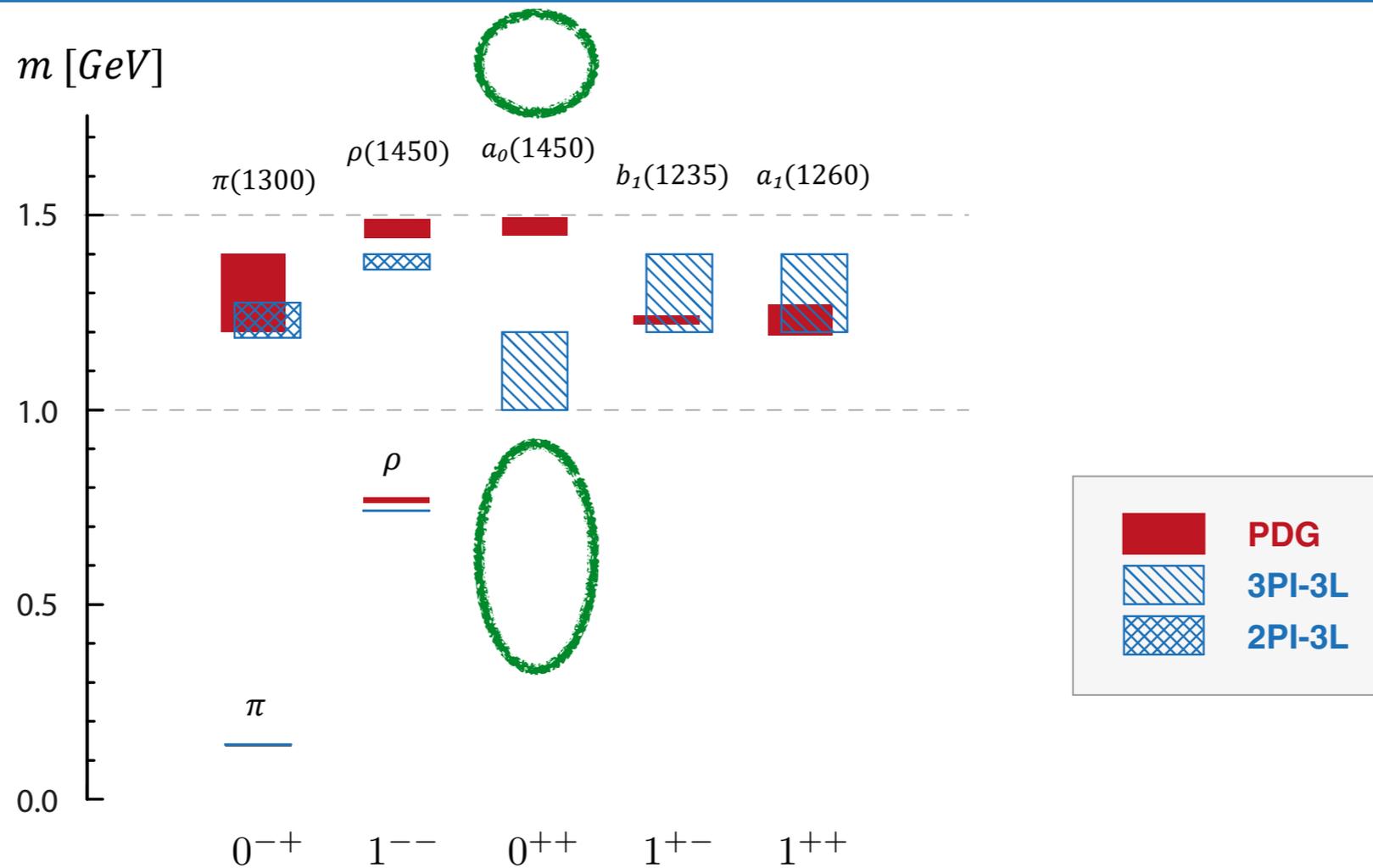




Confinement XV Stavanger 2022

Scalar states of QCD from functional methods



1. Conventional mesons

2. Pure Yang-Mills: glueballs

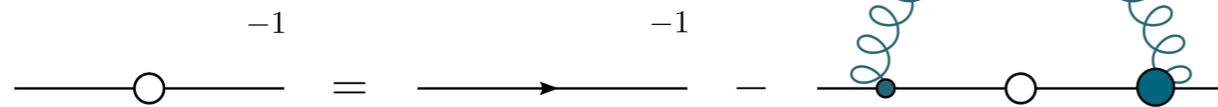
3. Light four-quark states: the $f_0(500)$

4. Heavy-light four-quark states

Dyson-Schwinger equations - “3PI vs RL”

$$Z_{QCD} = \int \mathcal{D}[\Psi, A] \exp \left\{ - \int d^4x \left(\bar{\Psi} (i\not{D} - m) \Psi - \frac{1}{4} (F_{\mu\nu}^a)^2 \right) \right\}$$

propagators

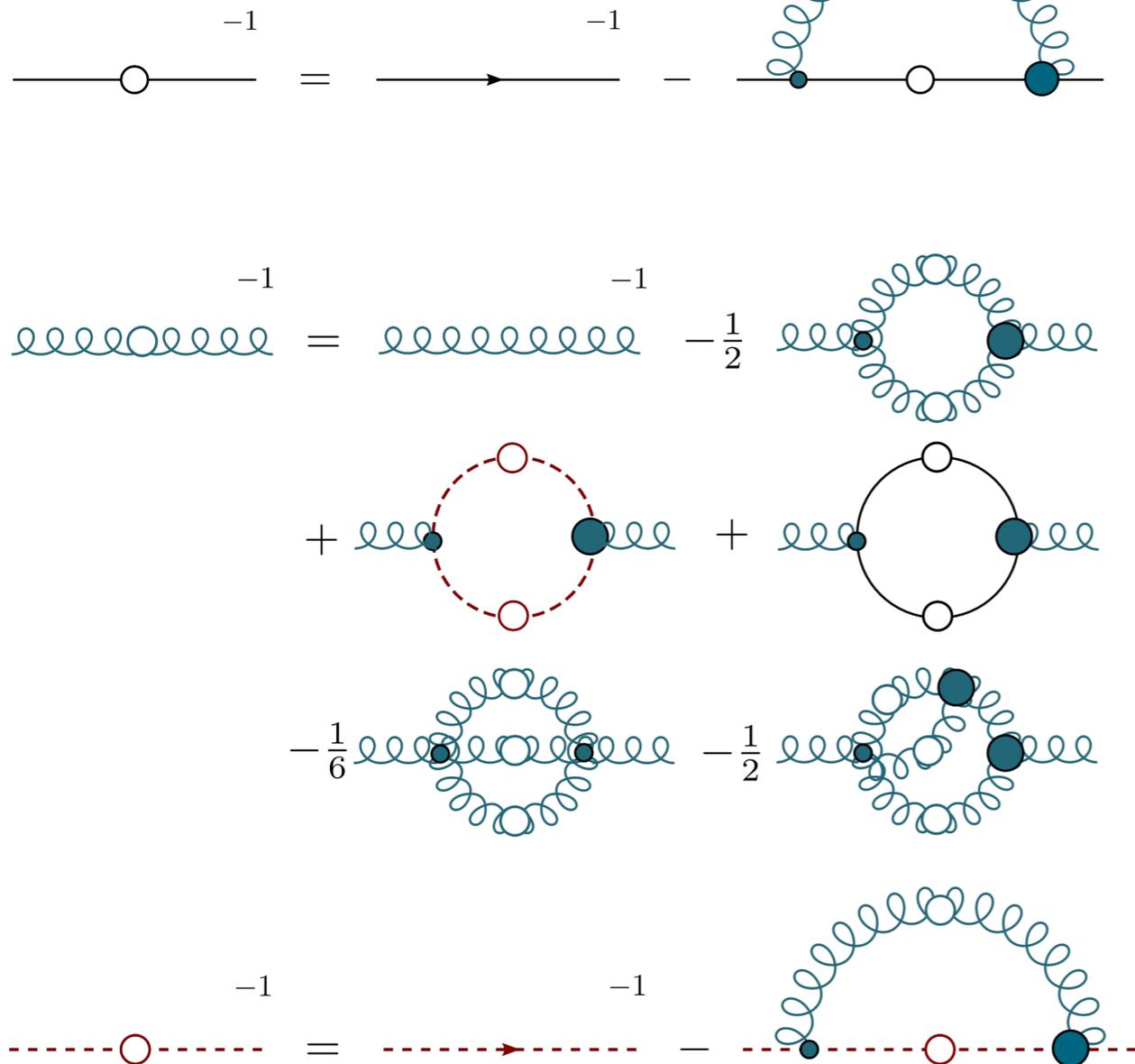


CF,Alkofer, PRD67 (2003) 094020
Williams, CF, Heupel, PRD93 (2016) 034026
Huber, PRD 101 (2020) 114009

Dyson-Schwinger equations - “3PI vs RL”

$$Z_{QCD} = \int \mathcal{D}[\Psi, A] \exp \left\{ - \int d^4x \left(\bar{\Psi} (i\not{D} - m) \Psi - \frac{1}{4} (F_{\mu\nu}^a)^2 \right) \right\}$$

propagators

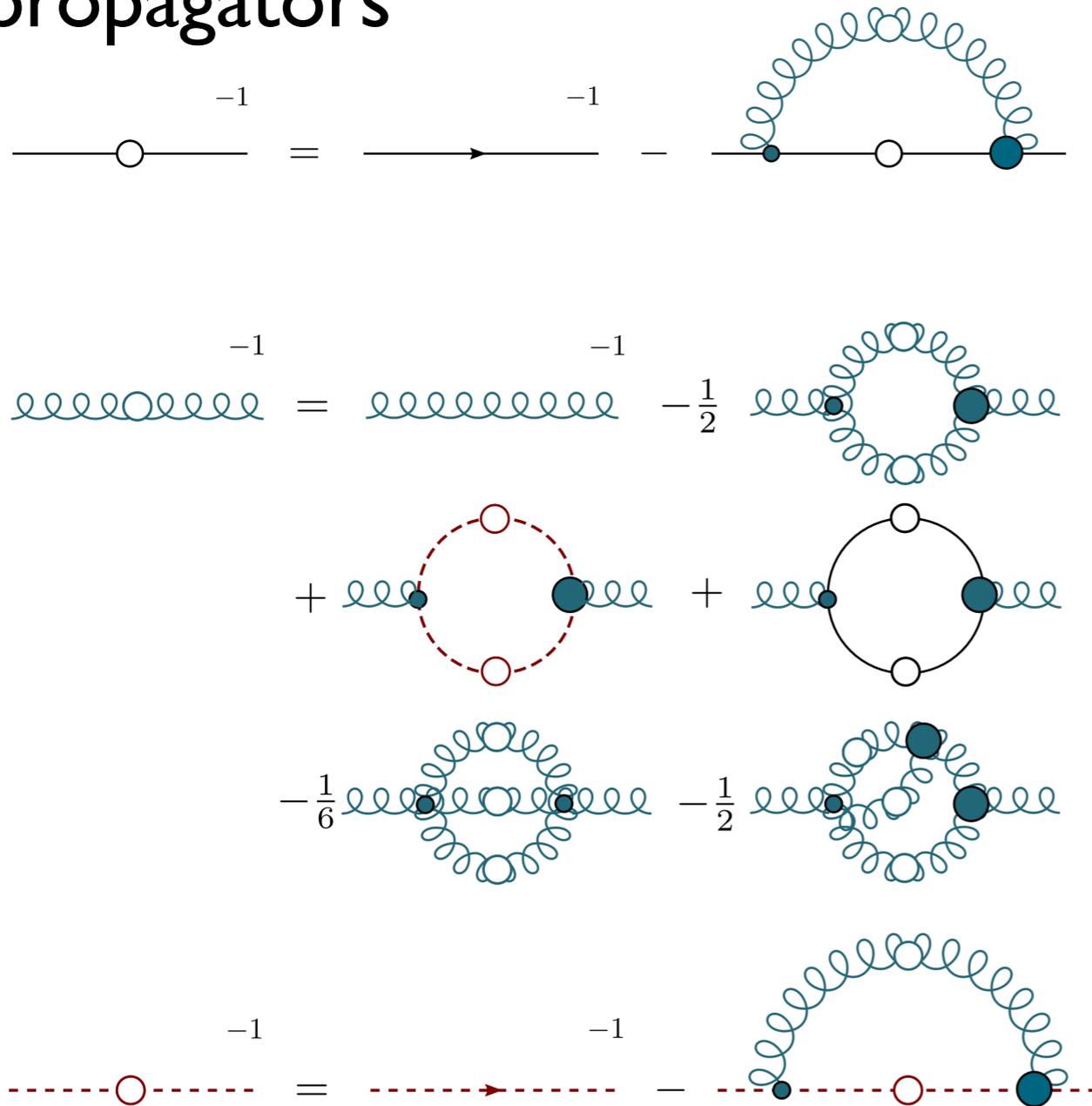


CF,Alkofer, PRD67 (2003) 094020
 Williams, CF,Heupel, PRD93 (2016) 034026
 Huber, PRD 101 (2020) 114009

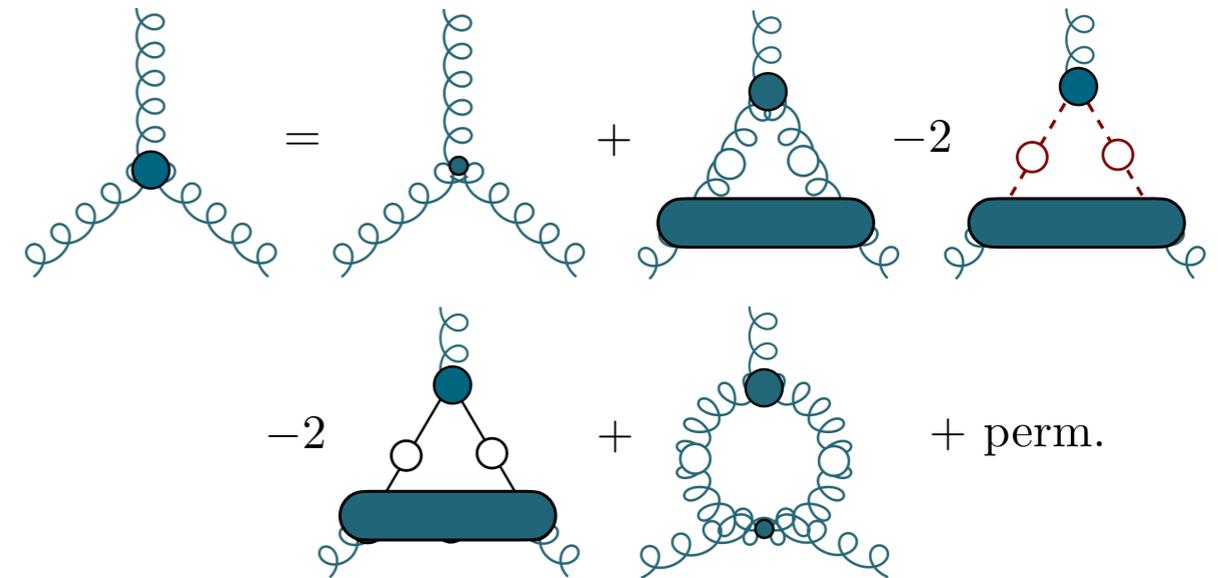
Dyson-Schwinger equations - “3PI vs RL”

$$Z_{QCD} = \int \mathcal{D}[\Psi, A] \exp \left\{ - \int d^4x \left(\bar{\Psi} (i\not{D} - m) \Psi - \frac{1}{4} (F_{\mu\nu}^a)^2 \right) \right\}$$

propagators



vertices

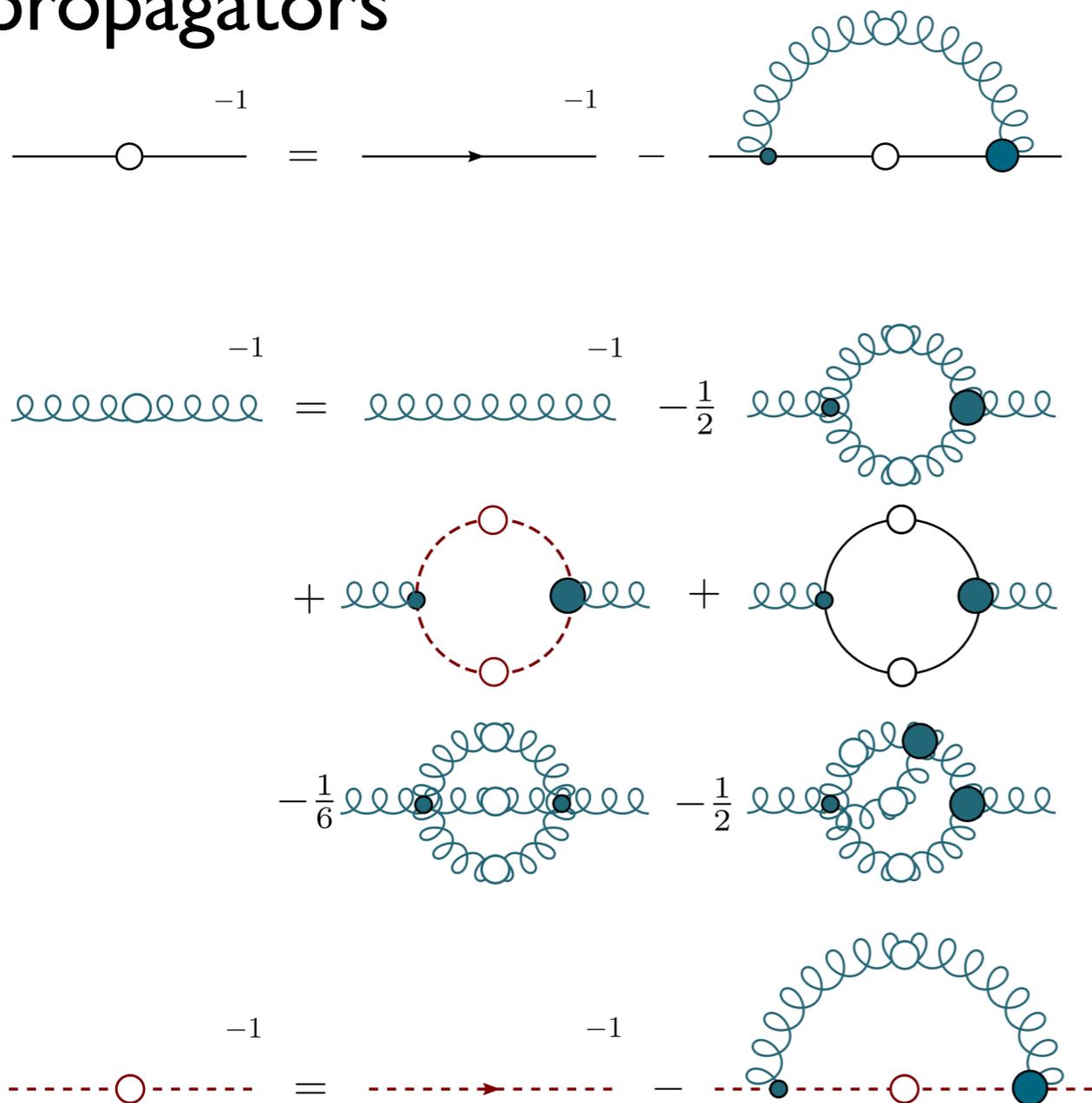


CF,Alkofer, PRD67 (2003) 094020
 Williams, CF,Heupel, PRD93 (2016) 034026
 Huber, PRD 101 (2020) 114009

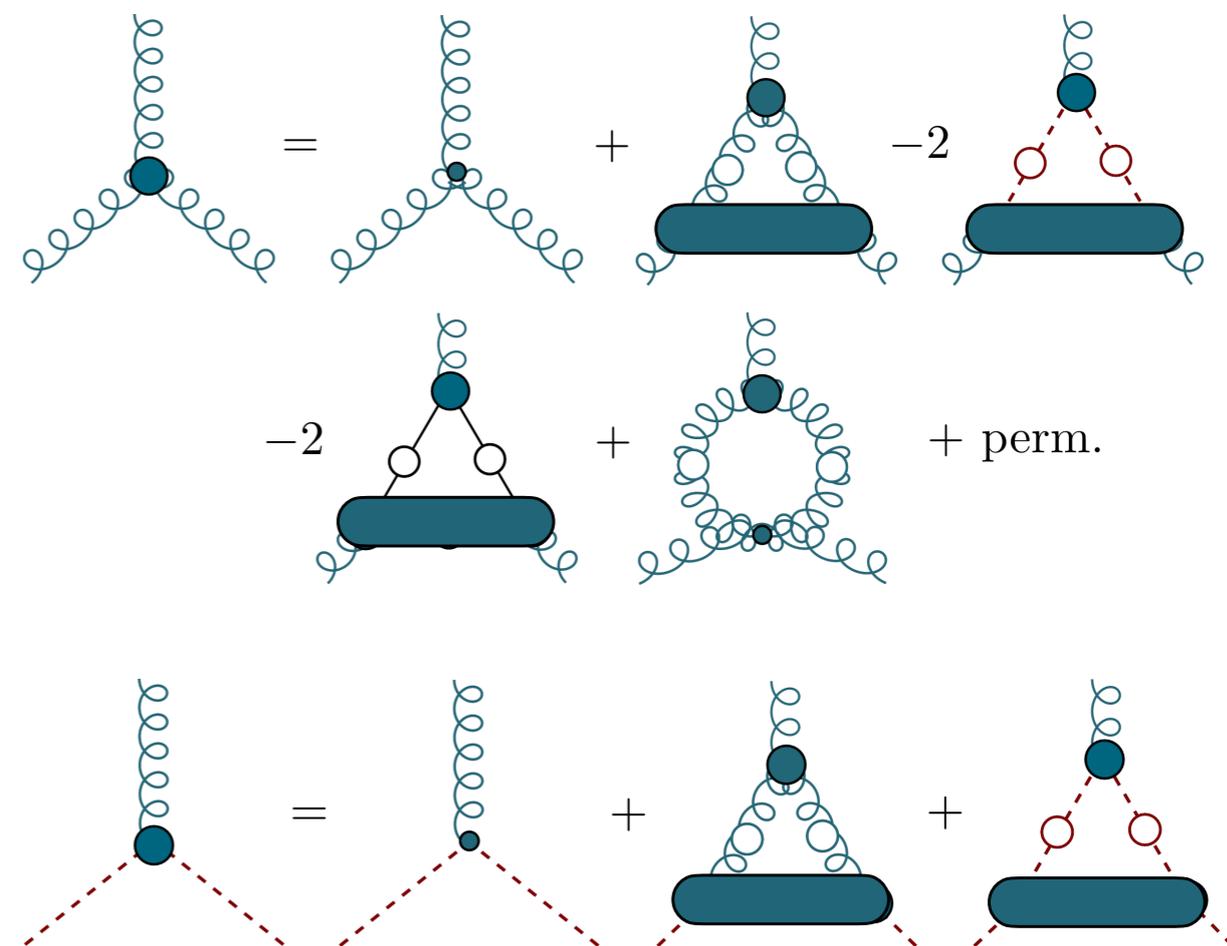
Dyson-Schwinger equations - “3PI vs RL”

$$Z_{QCD} = \int \mathcal{D}[\Psi, A] \exp \left\{ - \int d^4x \left(\bar{\Psi} (i\not{D} - m) \Psi - \frac{1}{4} (F_{\mu\nu}^a)^2 \right) \right\}$$

propagators



vertices

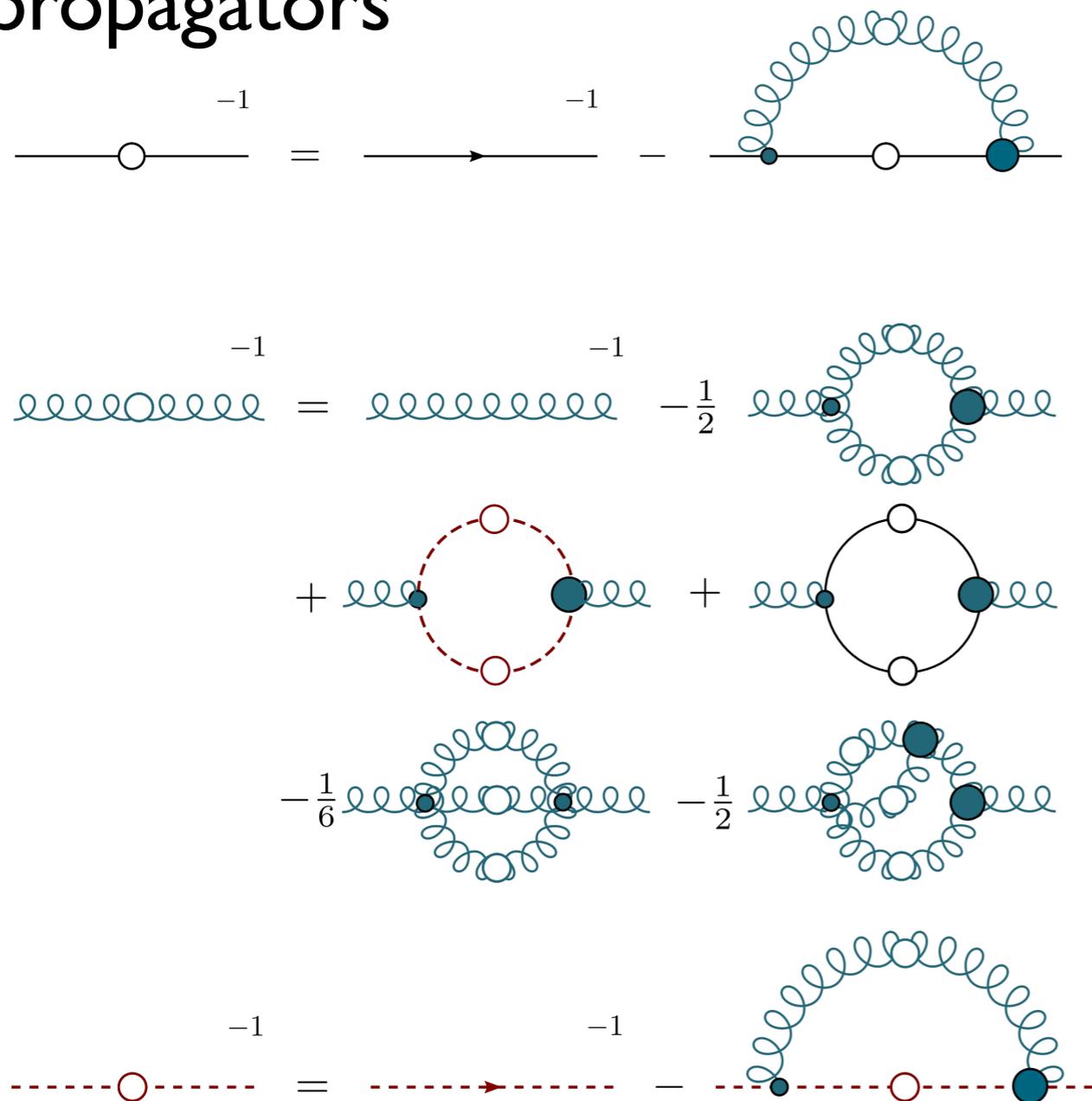


CF,Alkofer, PRD67 (2003) 094020
 Williams, CF,Heupel, PRD93 (2016) 034026
 Huber, PRD 101 (2020) 114009

