Excited States of “Elementary” Particles in Gauge-Higgs Theories

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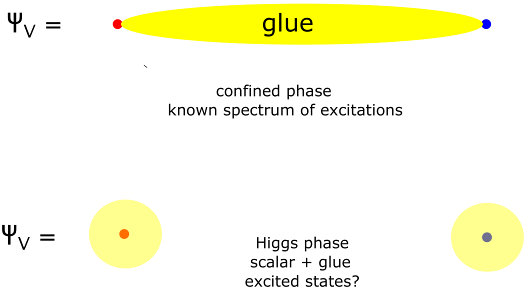
The question we address

Composite systems (molecules, atoms, nuclei, hadrons...) generally have a spectrum of excitations. What about non-composite systems: charged "elementary" particles like quarks and leptons?

If the particle is charged, then by Gauss's Law it is accompanied by a surrounding gauge (and possibly other) fields. If these surrounding fields interact with themselves, *could they not also exhibit a spectrum of excitations?* This would look like a mass spectrum of the isolated elementary particle.



“Real” gauge Higgs theories

Two gauge Higgs theories are known to describe reality:

1. Superconductivity. The effective theory, i.e. the Landau-Ginzburg model, is a non-relativistic version of an abelian Higgs theory with double-charged Higgs.
2. The Standard Model - the electroweak sector. (Possibly also an effective theory.)

So we would be looking for excitations of static charges in a superconductor, or mass excitations of quarks and leptons.

Perturbatively, no such thing is found. But the lattice supplies non-perturbative information.

The electroweak sector is a chiral gauge theory. The lattice formulation is so far problematic.

We therefore concentrate on superconductors.

Pseudomatter operators

Simplest case: free Maxwell field with a static charged source, infinite volume. The ground state is

$$|\Psi_{\mathbf{x}}\rangle = \overline{\psi}(\mathbf{x})\rho(\mathbf{x};A)|\Psi_0\rangle$$

where $\overline{\psi}(\mathbf{x})$, operating on the vacuum, creates the static charge, and

$$\rho(\mathbf{x};A) \quad = \quad \exp\left[-i\frac{e}{4\pi}\int d^3z\, A_i(\mathbf{z})\frac{\partial}{\partial z_i}\frac{1}{|\mathbf{x}-\mathbf{z}|}\right]$$

$\rho(\mathbf{x},A)$ is a **pseudomatter** operator. This is an operator which

- is a functional of the gauge field only
- transforms like a matter field at point $\mathbf{x}$ *except* under global gauge transformations in the center of the gauge group *which do not affect the gauge field*. In this case

$$g(x) = e^{i\theta} \quad , \quad \rho \to \rho \quad , \quad \Psi_{\mathbf{x}} \to e^{-i\theta}\Psi_{\mathbf{x}}$$

Stated without proof: In gauge Higgs theories

Spontaneous breaking of the global center subgroup of the gauge group distinguishes the Higgs phase from the massless and confining phases.

The symmetry breaking transition coincides with

1. in the Confinement $\rightarrow$ Higgs transition: A transition from *separation-of-charge* (S_c) confinement in the confining phase, to *color* (C) confinement in the Higgs phase. The two phases are physically distinct.
2. the massless $\rightarrow$ Higgs transition: Massless vector bosons acquire a mass.

The Higgs phase can be regarded as the *spin glass phase* of a gauge Higgs theory, with an order parameter closely analogous to the Edwards-Anderson order parameter for spin glasses.

But this is a long story. See *K. Matsuyama and JG, PRD 101 (2020)*.

Pseudomatter II

More examples:

- Any transformation to a physical gauge (e.g. axial, Coulomb) defined by $F[A] = 0$ is a pseudomatter field. $\rho(\mathbf{x}, A)$ is in fact the gauge transformation to Coulomb gauge in an abelian theory at infinite volume.

- Eigenstates $\zeta_n(\mathbf{x}; U)$ of the lattice Laplacian operator D^2 are pseudomatter operators

$$\sum_{\mathbf{y}} (-D^2)_{\mathbf{xy}} \zeta_n(\mathbf{y}) = \lambda_n \zeta(\mathbf{x})$$

  where

$$(-D^2)_{\mathbf{xy}} = \sum_{k=1}^{3} \left[2\delta_{\mathbf{xy}} - U_k(\mathbf{x})\delta_{\mathbf{y},\mathbf{x}+\hat{k}} - U_k^\dagger(\mathbf{x}-\hat{k})\delta_{\mathbf{y},\mathbf{x}-\hat{k}} \right]$$

