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Four-fermion deformations of the massless Schwinger model and confinement

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Schwinger model

Schwinger model - QED in the 1+1 spacetime dimensions - is a famous playground for the exploration of ideas about quark confinement and chiral symmetry breaking.

Schwinger '62; Coleman, Jackiw, Susskind '75

Originally the model was introduced as U(1) gauge theory with a single unit charge Dirac fermion. Our starting point will be the charge-N Schwinger model.

Hanson, Nielsen, Zahed '95 + many recent works

Its Euclidean action

$$S_{\text{standard}} = \int d^2x \left(\frac{1}{4e^2} f_{\mu\nu}^2 + \overline{\psi} \left[\gamma^\mu (\partial_\mu + i N a_\mu) \right] \psi \right) + m_\psi \overline{\psi}_L \psi_R + \text{h.c.}$$

contains U(1) gauge field a_μ, Dirac fermion ψ, the integer N is the charge of the fermion (in units of e). Heavy probe have the unit charge. The integer N keeps the group manifold compact.

We also add the topological θ term,

$$S_\theta = \frac{i\theta}{2\pi}\int_{M_2} da$$

Coleman observed that changing θ by 2π corresponds to inserting charges ± 1 at $x = \pm\infty$. It means the *k*-string tension can be written as

$$T_k(\theta) = \mathcal{E}(\theta + 2\pi k) - \mathcal{E}(\theta)\,,$$

where $\mathcal{E}(\theta)$ is the vacuum energy density. At $m_\psi = 0$ there is no θ dependence, so the massless theory does not confine integer test charges.

When $m_\psi \neq 0$

$$\mathcal{E}_k(\theta) = -m_\psi\langle\overline{\psi}_L\psi_R\rangle + \text{c.c}$$

$$\langle\overline{\psi}_L\psi_R\rangle = \frac{m_\gamma\, e^\gamma}{4\pi} e^{i\frac{\theta+2\pi k}{N}}, \qquad k = 1,\dots,N \qquad m_\gamma = Ne/\sqrt{\pi}$$

$$T_k(\theta) = -\frac{m_\psi m_\gamma e^\gamma}{2\pi}\left[\cos\left(\frac{\theta+2\pi k}{N}\right) - \cos\left(\frac{\theta}{N}\right)\right] + O(m_\psi^2)$$

For $\theta = 0$ it becomes

$$T_k(\theta = 0) = \frac{m_\psi m_\gamma e^\gamma}{\pi} \sin^2\left(\frac{\pi k}{N}\right) + O(m_\psi^2)\,.$$

Clearly, there is a finite tension, and hence confinement for charges $k \neq 0 \pmod N$, and charges that are multiples of N are screened.

$$\text{mass deformation}: \qquad \langle W_k(C)\rangle = \begin{cases} e^{-T_k A(C)}, & k \neq 0 \pmod N \\ e^{-MP(C)}, & k = 0 \pmod N \end{cases}.$$

For any value of m_ψ the model has $\mathbb{Z}_N$ 1-form symmetry. Generated by N topological operators $U_n(x)$ acting on Wilson loops:

$$\langle U_n(x)\, e^{iq\int_C a_\mu dx^\mu}\rangle = \exp\left(\frac{2\pi i n q}{N}\ell(C,x)\right)\langle e^{iq\int_C a_\mu dx^\mu}\rangle$$

where $\ell(C,x)$ is the linking number of C and x.

$\mathbb{Z}_N$ 1-form symmetry sharply defined the concept of confinement.

When $m_\psi = 0$ - at the classical level a U(1) axial symmetry broken by ABJ anomaly to the discrete $\mathbb{Z}_N$,

$$\psi(x) \to e^{2\pi i \gamma_5/(2N)} \psi(x)\,.$$

Under action of $\mathbb{Z}_N$, $\overline{\psi}_L \psi_R \to e^{2\pi i/N} \overline{\psi}_L \psi_R$ so is meaningful to discuss spontaneous symmetry breaking. The above $\mathbb{Z}_N$ 0-form chiral symmetry and the $\mathbb{Z}_N$ 1-form have a mixed 't Hooft anomaly Anber, Poppitz '18

Komargodski, Ohmori Roumpedakis, Seifnashri '21

At $m_\psi = 0$ both, axial $\mathbb{Z}_N$ 0-form and $\mathbb{Z}_N$ 1-form symmetries are spontaneously broken.

