

Confinement/deconfinement transition in D0-matrix model and appearance of M-theory

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University of Regensburg, Germany

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Based on: 2110.01312

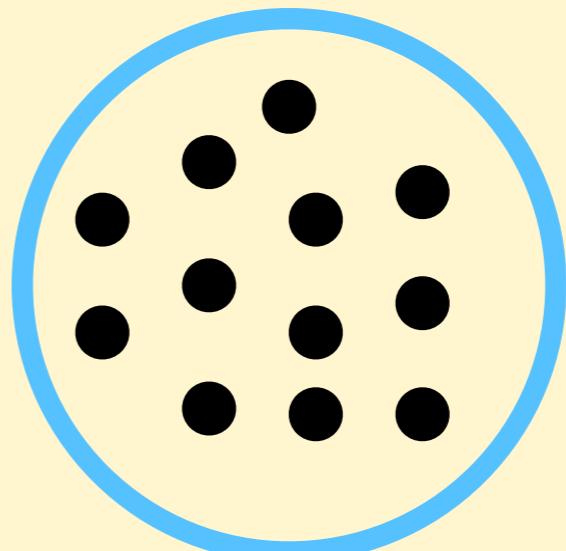
With: Bergner, Bodendorfer, Hanada, Rinaldi, Schäfer, Vranas, Watanabe (MCSMC)

Elitenetzwerk
Bayern



Plan of the talk

- Definition of the model
- Holography
- Relation with gravity
- Simulations
- Confinement in D0-matrix model



What is confined?

D0-matrix model (BFSS)

$$\mathcal{L} = \frac{1}{2g_{YM}^2} Tr \left\{ (D_t X_M)^2 + [X_M, X_N]^4 + i\bar{\psi}^\alpha D_t \psi^\beta + \bar{\psi}^\alpha \gamma_{\alpha\beta}^M [X_M, \psi^\beta] \right\}$$

X_M : $N \times N$ bosonic hermitian matrices with $M = 1, \dots, 9$ $D_t : D_t \mathcal{O} = \partial_t \mathcal{O} - i[A_t, \mathcal{O}]$

ψ_α : $N \times N$ fermionic hermitian matrices with $\alpha = 1, \dots, 16$ $\lambda = g_{YM}^2 N = [\text{energy}]^3$

- Dimensional reduction of 4D $\mathcal{N} = 4$ / 10D $\mathcal{N} = 1$
- Matrix regularisation of 11D supermembrane De Wit-Hoppe-Nicolai, 1988
- Matrix model of M-theory (BFSS) Banks-Fischler-Shenker-Susskind, 1996
- Dual to type IIA black 0-brane near 't Hooft limit Itzhaki- Maldacena-Sonnenschein-Yankielowicz, 1998

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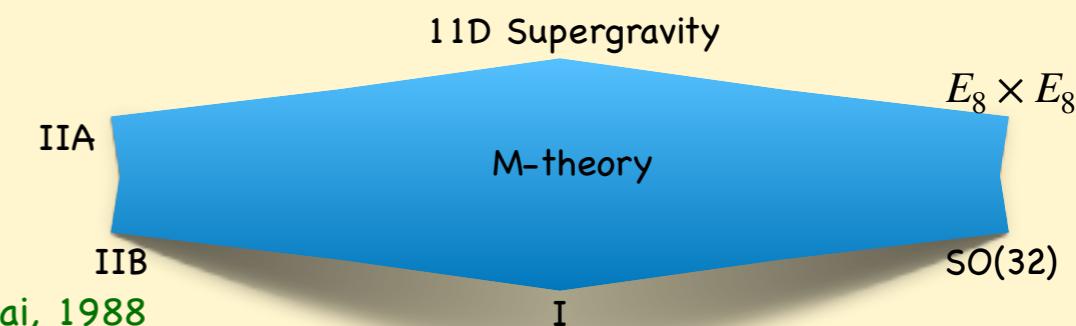
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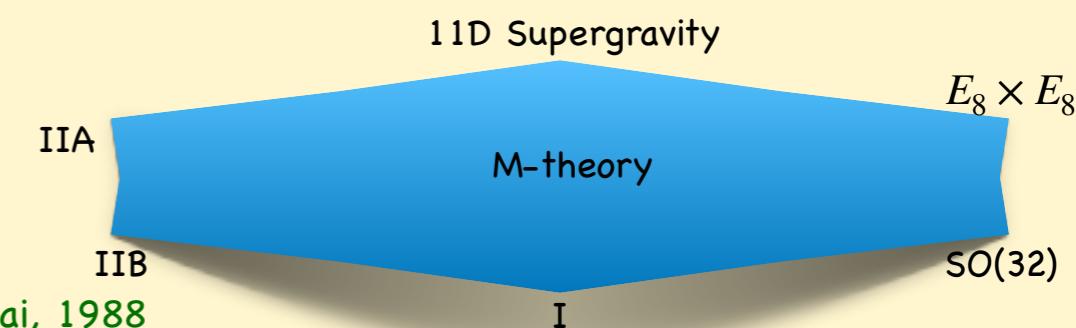
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Gauge/gravity duality in string theory

String theory contains strings and Dp-branes



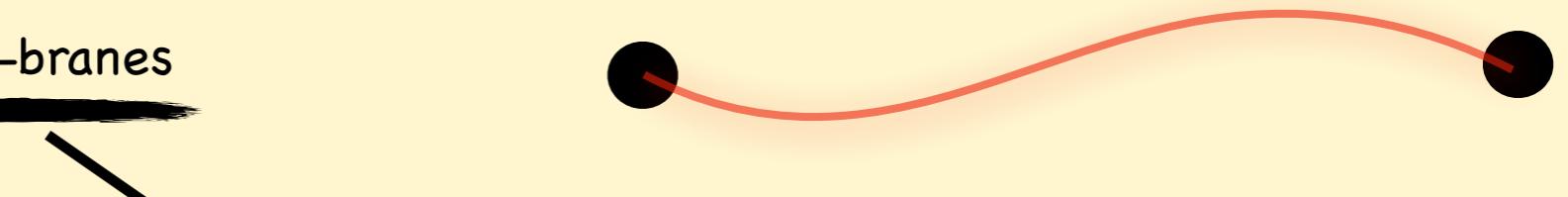
Dynamical and non-perturbative



$$\mathcal{T}_0 = \frac{\sqrt{2\pi}}{g_s}$$

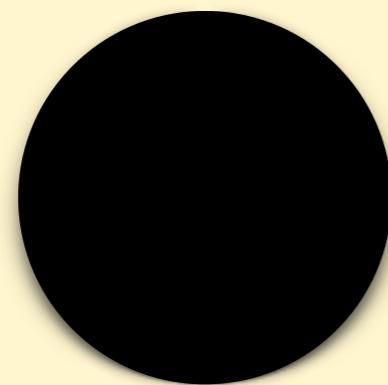
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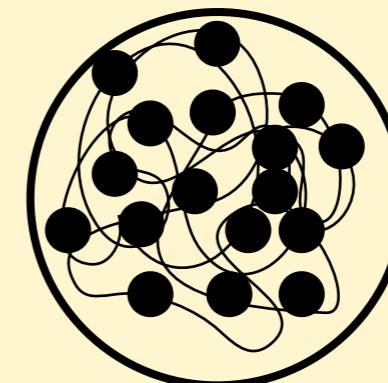
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Black p-brane
in IIA/IIB string



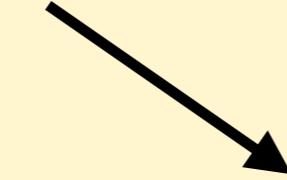
dual



(p+1)-d U(N) SYM
(D_p-branes + strings)

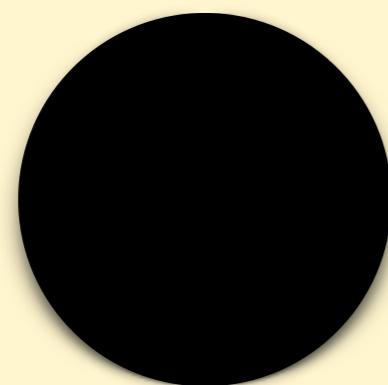
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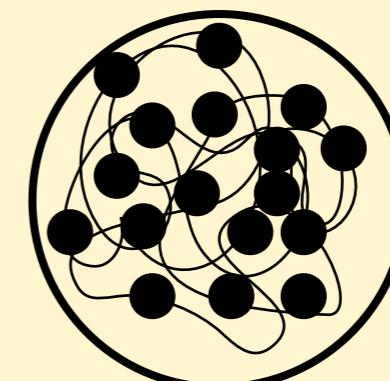
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Black p-brane
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In this talk p=0

The curious case of p=0

$$\lambda = g_{YM}^2 N = [\text{energy}]^3 \quad \Rightarrow \quad g_{\text{eff}} = \frac{\lambda}{E^3}$$