The Landau-Ginzburg model

This is an effective model of superconductivity. It can be derived from the underlying microscopic BCS theory.

$$S = \int d^4x \left\{ \frac{1}{2}\rho_s \left(\frac{1}{\upsilon^2}(\partial_0\xi + 2eA_0)^2 + (\partial_k\xi - 2eA_k)^2 \right) + \frac{1}{2}(E^2 - B^2) \right\}$$

The factor of $2e$ indicates that the scalar field is double-charged (Cooper pairs). On the lattice

$$\begin{aligned} S_{eff} &= -\beta \sum_{plaq} \mathrm{Re}[UUU^*U^*] - \gamma \sum_x \mathrm{Re} \sum_{k=1}^{3} \phi^*(x) U_k^2(x) \phi(x+\hat{k})] \\ &\quad - \frac{\gamma}{\upsilon^2} \sum_x \mathrm{Re}[\phi^*(x) U_0^2(x) \phi(x+\hat{t})] \end{aligned}$$

where

$$\phi(x) = e^{i\xi(x)} \quad , \quad \beta = \frac{1}{e^2} = 10.9 \ , \ \ \upsilon \sim 10^{-2}$$

Physical States

We consider physical states with $+/-$ static charges at points $\mathbf{x}, \mathbf{y}$ of the form

$$|\Phi_n(\mathbf{x},\mathbf{y})\rangle = Q_n(\mathbf{x},\mathbf{y})|\Psi_0\rangle$$

with

$$\begin{aligned} Q_{2n-1} &= \overline{\psi}(\mathbf{x})\zeta_n(\mathbf{x})\zeta_n^*(\mathbf{y})\psi(\mathbf{y}) \\ Q_{2n} &= \overline{\psi}(\mathbf{x})\phi(\mathbf{x})\zeta_n^*(\mathbf{x})\zeta_n(\mathbf{y})\phi^*(\mathbf{y})\psi(\mathbf{y}) \end{aligned}$$

The idea is to diagonalize the (rescaled) transfer matrix $\mathcal{T}$ in a subspace spanned by N states $\{|\Phi_n(\mathbf{x},\mathbf{y})\rangle\}$, and check whether the eigenstates in the subspace are close to eigenstates in the full Hilbert space. If so, we can obtain the low-lying spectrum of the +/- charges in the superconductor.

The numerical calculation

$$
\begin{aligned}
[\mathcal{T}]_{\alpha\beta}(R) &= \langle \Phi_\alpha | \mathcal{T} | \Phi_\beta \rangle = \langle Q^\dagger_\alpha(R,1) Q(R,0) \rangle \\
[O]_{\alpha\beta}(R) &= \langle \Phi_\alpha | \Phi_\beta \rangle = \langle Q^\dagger_\alpha(R,0) Q(R,0) \rangle
\end{aligned}
$$

We obtain the eigenstates of $\mathcal{T}$ in the subspace by solving the generalized eigenvalue problem,

$$
\begin{aligned}
[\mathcal{T}]\vec{\upsilon}_n &= \lambda_n [O] \vec{\upsilon}^{(n)} \\
|\Psi_n(R)\rangle &= \sum_{\alpha=1}^{N} \upsilon^{(n)}_\alpha |\Phi_\alpha(R)\rangle
\end{aligned}
$$

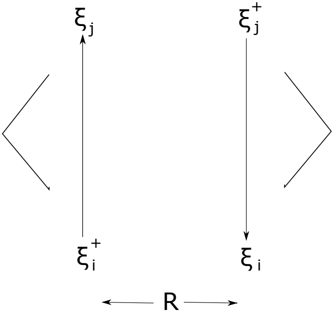
and consider evolving these states in Euclidean time

$$
\begin{aligned}
\mathcal{T}_{nn}(R,T) &= \langle \Psi_n | \mathcal{T}^T | \Psi_n \rangle \\
&= \upsilon^{(n)*}_i \langle \Phi_i | \mathcal{T}^T | \Phi_j \rangle \upsilon^{(n)}_j
\end{aligned}
$$

Integrating out the massive (i.e. static) fermion fields generates a pair of Wilson lines.