The second breaking implies that the test unit charges are not confined and the Wilson loop has a perimeter behavior. Confinement appears at $m_\psi \neq 0$.

Can we modify the model to get confinement at $m_\psi = 0$?

Four-fermion modifications

The Schwinger model is super-renormalizable, approaches a free field CFT at high energies. There is an unique exactly marginal four-fermion operator

$$\mathcal{O}_{\rm jj} = j_\mu j^\mu = \overline{\psi}\gamma_\mu\psi\,\overline{\psi}\gamma^\mu\psi = -4\overline{\psi}_R\psi_L\overline{\psi}_L\psi_R\,.$$

This 'Thirring model' operator with $\Delta_{\rm jj} = 2$ preserves all the symmetries of the model. Adding it with a dimensionless coefficient g we come to what we call the Schwinger-Thirring (ST) model:

$$S_{\rm ST} = S_{\rm standard} + g\int d^2x\,\mathcal{O}_{jj}$$

It is important that when $m_\psi = 0$ the parameter g remains exactly marginal even in the presence of gauge interaction.

The only limitation for value of g is $g > g_* = -\pi/2$
Unitarity is broken below g_*.

While the model still approaches a free field CFT point at high energies it becomes an interacting fixed point — scaling dimensions of operators do not coincide with canonical ones.

The lowest dimension four-fermion operator breaking chiral symmetry is .

$$\mathcal{O}_\chi = \overline{\psi}_L \psi_R (D_\mu \overline{\psi}_L)(D^\mu \psi_R)\,.$$

For its scaling dimension Δ_χ bosonization implies .

$$\Delta_\chi = \frac{4}{1+2g/\pi}\,.$$

Thus, $\mathcal{O}_\chi$ is RG irrelevant at $g = 0$, but becomes relevant when $g > \pi/2$.

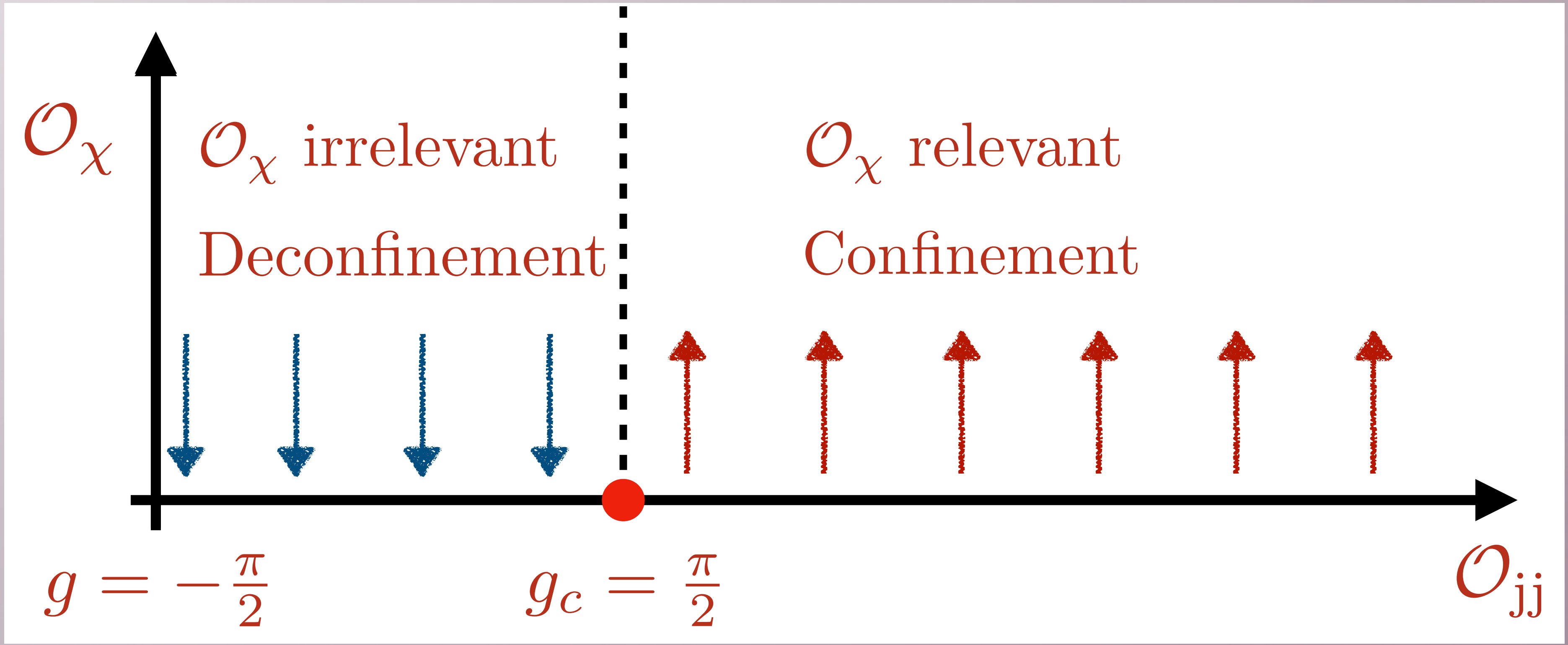
What happens to the low-energy physics once we add $\mathcal{O}_\chi$?