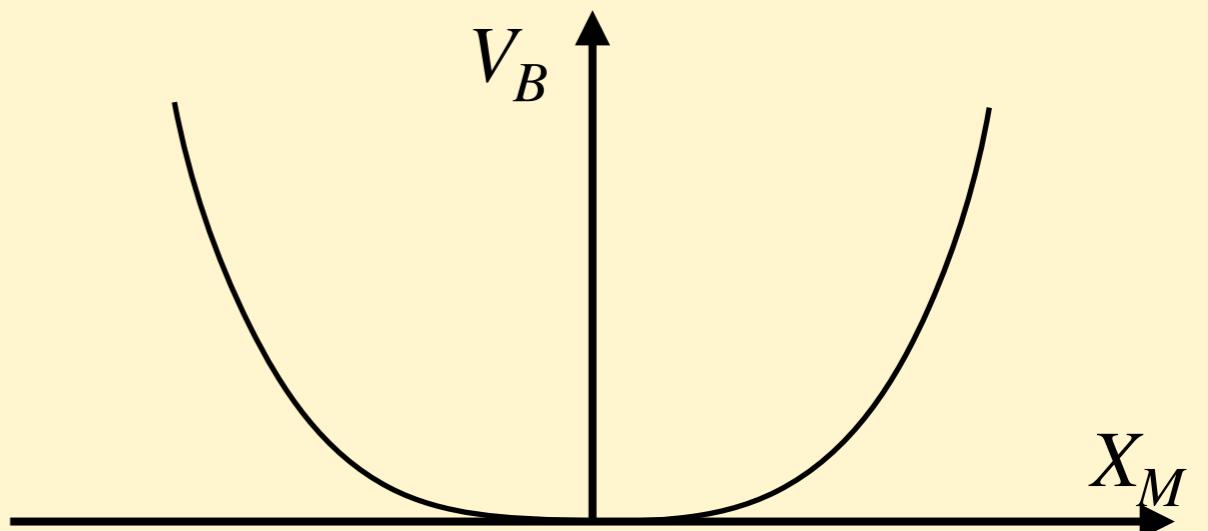
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$$V_B = [X_M, X_N]^2 \sim X_M^4$$



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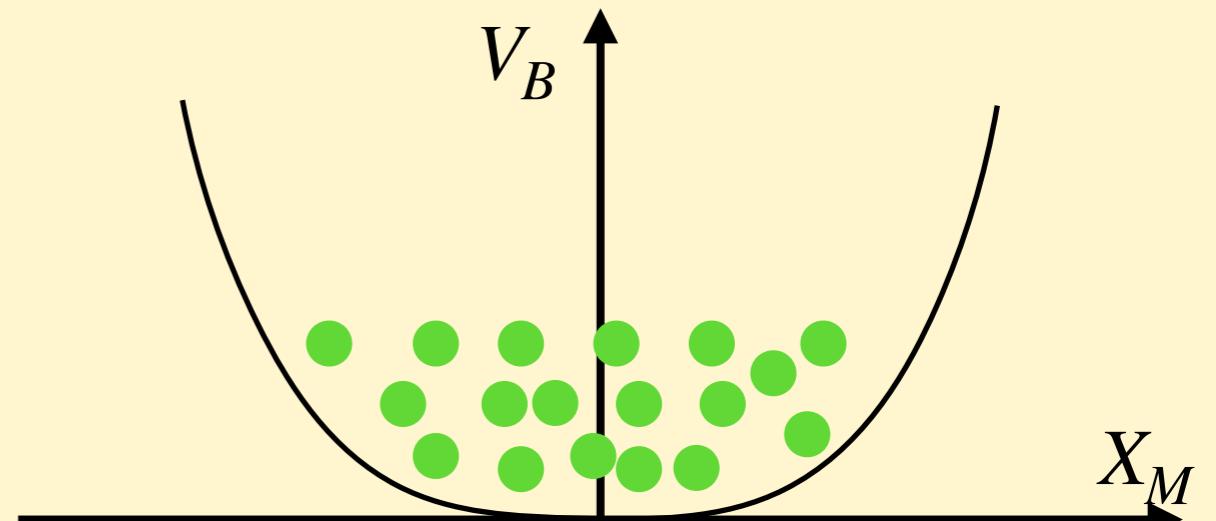
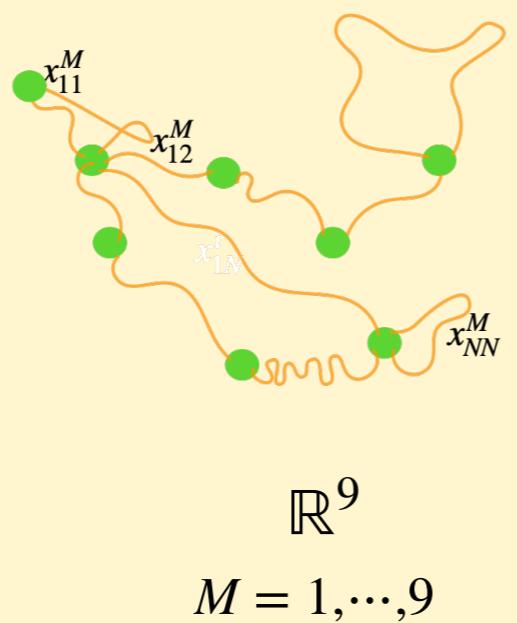
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$$X^M = \begin{pmatrix} & x_{11}^M & x_{12}^M & \cdots & x_{1N}^M \\ x_{21}^M & & & & \\ & \vdots & & & \\ & x_{N1}^M & & & x_{NN}^M \end{pmatrix}_{N \times N}$$

Witten 1995

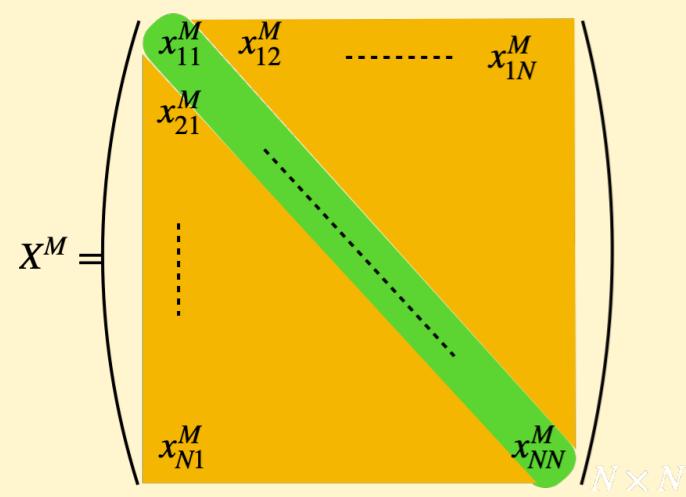


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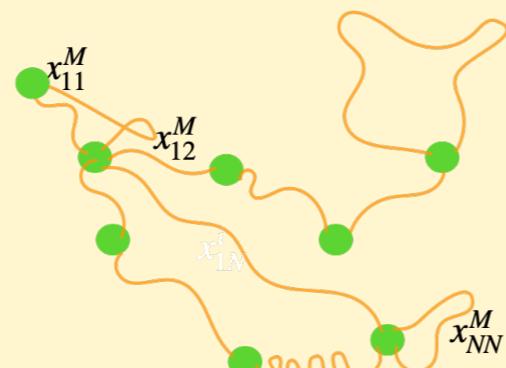
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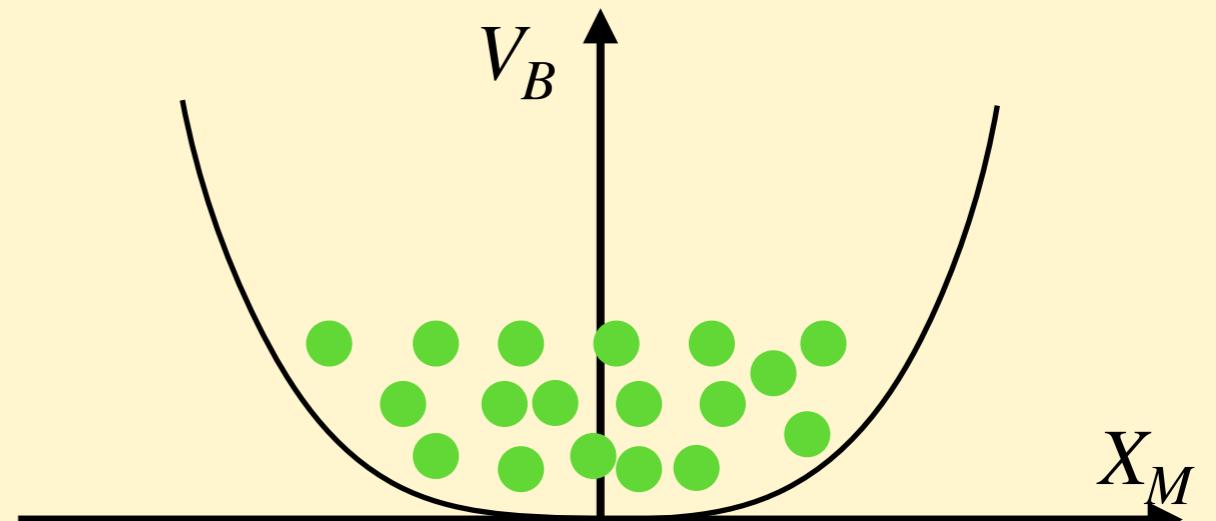


Witten 1995



$$\mathbb{R}^9$$

$$M = 1, \dots, 9$$



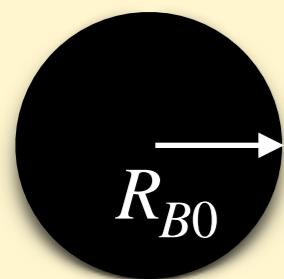
What does this correspond to in the gravity side?