The numerical computation of $\langle \Phi_i | \mathcal{T}^T | \Phi_j \rangle$ involves expectation values of products of Wilson lines, terminated by matter or pseudomatter fields:

$$\langle \Phi_i | \mathcal{T}^T | \Phi_j \rangle \quad =$$



From the $\langle \Phi_i | \mathcal{T}^T | \Phi_j \rangle$ we can determine the $\mathcal{T}_{nn}(R,T)$, and on general grounds

$$\begin{aligned} \mathcal{T}_{nn}(R,T) &\equiv \langle \Psi_n | \mathcal{T}^T | \Psi_n \rangle \\ &= \sum_k |c_k(R)|^2 e^{-E_k(R)T} \end{aligned}$$

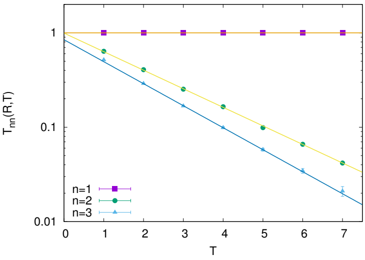
If $\Psi_n(R)$ has a large overlap with one excited energy eigenstate Ψ_i^{exact}, and very small overlap with other energy eigenstates, then we may expect that for some range of $T_{min} \le T \le T_{max}$

$$\mathcal{T}_{nn}(R,T) \approx |c_i(R)|^2 e^{-E_i(R)T}$$

and in that case we may extract the excitation energy $E_i(R)$ from a logarithmic plot of $\mathcal{T}_{nn}(R,T)$.

Excitations

Here is a log plot of $\mathcal{T}_{nn}(R,T)$ vs. T at $R = 6.0$, $\gamma = 0.6$, for $n = 1, 2, 3$, on a $12^3 \times 36$ lattice.



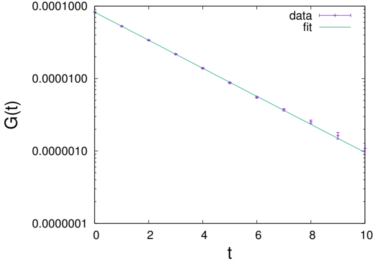
From the slopes, we deduce that $E_1 = 0, E_2 = 0.46, E_3 = 0.55$.

photon mass

We compute the photon mass from the time correlator

$$
\begin{aligned}
G(t) &= \frac{1}{3}\sum_{i=1}^{3}\langle \mathcal{A}_i(0)\mathcal{A}_i(t)\rangle \\
\mathcal{A}(t) &= \frac{1}{L^3}\sum_{\mathbf{x}} \mathrm{Im}[\phi^\dagger(\mathbf{x},t)U_i^2(\mathbf{x},t)\phi(\mathbf{x}+\hat{i})]
\end{aligned}
$$

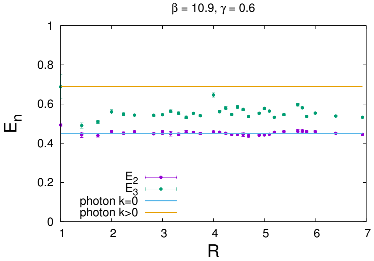
At $\beta = 1/e^2, \gamma = 0.6$ on a $16^3 \times 36$ lattice volume



From the slope, we find $m_{ph} = 0.446(3)$ in lattice units.

Excitations II

Here is a plot of E_1, E_2, E_3 vs. R.

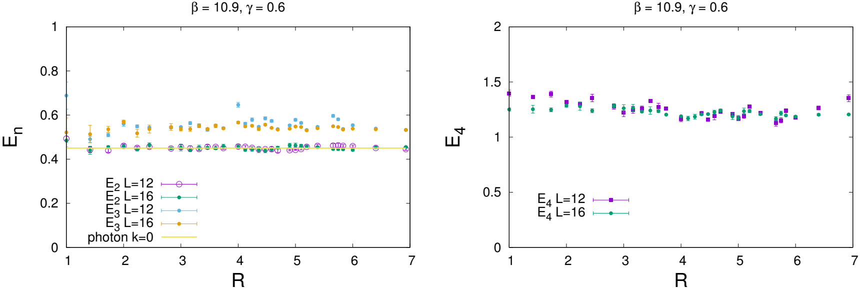


We see that E_2 coincides with the mass of a photon. The upper solid line is next higher energy of a massive photon on the $12^3 \times 36$ lattice, with lowest non-zero momentum $|k| = 2\pi/12$, and this is $E_\gamma = 0.687$. The $E_3 \approx 0.56$ values lie well below this number.

Therefore, *E_3 cannot be interpreted as the ground state + a photon of non-zero momentum.*

It seems to be an excited state of the static charges.