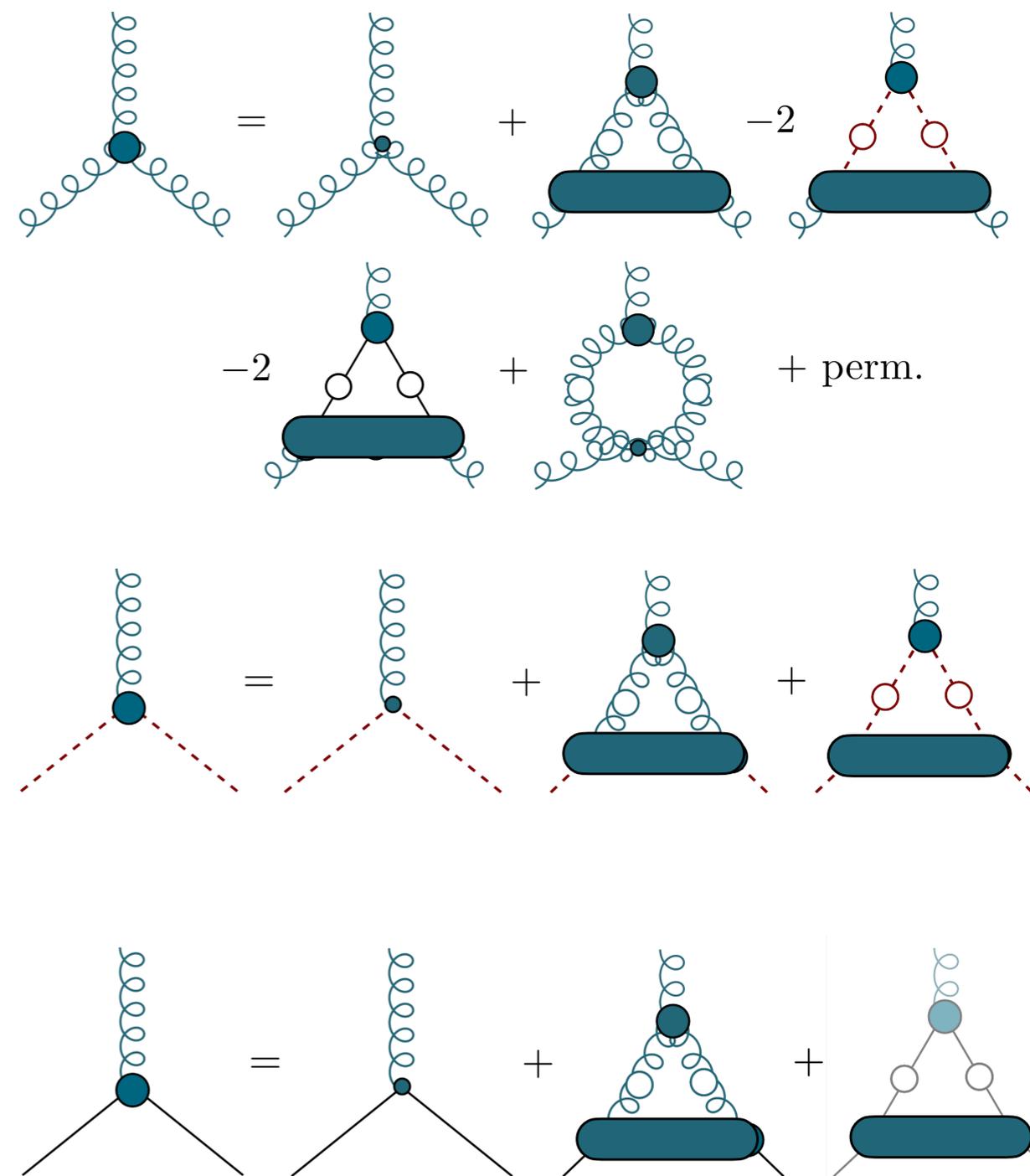
Dyson-Schwinger equations - “3PI vs RL”

$$\mathcal{Z}_{QCD} = \int \mathcal{D}[\Psi, A] \exp \left\{ - \int d^4x \left(\bar{\Psi} (i\not{D} - m) \Psi - \frac{1}{4} (F_{\mu\nu}^a)^2 \right) \right\}$$

propagators



vertices

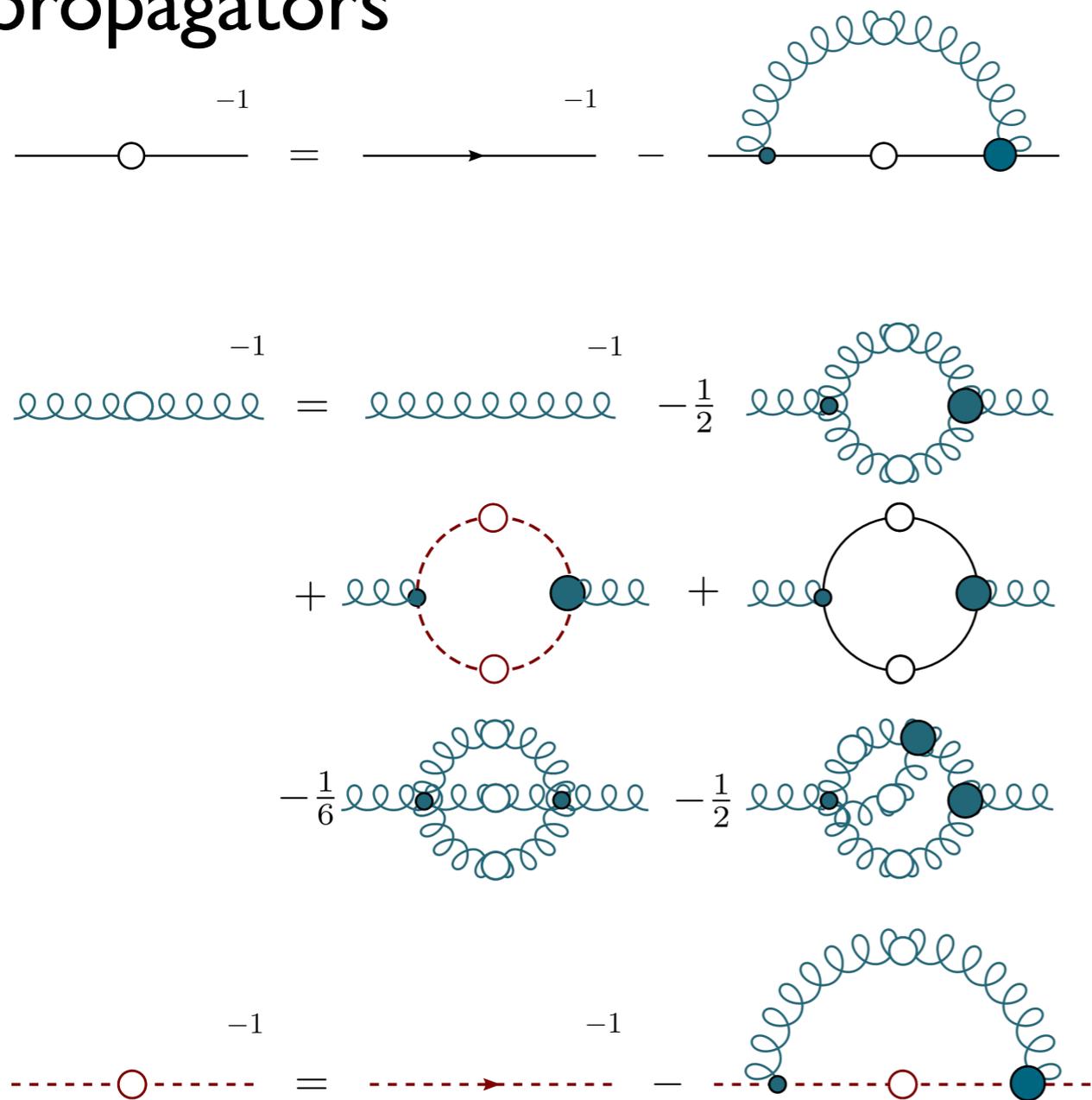


CF, Alkofer, PRD67 (2003) 094020
 Williams, CF, Heupel, PRD93 (2016) 034026
 Huber, PRD 101 (2020) 114009

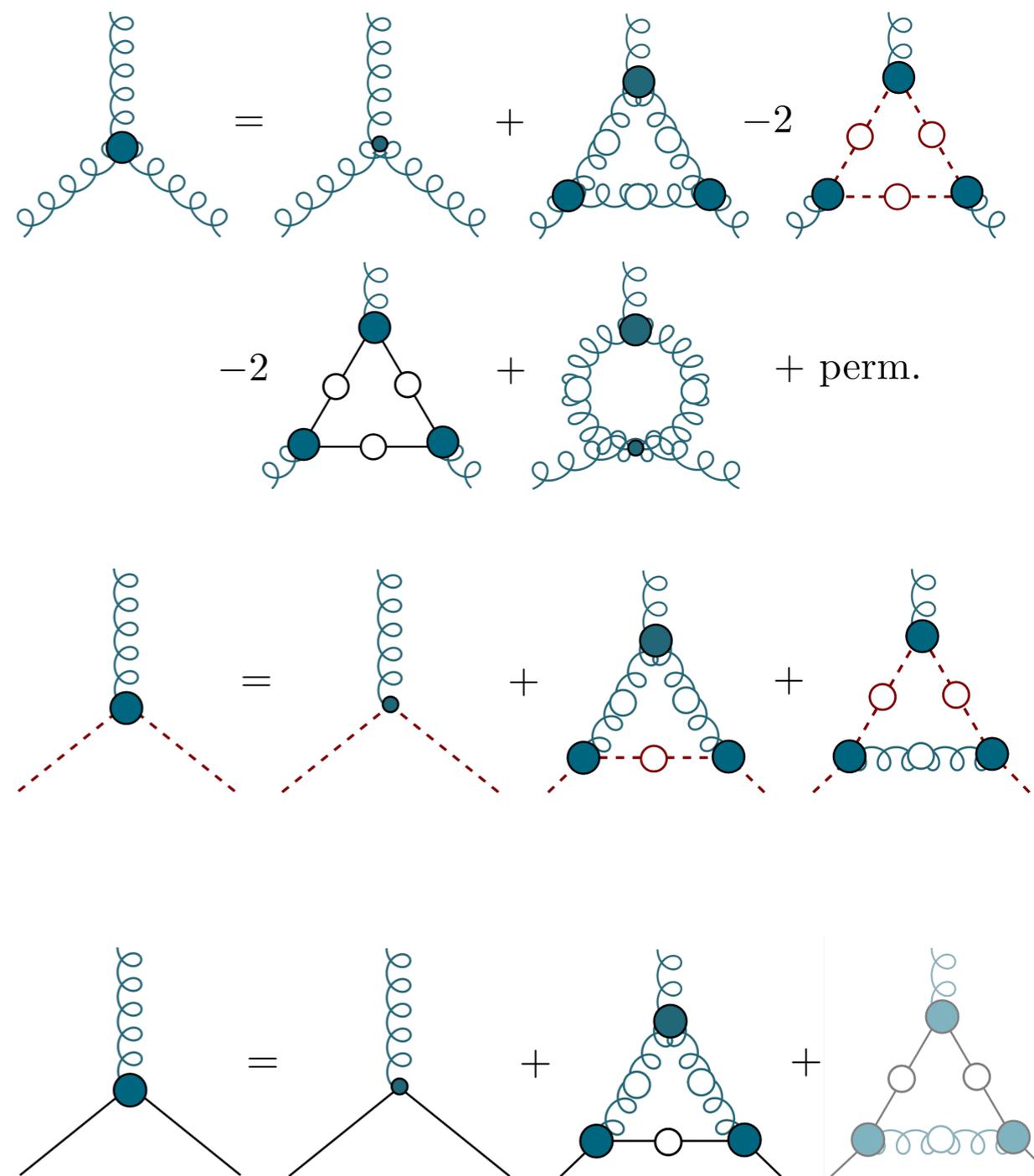
Dyson-Schwinger equations - “3PI vs RL”

$$Z_{QCD} = \int \mathcal{D}[\Psi, A] \exp \left\{ - \int d^4x \left(\bar{\Psi} (i\not{D} - m) \Psi - \frac{1}{4} (F_{\mu\nu}^a)^2 \right) \right\}$$

propagators



vertices

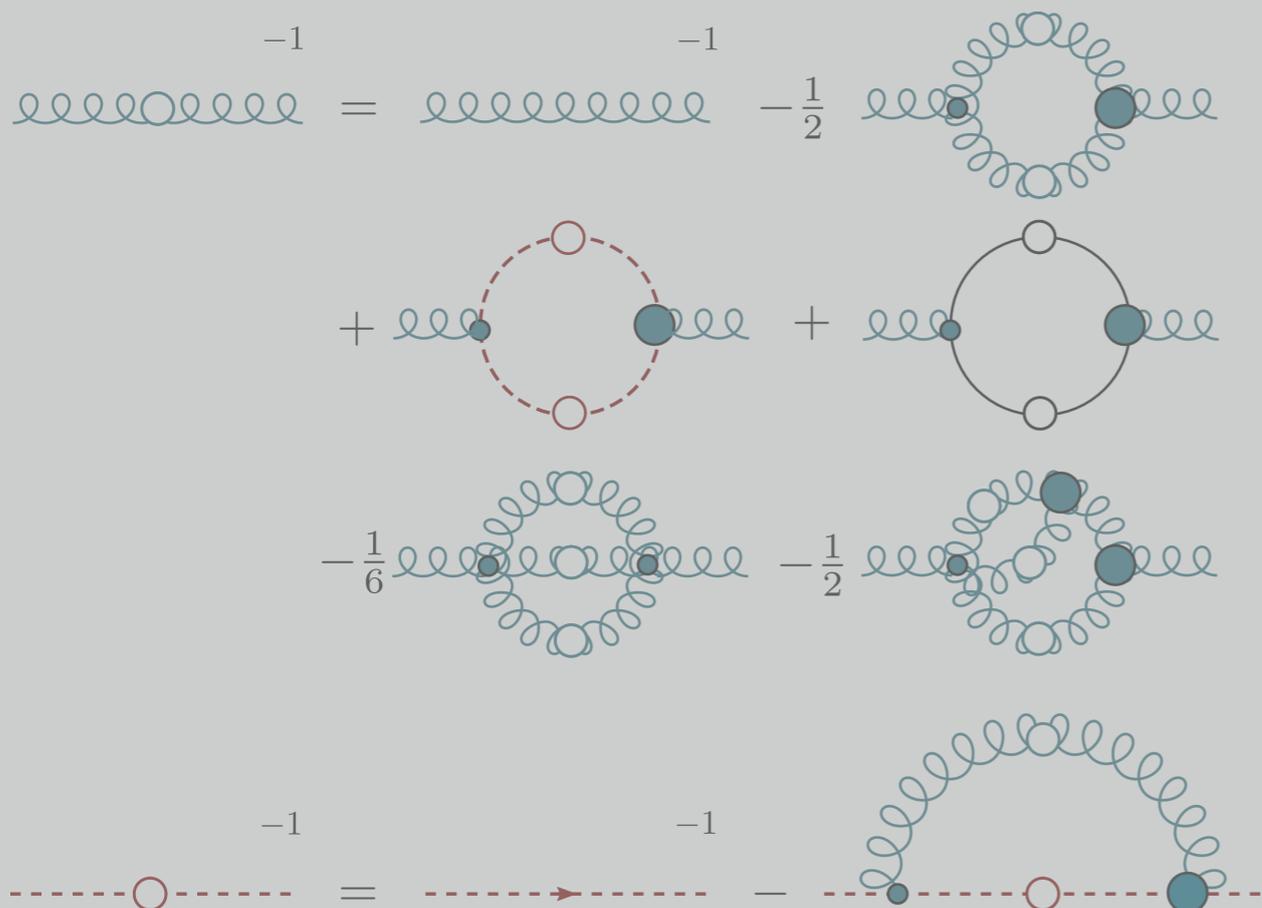
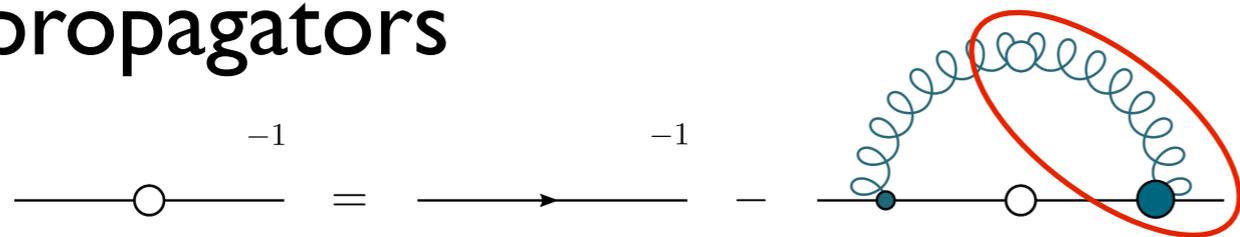


CF,Alkofer, PRD67 (2003) 094020
 Williams, CF,Heupel, PRD93 (2016) 034026
 Huber, PRD 101 (2020) 114009

Dyson-Schwinger equations - “3PI vs RL”

$$Z_{QCD} = \int \mathcal{D}[\Psi, A] \exp \left\{ - \int d^4x \left(\bar{\Psi} (i\not{D} - m) \Psi - \frac{1}{4} (F_{\mu\nu}^a)^2 \right) \right\}$$

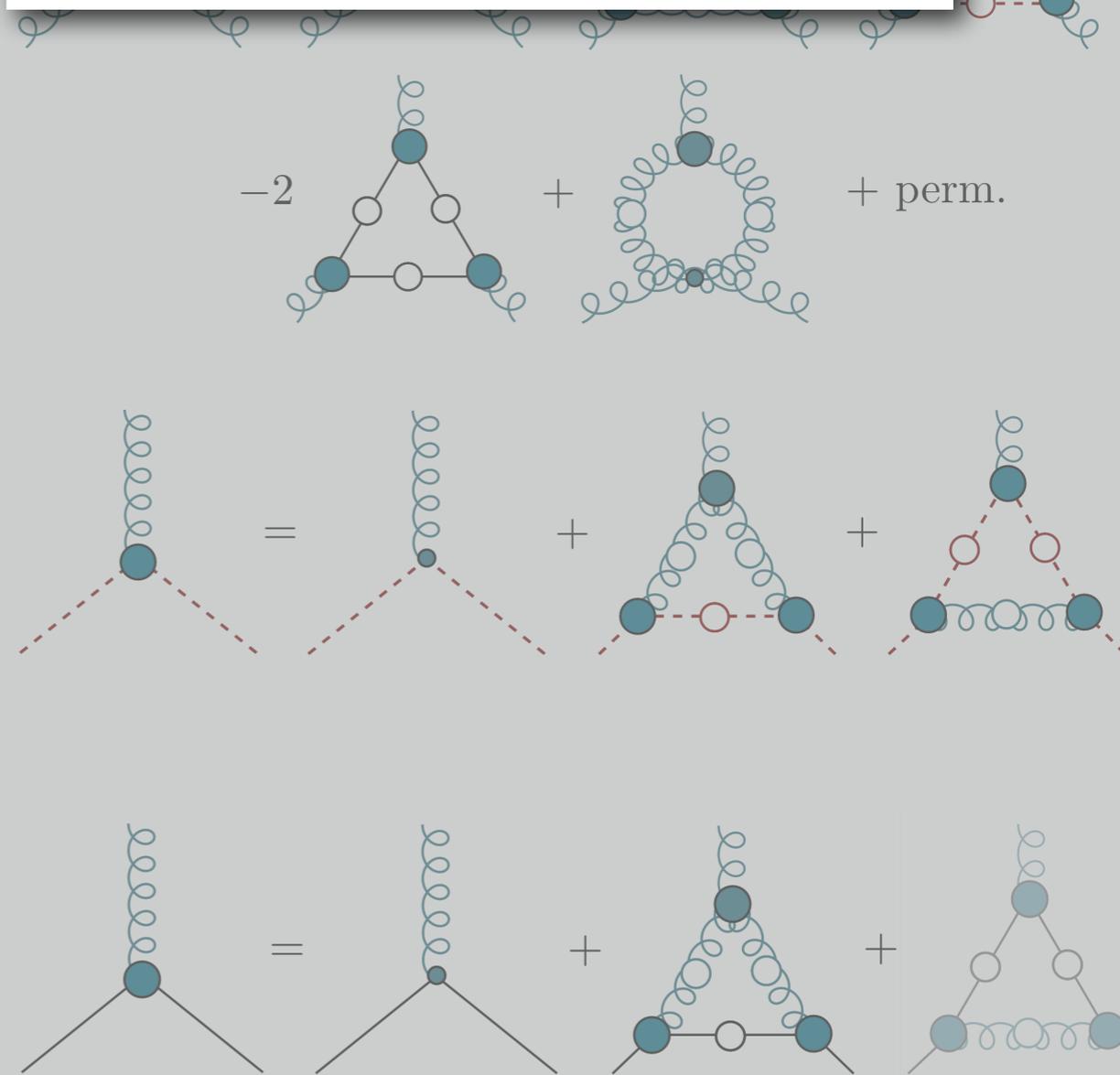
propagators



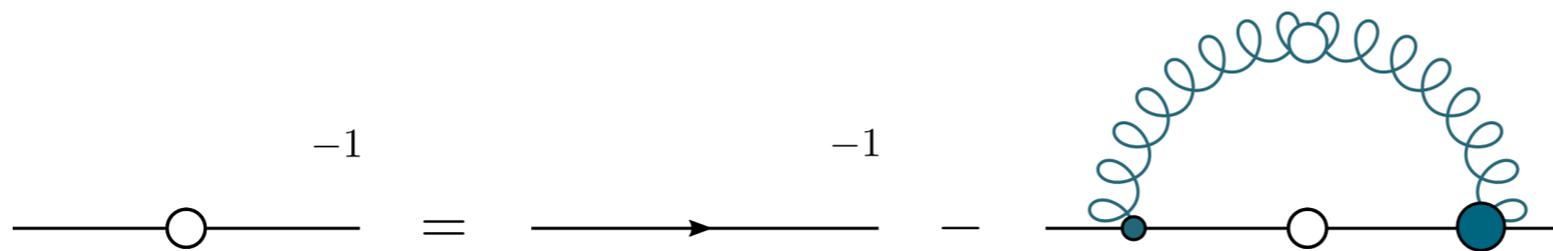
CF,Alkofer, PRD67 (2003) 094020
 Williams, CF, Heupel, PRD93 (2016) 034026
 Huber, PRD 101 (2020) 114009