The curious case of p=0

$$g_{\text{eff}} = \frac{\lambda}{E^3}$$

Low energies \longleftrightarrow Strong coupling

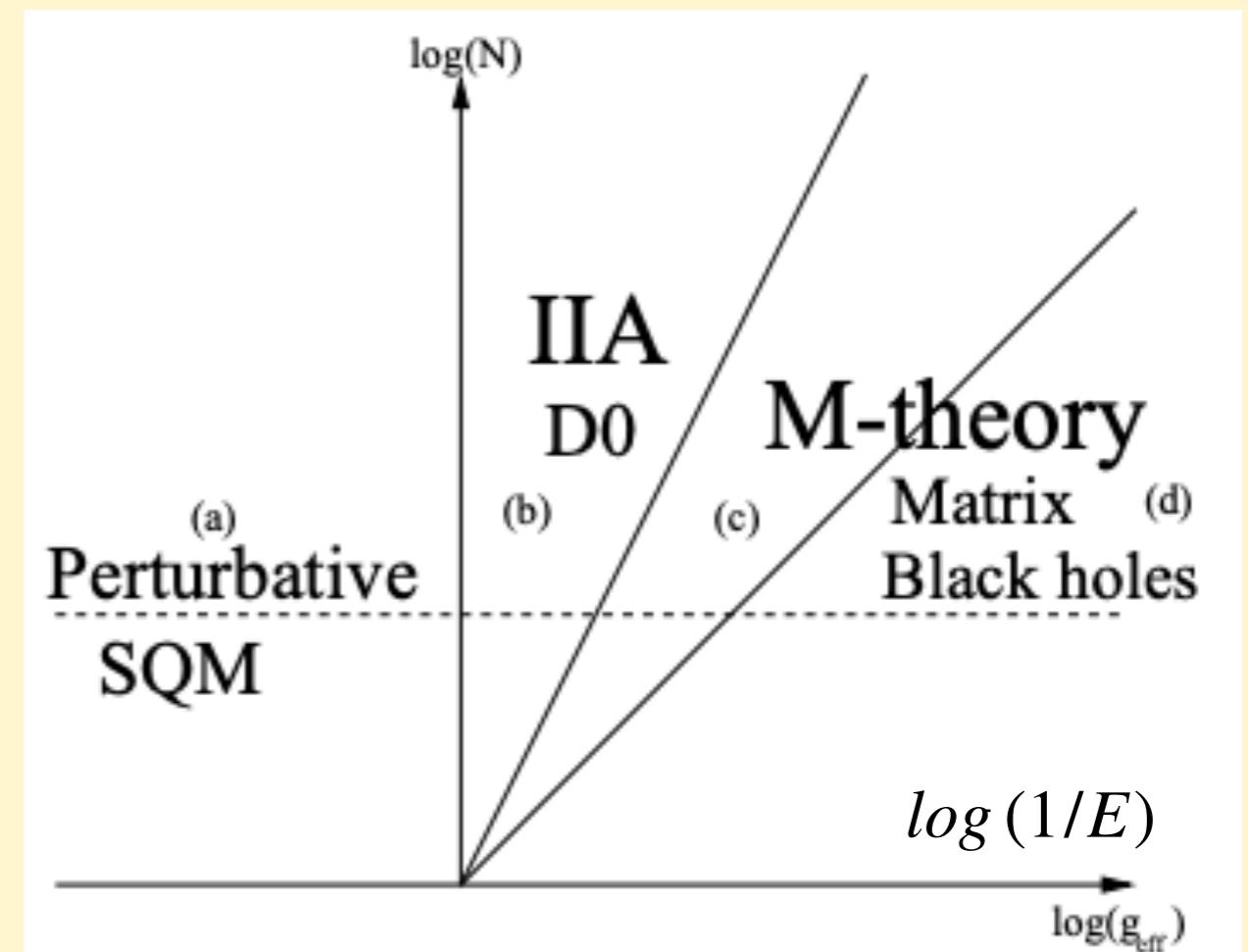
Black zero-brane in IIA SUGRA



$$E = 7.41 N^2 \lambda^{-3/5} T^{14/5}$$

$$\frac{R_{B0}^2}{\alpha'} \sim g_{\text{eff}}^{\frac{1}{2}} \sim \sqrt{\frac{\lambda}{E^3}}$$

$$e^\phi \Big|_{\text{horizon}} \sim \frac{g_{\text{eff}}^{7/4}}{N}$$



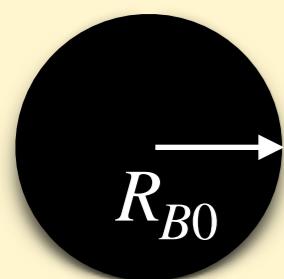
Itzhaki-Maldacena-Sonnenschein-Yankielowicz, 1998

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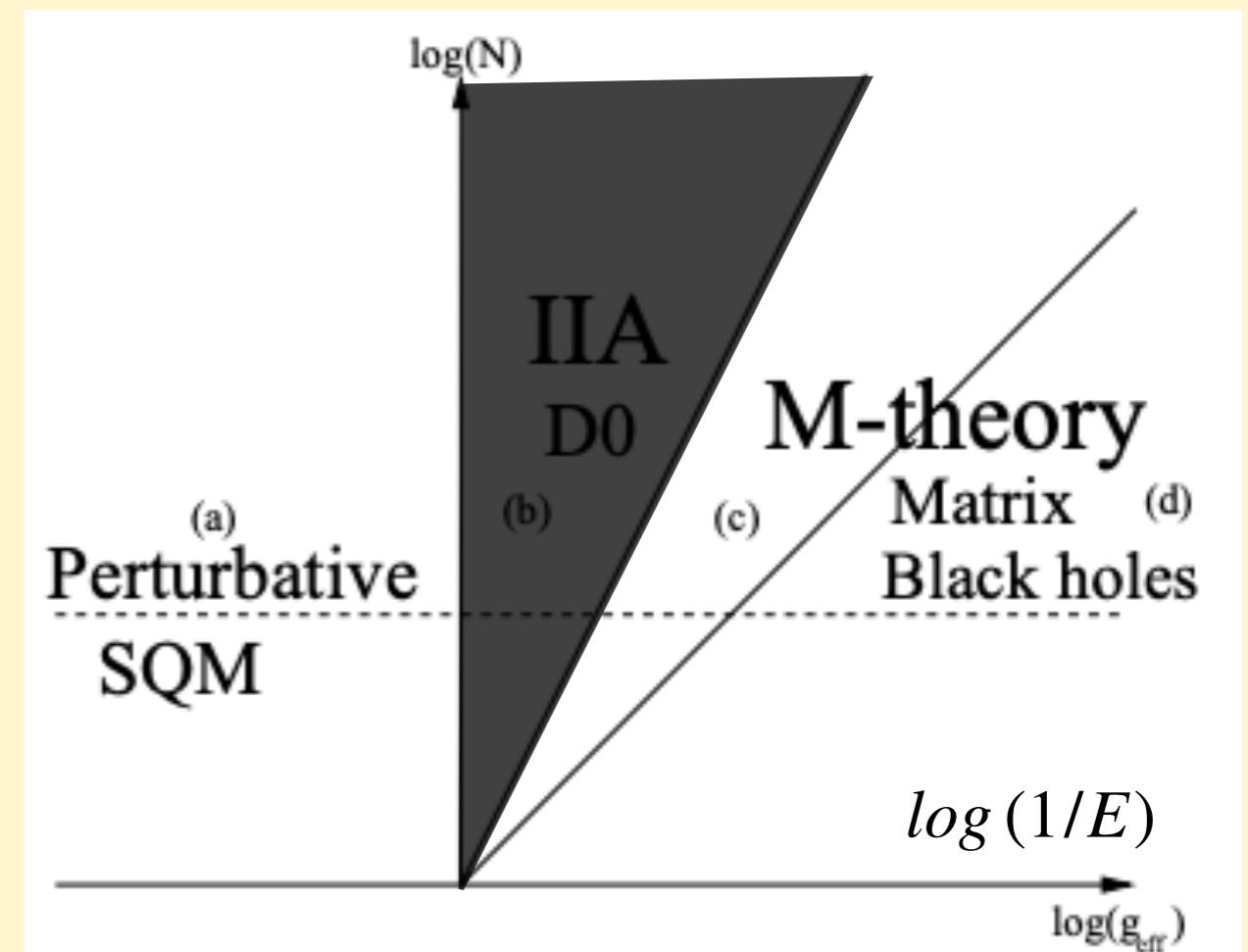
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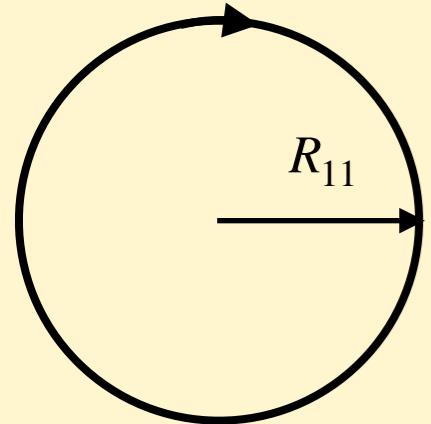
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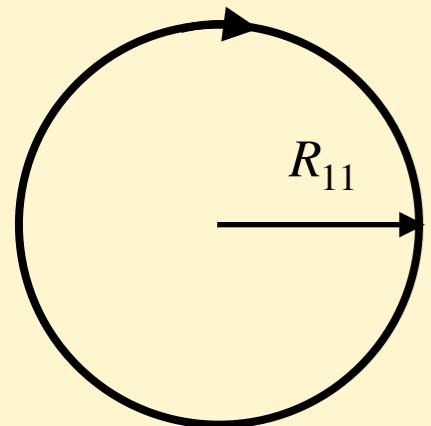
Type IIA string theory is defined as M-theory compactified on a circle S^1