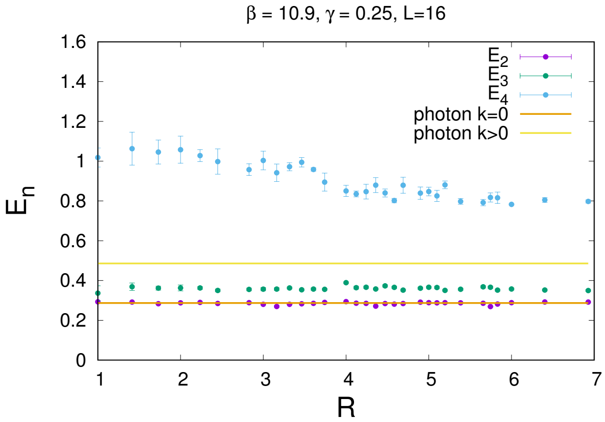
Larger volume, higher excitation



Larger volume doesn't change the energies much, but there is less scatter in the higher excitations E_3, E_4 at the higher volume.

Smaller γ

This is the result at $\gamma = 0.25$, still at $\beta = 10.9$, on a $16^3 \times 36$ lattice volume.

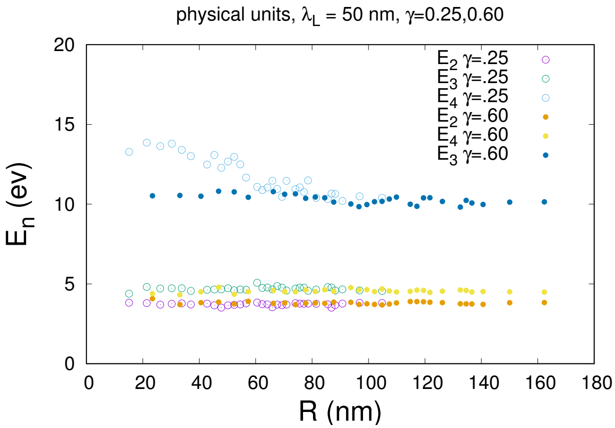


Scaling

The scale is set by the London penetration depth λ_L, typically $\sim$ 50 nm. The lattice spacing is given by

$$a = 2e\sqrt{\gamma}\lambda_L$$

and from this we can convert to physical units. The figure below shows E_n in ev vs. R in nm, computed on a $16^3 \times 36$ lattice, for both $\gamma = 0.25$ and $\gamma = 0.60$.

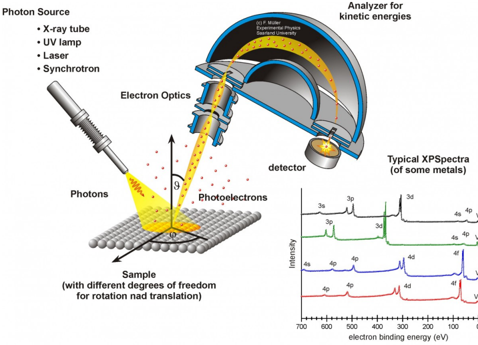


Is this observable?

Maybe.

We are a long way from accurate predictions.

Still, if the effect is there, it might show up in photoelectron spectroscopy, comparing core level electron spectra in the normal and superconducting states.

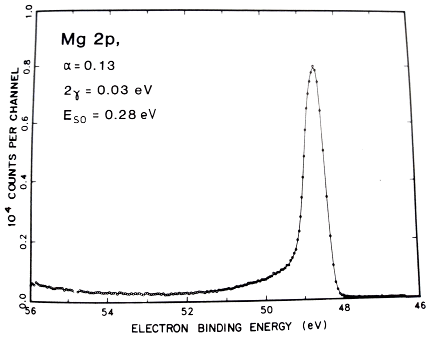


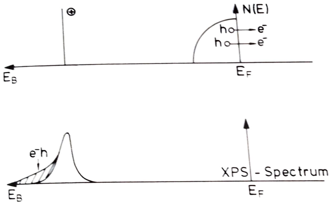
One can argue that a similar effect – excitations of the field surrounding static charges – has *already* been seen in normal metals.

Normal metals - core electron spectra

- A photon knocks out a core electron (bound to an atom), suddenly creating an isolated charge.
- In a normal metal, conduction electrons respond by screening the charge.
- In the screening response there is a *near-continuum of excitations* of the Fermi sea above the ground state of the screened charge.
- If the Fermi sea is left in an excitation above the ground state, this reduces the energy of the emitted electron.
- This in turn affects the line shape in the photoelectron spectrum.