$$S = S_{\rm ST} + \Lambda^{2-\Delta_\chi} \int d^2x\, (\mathcal{O}_\chi + \mathcal{O}_\chi^\dagger)$$

$$S_{\rm ST} = S_{\rm standard} + g \int d^2x\, \mathcal{O}_{jj}$$

We limit ourselves by even N . In this case $\mathbb{Z}_N$ chiral symmetry is broken by $\mathcal{O}_\chi$ to $\mathbb{Z}_2$. This $\mathbb{Z}_2$ is sufficient to forbid an emergence of mass when we start with $m_\psi = 0$

The result is illustrated at the figure.

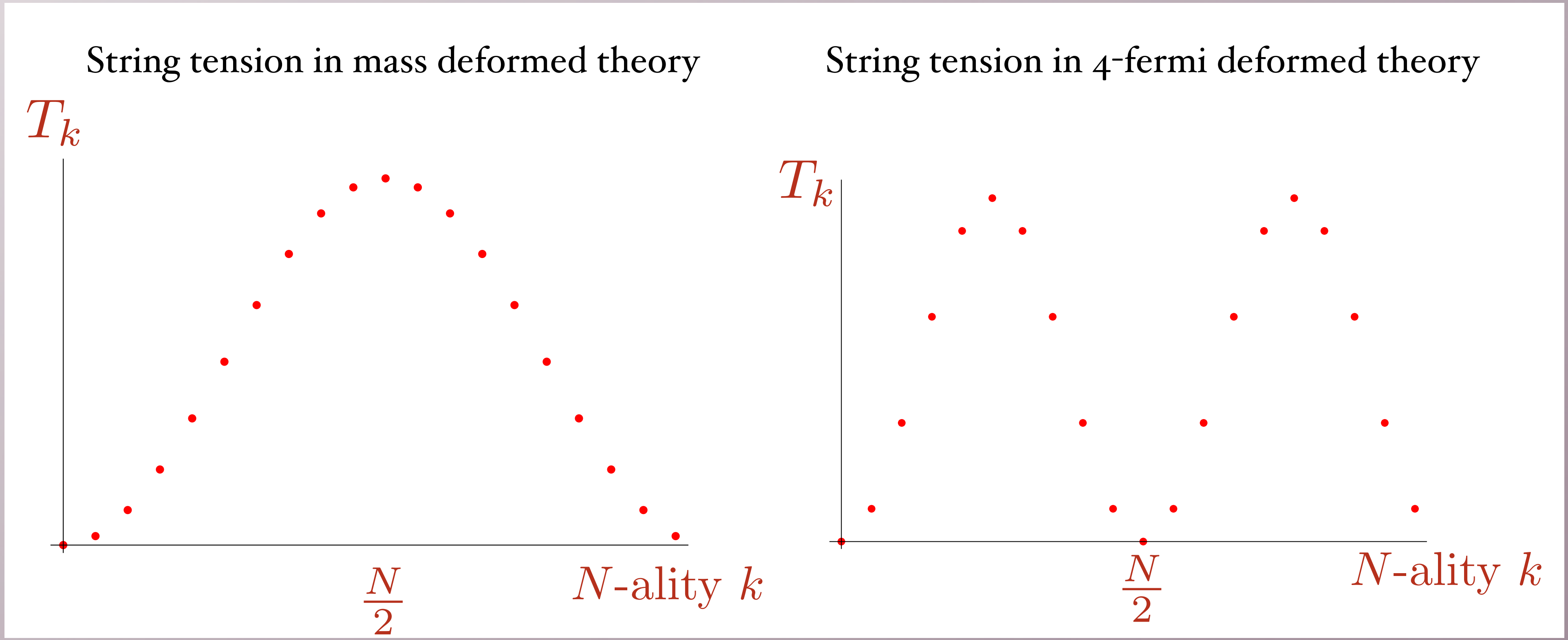


Note that $\mathcal{O}_\chi$ deformation introduces θ dependence even in the massless case, $\mathcal{O}_\chi \mapsto e^{2i\theta/N}\mathcal{O}_\chi$. It leads to

$$T_k \sim -\Lambda^{2-\Delta_\chi}\left[\cos\left(\frac{2(\theta+2\pi k)}{N}\right)-\cos\left(\frac{2\theta}{N}\right)\right]+\mathcal{O}(\Lambda^{2(2-\Delta_\chi)})$$

and at $\theta=0$

$$T_k \sim \Lambda^{2-\Delta_\chi}\sin^2\left(\frac{2\pi k}{N}\right)+\mathcal{O}(\Lambda^{2(2-\Delta_\chi)})\,.$$



Bosonizations

The compact boson action corresponding to free-fermion theory is

$$S_{\varphi,\mathrm{free}} = \int_{M_2} \frac{1}{8\pi} \|d\varphi\|^2$$

The conserved currents are related via

$$j_V \leftrightarrow -\frac{1}{2\pi} \star d\varphi\,, \quad j_A \leftrightarrow \frac{i}{4\pi} d\varphi\,.$$

Chiral symmetry acts via $e^{i\varphi} \to e^{2\pi i/N} e^{i\varphi}$. So

$$\overline{\psi}_L(x)\psi_R(x) \leftrightarrow -\frac{\mu e^{\gamma}}{2\pi} e^{i\varphi(x)}$$

Change of the renormalization scale

$$e^{ik\varphi} \to \left|\frac{\mu'}{\mu}\right|^{k^2} e^{ik\varphi}$$