vertices

“rainbow-ladder” (RL) :
 model for gluon+vertex

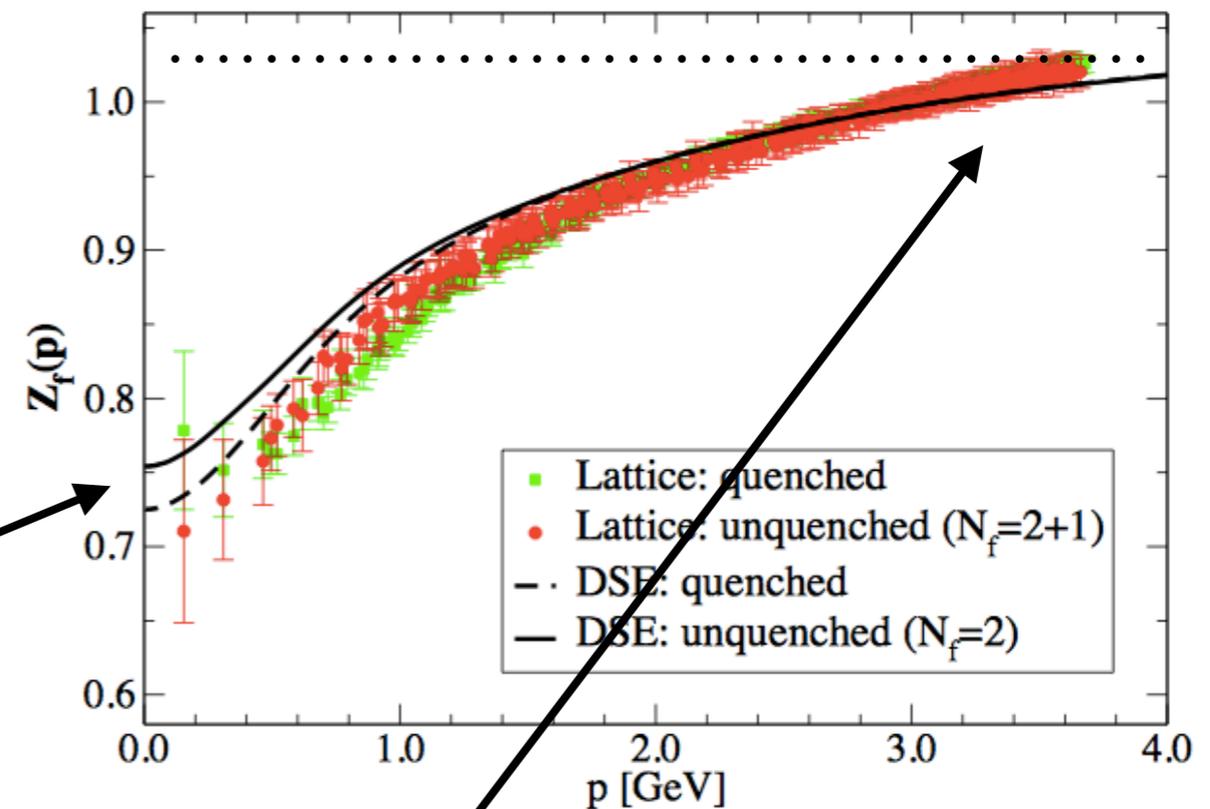
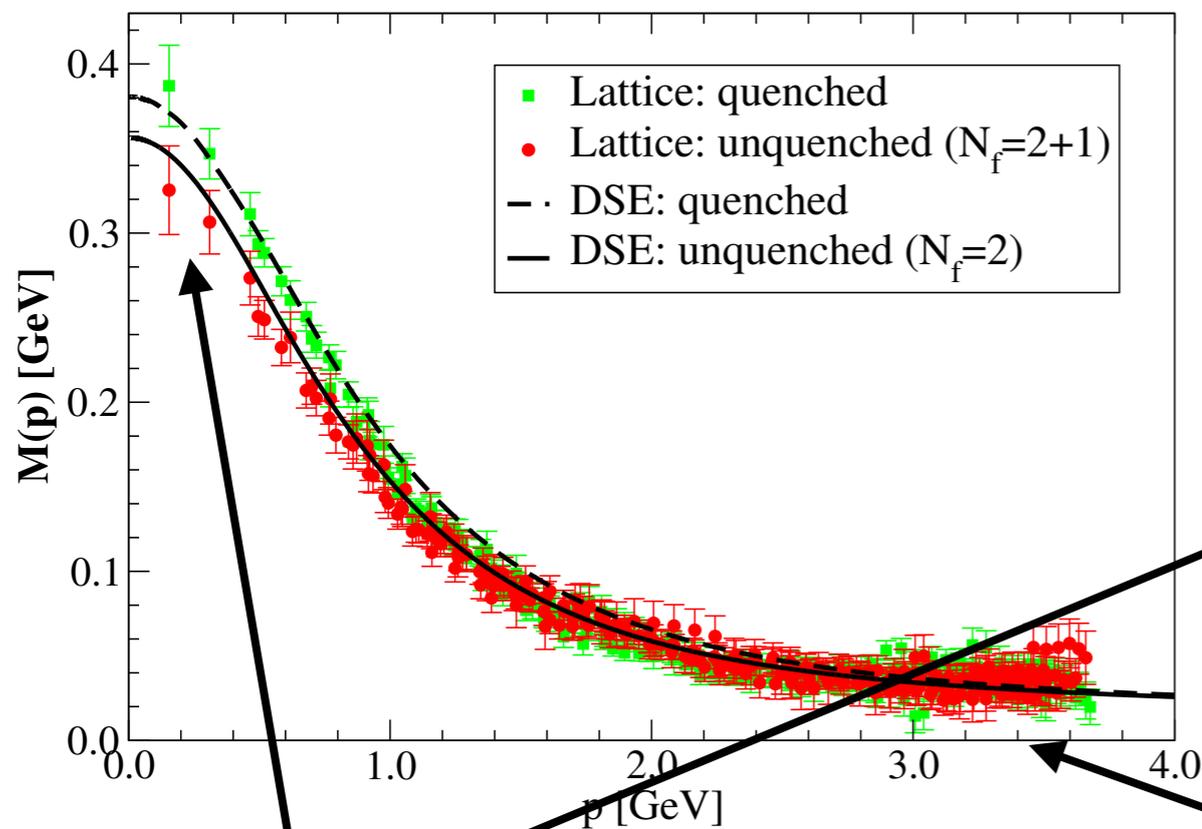


Quarks: mass from interaction



$$S(p) = Z_f(p^2) \frac{-i\not{p} + M(p^2)}{p^2 + M^2(p^2)}$$

DSE: CF, Nickel, Williams, EPJ C 60 (2009) 47
 Williams, CF, Heupel, PRD 93 (2016) 034026
 Lattice: P. O. Bowman, et al PRD 71 (2005) 054507

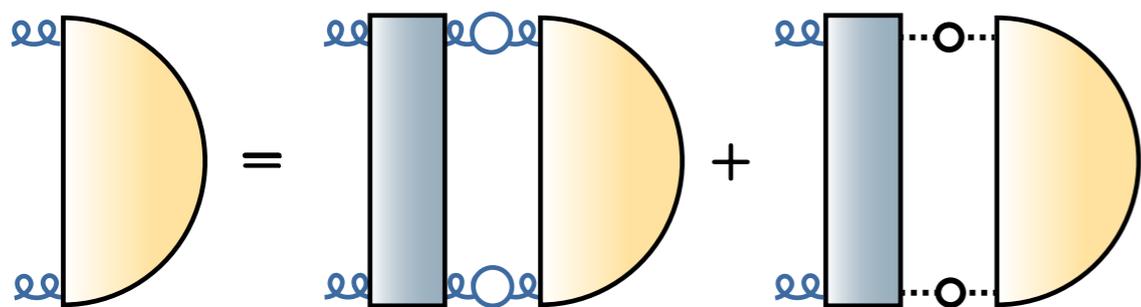
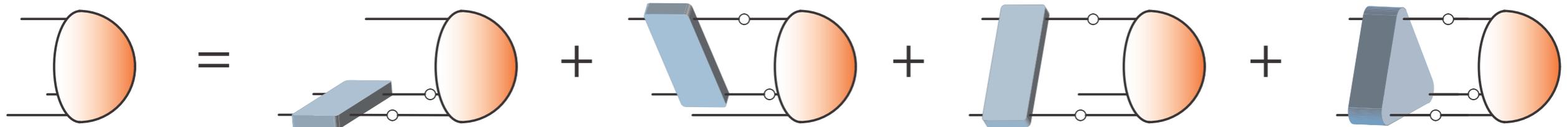
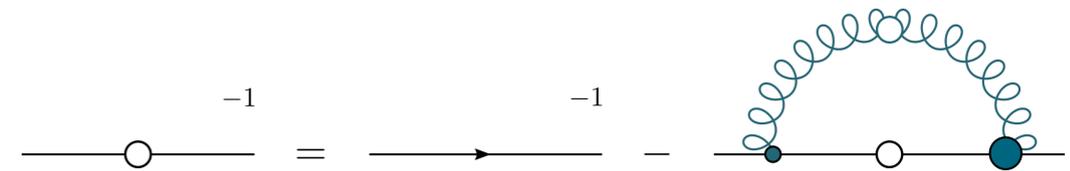
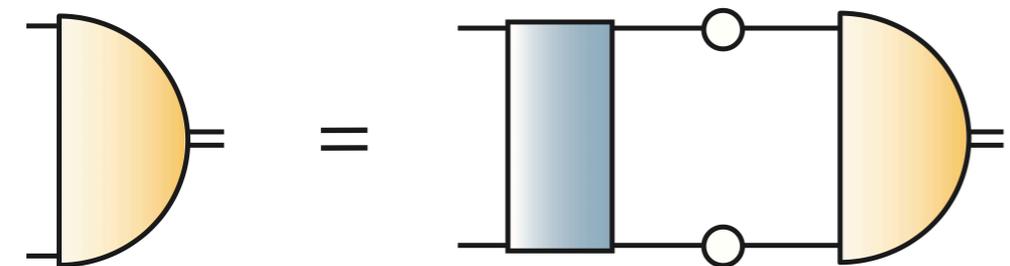


‘constituent quark’:
 large mass; very composite

‘current quark’:
 - small mass; non-composite

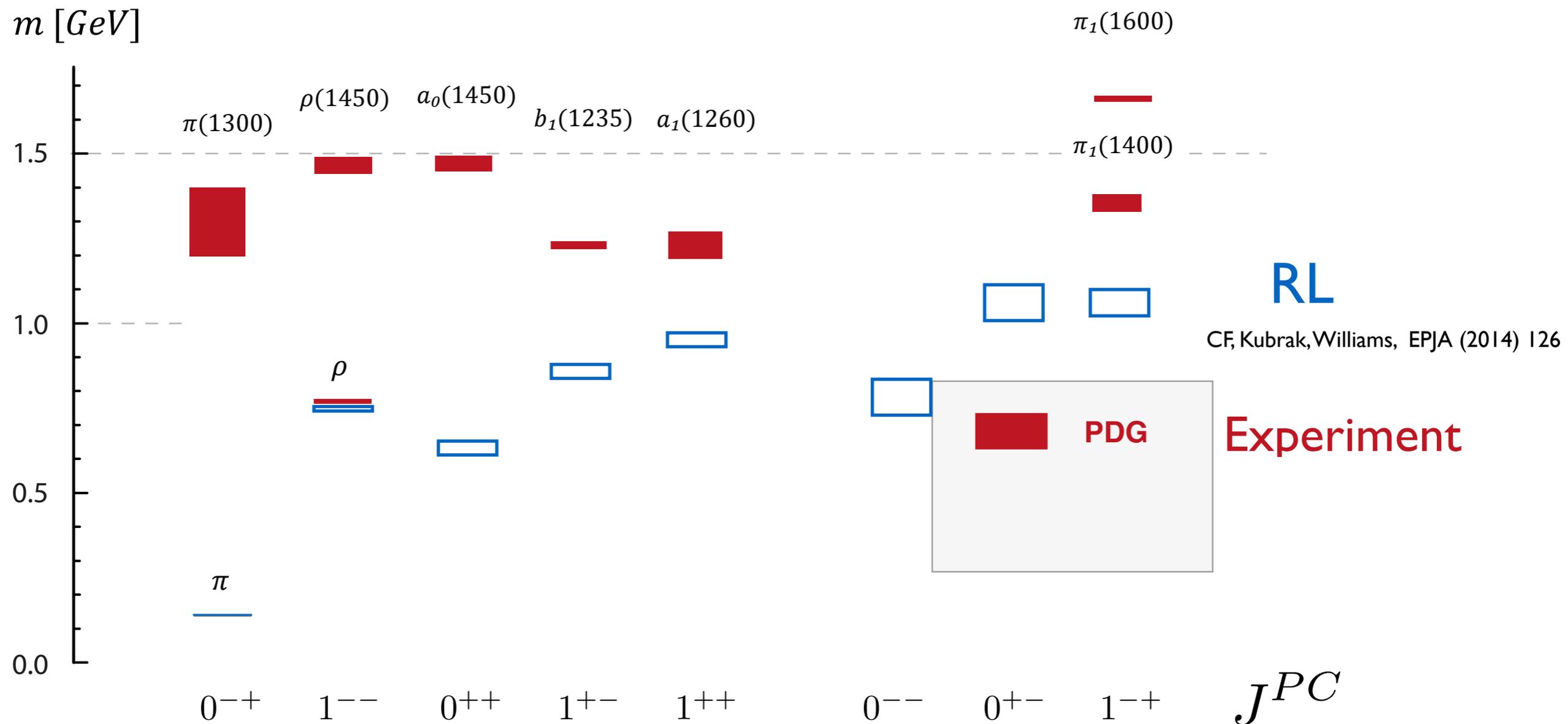
Bound states and Bethe-Salpeter equations

BSEs:

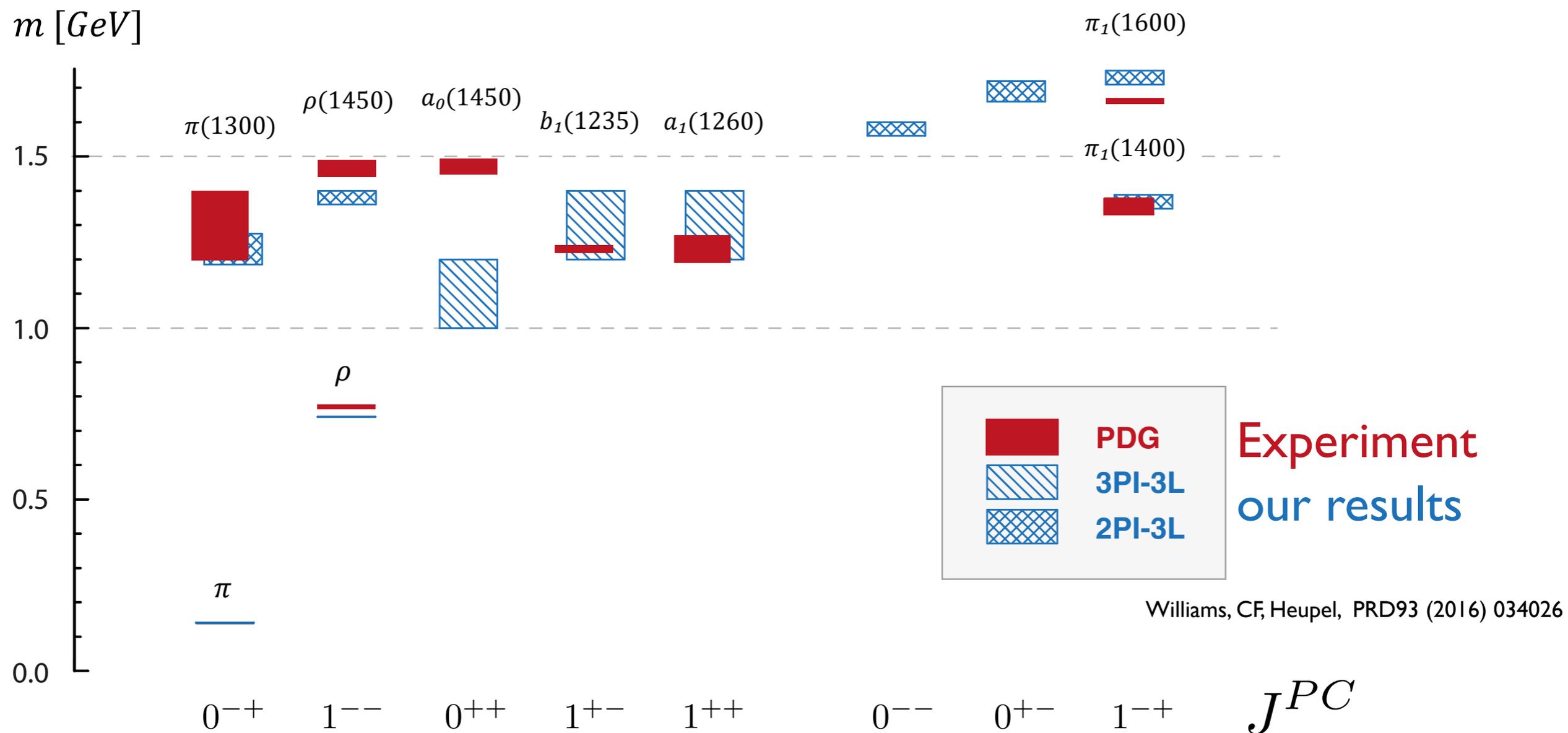


Eigenvalue equations: masses and wave functions

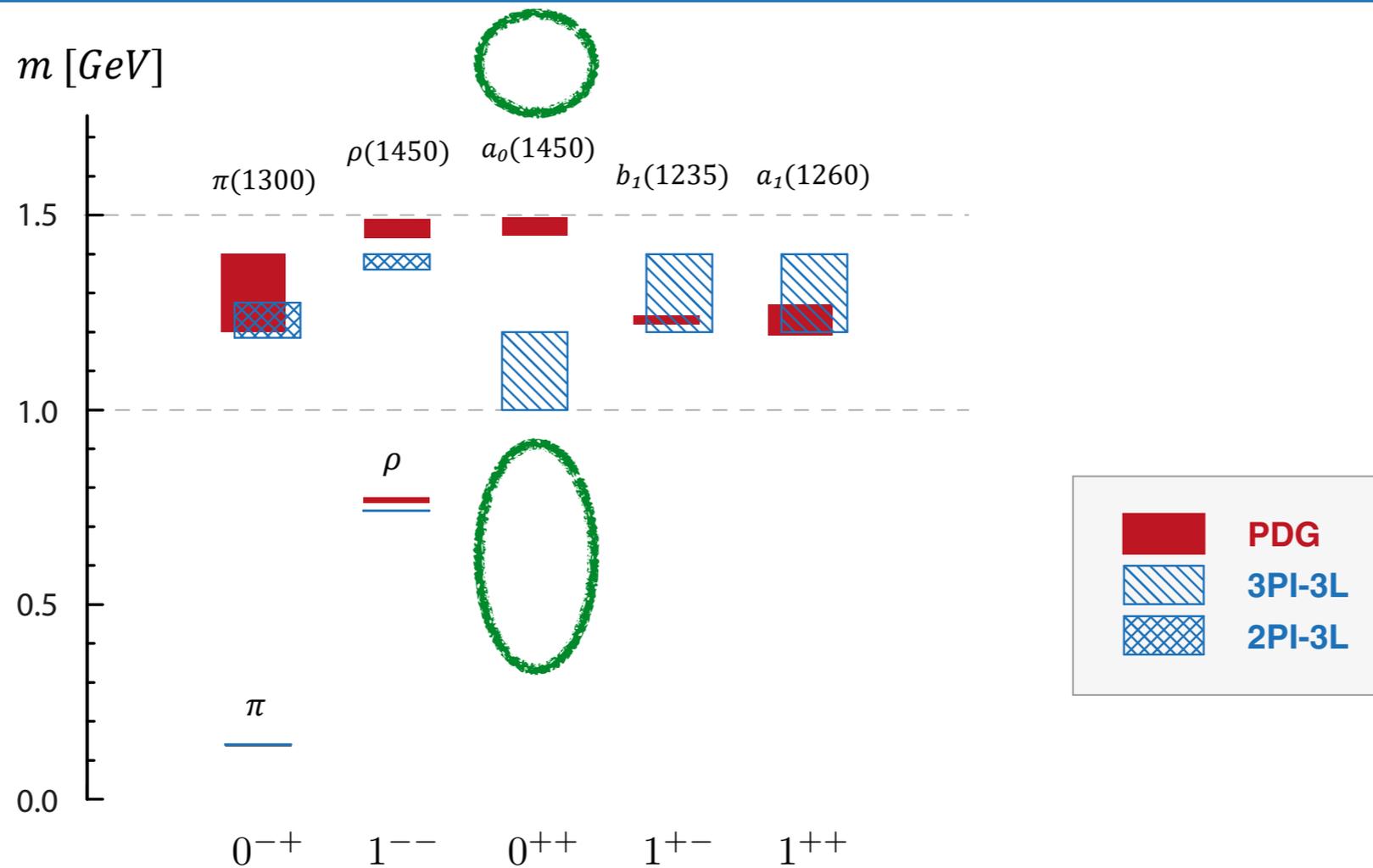
Light meson spectrum - full 3PI-calculation



Light meson spectrum - full 3PI-calculation



- good agreement with experiment in most channels
- special channels:
 pseudoscalar 0^{-+} : (pseudo-) Goldstone bosons
 scalar 0^{++} : complicated channel...



