0 \hat{T}_l^{-1} \implies \hat{P}_0 |e_3\rangle = |-e_3\rangle$$

Recall: e_3 is defined $\mathrm{mod}\, N$

$\boldsymbol{N}$ **odd**: $e_3 = 0$ invariant

$\boldsymbol{N}$ **even**: $e_3 = 0, \dfrac{N}{2}$ invariant

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At $\boldsymbol{\theta = \pi}$:

$$\hat{T}_l \hat{P}_\pi = e^{2\pi i m_l/N} \hat{P}_\pi \hat{T}_l^{-1} \implies \hat{P}_\pi |e_3\rangle = |m_3 - e_3\rangle$$

Recall: e_3 and m_3 are defined $\mathrm{mod}\, N$

$\boldsymbol{N}$ **odd**: $e_3 = \dfrac{N+1}{2}$ invariant

$\boldsymbol{N}$ **even**: no invariant states

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No anomaly, but a global inconsistency

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Candidates for unique ground state

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N **odd**: $e_3 = \dfrac{N+1}{2}$ invariant

No anomaly, but a global inconsistency

N **even**: no invariant states

Candidates for unique ground state

No unique ground state, at least doubly degenerate!

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Examples

Take $\vec{m} = (0,0,1)$, label ground states by e_3 only

$N = 4$:

$$\underbrace{\begin{array}{c} |0\rangle \longleftrightarrow |0\rangle \\ |2\rangle \longleftrightarrow |2\rangle \\ |1\rangle \longleftrightarrow |3\rangle \end{array}}_{\theta = 0} \qquad \underbrace{\begin{array}{c} |0\rangle \longleftrightarrow |1\rangle \\ |2\rangle \longleftrightarrow |3\rangle \end{array}}_{\theta = \pi}$$

$N = 3$:

$$\underbrace{\begin{array}{c} |0\rangle \longleftrightarrow |0\rangle \\ |1\rangle \longleftrightarrow |2\rangle \end{array}}_{\theta = 0} \xrightarrow{\text{Global inconsistency}} \underbrace{\begin{array}{c} |0\rangle \longleftrightarrow |1\rangle \\ |2\rangle \longleftrightarrow |2\rangle \end{array}}_{\theta = \pi}$$

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Summary and Discussion of Results

N odd:

- Candidates for unique ground states for both $\theta = 0,\ \pi$
- Ground states are different - global inconsistency (no anomaly)

N even:

- Only unique ground state at $\theta = 0$
- States at $\theta = \pi$ at least doubly degenerate

All results are **exact** at **any size torus**

Infinite volume limit: if center is preserved (confining phase), parity must be spontaneously broken

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All Gauge Groups

Recall: $\hat{T}_l \hat{P}_\pi = e^{2\pi i Q[T_l]} \hat{P}_\pi \hat{T}_l^{-1}$

Replace $e^{2\pi i\, m_l/N}$ in phases by $e^{2\pi i\, Q_{top}[\vec{m}\cdot\vec{k}=1]}$

Group	Center	Q_{top} (mod 1)	Parity Broken
$SU(N)$	$\mathbb{Z}_N$	$\frac{1}{N}\vec{m}\cdot\vec{k}$	N even
$Sp(N)$	$\mathbb{Z}_2$	$-\frac{N}{2}\vec{m}\cdot\vec{k}$	N odd
$Spin(8N)$	$\mathbb{Z}_2^+ \times \mathbb{Z}_2^-$	$-\frac{1}{2}(\vec{m}^+\cdot\vec{k}^- + \vec{m}^-\cdot\vec{k}^+)$	Always
$Spin(8N+4)$	$\mathbb{Z}_2^+ \times \mathbb{Z}_2^-$	$-\frac{1}{2}(\vec{m}^+\cdot\vec{k}^+ + \vec{m}^-\cdot\vec{k}^-)$	Always
$Spin(4N+2)$	$\mathbb{Z}_4$	$-\frac{1+2N}{4}\vec{m}\cdot\vec{k}$	Always
$Spin(2N+1)$	$\mathbb{Z}_2$	0	Never
E_6	$\mathbb{Z}_3$	$-\frac{1}{3}\vec{m}\cdot\vec{k}$	Never
E_7	$\mathbb{Z}_2$	$-\frac{1}{2}\vec{m}\cdot\vec{k}$	Always

Note that none of $Sp(2N)$, $Spin(2N+1)$, or E_6 have a center-parity anomaly

Center-Chiral Anomaly: QCD (adj)

QCD (adj)

Introduction

Same setup as pure Yang-Mills (gauge fields, spacetime, etc)

Introduce $n_f \leq 5$ adjoint Weyl fermions

Classical 0-form $U(n_f)$ chiral symmetry broken by anomaly to $\dfrac{\mathbb{Z}_{2n_fN} \times SU(n_f)}{\mathbb{Z}_{n_f}}$

Focus on anomaly between $\mathbb{Z}_{2n_fN}$ discrete chiral symmetry and center symmetry

In a general gauge group $\mathbb{Z}_{2n_fN}$ is replaced by $\mathbb{Z}_{2n_fc_2}$ where c_2 is the dual Coxeter number of the gauge group

QCD (adj)

Discrete Chiral Symmetry

Implemented on Hilbert space by $\hat{X}_{\mathbb{Z}^{(0)}_{2n_f N}} = e^{\frac{2\pi i}{2n_f N}\int \mathrm{d}^3x\, \hat{j}^0_f} \hat{V}^{-1}_{2\pi}$

$\hat{j}^\mu_f = \hat{\lambda}^{a\,\dagger}\bar{\sigma}^\mu\hat{\lambda}^a$ is the classical $U(1)$ chiral current