The asymmetric Doniach-Sunjic lineshape





From **Doniach and Sunjic, 1969** (3100+ citations):

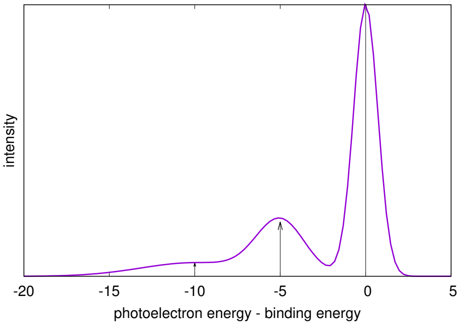
- *"Thus the maximum photoelectron energy corresponds to the ground state of the hole +metal, while photoelectrons emitted below the maximum correspond to events in which the hole + Fermi sea is left in an excited state. Excited states with energies very close (a fraction of an electron-volt) to the ground state are those in which the Fermi sea is excited by the creation of low energy conduction electron-hole pairs (i.e. charge density fluctuations)."*

Observable II

In the superconductor phase, we would expect screening from the Cooper pair condensate, and excitations in this condensate would lead to additional peaks in the spectrum, separated by a few ev from the main peak.

We don't know the height or width of those extra peaks. But, given λ_L, *we do know their locations*.

Schematically, we expect something like this:



What is needed is a *comparison of core emission spectra* above and below the superconducting transition,

Surprisingly, this has not been done yet.

Did you look at other gauge Higgs theories?

Other gauge Higgs theories

This is from earlier, published work:

1. SU(3) gauge Higgs theory. The Higgs scalar is in the fundamental representation.
   J.G., PRD 102 (2020) 5, 054504 , arXiv: 2007.11616 [hep-lat]
2. The $q = 2$ abelian Higgs model. The Higgs scalar has charge $q = 2$.
   K. Matsuyama, PRD 103 (2021) 7, 074508 , arXiv: 2012.13991 [hep-lat]
3. Chiral U(1) gauge Higgs theory (Smit-Swift formulation). The Higgs scalar has charge $q = 1$.
   J.G., PRD 104 (2021) 3, 034508 , arXiv: 2104.12237 [hep-lat]

In each of these models we impose a unimodular constraint $|\phi| = 1$ for simplicity.

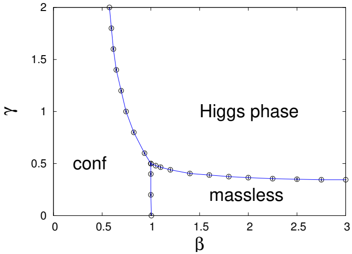
$q = 2$ abelian Higgs model

A similar result has recently been found in the $q = 2$ abelian Higgs model by K. Matsuyama.

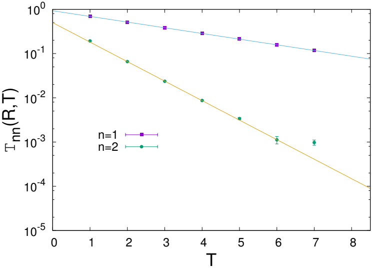
This is a version of the abelian Higgs model in which the scalar field (like Cooper pairs) carries two units of electric charge, and Matsuyama considers also $q = 2$ static charges.

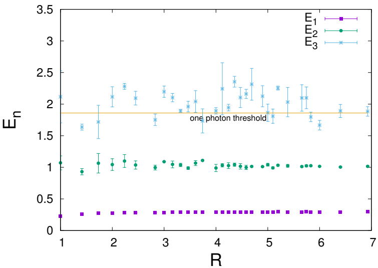
$$S = -\beta \sum_{plaq} \mathrm{Re}[U_\mu(x)U_\nu(x+\hat{\mu})U^*_\mu(x+\hat{\nu})U^*_\nu(x)] - \gamma \sum_{x,\mu} \mathrm{Re}[\phi^*(x)U^2_\mu(x)\phi(x+\widehat{\mu})]$$

The phase diagram is on the right. Matsuyama works just inside the Higgs phase, at $\beta = 3.0, \gamma = 0.5$.



K.M. considers charge $q = 2$ sources. Following along the same lines of diagonalizing $\mathcal{T}$ in a small subspace, and judging by the fit to $\mathcal{T}_{11}(R,T)$ and $T_{22}(R,T)$, the lowest two states $\Psi_{1,2}$ seem to be very nearly exact eigenstates of the system, even at small T:





Note that the first excitation is well below the threshold, and is therefore stable against massive photon emission.

SU(3) gauge Higgs theory

We have computed $E_n(R,T)$ for SU(3) gauge theory with a unimodular Higgs field on a $14^3 \times 32$ lattice volume, at $\beta = 5.5$ with $\gamma = 0.5$ and $\gamma = 3.5$, in the confinement and Higgs phases respectively. The action is

$$S = -\frac{\beta}{3}\sum_{plaq} \mathrm{ReTr}[U_\mu(x)U_\nu(x+\hat{\mu})U_\mu^\dagger(x+\hat{\nu})U_\nu^\dagger(x)] - \gamma \sum_{x,\mu} \mathrm{Re}[\phi^\dagger(x)U_\mu(x)\phi(x+\widehat{\mu})]$$

But this time we can also create color-neutral states using the Higgs field.