1. Conventional mesons

2. Pure Yang-Mills: glueballs

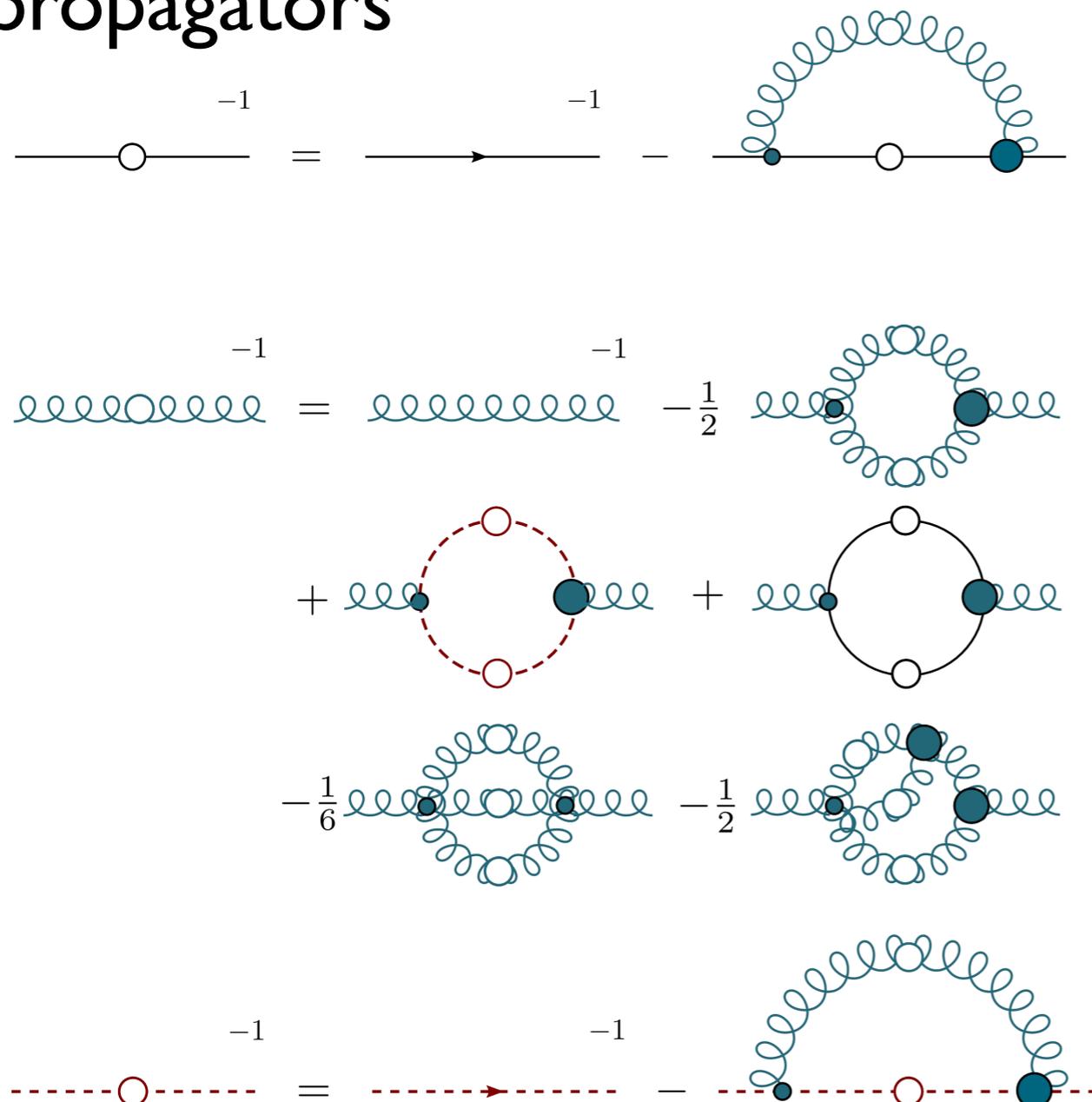
3. Light four-quark states: the $f_0(500)$

4. Heavy-light four-quark states

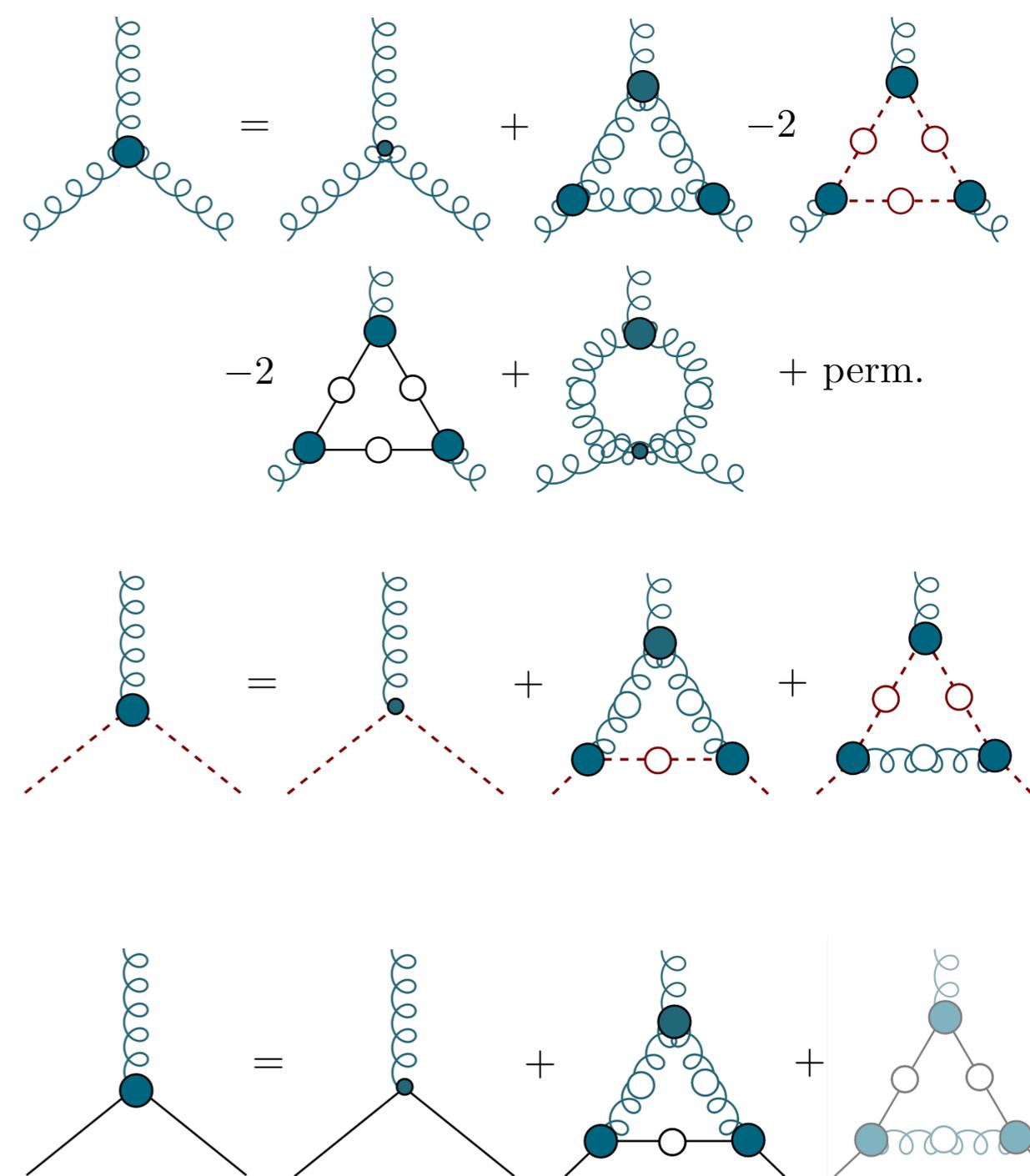
Dyson-Schwinger equations

$$Z_{QCD} = \int \mathcal{D}[\Psi, A] \exp \left\{ - \int d^4x \left(\bar{\Psi} (i \not{\partial} - m) \Psi + \frac{1}{4} F_{\mu\nu}^2 + \bar{\Psi} g \not{A} \Psi \right) \right\}$$

propagators



vertices



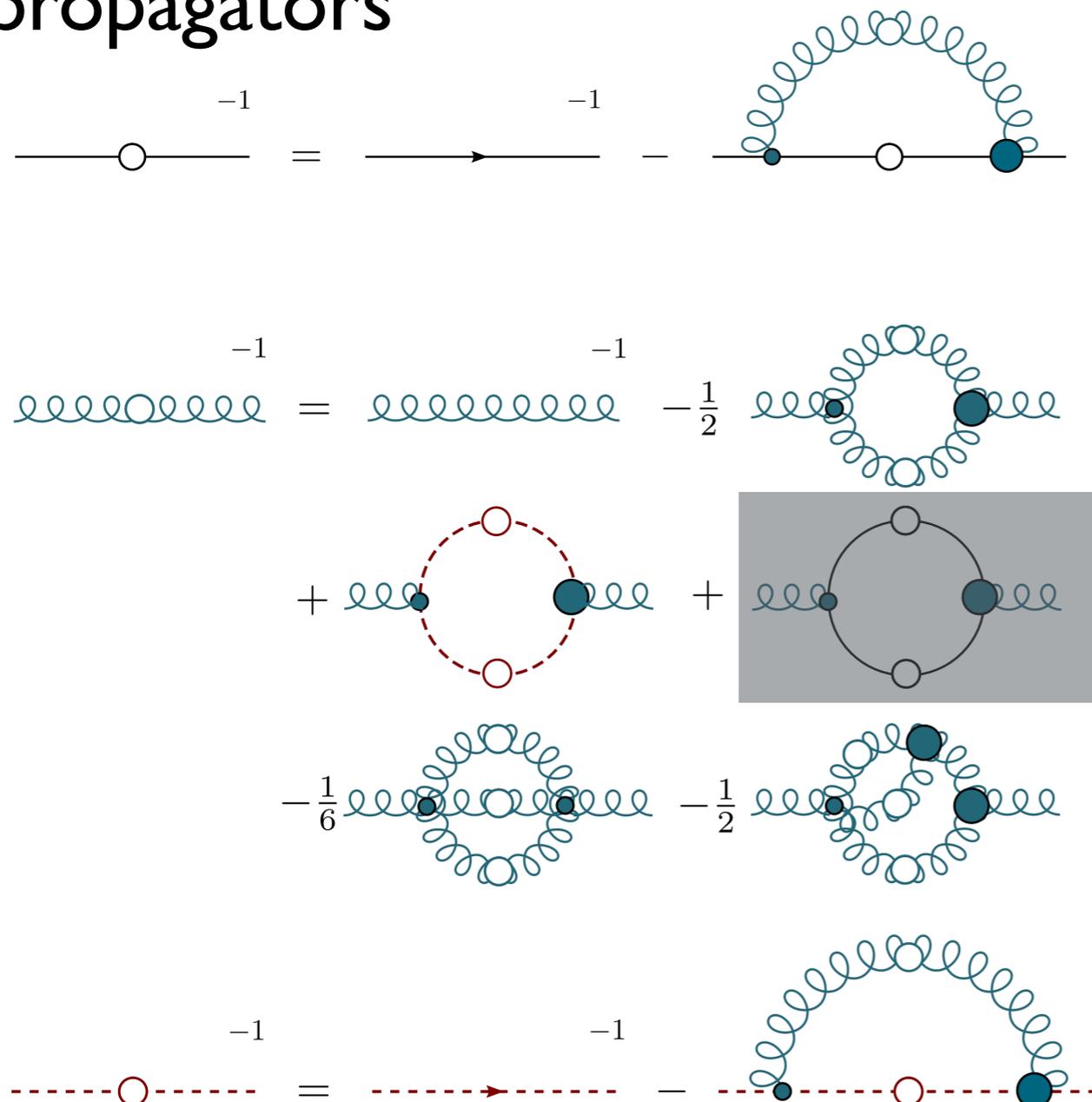
CF,Alkofer, PRD67 (2003) 094020
 Williams, CF,Heupel, PRD93 (2016) 034026
 Huber, PRD 101 (2020) 114009

Dyson-Schwinger equations

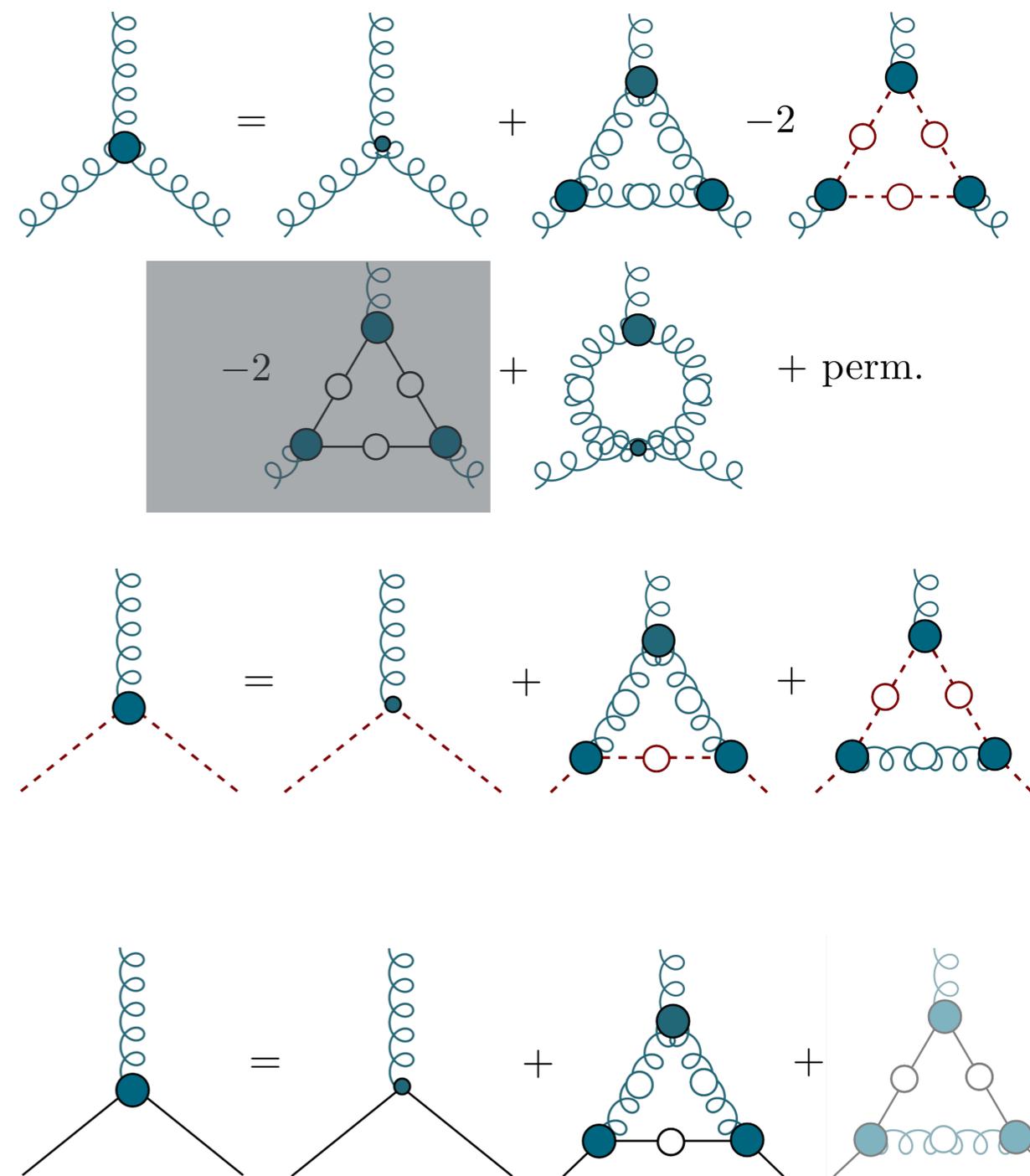
“quenched”

$$Z_{QCD} = \int \mathcal{D}[\Psi, A] \exp \left\{ - \int d^4x \left(\bar{\Psi} (i \not{D} - m) \Psi + \frac{1}{4} F_{\mu\nu}^2 \right) \right\}$$

propagators



vertices



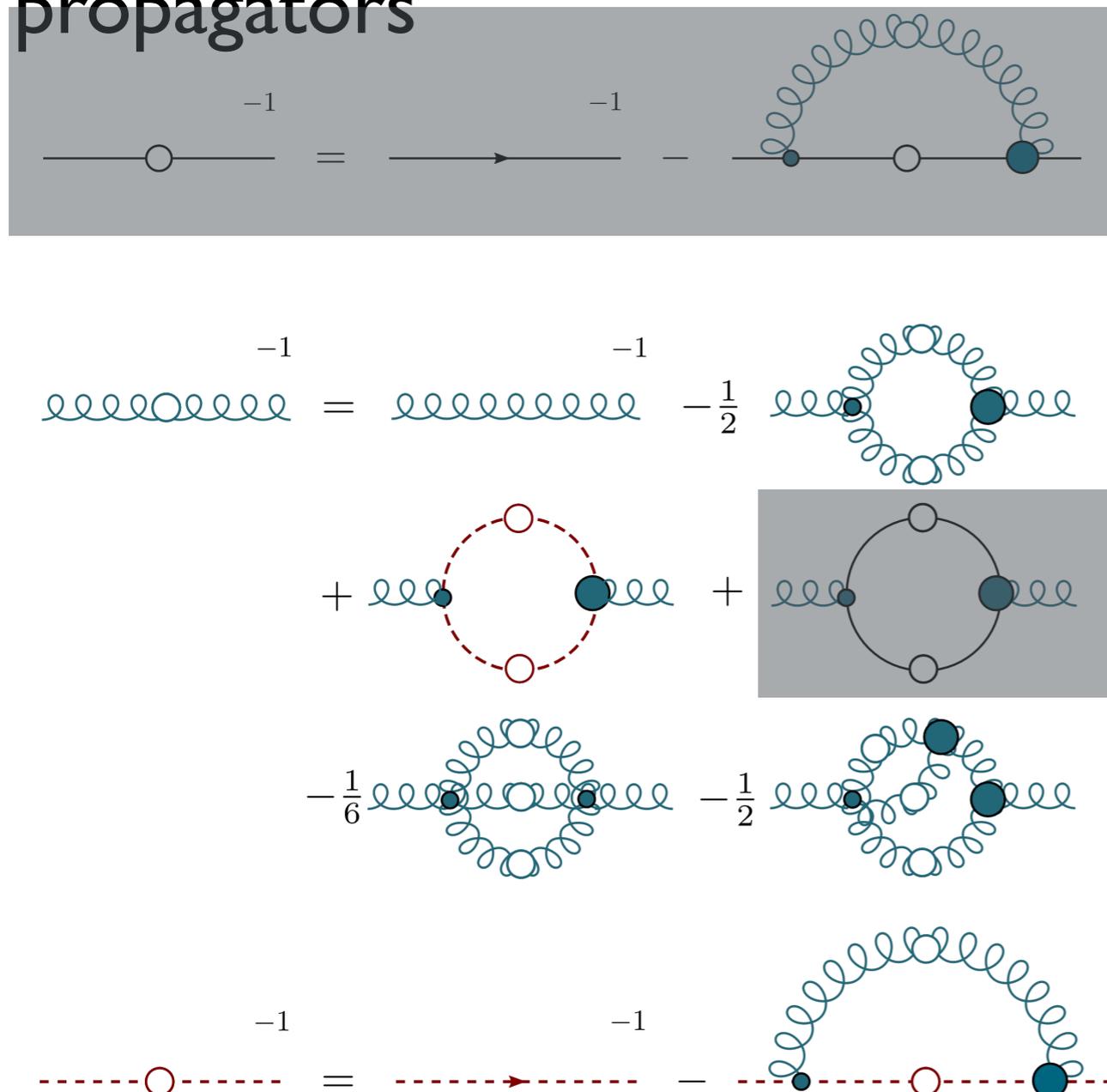
CF,Alkofer, PRD67 (2003) 094020
 Williams, CF,Heupel, PRD93 (2016) 034026
 Huber, PRD 101 (2020) 114009

Dyson-Schwinger equations

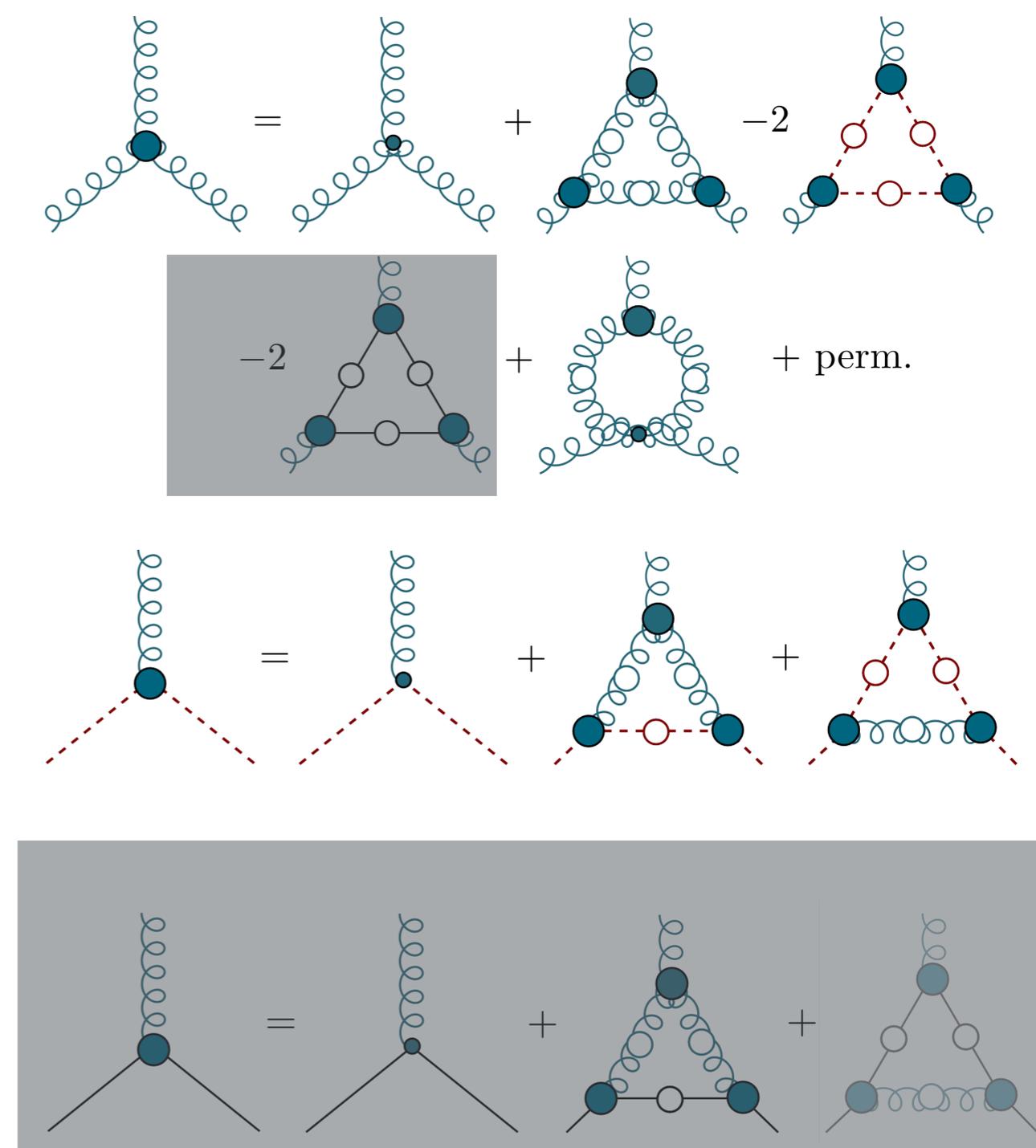
$$Z_{QCD} = \int \mathcal{D}[\Psi, A] \exp \left\{ - \int d^4x \left(\bar{\Psi} (i \not{\partial} - m) \Psi + \frac{1}{4} F_{\mu\nu}^2 \right) \right\}$$