Note: $\hat{j}^0_f$ depends only on fermion operators $\implies \left[\hat{j}^0_f, \hat{T}_l\right] = 0$

Center-chiral algebra determined by $\hat{T}_l$, $\hat{V}_{2\pi}$ algebra!

$$\boxed{\text{Result: } \hat{T}_l \hat{X}_{\mathbb{Z}^{(0)}_{2n_f N}} = e^{-2\pi i \frac{m_l}{N}} \hat{X}_{\mathbb{Z}^{(0)}_{2n_f N}} \hat{T}_l}$$

QCD (adj)

Discrete Chiral Symmetry

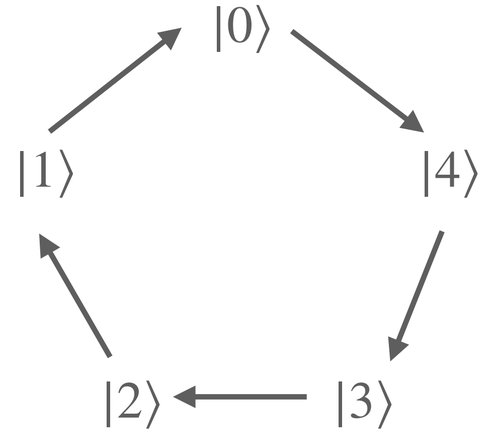
Take $\vec{m} = (0,0,1)$ as example

Again use electric flux, energy eigenstates $|E, e_3\rangle$

$$\hat{X}_{\mathbb{Z}^{(0)}_{2n_f N}} |E, e_3\rangle = |E, e_3 - 1\rangle \implies \text{at least } N\text{-fold degeneracy for all } N$$

$$\implies \text{spontaneous breaking } \mathbb{Z}_{2n_f N} \to \mathbb{Z}_{2n_f}$$

$N = 5$:



QCD (adj)

All Gauge Groups

Recall: $\hat{T}_l \hat{X}_{\mathbb{Z}^{(0)}_{2n_f N}} = e^{-2\pi i \frac{m_l}{N}} \hat{X}_{\mathbb{Z}^{(0)}_{2n_f N}} \hat{T}_l$

Replace $e^{2\pi i\, m_l/N}$ in phases by $e^{2\pi i\, Q_{top}[\vec{m}\cdot\vec{k}=1]}$

Group	Center	Q_{top} (mod 1)	Minimum Chiral Breaking	Minimum Degeneracy
$SU(N)$	$\mathbb{Z}_N$	$\frac{1}{N}\vec{m}\cdot\vec{k}$	$\mathbb{Z}_{2n_f N} \to \mathbb{Z}_{2n_f}$	N
$Sp(2N)$	$\mathbb{Z}_2$	0	$\mathbb{Z}_{2n_f(2N+1)} \to \mathbb{Z}_{2n_f(2N+1)}$	1
$Sp(2N+1)$	$\mathbb{Z}_2$	$-\frac{1}{2}\vec{m}\cdot\vec{k}$	$\mathbb{Z}_{2n_f(2N+2)} \to \mathbb{Z}_{2n_f(N+1)}$	2
$Spin(8N)$	$\mathbb{Z}_2^+ \times \mathbb{Z}_2^-$	$-\frac{1}{2}(\vec{m}^+\cdot\vec{k}^- + \vec{m}^-\cdot\vec{k}^+)$	$\mathbb{Z}_{2n_f(8N-2)} \to \mathbb{Z}_{2n_f(4N-1)}$	2
$Spin(8N+4)$	$\mathbb{Z}_2^+ \times \mathbb{Z}_2^-$	$-\frac{1}{2}(\vec{m}^+\cdot\vec{k}^+ + \vec{m}^-\cdot\vec{k}^-)$	$\mathbb{Z}_{2n_f(8N+2)} \to \mathbb{Z}_{2n_f(4N+1)}$	2
$Spin(4N+2)$	$\mathbb{Z}_4$	$-\frac{1+2N}{4}\vec{m}\cdot\vec{k}$	$\mathbb{Z}_{8n_f N} \to \mathbb{Z}_{2n_f N}$	4
$Spin(2N+1)$	$\mathbb{Z}_2$	0	$\mathbb{Z}_{2n_f(2N-1)} \to \mathbb{Z}_{2n_f(2N-1)}$	1
E_6	$\mathbb{Z}_3$	$-\frac{1}{3}\vec{m}\cdot\vec{k}$	$\mathbb{Z}_{24n_f} \to \mathbb{Z}_{8n_f}$	3
E_7	$\mathbb{Z}_2$	$-\frac{1}{2}\vec{m}\cdot\vec{k}$	$\mathbb{Z}_{36n_f} \to \mathbb{Z}_{18n_f}$	2

Notice: breaking relatively mild for non $SU(N)$ groups

Expect higher degeneracy for (at least) $n_f = 1$ case (SYM)

- Perhaps non-invertible symmetries?
- Discrete chiral/gravity anomaly implies more breaking (for all n_f)

Conclusion

Conclusions and Outlook

Summary of Results

All results are **exact** for **any** torus size

Pure Yang-Mills

Exact degeneracy of ground state for all but $SU(2N+1)$, $Sp(2N)$, $Spin(2N+1)$, and E_6

Usually expect tunnelling to lift degeneracy at finite volume - there must be some delicate cancellation

Interesting implications for both lattice and semiclassics

QCD (adj)

At least N-fold degeneracy for $SU(N)$

All other groups have much smaller minimal degeneracy - expect higher

Perhaps breaking of non-invertible symmetries?

Discrete chiral/gravity anomaly implies further breaking then center/chiral

Thanks for Listening!