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Strong coupling/low energies corresponds to the M-theory region

To probe M-theory region $E \ll 1$ ($E = 7.41 N^2 \lambda^{-3/5} T^{14/5}$) \longrightarrow Low temperatures

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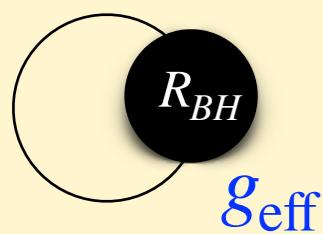


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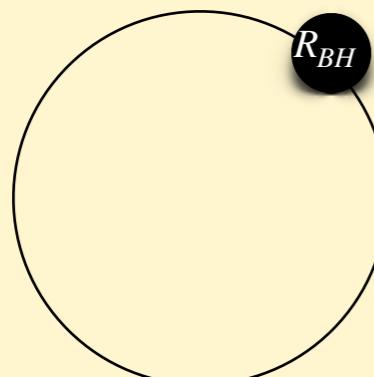
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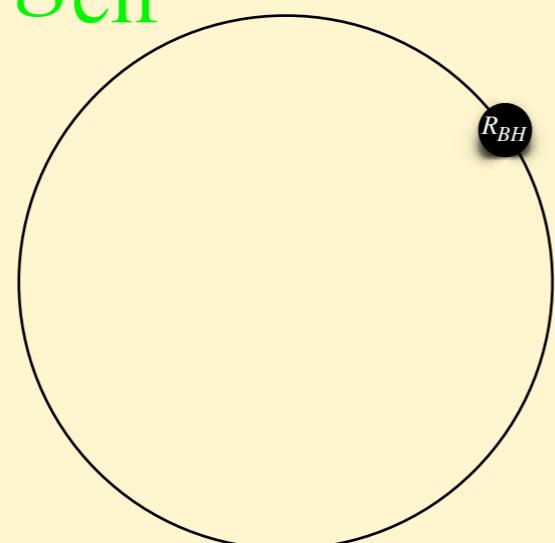
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R_{BH} small \leftrightarrow g_{eff} large

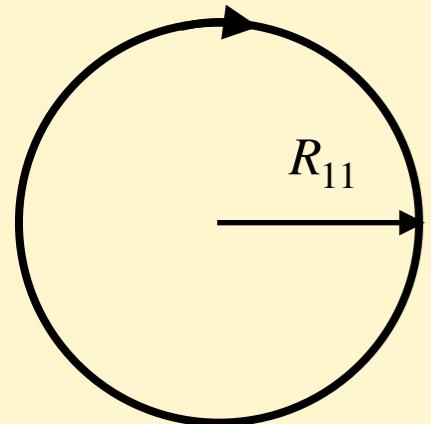


g_{eff}



g_{eff}

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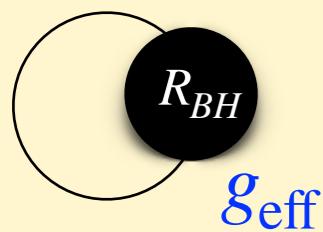


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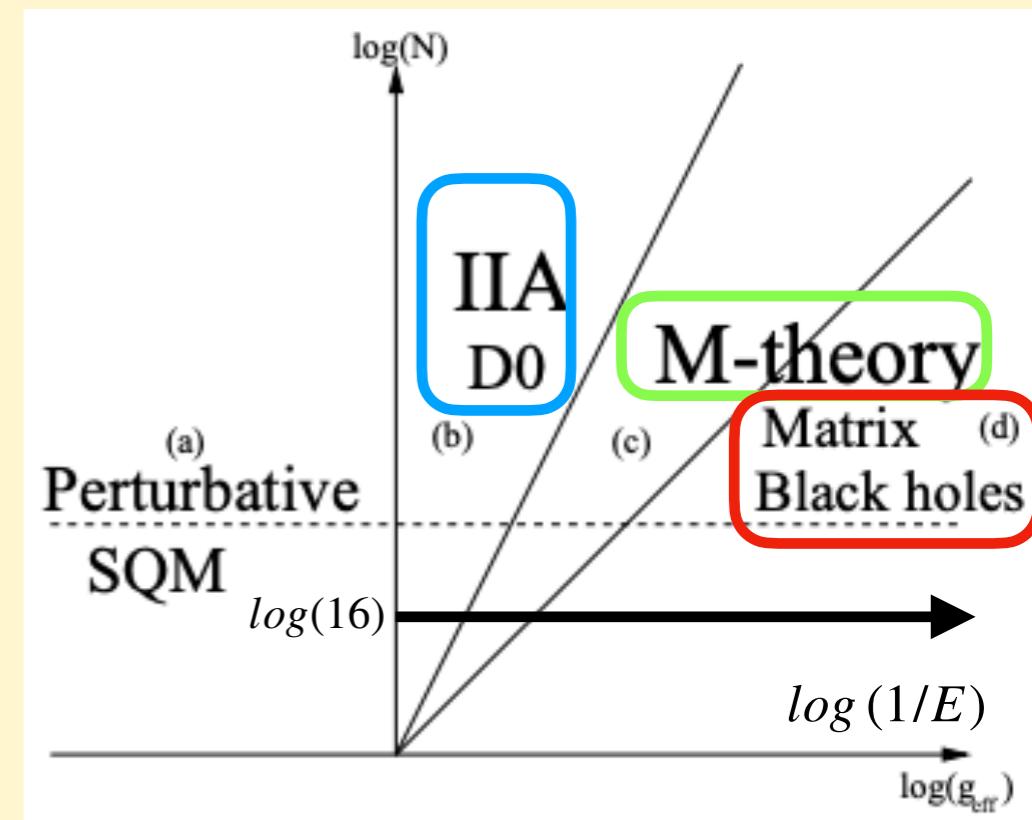
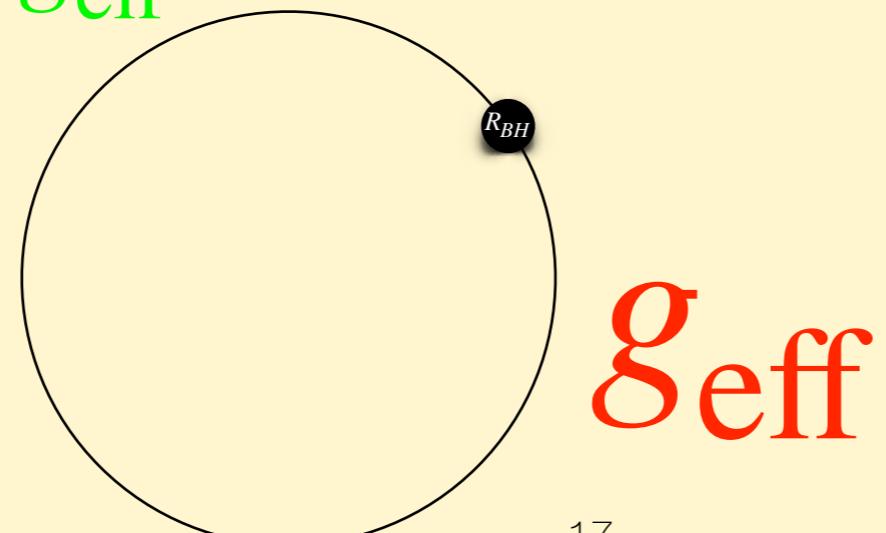
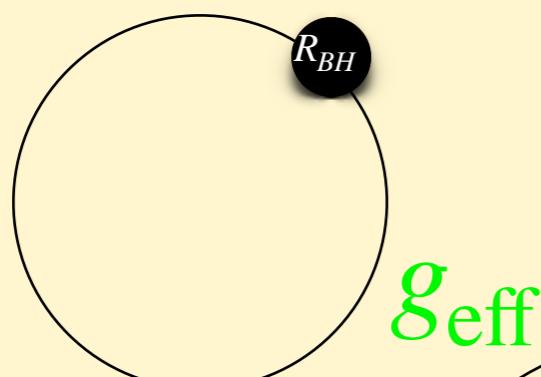
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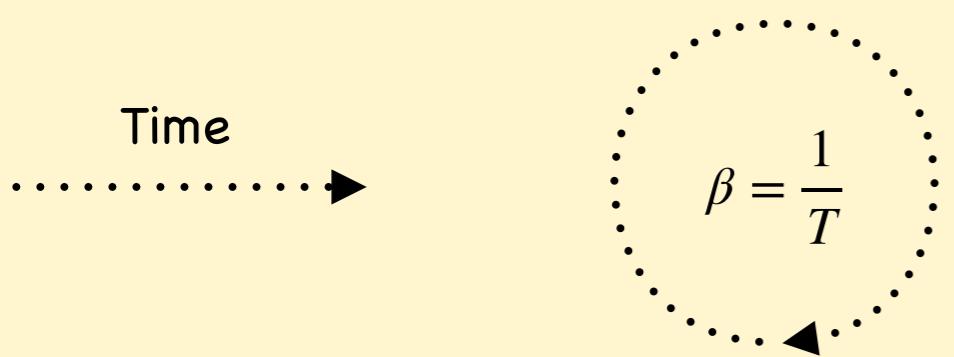