pure YM-Theory

propagators

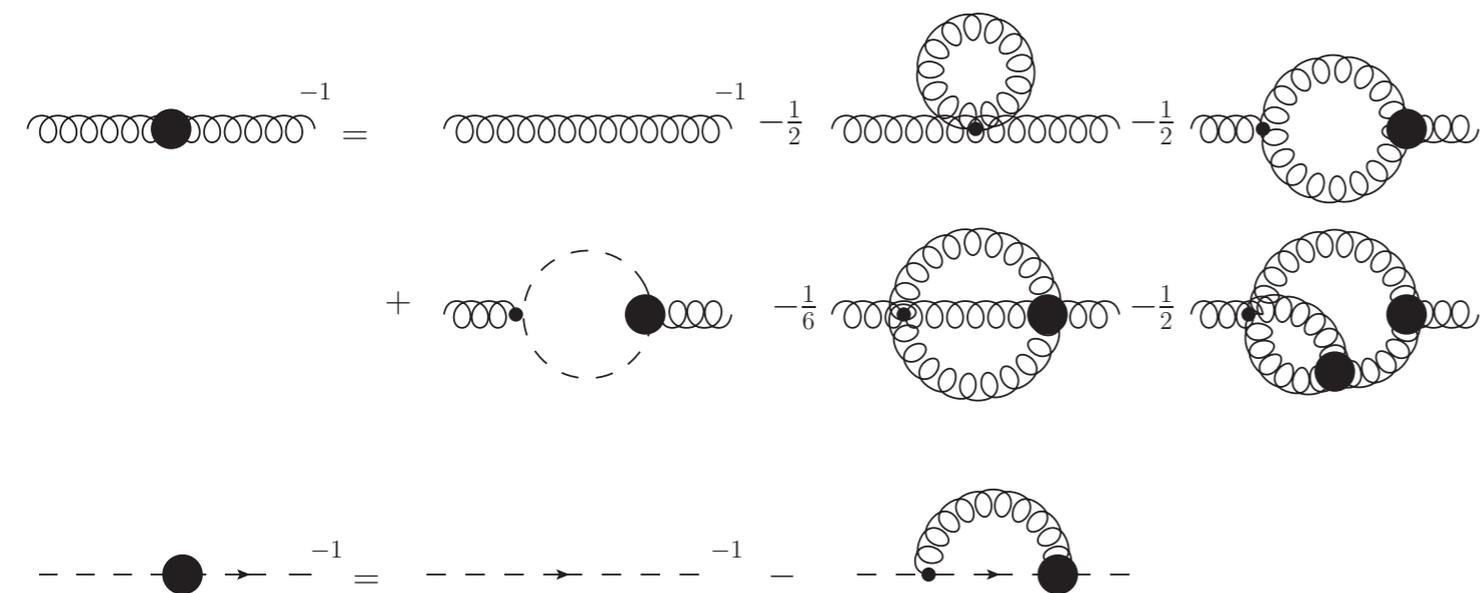


vertices



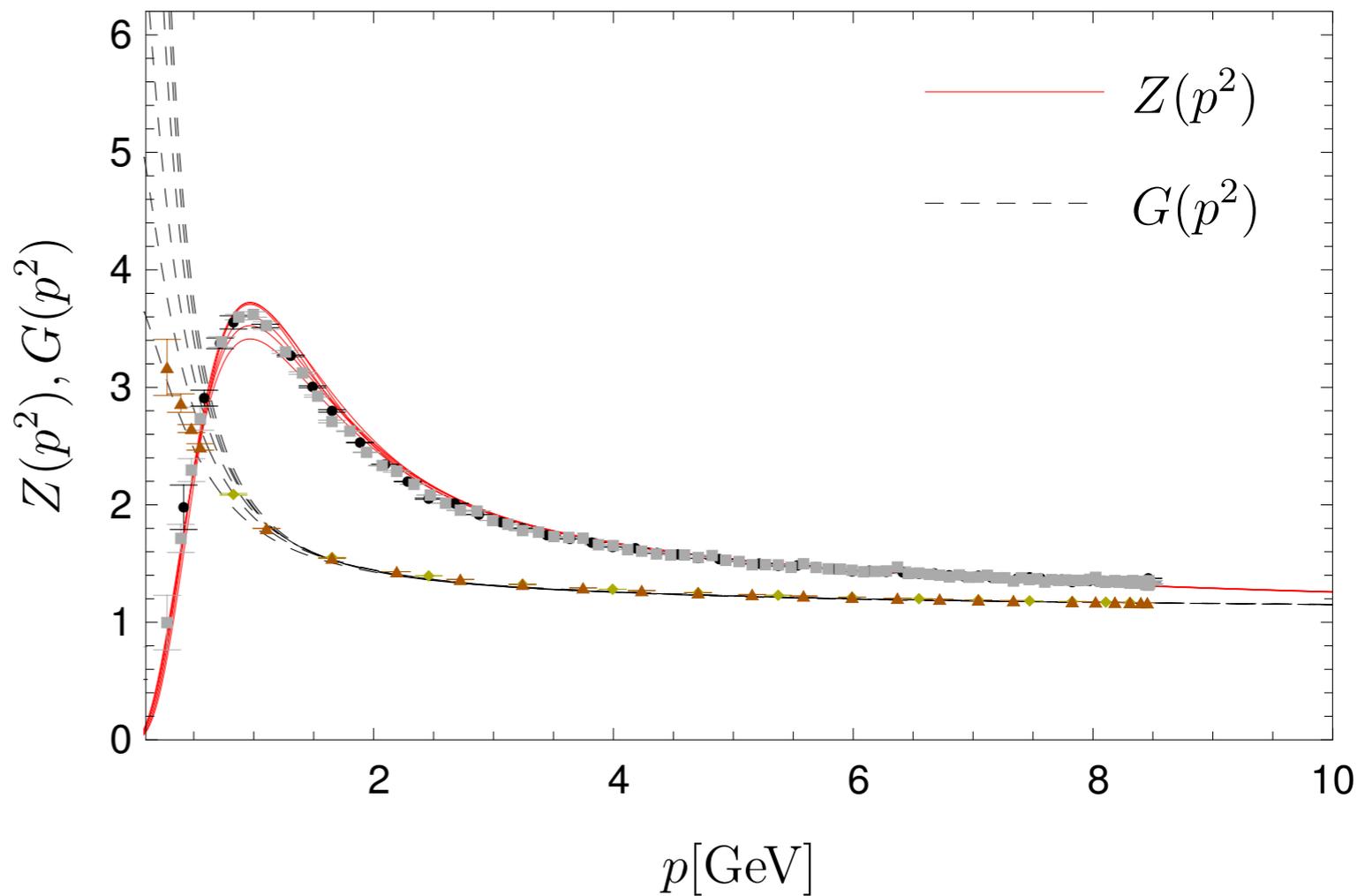
CF,Alkofer, PRD67 (2003) 094020
 Williams, CF,Heupel, PRD93 (2016) 034026
 Huber, PRD 101 (2020) 114009

Landau gauge gluon propagator



$$D_{\mu\nu}(p) = \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) \frac{Z(p^2)}{p^2}$$

$$D_G(p) = \frac{-G(p^2)}{p^2}$$



- spacelike momenta:
good agreement with lattice
- fully dressed gluon appears massive

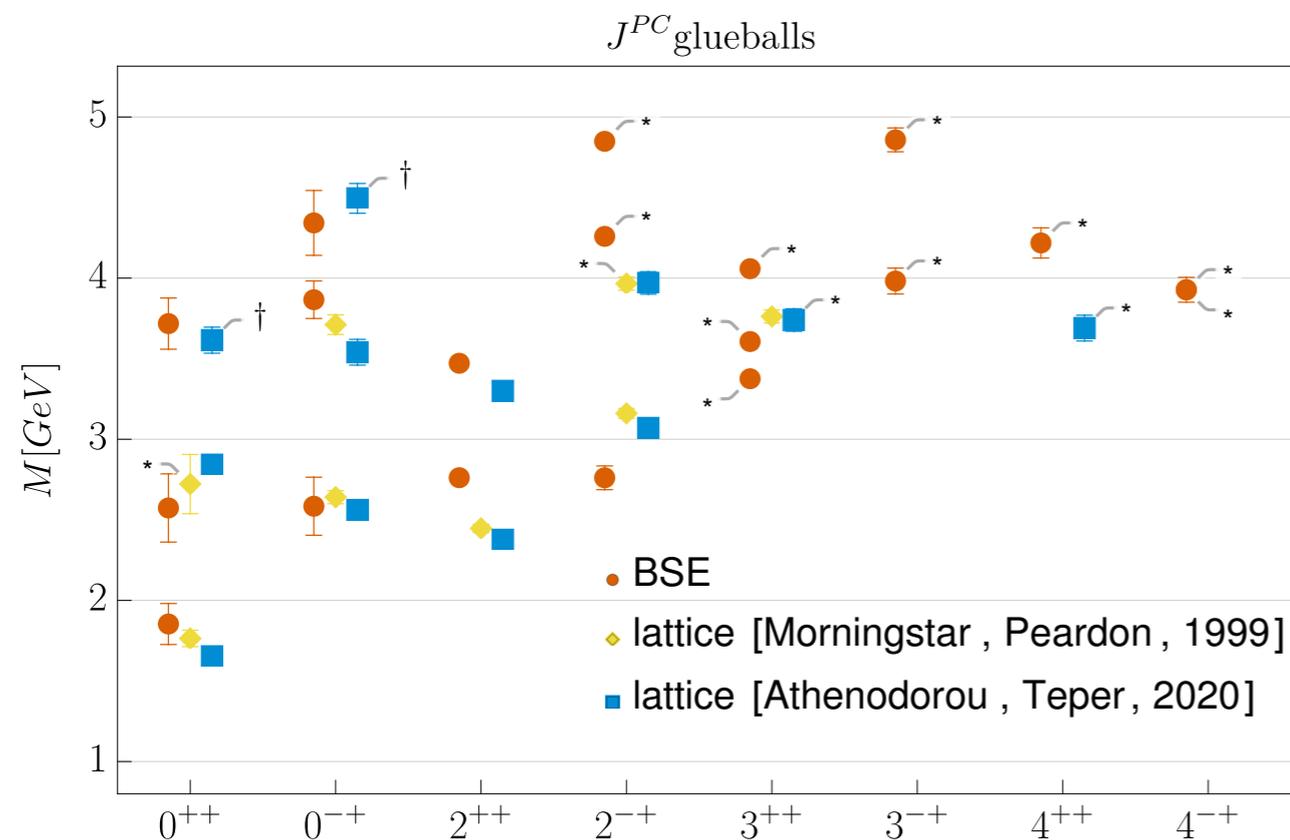
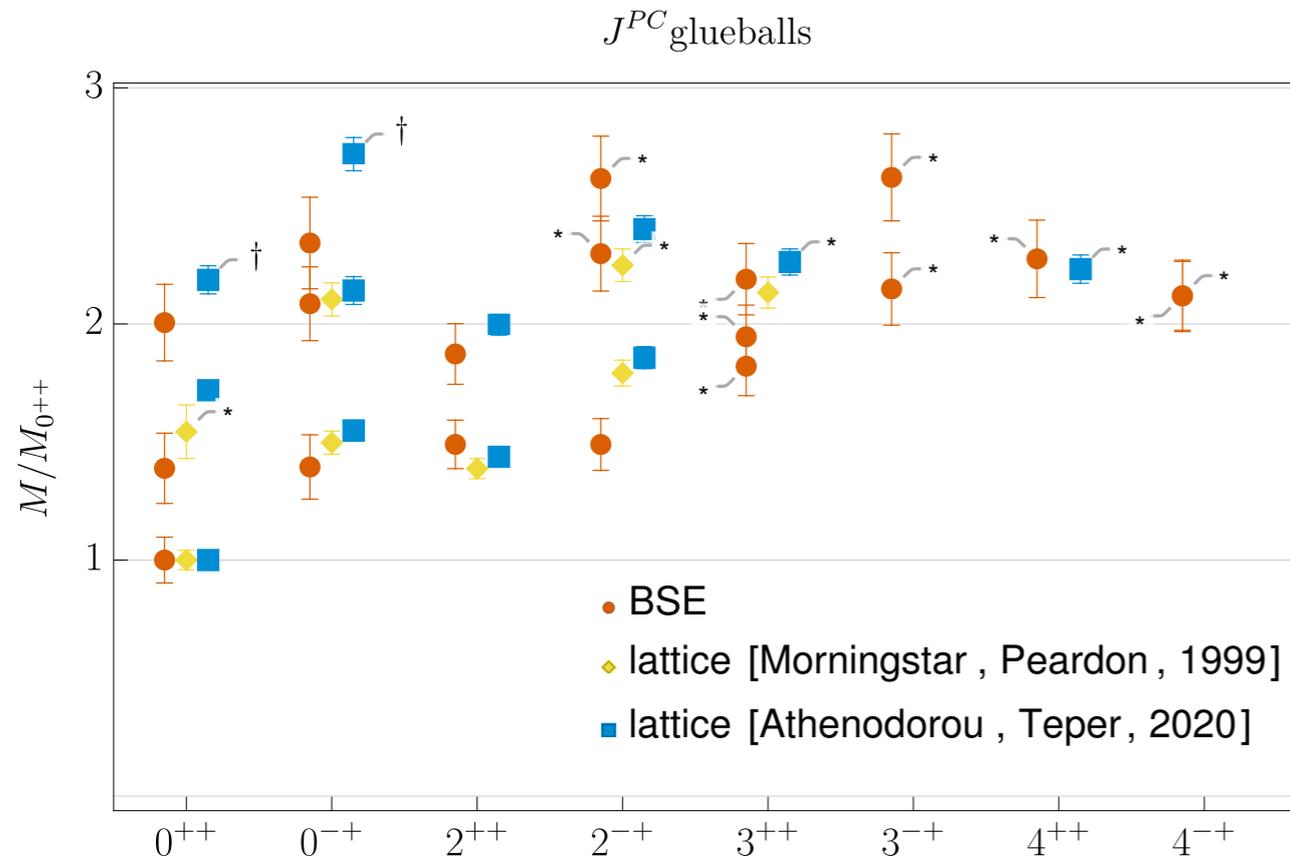
Cornwall PRD 26 (1982);
 Cucchieri, Mendes PoS Lat2007 297
 Aguilar, Binosi, Papavassiliou, PRD 78, 025010 (2008);
 Boucaud et al. JHEP 0806 (2008) 099;
 CF, Maas, Pawłowski, Annals Phys. 324 (2009) 2408

- time-like momenta: work in progress

CF, Huber, PRD 102 (2020) 094005, arXiv:2007.11505

DSE: Huber, PRD 101 (2020) 114009, arXiv:2003.13703
 Lattice: Sternbeck, Müller-Preussker, PLB 726 (2013)

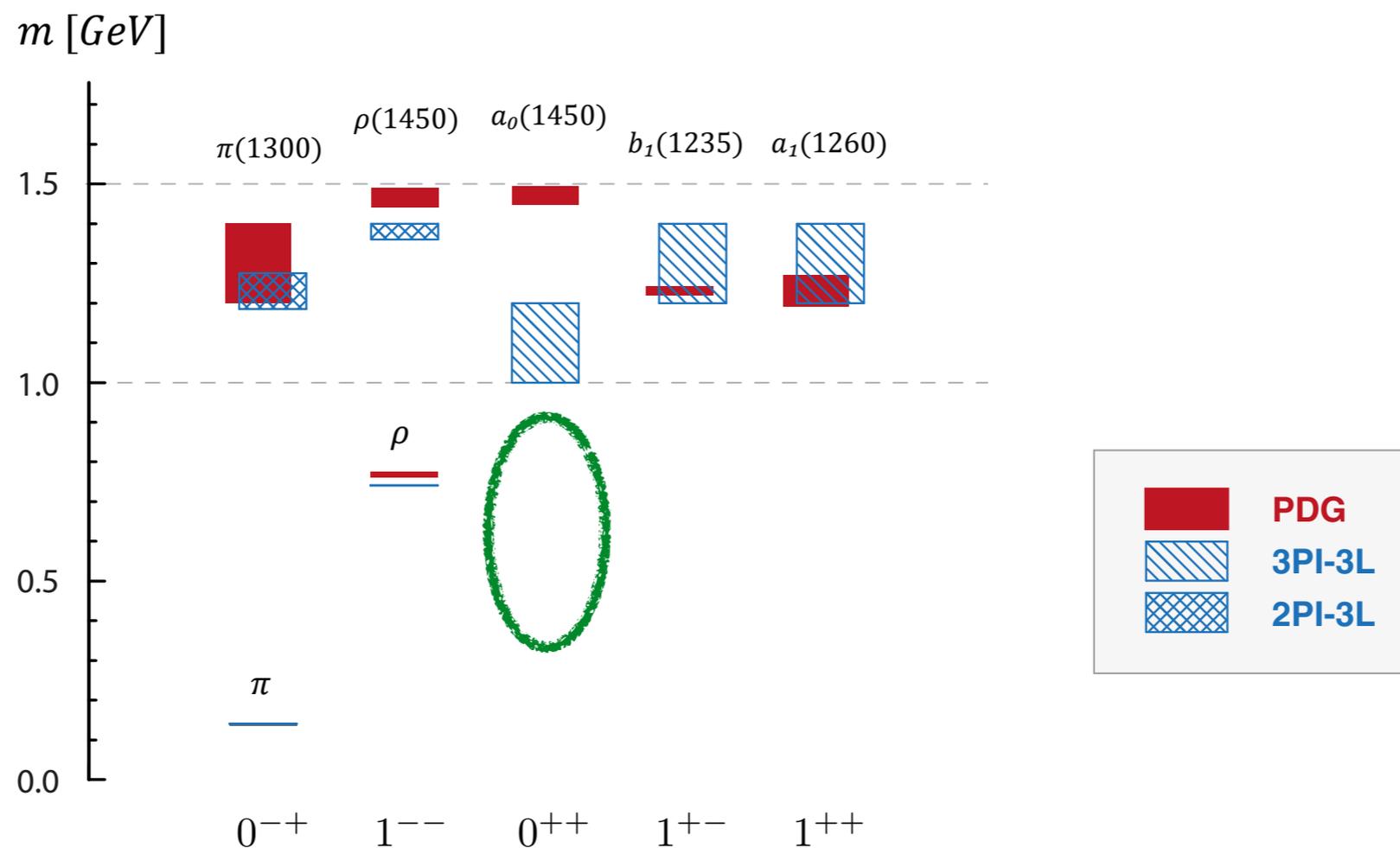
Glueballs: results for different quantum numbers



see talk of Markus Huber

- spectrum:
very good agreement
lattice vs. DSE/BSE
- predictions for some
channels

CF, Huber, Sanchis-Alepuz, EPJC 80 (2020) [arXiv:2004.00415]
 Huber, CF, Sanchis-Alepuz, EPJC 81 (2021) [arXiv:2110.09180]



1. Conventional mesons

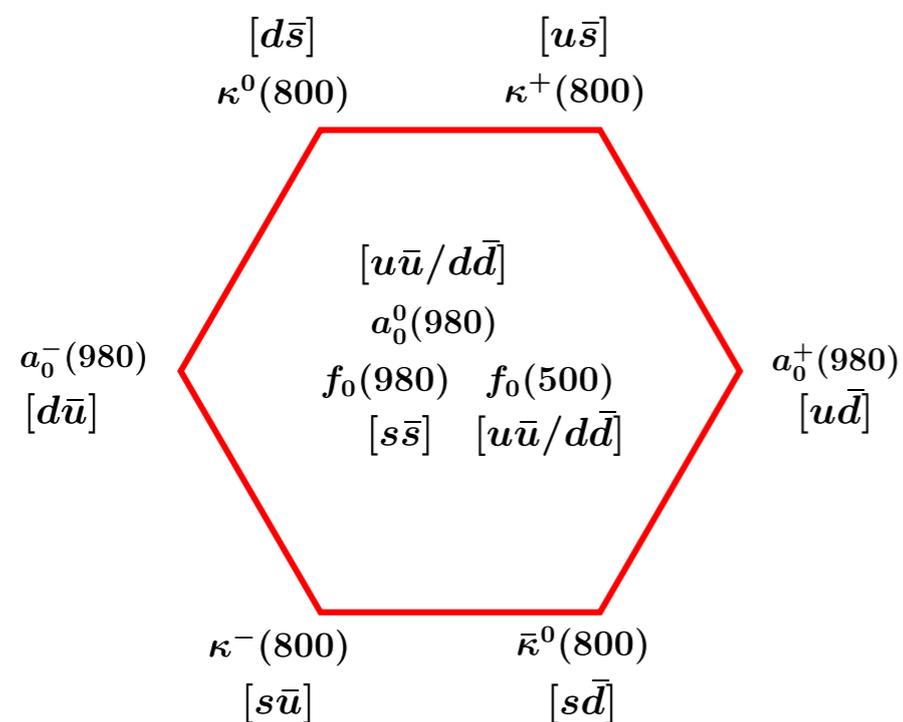
2. Pure Yang-Mills: glueballs

3. Light four-quark states: the $f_0(500)$

4. Heavy-light four-quark states

Light mesons with $\bar{q}q$ -content

Light scalar mesons:

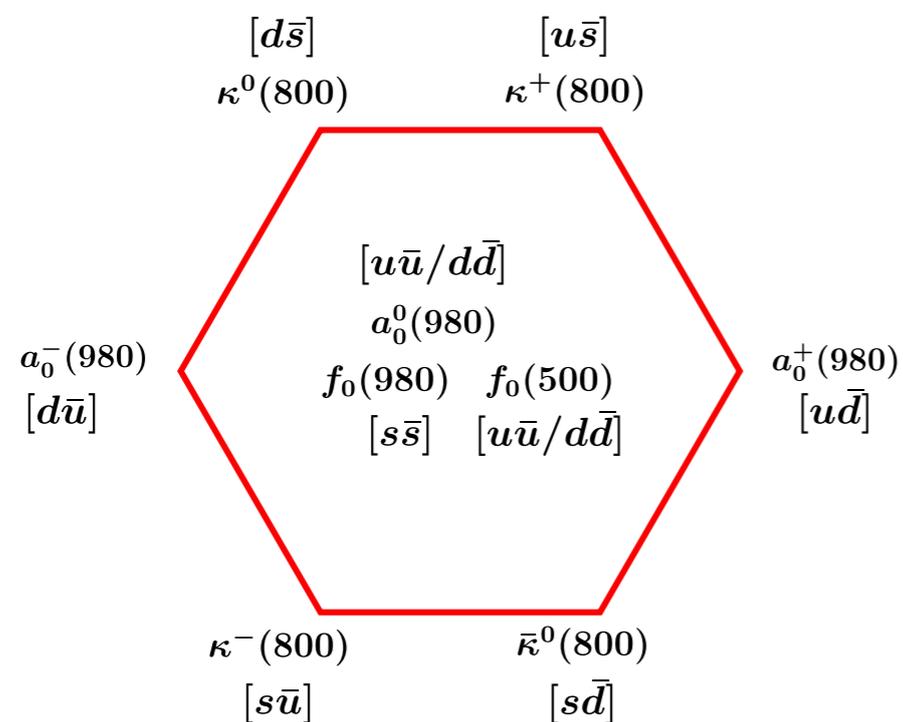


wrong level ordering

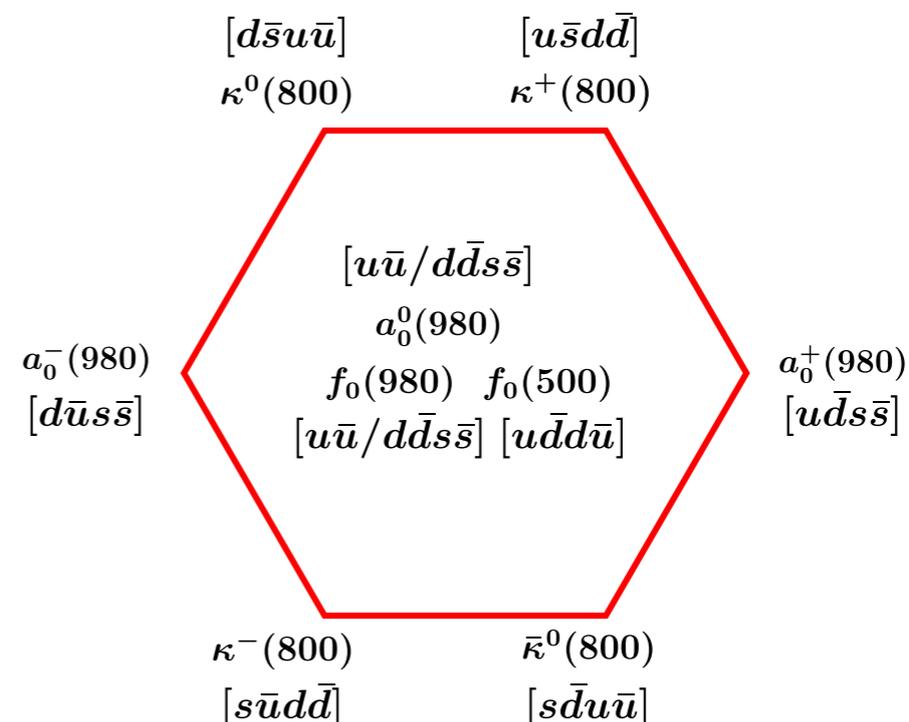
R. L. Jaffe, Phys. Rev. D 15, 267 (1977)

Light mesons with $\bar{q}q$ -content

Light scalar mesons:



wrong level ordering



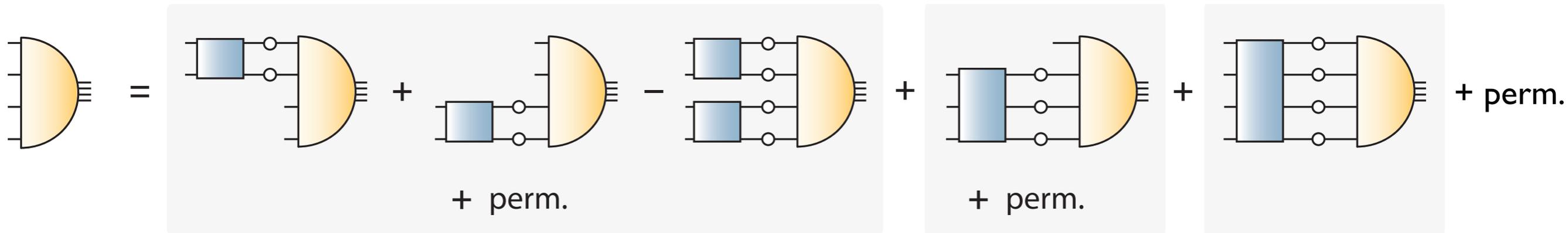
correct level ordering

R. L. Jaffe, Phys. Rev. D 15, 267 (1977)

Tetraquarks from the four-body equation

Exact equation:

Kvinikhidze & Khvedelidze, Theor. Math. Phys. 90 (1992)
Heupel, Eichmann, CF, PLB 718 (2012) 545-549
Eichmann, CF, Heupel, PLB 753 (2016) 282-287