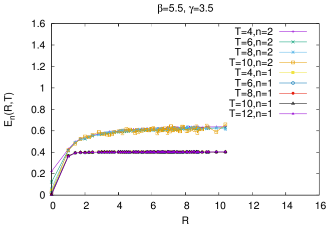
$$\begin{aligned} \Phi_n(R) &= [\overline{q}^a(\mathbf{x})\zeta_n^a(\mathbf{x})] \times [\zeta_n^{\dagger b}(\mathbf{y})q^b(\mathbf{y})]\,\Psi_0 \quad (n=1,2,3) \\ \Phi_4(R) &= [\overline{q}^a(\mathbf{x})\phi^a(\mathbf{x})] \times [\phi^{\dagger b}(\mathbf{y})q^b(\mathbf{y})]\,\Psi_0 \end{aligned}$$

Same procedure. Diagonalize $\mathcal{T}$ in the four-dimensional subspace of Hilbert Space, and compute

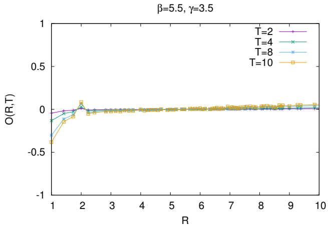
$$\begin{aligned} \mathcal{T}_{mn}^T(R) &= \langle \Psi_n|\mathcal{T}^T|\Psi_n\rangle \\ E_n(R,T) &= -\log\left[\frac{\mathcal{T}_{nn}^T(R)}{\mathcal{T}_{nn}^{T-1}(R)}\right] \end{aligned}$$

Energies in SU(3) gauge Higgs

Now we show $E_n(R,T)$ and the overlap for $\Psi_1(R)$, $\Psi_2(R)$ and $T = 4 - 12$.



(a) Energies



(b) Overlap

There seems to be clear evidence of a metastable excited state in the spectrum, orthogonal to the ground state.

The energy gap is far smaller than the threshold for vector boson creation.

Excitations in chiral gauge theories

No known lattice formulation of chiral non-abelian gauge theories with a continuum limit. There is a formulation for U(1) gauge theories due to Lüscher, involving overlap fermions. Difficult to implement numerically.

In this exploratory work, we chose a simpler option.

For *static* fermions, work instead with a quenched version, at fixed lattice spacing, of the Smit-Swift lattice action, U(1) gauge group, with oppositely charged right and left-handed fermions.

Doublers restore chiral symmetry, so the idea was to use a Wilson-style non-local mass term so that the mass of the doublers is infinite in the continuum limit.

The continuum limit doesn't work...Smit-Swift is not a true chiral gauge theory. Moreover, the positivity of the transfer matrix is unproven. But at least there is a mass asymmetry, between the desired states and the doublers, in part of the phase diagram. We can try it.

Smit-Swift model

The action of the Smit-Swift model with a U(1) gauge group and opposite charged right and left-handed fermions:

$$
\begin{aligned}
S \quad = \quad & -\beta \sum_x \sum_{\mu<\nu} \Re[U_\mu(x)U_\nu(x+\hat{\nu})U^*_\mu(x+\hat{\mu})U^*_\nu(x)] - \gamma \sum_x \sum_\mu \Re[\phi^*(x)U_\mu(x)\phi(x+\hat{\mu})] \\
& + M \sum_x [\overline{\psi}_L(x)\varphi(x)\psi_R(x) + \overline{\psi}_R(x)\varphi^*(x)\psi_L(x)] \\
& - \frac{1}{2} \sum_x \sum_\mu \left[\overline{\psi}_R(x), \overline{\psi}_L(x)\right] \mathbf{D}_{\mu+}(x) \left[\begin{array}{c} \psi_L(x+\hat{\mu}) \\ \psi_R(x+\hat{\mu}) \end{array} \right] \\
& - \frac{1}{2} \sum_x \sum_\mu \left[\overline{\psi}_R(x), \overline{\psi}_L(x)\right] \mathbf{D}_{\mu-}(x) \left[\begin{array}{c} \psi_L(x-\hat{\mu}) \\ \psi_R(x-\hat{\mu}) \end{array} \right]
\end{aligned}
$$