R_{BH} small \leftrightarrow g_{eff} large



Simulations

- We can do Monte Carlo simulations
- Borrow techniques from lattice QCD
- (0+1)-d matrix quantum mechanics



- Parameters

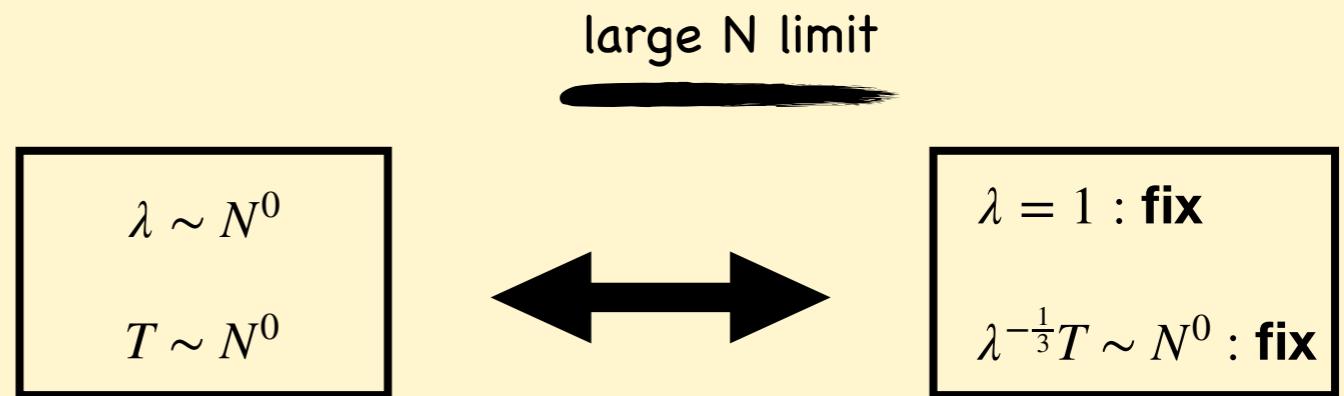
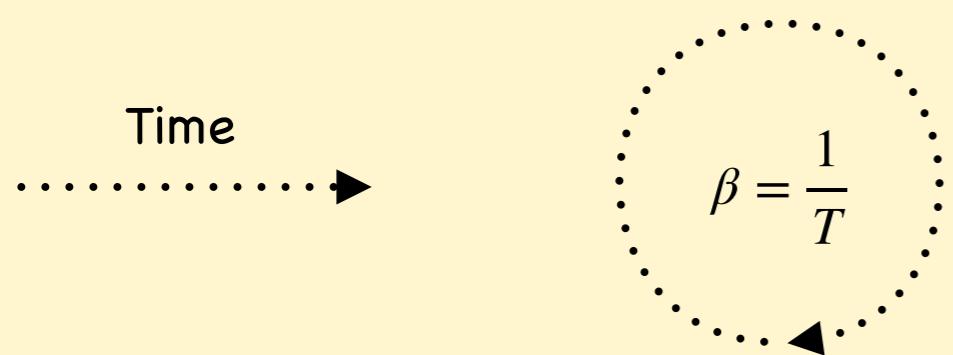
$$N \longrightarrow X_{N \times N}$$

$$S \longrightarrow \text{Lattice points}$$

$$T \longrightarrow \text{Temperature}$$

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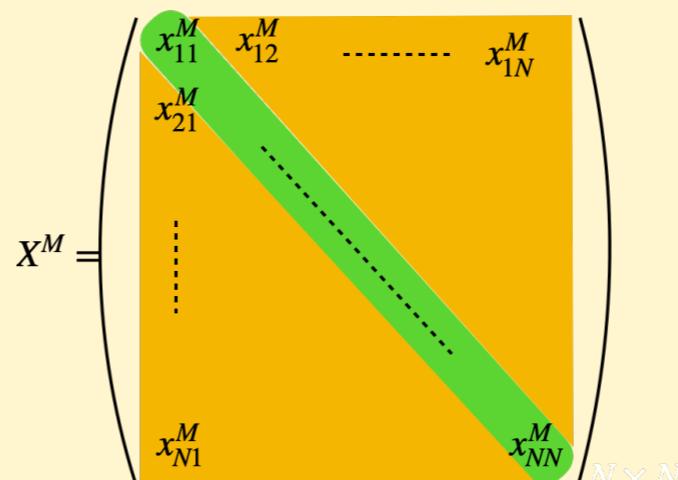
$$\longrightarrow X_{N \times N}$$

S

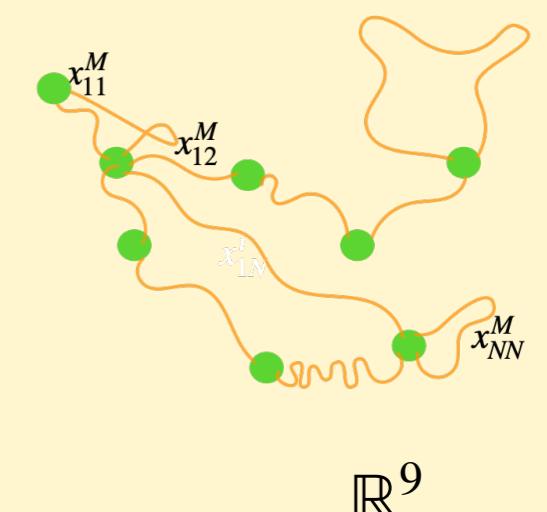
\longrightarrow Lattice points

T

\longrightarrow Temperature



Witten 1995



$$M = 1, \dots, 9$$

Confinement/deconfinement

- Polyakov loop

$$P = \frac{1}{N} \text{Tr} \left(\mathcal{P} \exp \left(i \int_0^\beta dt A_t \right) \right) = \frac{1}{N} \sum_{j=1}^N e^{i\theta_j}$$

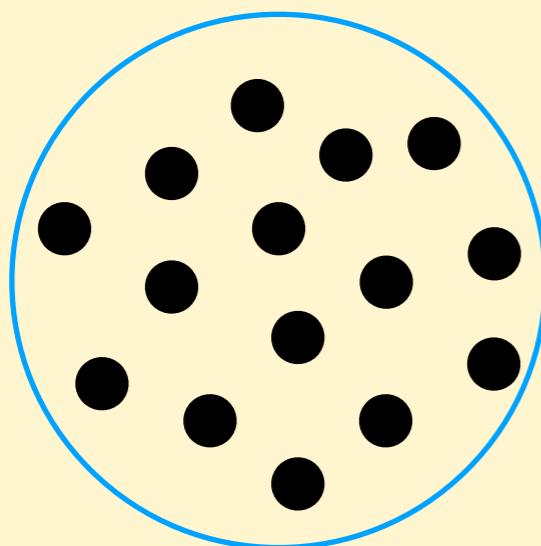
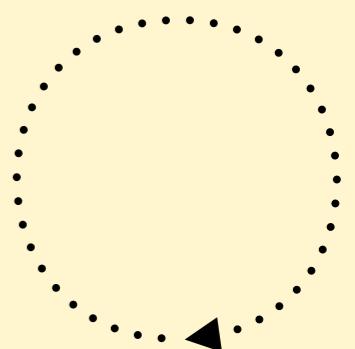
On the lattice

- Restoration/breaking of $U(1)$ symmetry



See talk on Friday from M. Hanada [140]