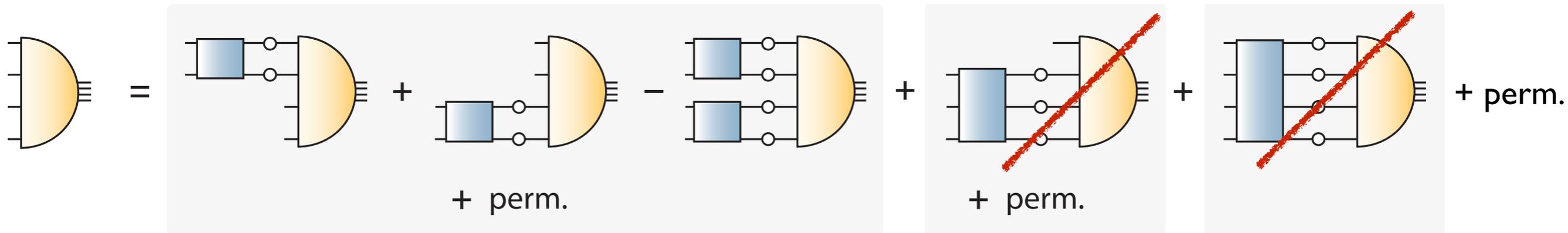
Two-body interactions

Three- and four-body interactions

Tetraquarks from the four-body equation

Exact equation:

Kvinikhidze & Khvedelidze, Theor. Math. Phys. 90 (1992)
Heupel, Eichmann, CF, PLB 718 (2012) 545-549
Eichmann, CF, Heupel, PLB 753 (2016) 282-287



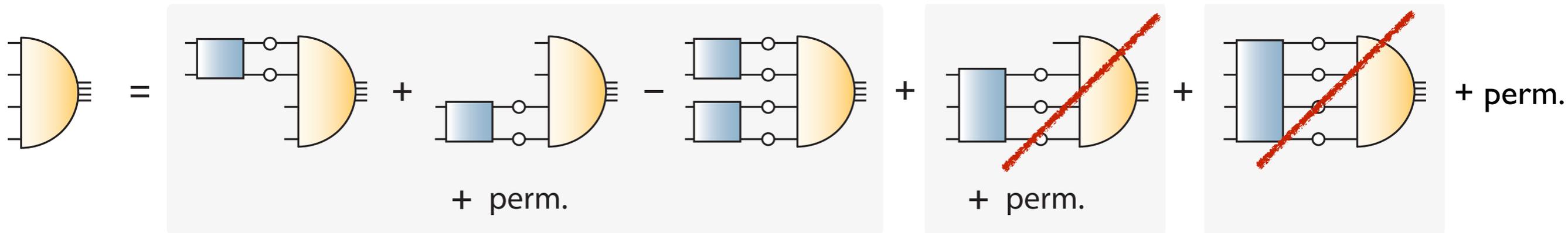
Two-body interactions

Three- and four-body interactions

Tetraquarks from the four-body equation

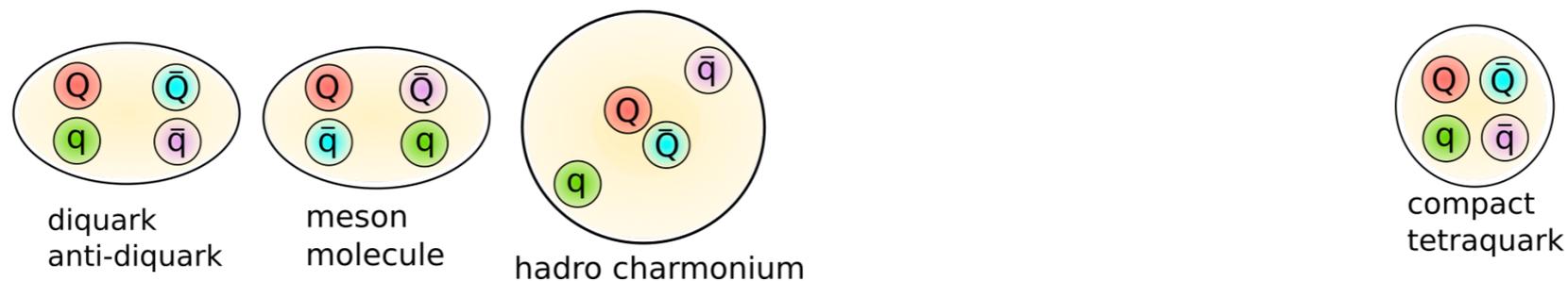
Exact equation:

Kvinikhidze & Khvedelidze, Theor. Math. Phys. 90 (1992)
 Heupel, Eichmann, CF, PLB 718 (2012) 545-549
 Eichmann, CF, Heupel, PLB 753 (2016) 282-287



Two-body interactions

Three- and four-body interactions

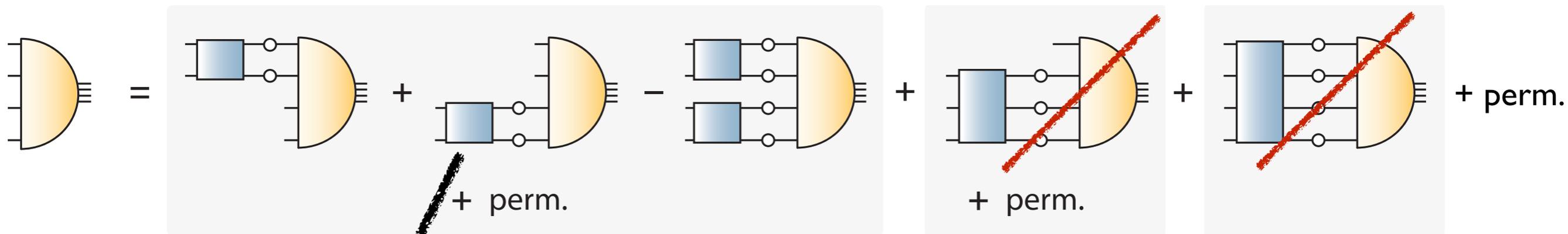


- Two-body interactions: allow for **internal clustering**
- use rainbow-ladder approximation...

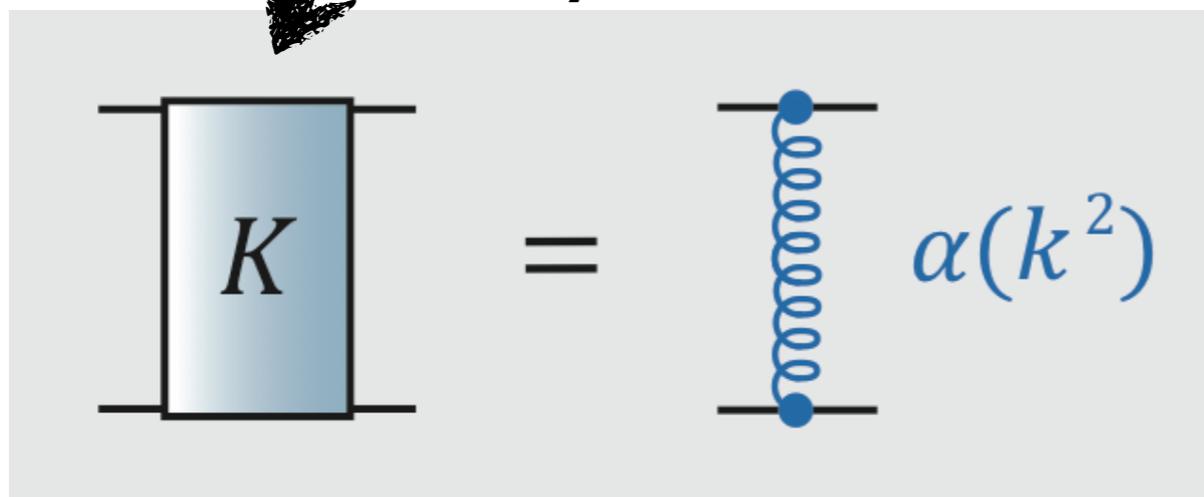
Tetraquarks from the four-body equation

Exact equation:

Kvinikhidze & Khvedelidze, Theor. Math. Phys. 90 (1992)
 Heupel, Eichmann, CF, PLB 718 (2012) 545-549
 Eichmann, CF, Heupel, PLB 753 (2016) 282-287



Two-body interactions



Three- and four-body interactions

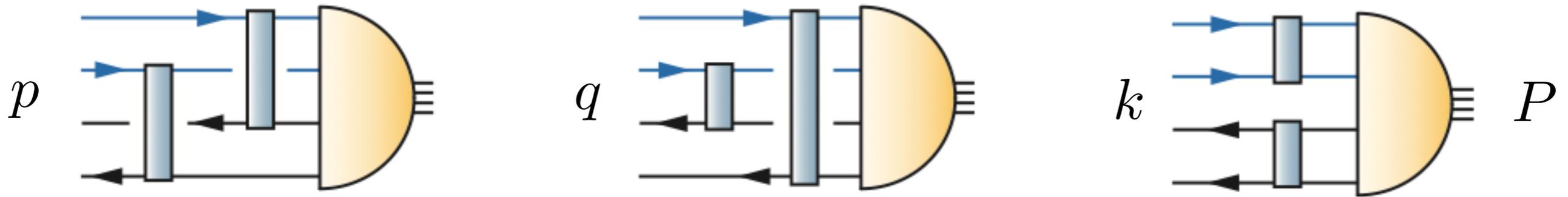
- Input: Non-perturbative quark, quark-gluon interaction



$$\alpha(k^2) = \pi\eta^7 \left(\frac{k^2}{\Lambda^2} \right) e^{-\eta^2 \left(\frac{k^2}{\Lambda^2} \right)} + \alpha_{UV}(k^2)$$

Structure of the amplitude

Scalar tetraquark:



$$\Gamma(P, p, q, k) = \sum_i f_i(s_1, \dots, s_9) \times \tau_i(P, p, q, k) \times color \times flavor$$

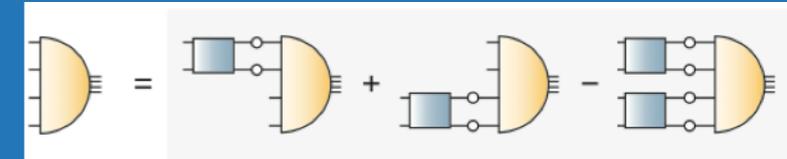
9 Lorentz scalars
(built from P, p, q, k)

256 tensor
structures
(scalar tetra)

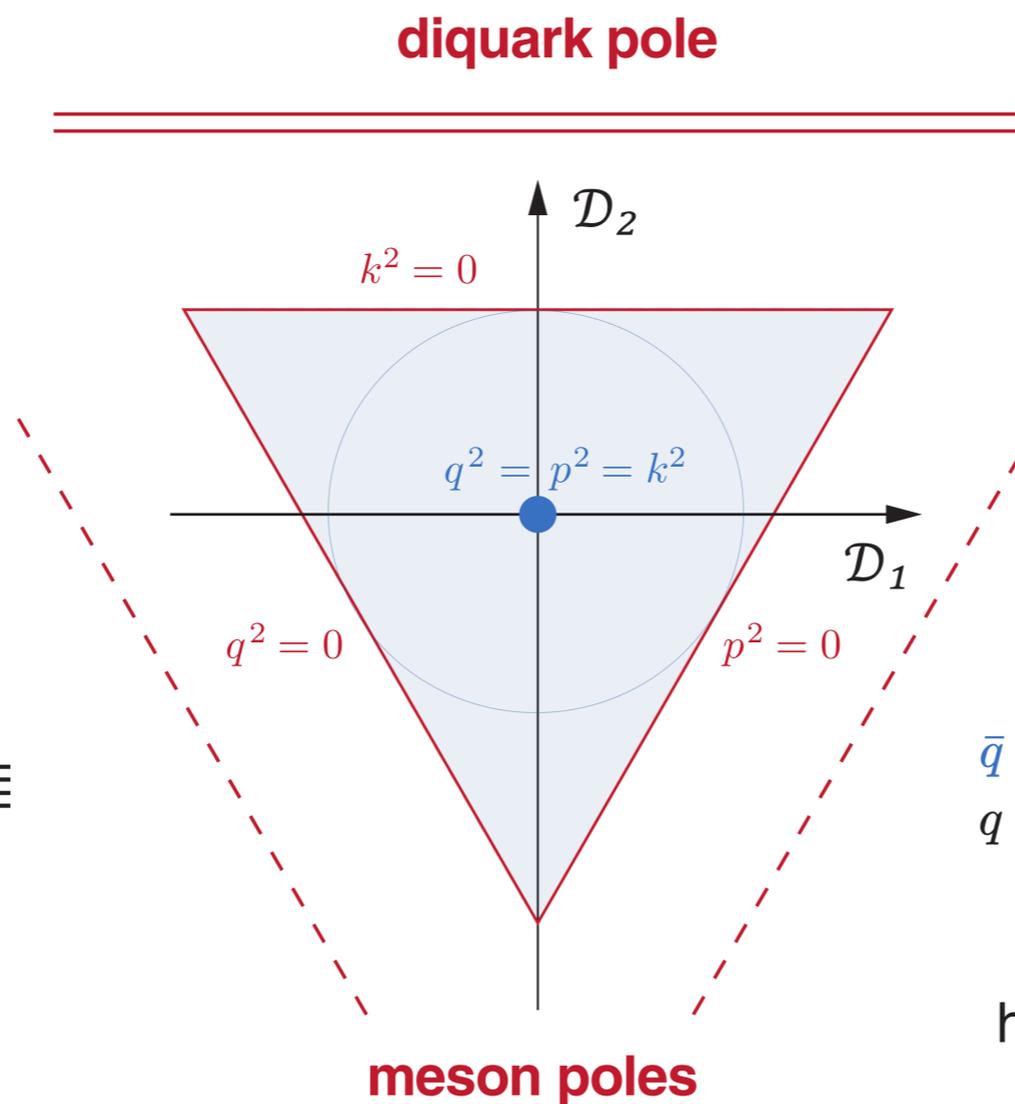
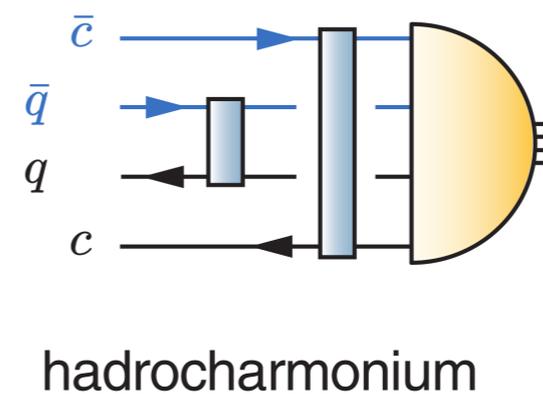
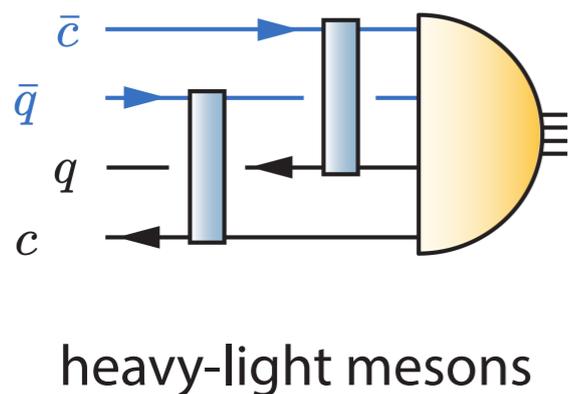
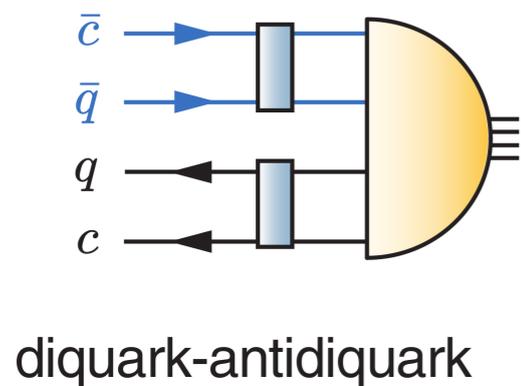
$3 \otimes \bar{3}, 6 \otimes \bar{6}$ or
 $1 \otimes 1, 8 \otimes 8$

- reduce # tensor structures guided by physics:
→ ~20 tensor structures

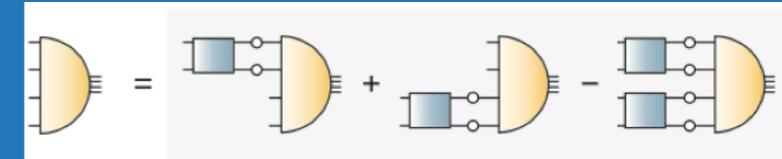
Four-body equation: permutations



- **Singlet:** $S_0 = (p^2 + q^2 + k^2)/4$ p, q, k : relative momenta
- **Doublet:** $\mathcal{D}_1 \sim p^2 + q^2 - 2k^2$
 $\mathcal{D}_2 \sim q^2 - p^2$



Four-body equation: permutations



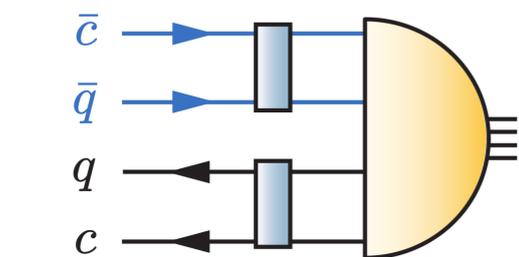
- **Singlet:** $S_0 = (p^2 + q^2 + k^2)/4$

p, q, k : relative momenta

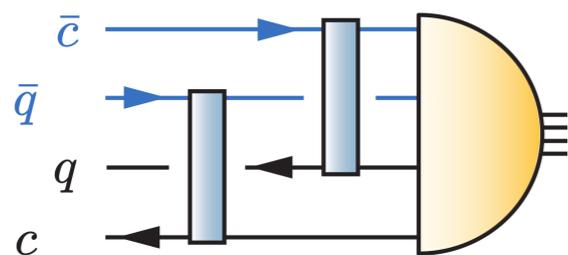
- **Doublet:** $\mathcal{D}_1 \sim p^2 + q^2 - 2k^2$

$$\mathcal{D}_2 \sim q^2 - p^2$$

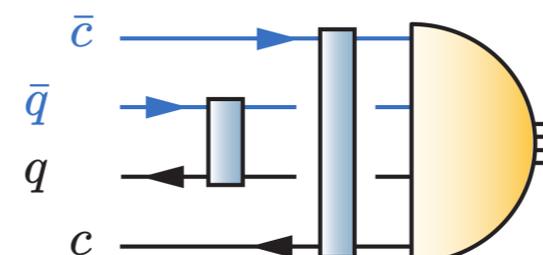
- **model independent:**
heavy-light meson poles
more important than
diquark poles
(color factor !)



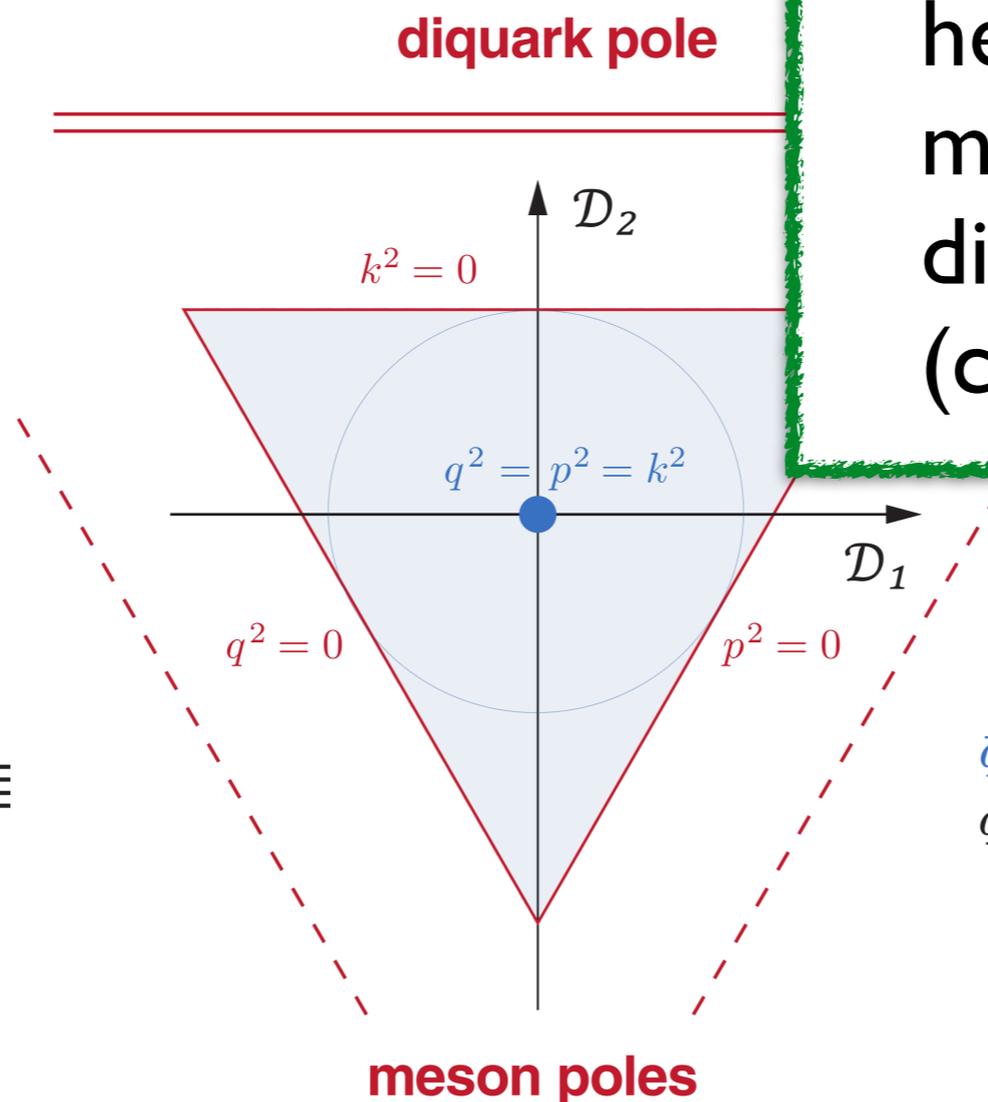
diquark-antidiquark



heavy-light mesons

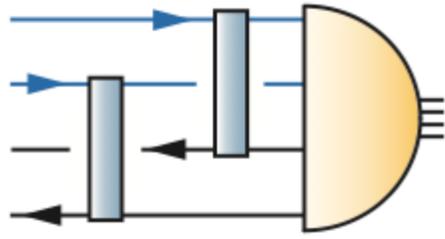


hadrocharmonium



meson poles

Bound state vs resonance: scalar four-quark states



$$\Gamma(S_0, \cancel{s}, \cancel{a}, \dots)$$

without twobody-clustering

0

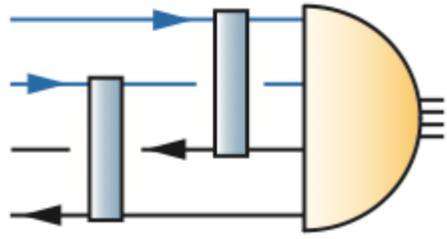
1200

$M_{\text{Tetra}} [MeV]$

Bound state of
four massive quarks

Eichmann, CF, Heupel, PLB 753 (2016) 282-287
Santowsky, CF, PRD 105 (2022) 4,313

Bound state vs resonance: scalar four-quark states



$$\Gamma(S_0, s, a, \dots)$$

without twobody-clustering

0

300-400

1200

$M_{\text{Tetra}} [MeV]$

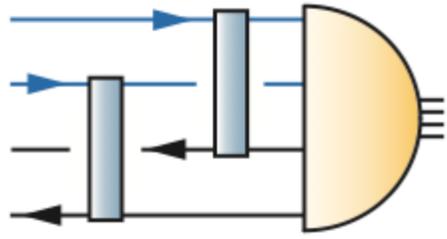
with π -clustering

Two-pion resonance

Bound state of
four massive quarks

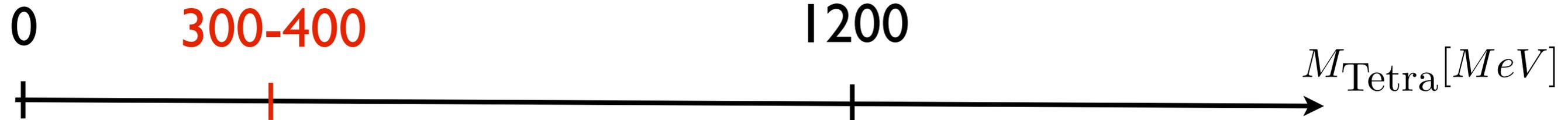
Eichmann, CF, Heupel, PLB 753 (2016) 282-287
Santowsky, CF, PRD 105 (2022) 4,313