For four-fermion operator $\mathcal{O}_{jj}$

$$|j_V|^2 = \overline{\psi}\gamma_\mu\psi\overline{\psi}\gamma^\mu\psi \leftrightarrow \frac{1}{4\pi^2}|d\varphi|^2 .$$

Bozonized version of Schwinger -Thirring model

$$S_{\text{bosonized ST}} = \int_{M_2}\left[\frac{1}{2e^2}\|da\|^2 + \frac{1}{2}R^2\|d\varphi\|^2 - m\mu\cos(\varphi) - \frac{iN}{2\pi}d\varphi\wedge a\right],$$

$$m = e^\gamma m_\psi/2\pi, \qquad R^2 = \frac{1}{4\pi}\left(1+\frac{2g}{\pi}\right).$$

$$e^{ik\varphi} \to \left|\frac{\mu'}{\mu}\right|^{k^2/4\pi R^2} e^{ik\varphi} \qquad \Delta_k \equiv \Delta[e^{ik\varphi}] = \frac{k^2}{1+2g/\pi}$$

$$\overline{\psi}_L(x)\psi_R(x) \leftrightarrow -\frac{\mu^{\Delta_1}e^\gamma}{2\pi}e^{i\varphi(x)} \qquad \mathcal{O}_\chi + \mathcal{O}_\chi^\dagger \longleftrightarrow c\mu^{\Delta_2}\cos(2\varphi)$$

Thus, at $m_\psi = 0$

$$S = \int_{M_2}\left[\frac{1}{2}R^2\|d\varphi\|^2 + m_\gamma^{\Delta_2}\Lambda^{2-\Delta_2}\cos(2\varphi) + \frac{1}{2e^2}\|da\|^2 - \frac{i}{2\pi}N d\varphi\wedge a\right].$$

Conclusions

We analyzed bosonized form in $\mathbb{R}^2$ and $\mathbb{R} \times S^1$ geometries.

It allows for explicit construction of 0-form and 1-form operators. Without $\mathcal{O}_\chi$ modification 't Hooft anomaly in their mixing explains deconfinement in the model.

Adding $\mathcal{O}_\chi$ modification leads to the breaking of chiral symmetry without emergence of the fermion mass and confinement arises. It makes the situation more analogous to 4d confinement.