- Intuition from $AdS_5 \times S^5$

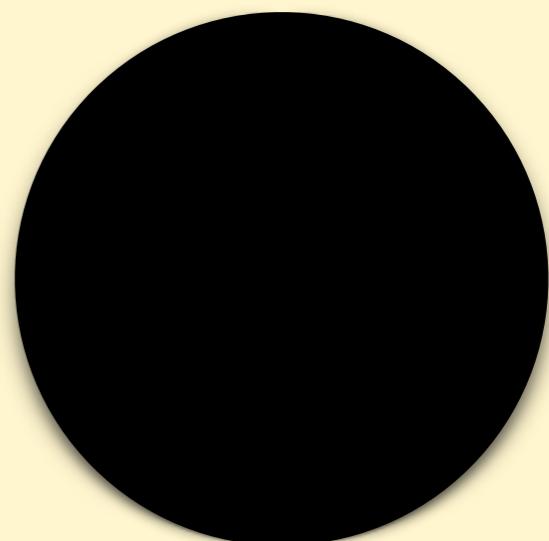


$$E, P = 0$$

Confinement

Graviton gas

Gravity side



$$E \neq 0, P \gtrsim \frac{1}{2}$$

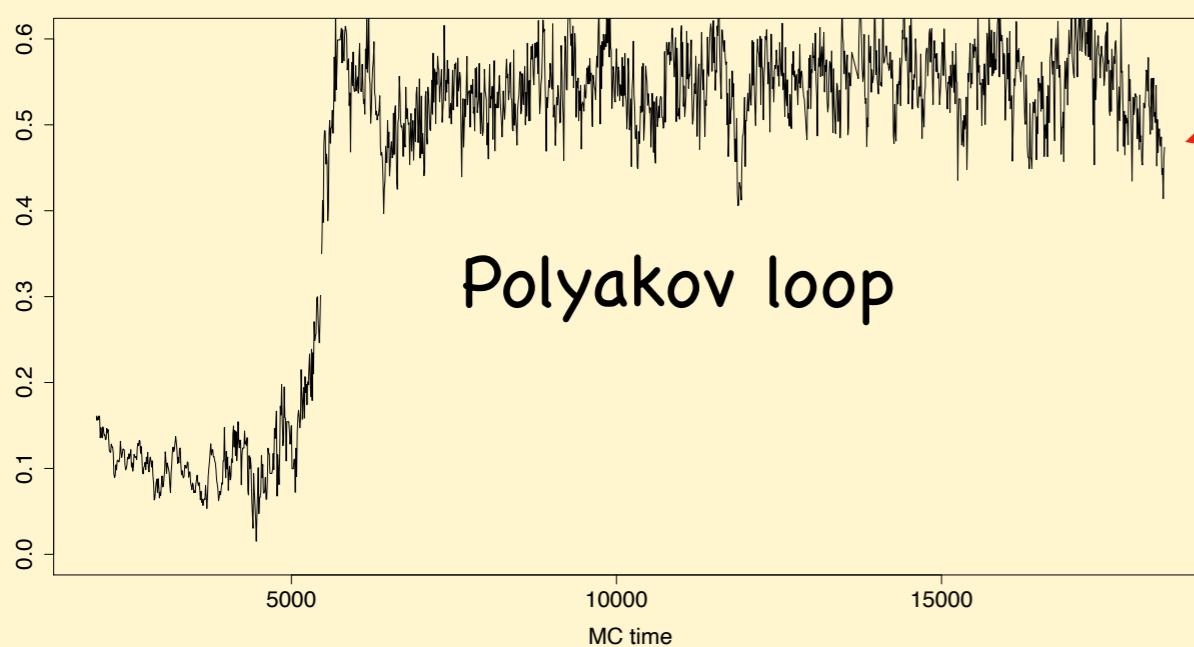
Deconfinement

Black holes

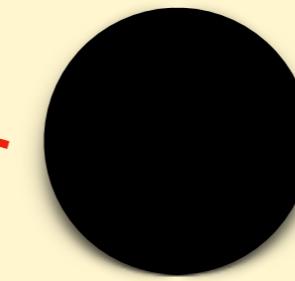
MAGOO, Witten, Sundborg

Aharony-Marsano-Minwalla-Papadodimas-Van Raamsdonk, 2003

Confinement/deconfinement

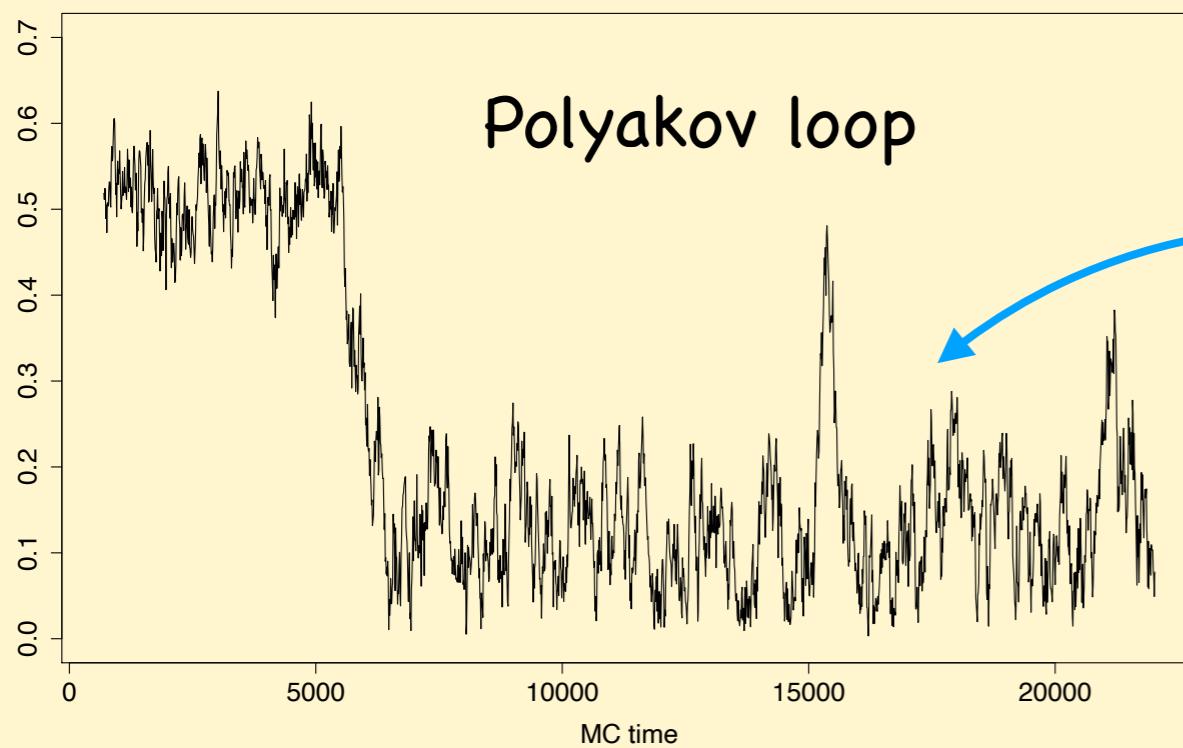


Polyakov loop



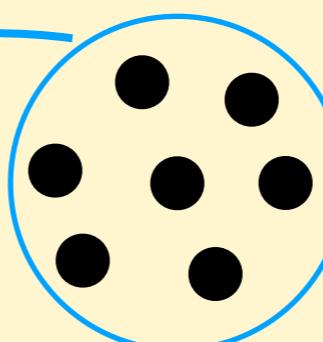
deconfined

$$P \gtrsim \frac{1}{2}$$



Polyakov loop

Confined



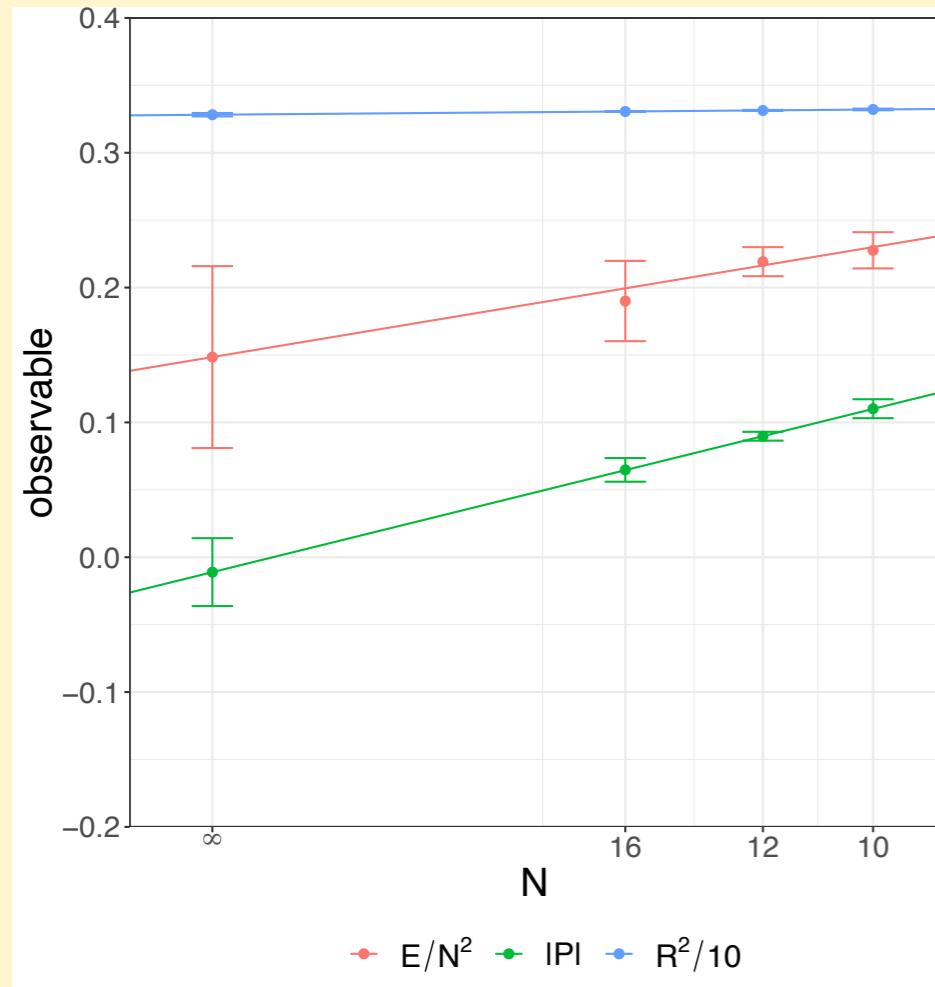
$$P \approx 0$$

The gravity theory predicts
always **deconfinement**
($E = 7.41N^2\lambda^{-3/5}T^{14/5}$)