Bound state vs resonance: scalar four-quark states



$$\Gamma(S_0, s, a, \dots)$$

without twobody-clustering



with π -clustering

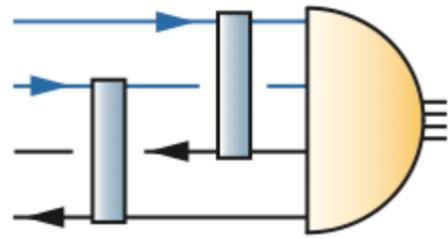
Two-pion resonance

Bound state of
four massive quarks

→ identify with $f_0(500)$ (' σ -meson')

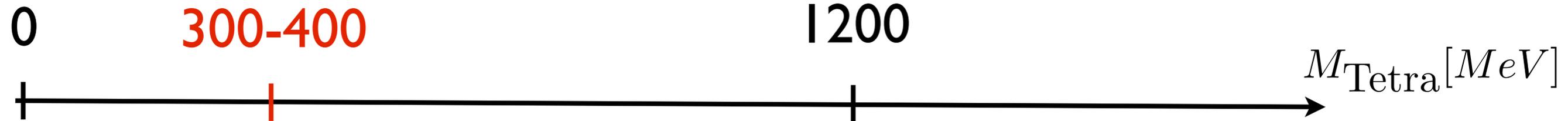
Eichmann, CF, Heupel, PLB 753 (2016) 282-287
Santowsky, CF, PRD 105 (2022) 4,313

Bound state vs resonance: scalar four-quark states



$$\Gamma(S_0, s, a, \dots)$$

without twobody-clustering



with π -clustering

Two-pion resonance

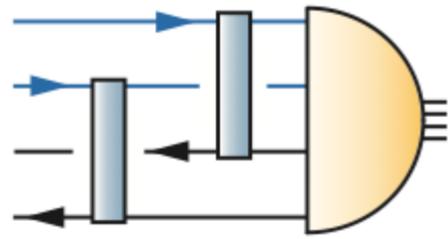
Bound state of
four massive quarks

→ identify with $f_0(500)$ (' σ -meson')

with strange quarks: $m(a_0, f_0) \approx 1 GeV$

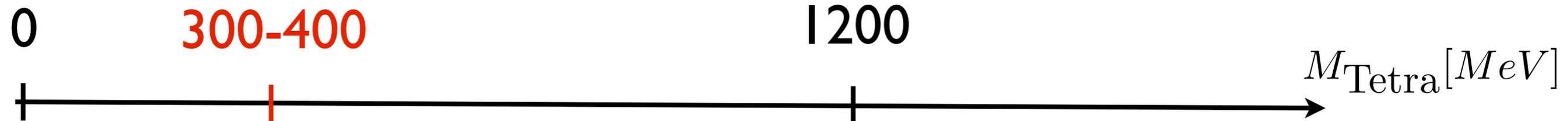
Eichmann, CF, Heupel, PLB 753 (2016) 282-287
Santowsky, CF, PRD 105 (2022) 4,313

Bound state vs resonance: scalar four-quark states



$$\Gamma(S_0, s, a, \dots)$$

without twobody-clustering



with π -clustering

Two-pion resonance

Bound state of
four massive quarks

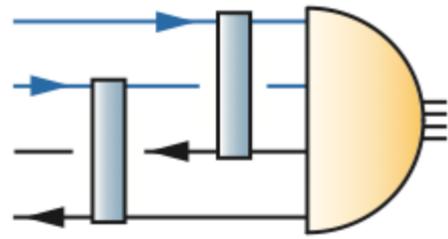
→ identify with $f_0(500)$ (' σ -meson')

with strange quarks: $m(a_0, f_0) \approx 1 GeV$

Eichmann, CF, Heupel, PLB 753 (2016) 282-287
Santowsky, CF, PRD 105 (2022) 4,313

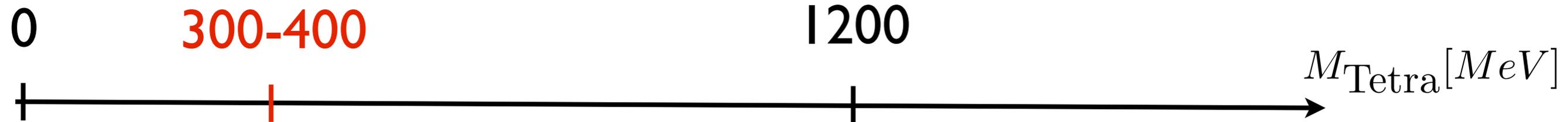
Meson-meson components dominate over diquarks !

Bound state vs resonance: scalar four-quark states



$$\Gamma(S_0, s, a, \dots)$$

without twobody-clustering



with π -clustering

Two-pion resonance

Bound state of
four massive quarks

→ identify with $f_0(500)$ (' σ -meson')

with strange quarks: $m(a_0, f_0) \approx 1 GeV$

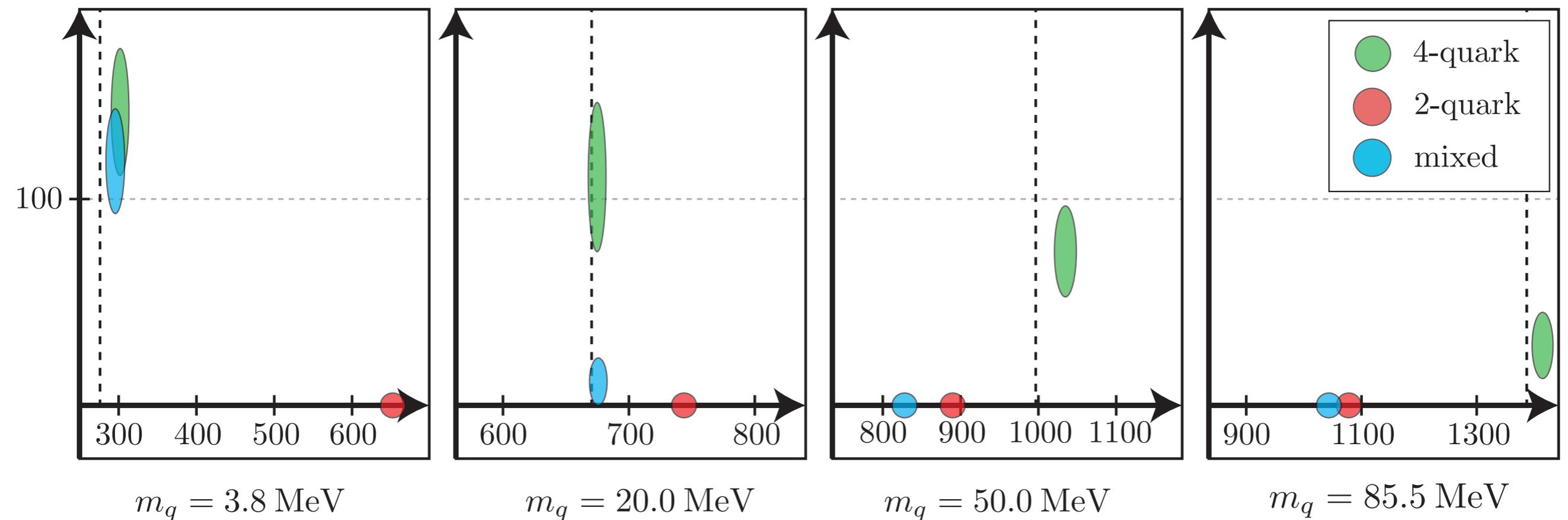
Eichmann, CF, Heupel, PLB 753 (2016) 282-287
Santowsky, CF, PRD 105 (2022) 4,313

Meson-meson components dominate over diquarks !

Mixing with $q\bar{q}$: small effect

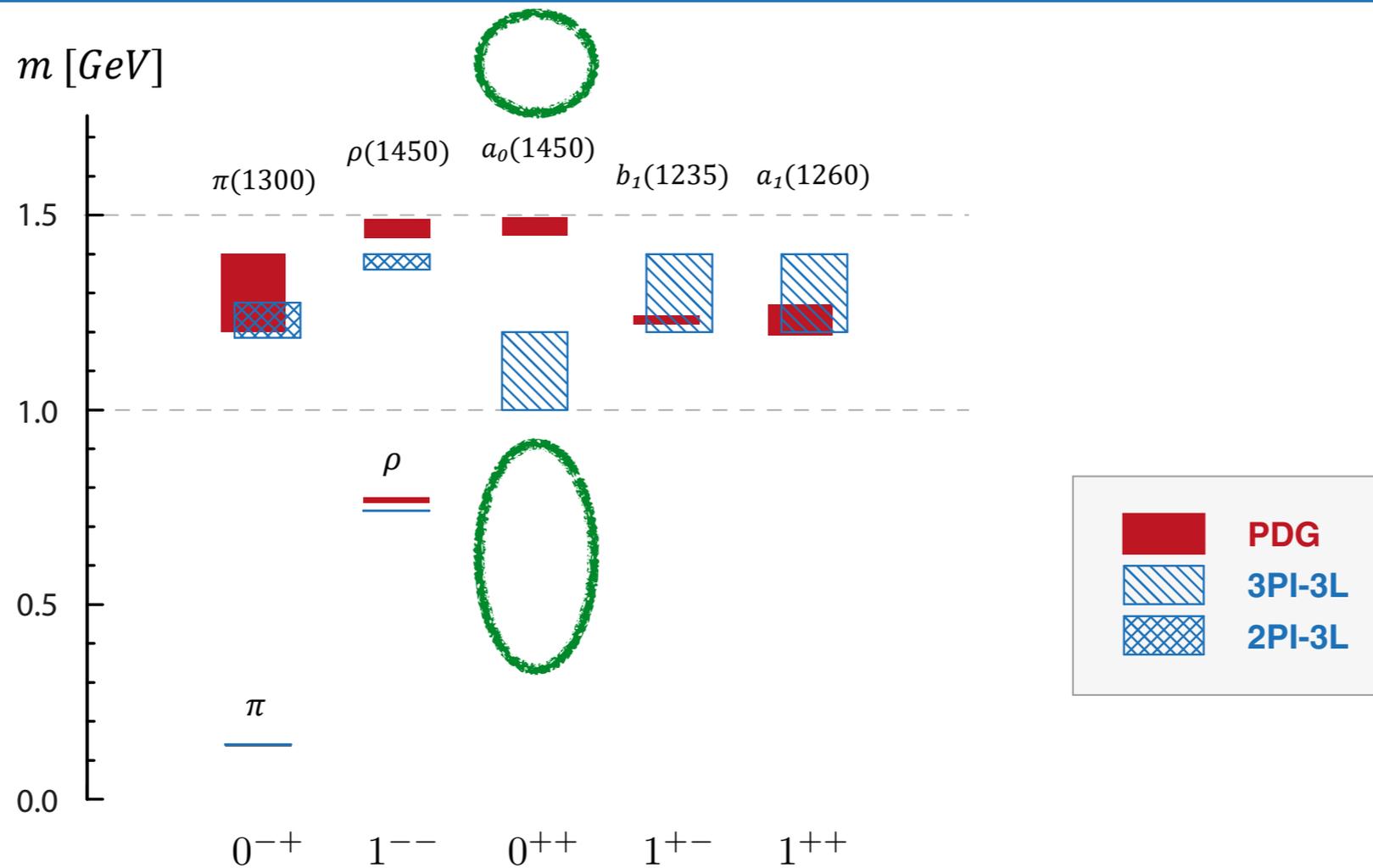
Santowsky, Eichmann, CF, Wallbott and Williams, PRD 102 (2020) no.5, 056014
Santowsky, CF, PRD 105 (2022) 4,313

Mass evolution of four-quark state



- mixed state becomes qq-dominated for large m_q
- dynamical decision !

Santowsky, CF, PRD 105 (2022) 4,313; arXiv:2109.00755



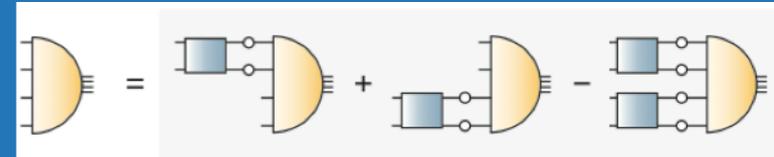
1. Conventional mesons

2. Pure Yang-Mills: glueballs

3. Light four-quark states: the $f_0(500)$

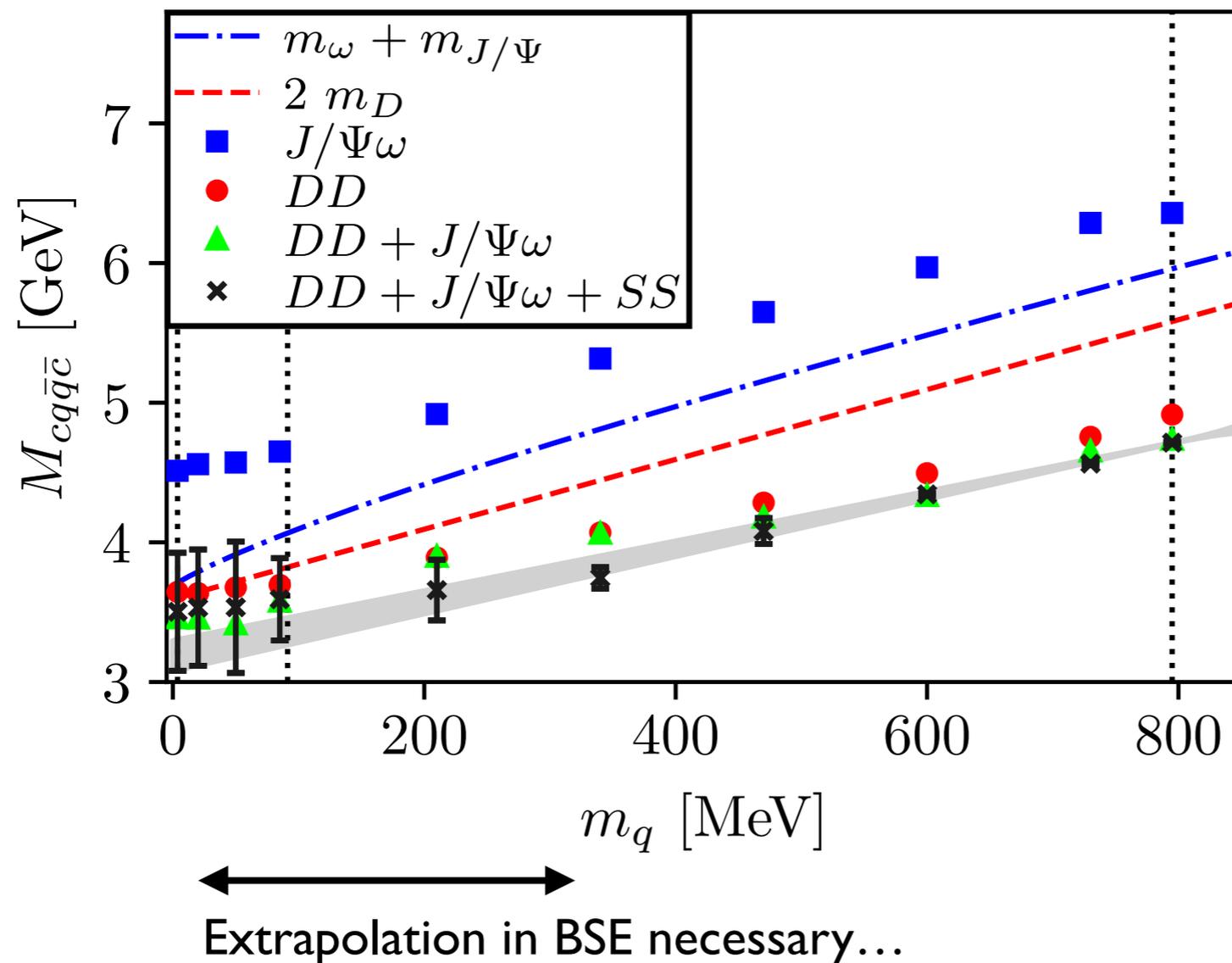
4. Heavy-light four-quark states

$$I(J^{PC}) = 0(0^{++})$$



$cu\bar{u}\bar{c}$ $cs\bar{s}\bar{c}$

$cc\bar{c}\bar{c}$



m_c fixed
 m_q varied

- DD^* components dominate !

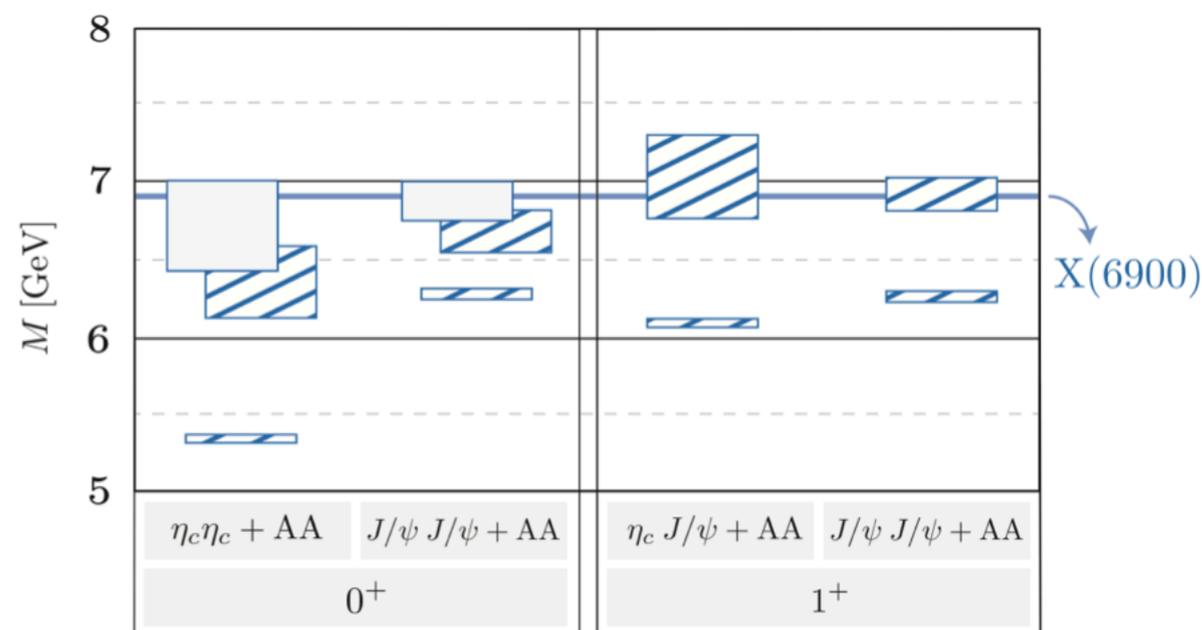
$$M_{0^{++}}^{cq\bar{q}\bar{c}} = 3195(107) \rightarrow ?$$

Wallbott, Eichmann and CF, PRD100 (2019) 014033, [1905.02615]
Wallbott, Eichmann and CF, PRD102 (2020) 051501, [2003.12407]

Heavy four-quark states from DSE/BSEs

	$I(J^{PC})$	four-quark	effective two-body	Exp.
hidden charm ($\bar{c}c\bar{q}q$)	$0(0^{++})$	3.20 (11)	3.49 (25)	
	$0(1^{++})$	3.92 (7)	3.85 (18)	X(3872)
	$1(1^{+-})$	3.74 (9)	3.79 (31)	Z _c (3900)
	$1(0^{++})$		3.20 (31)	
open charm ($cc\bar{q}\bar{q}$)	$1(0^+)$	3.80 (10)	3.21 (2)	
	$0(1^+)$	3.90 (8)	3.49 (48)	T _{cc} (3875)
	$1(1^+)$	4.22 (44)	3.47 (24)	

all charm
($cc\bar{c}\bar{c}$)



Wallbott, Eichmann and CF, PRD100 (2019) 014033, [1905.02615]

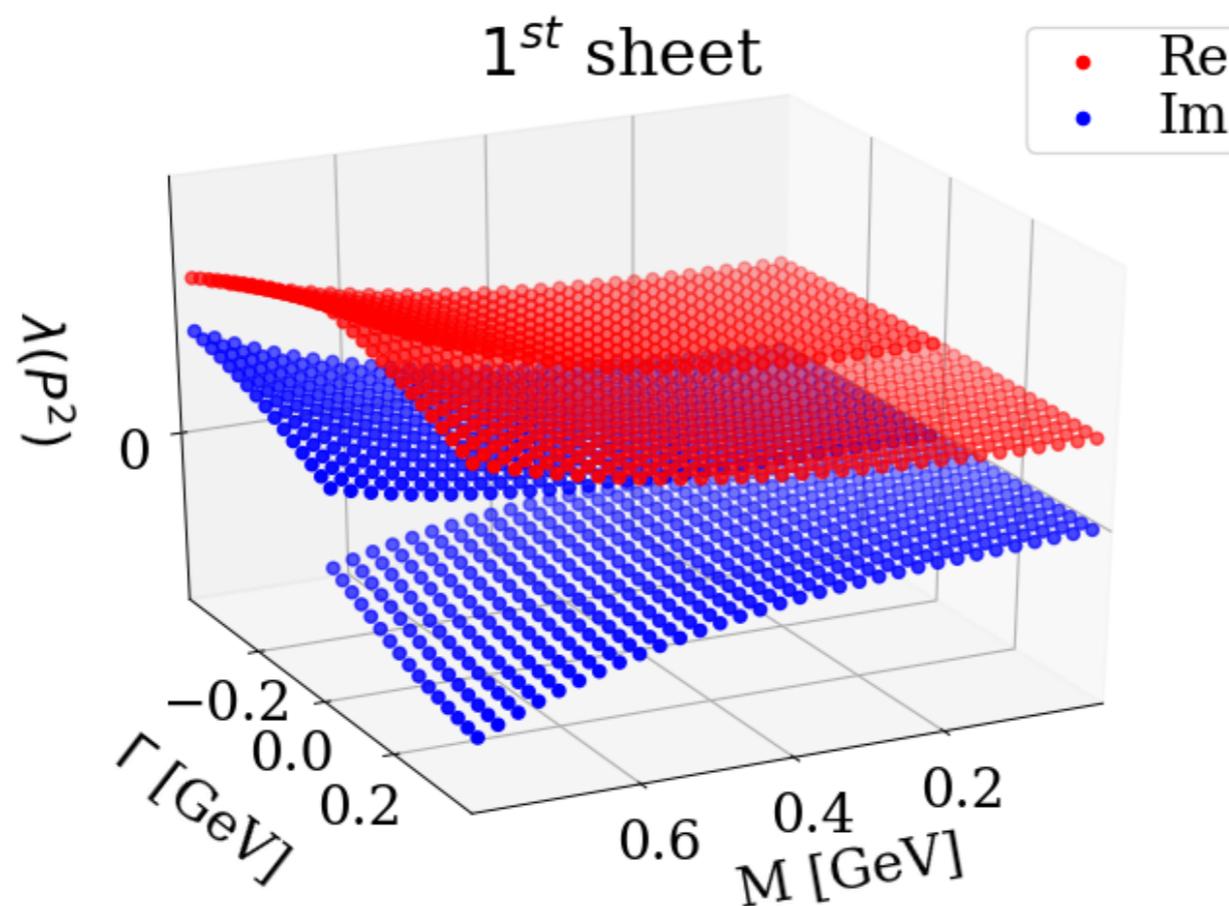
Wallbott, Eichmann and CF, PRD102 (2020), 051501, [2003.12407]

Santowsky, CF, EPJC 82 (2022) 4, 313 [2111.15310]

Work to do...

- improve two-body interactions
- further study mixing with $q\bar{q}$ in $I=0$ sector
- solve four-body BSE in the complex momentum plane

Santowsky, Eichmann, CF, Wallbott and Williams,
PRD 102 (2020) no.5, 056014, arXiv:2007.06495.



successful for ρ -meson:

Williams, PLB 798 (2019) 134943, [arXiv:1804.11161]

Internal dynamics very important !!