with

$$
|\phi(x)| = 1 \quad , \quad \varphi(x) = \phi^2(x)
$$

Chiral I

$$\mathbf{D}_{\mu+}(x) = \begin{bmatrix} \frac{1}{2}r[\varphi^*(x)U_\mu(x) + U^*_\mu(x)\varphi^*(x+\hat{\mu})] & -\eta^R_\mu U^*_\mu(x) \\ -\eta^L_\mu U_\mu(x) & \frac{1}{2}r[\varphi(x)U^*_\mu(x) + U_\mu(x)\varphi(x+\hat{\mu})] \end{bmatrix}$$

$$\mathbf{D}_{\mu-}(x) = \begin{bmatrix} \frac{1}{2}r[\varphi^*(x)U^*_\mu(x-\hat{\mu}) + U_\mu(x-\hat{\mu})\varphi^*(x-\hat{\mu})] & \eta^R_\mu U_\mu(x-\hat{\mu}) \\ \eta^L_\mu U^*_\mu(x-\hat{\mu}) & \frac{1}{2}r[\varphi(x)U_\mu(x-\hat{\mu}) + U^*_\mu(x-\hat{\mu})\varphi(x-\hat{\mu})] \end{bmatrix}$$

Here we have defined

$$\begin{aligned} \eta^R_k &= -\eta^L_k = -i\sigma_k \quad (k=1,2,3) \\ \eta^R_4 &= \eta^L_4 = \mathbb{1}_2 \end{aligned}$$

The diagonal terms in $\mathbf{D}_{\mu\pm}(x)$ are analogous to Wilson non-local mass terms. This particular choice is not unique, e.g. in a different construction one can dispense with link variable.

The left and right-handed fermion operators transform differently under a U(1) gauge transformations $g(x) = \exp(i\theta(x))$, which transform fields according to

$$\begin{aligned} \psi_L(x) &\to g(x)\psi_L(x) \quad , \quad \psi_R(x) \to g^*(x)\psi_R(x) \\ \overline{\psi}_L(x) &\to g^*(x)\overline{\psi}_L(x) \quad , \quad \overline{\psi}_R(x) \to g(x)\overline{\psi}_R(x) \\ \phi(x) &\to g(x)\phi(x) \quad , \quad \varphi(x) \to g^2(x)\varphi(x) \\ U_\mu(x) &\to g(x)U_\mu(x)g^*(x+\hat{\mu}) \end{aligned}$$

Chiral II

Regarding the local “mass” term in the action

$$S_M = M \sum_x [\overline{\psi}_L(x)\varphi(x)\psi_R(x) + \overline{\psi}_R(x)\varphi^*(x)\psi_L(x)]$$

as a vertex between, e.g., a right-handed fermion and a composite left-handed fermion + Higgs state of the same U(1) charge, then we may construct $q = \pm 1$ massive fermions from a combination of the corresponding local operators,

$$\begin{aligned}
a^\dagger(x) &= \frac{1}{\sqrt{2m}}(\overline{\psi}_L(x)\varphi(x) + \overline{\psi}_R(x)) \quad , \quad a(x) = \frac{1}{\sqrt{2m}}(\psi_R(x) + \psi_L(x)\varphi^\dagger(x)) \\
b^\dagger(x) &= \frac{1}{\sqrt{2m}}(\psi_L(x)\varphi^\dagger(x) - \psi_R(x)) \quad , \quad b(x) = \frac{1}{\sqrt{2m}}(\overline{\psi}_R(x) - \overline{\psi}_L(x)\varphi(x))
\end{aligned}$$

In the same way one can construct operators transforming covariantly with opposite charge, by combining the right instead of left-handed fermion operators with the squared Higgs field.

Chiral III

Construct a set of states which span a small subspace of the Hilbert space containing a static fermion-antifermion pair:

$$|\Phi_i(R)\rangle = \{a^\dagger(\mathbf{x})\zeta_i^*(\mathbf{x};U)\}\,\{b^\dagger(\mathbf{y})\zeta_i(\mathbf{y};U)\}\,|\Psi_0\rangle$$

with

$$\zeta_i(\mathbf{x};U) = \left\{ \begin{array}{cc} \xi_i(\mathbf{x};U) & i \le n_{ev} \\ \varphi(x)\xi^*_{i-n_{ev}}(\mathbf{x};U) & n_{ev}+1 \le i \le 2n_{ev} \\ \phi(\mathbf{x}) & i = 2n_{ev}+1 \end{array} \right. .$$

where the $\{\xi_i,\ i = 1, 2, .., n_{ev}\}$ are eigenstates of the covariant Laplacian.

Then proceed as before, looking for the ground and excited states. This time we compute the expectation value of products of $\mathbf{D}_\pm$, rather than simply Wilson lines at $\mathbf{x}, \mathbf{y}$.