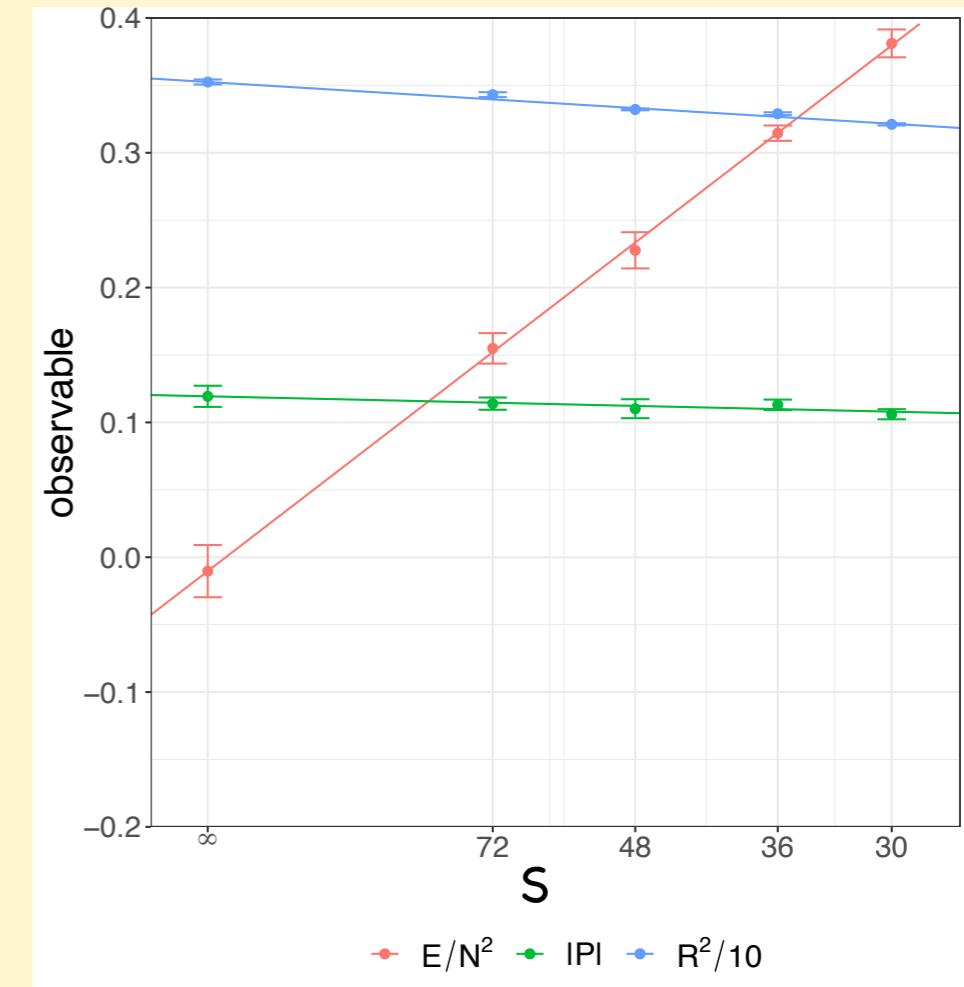
How to understand this?