Glueballs:

- First quantitatively reliable results using very involved truncation

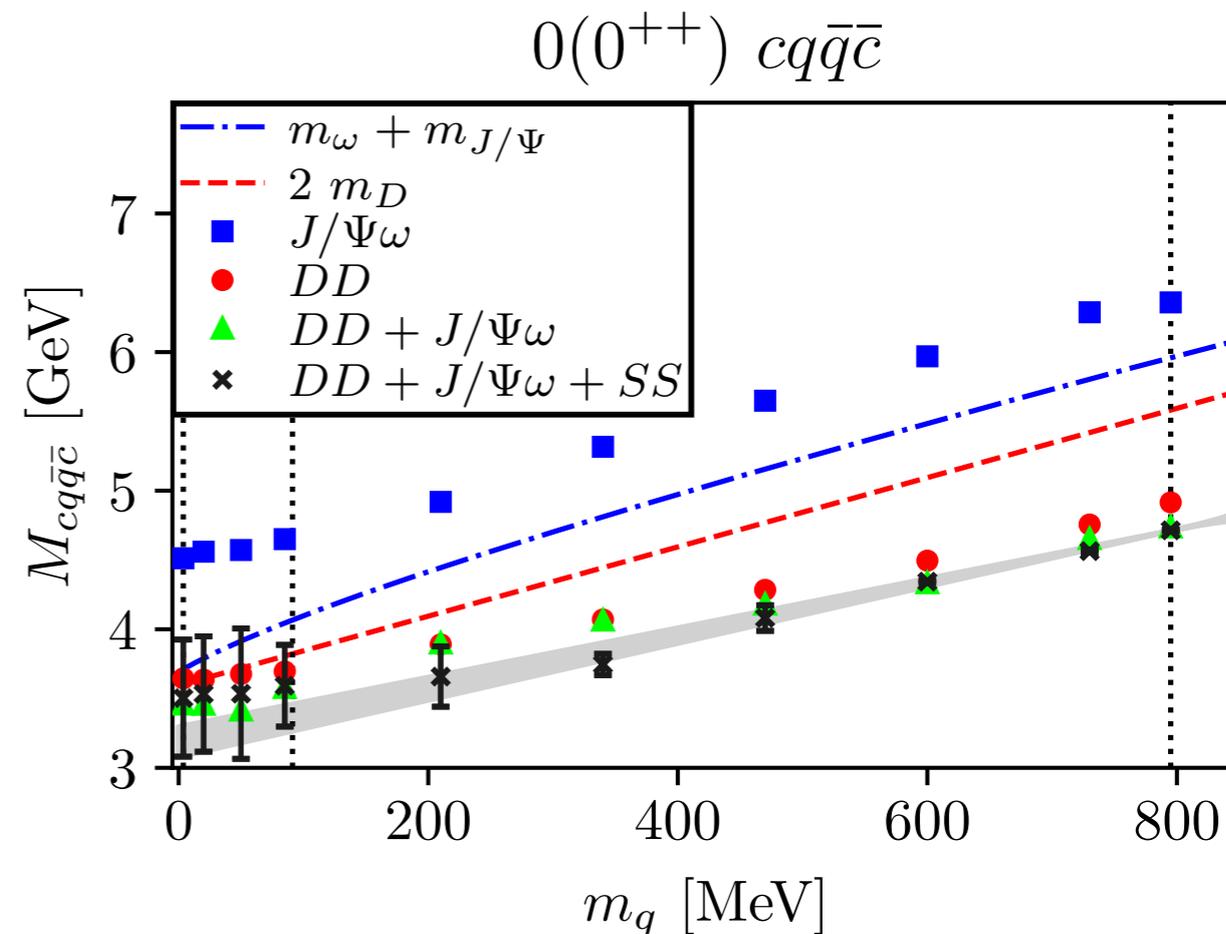
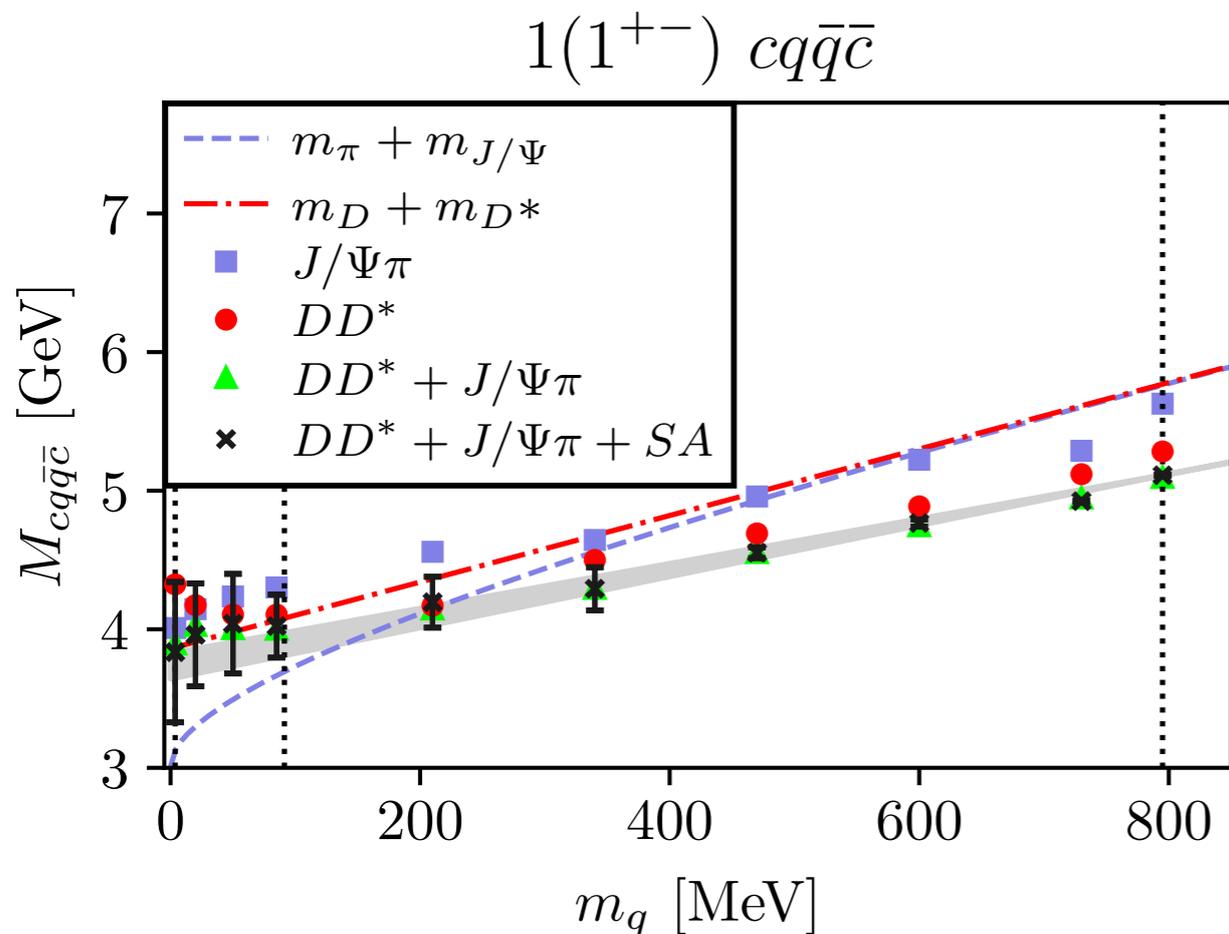
CF, Huber, Sanchis-Alepuz, EPJC 80 (2020) [arXiv:2004.00415]
Huber, CF, Sanchis-Alepuz, EPJC 81 (2021) [arXiv:2110.09180]

Four-quark states:

- Dynamical description of σ : π - π resonance Eichmann, CF, Heupel, PLB 753 (2016) 282-287
- Dynamical description of $X(3872)$ and $Z(3900)$: DD^* dominated
- First results in open charm channels Wallbott, Eichmann and CF, PRD100 (2019) 014033, [1905.02615]
Wallbott, Eichmann and CF, PRD102 (2020) 051501, [2003.12407]
- Mixing with $q\bar{q}$ studied for light mesons Santowsky, Eichmann, CF, Wallbott and Williams,
PRD 102 (2020) no.5, 056014, [2007.06495].

Mini-Review: Eichmann, CF, Heupel, Santowsky, Wallbott, FBS 61 (2020) 4 38, [2008.10240]

$J^{PC} = 1^{+-}$ and 0^{++}

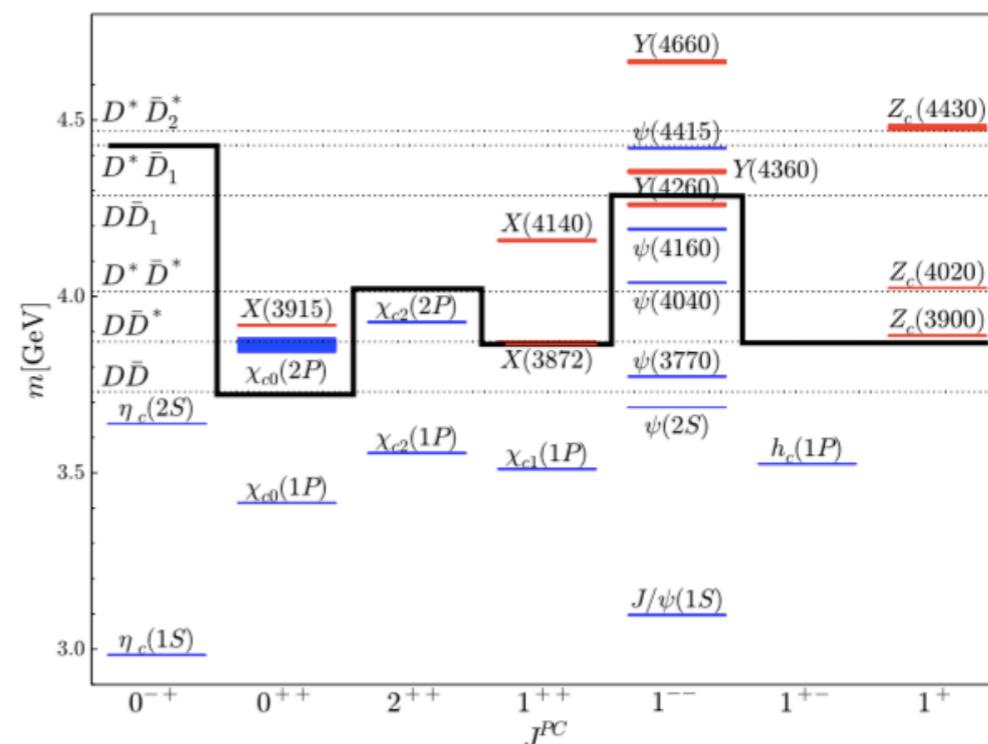


Wallbott, Eichmann and CF, PRD 102 (2020) no.5, 051501, arXiv:2003.12407

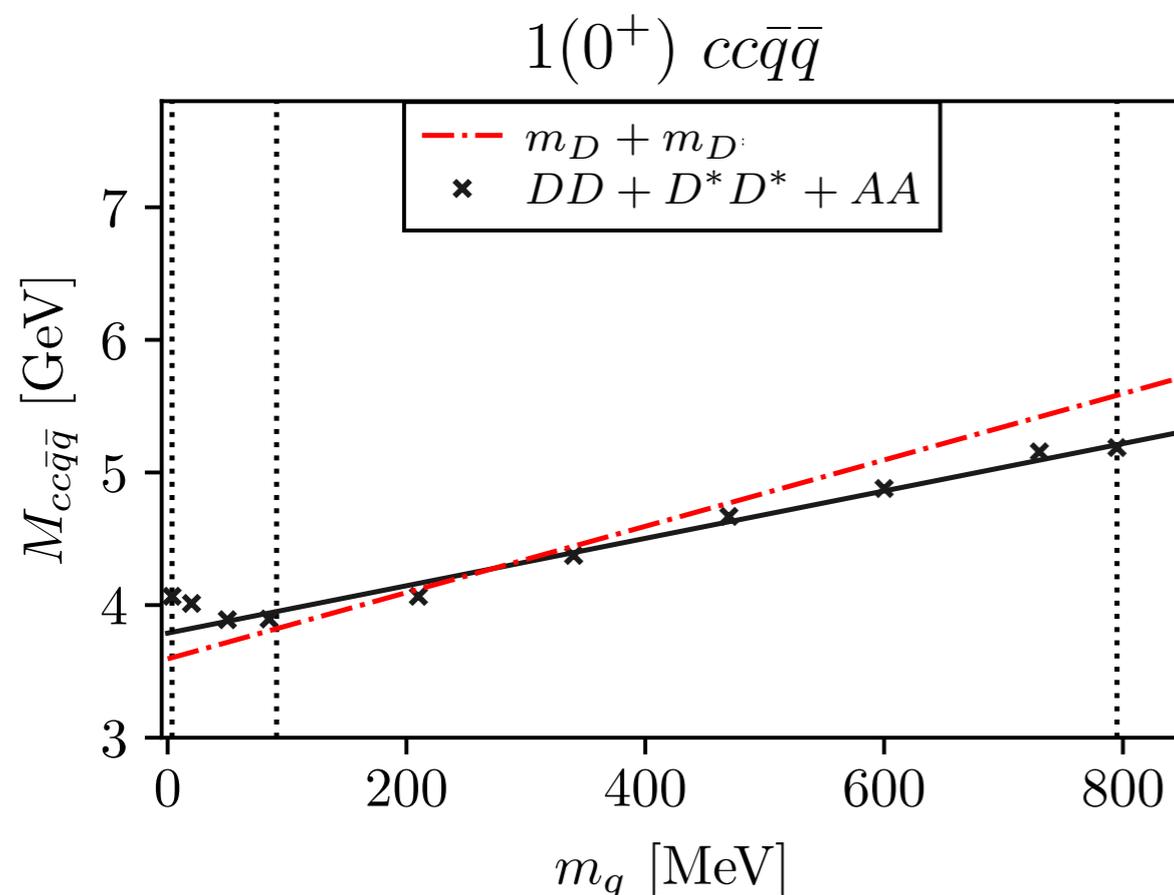
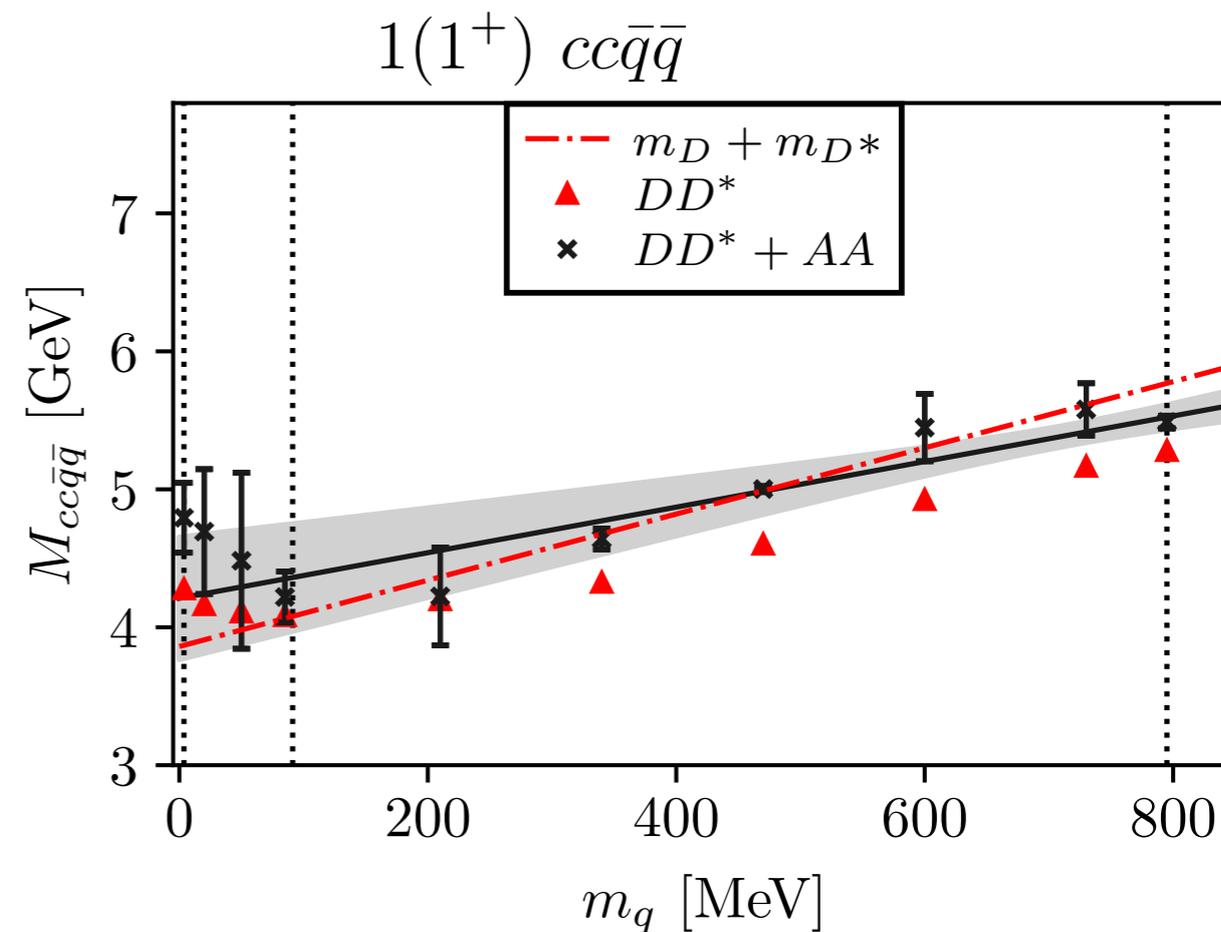
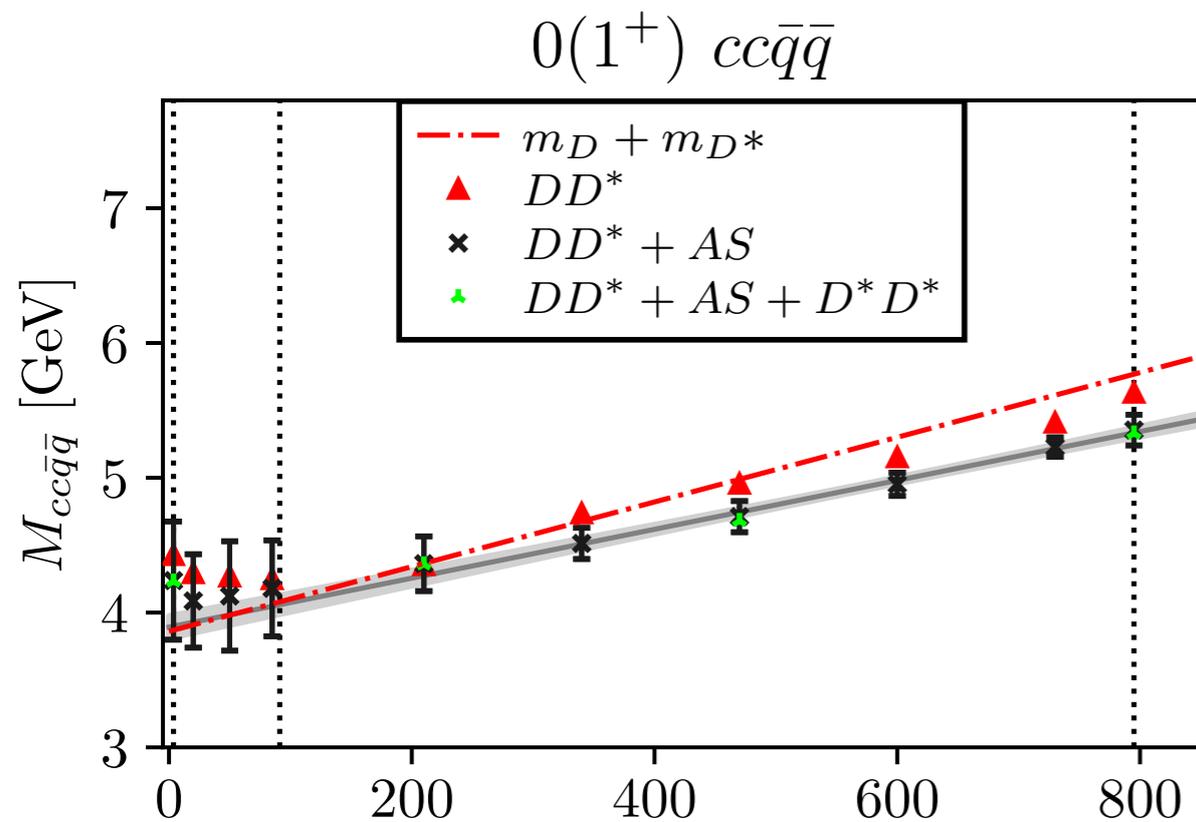
$$M_{1^{+-}}^{cq\bar{q}\bar{c}} = 3741(91) \rightarrow Z(3900)$$

$$M_{0^{++}}^{cq\bar{q}\bar{c}} = 3195(107) \rightarrow ?$$

mass pattern matches molecule picture of Cleven et al. PRD 92 (2015) 014005:



Open charm four-quark states



● **DD(*) and diquarks important!**

Wallbott, Eichmann and CF, PRD102 (2020)no.5, 051501, arXiv:2003.12407

Rainbow-ladder model for quark-gluon interaction



Combine **gluon** with **quark-gluon vertex**:

$$\Gamma^\mu(p, k) = \sum_{i=1,12} \tau_i(p, k) T_i^\mu$$

$$\sim \gamma^\mu \tau(k^2)$$

“approximation” !

$$D^{\mu\nu}(k) = \left(\delta^{\mu\nu} - \frac{k^\mu k^\nu}{k^2} \right) \frac{Z(k^2)}{k^2}$$

$$\frac{g^2}{4\pi} \tau(k^2) Z(k^2) \sim \alpha(k^2)$$

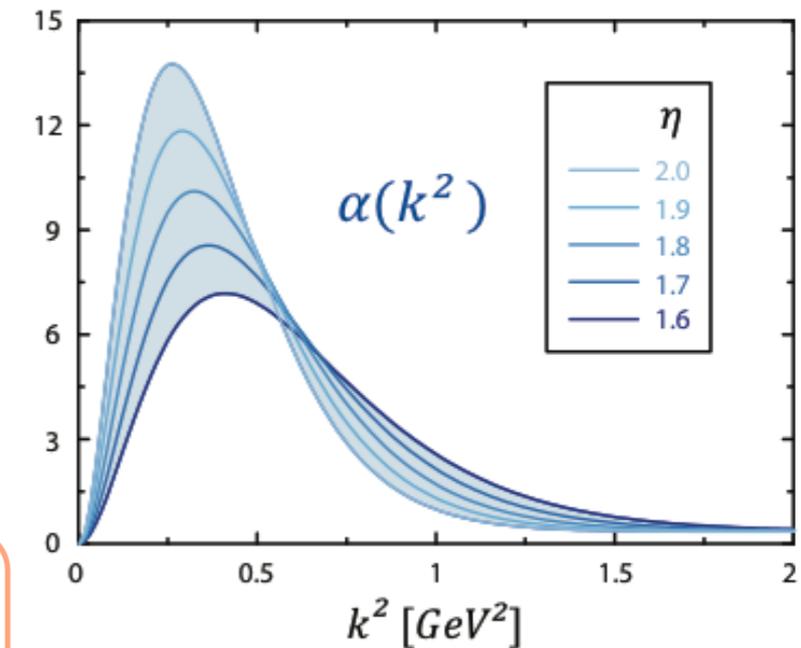
Rainbow-ladder model for quark-gluon interaction



Combine **gluon** with **quark-gluon vertex**:

effective coupling

$$\alpha(k^2) = \pi\eta^7 \left(\frac{k^2}{\Lambda^2} \right) e^{-\eta^2 \left(\frac{k^2}{\Lambda^2} \right)} + \alpha_{UV}(k^2)$$



Maris, Roberts, Tandy, PRC 56 (1997), PRC 60 (1999)

- scale Λ from f_π , masses $m_u=m_d, m_s$ from m_π, m_K
- α_{UV} from perturbation theory
- parameter η : results almost independent
- qualitatively similar to explicit calc.

Williams, EPJA 51 (2015) 5, 57.
 Sanchis-Alepuz, Williams, PLB 749 (2015) 592;
 Mitter, Pawłowski and Strodthoff, PRD 91 (2015) 054035
 Williams, CF, Heupel, PRD93 (2016) 034026, and refs. therein