Chiral IV

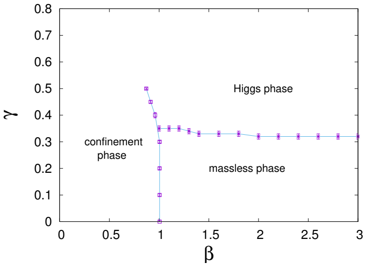
Explicitly, we compute numerically

$$[\mathcal{T}^T]_{ji}(R) \quad = \quad \langle \Phi_j | \mathcal{T}^T | \Phi_i \rangle = \langle Q_{ji}^{+T}(\mathbf{x}, t) Q_{ji}^{-T}(\mathbf{y}, t) \rangle$$

where

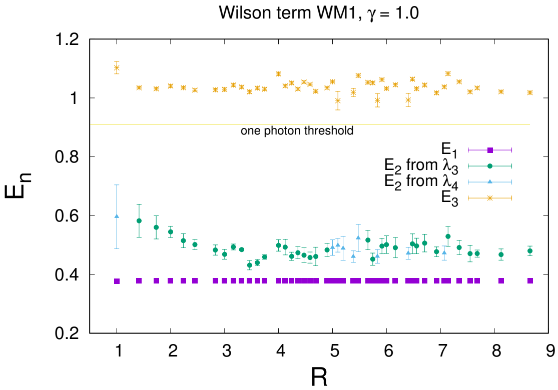
$$\begin{aligned}
Q_{ji}^{+T}(\mathbf{x}, t) \quad &= \quad \Big[\zeta_i(\mathbf{x}, t), -\varphi^*(\mathbf{x}, t)\zeta_i(\mathbf{x}, t) \Big] \, \mathbf{D}_{4+}(\mathbf{x}, t) \\
&\quad \left(\prod_{\tau=1}^{T-1} \mathbf{F}(\mathbf{x}, t+\tau) \mathbf{D}_{4+}(\mathbf{x}, t+\tau) \right) \begin{bmatrix} \varphi(\mathbf{x}, t+T)\zeta_j^*(\mathbf{x}, t+T) \\ -\zeta_j^*(\mathbf{x}, t+T) \end{bmatrix} \\
Q_{ji}^{-T}(\mathbf{x}, t) \quad &= \quad \Big[\zeta_j(\mathbf{x}, t+T), \varphi^*(\mathbf{x}, t+T)\zeta_j(\mathbf{x}, t+T) \Big] [\, \mathbf{D}_{4-}(\mathbf{x}, t+T) \\
&\quad \left(\prod_{\tau=1}^{T-1} \mathbf{F}(\mathbf{x}, t+T-\tau) \mathbf{D}_{4-}(\mathbf{x}, t+T-\tau) \right) \begin{bmatrix} \varphi(\mathbf{x}, t)\zeta_i^*(\mathbf{x}, t) \\ \zeta_i^*(\mathbf{x}, t) \end{bmatrix} \\
\mathbf{F}(x) \quad &= \quad \begin{bmatrix} \varphi(x) & 0 \\ 0 & \varphi^*(x) \end{bmatrix}
\end{aligned}$$

The (thermodynamic) phase diagram of U(1) gauge Higgs theory with a $q = 1$ Higgs field



The numerical simulation is carried out on a $14^3 \times 32$ lattice volume at $\beta = 3.0, \ \gamma = 1.0$ using $n_{ev} = 4$ Laplacian eigenstates.

Excitations



Energies E_1, E_2, E_3 vs. R at $\beta = 3, \gamma = 1$, shown together with the one photon threshold.

Conclusions

- The gauge+Higgs fields surrounded a charged static fermion have a spectrum of localized excitations, which cannot be interpreted as simply the ground state plus some massive bosons.
- **This means that charged "elementary" particles can have a mass spectrum in gauge Higgs theories.**
- This conclusion seems robust. We see it in Landau-Ginzburg, abelian Higgs, SU(3) gauge Higgs, and chiral U(1) models.
- Excitations of screened ions have already seen in normal metals. They *might* be observable in the superconducting phase, via photoelectron spectroscopy, by comparing core-level spectra above and below the superconducting transition.

- Given a lattice formulation of chiral gauge theories, with a positive transfer matrix and a sensible continuum limit, we could figure out the excitations of quarks and leptons in the electroweak theory.

- Going by previous results, the excitation energies should be O(M_Z) above the ground state masses of quarks and leptons.

- It would be nice to have that lattice formulation...

Takk for at du lyttet!

Er det noen spoersmaal?