Confined studies of D0-matrix model

Large N and S=48 @ T=0.2



Continuum and N=10 @ T=0.2



Deconfined phase

$$\frac{E}{N^2} \simeq 7.41 T^{\frac{14}{5}} \simeq 0.0818 \quad @ T=0.2$$

$$P \simeq 0.5 \quad @ T=0.2$$

Confined phase

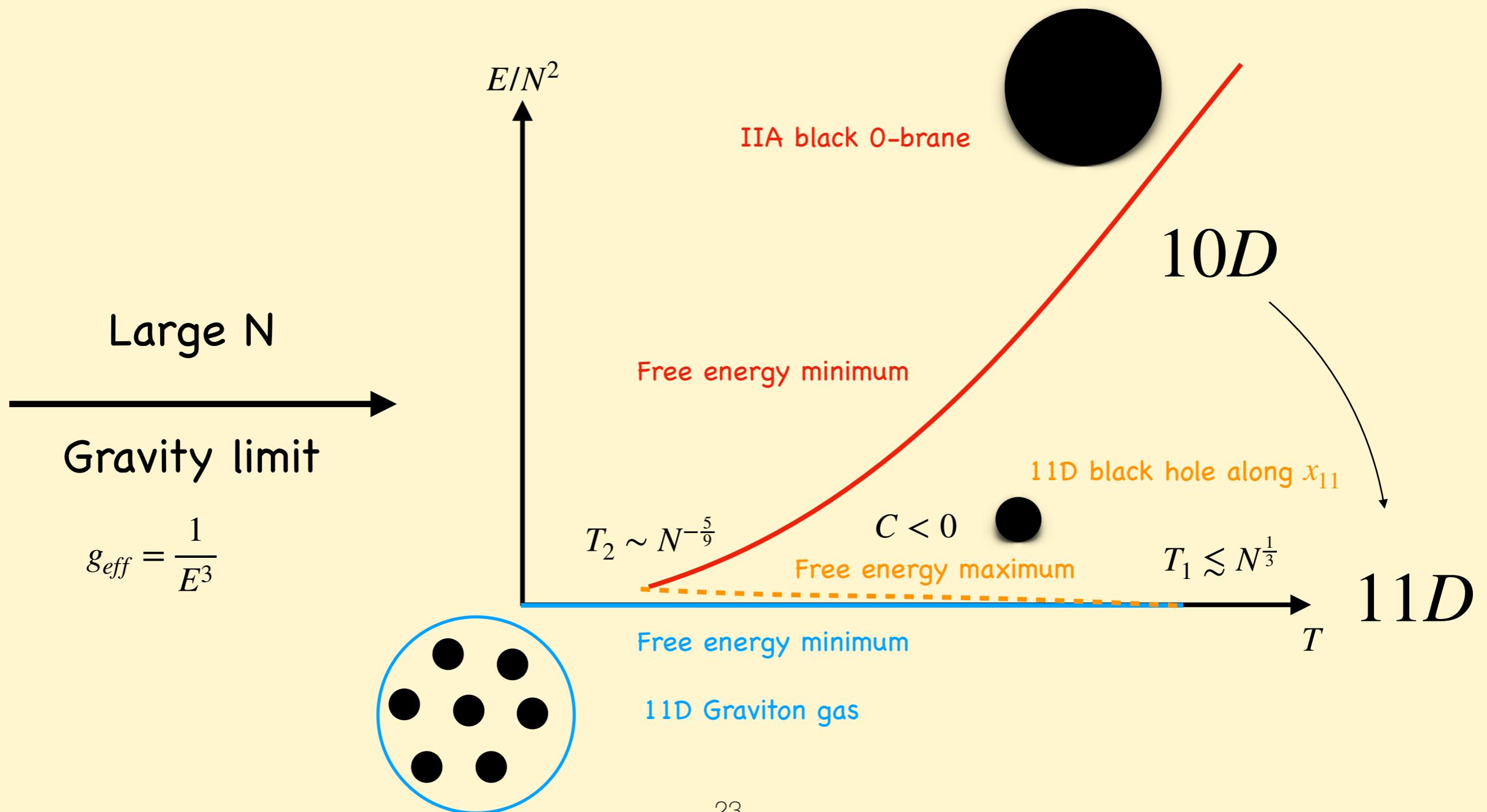
$$E \simeq 0$$

$$P \simeq 0$$

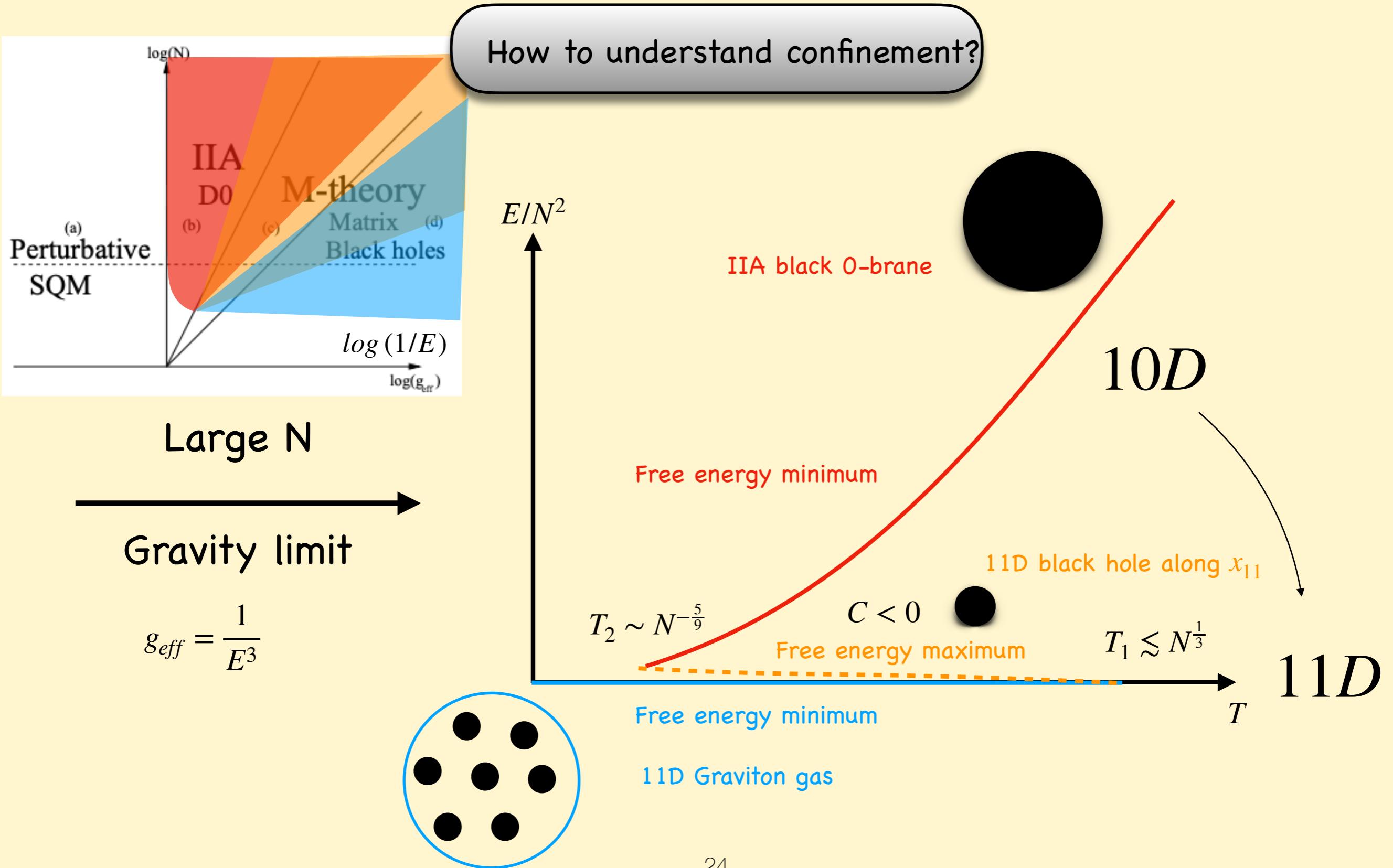
@ large N and continuum

Confined studies of D0-matrix model

How to understand confinement?

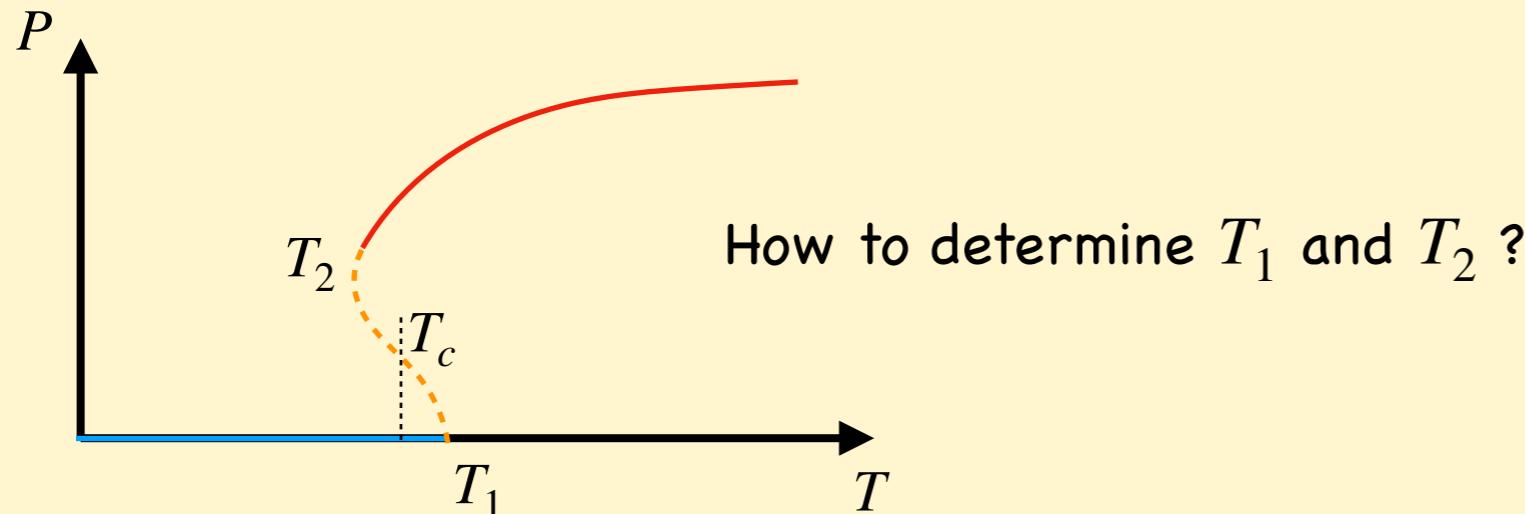


Confined studies of D0-matrix model

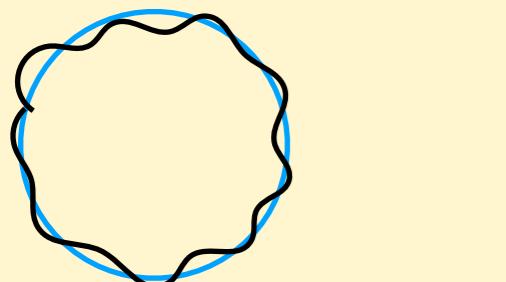


$$g_{\text{eff}} = \frac{1}{E^3}$$

Confined studies of D0-matrix model

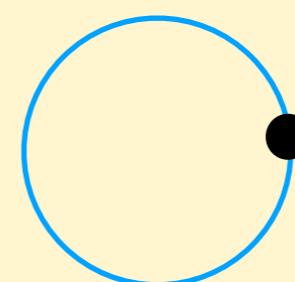


- T_2 corresponds to Gregory-Laflamme (GL) transition



A black string wrapping S^1

Collapses to a BH localised along S^1



Gregory-Laflamme 1994, Gubser-Mitra 2001,
Harmark-Niarchos-Obers 2004,
Kudoh-Wiseman 2004,
Horowitz-Martinec 1997, ...

$$T_2 \sim N^{-5/9}$$

- T_1 corresponds to maximum/minimum confinement/deconfinement temperature

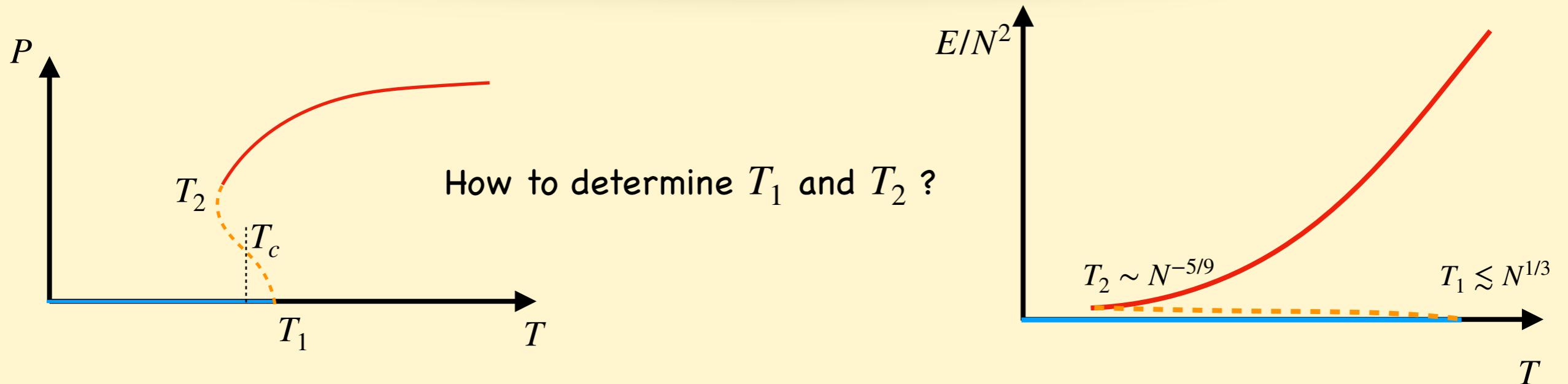
Schwarzschild BH in 11D with $M = M_{Pl}$



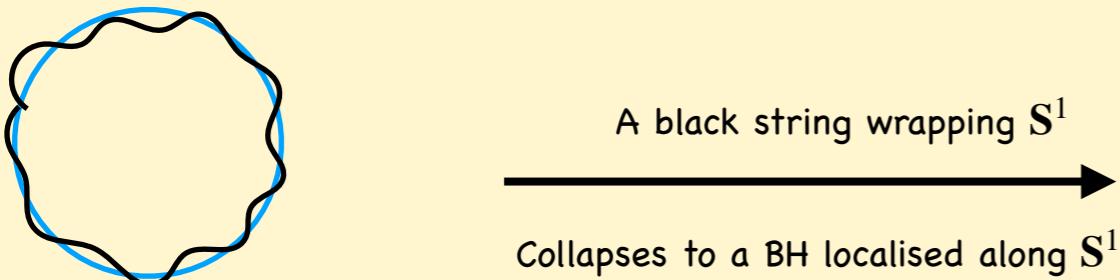
$$T_1 \lesssim N^{1/3}$$

[MCSMC, 2021]

Confined studies of D0-matrix model



- T_2 corresponds to Gregory-Laflamme (GL) transition

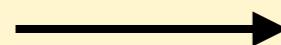


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Kudoh-Wiseman 2004,
Horowitz-Martinec 1997, ...

$$T_2 \sim N^{-5/9}$$

- T_1 corresponds to maximum/minimum confinement/deconfinement temperature

Schwarzschild BH in 11D with $M = M_{Pl}$



$$T_1 \lesssim N^{1/3}$$

[MCSMC, 2021]

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Thank you