Unitarized one-loop graviton-graviton scattering

Rafael L. Delgado

UNIVERSIDAD POLITÉCNICA DE MADRID

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XV$^{\text{th}}$ Quark Confinement and the Hadron Spectrum, Stavanger (Norway)

Graviton-graviton scattering

We use the usual Einstein-Hilbert (EH) Lagrangian with the usual normalization [PRD**50**, 3874; gr-qc/9512024; EPJA**56**, no.3, 86; Act.Phys.Polon.B**40**, 3409],

$$\mathcal{L} = \frac{M_P^2}{16\pi}\sqrt{-g}\mathcal{R} + \dots, \quad M_P^2 = \frac{1}{G}, \quad \kappa^2 = \frac{32\pi}{M_P^2} = 32\pi G$$

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}, \quad g^{\mu\nu} = \eta^{\mu\nu} - \kappa h^{\mu\nu} + \kappa^2 h^{\mu\lambda} h^{\nu}_{\lambda} + \dots$$

$$\mathcal{R} = \kappa[\Box h^{\lambda}_{\lambda} - \partial_\mu \partial \nu h^{\mu\nu}] + \mathcal{O}(h^2)$$

and the NLO graviton-graviton scattering amplitudes computed by Dunbar et.al. [Nucl.Phys.B **433** (1995) 181-208]. At the tree level, the results are:

$$T_{++++}(s,t,u) = \frac{8\pi}{M_P^2}\frac{s^3}{tu}$$

$$T_{+++-}(s,t,u) = T_{++--}(s,t,u) = 0$$

with the usual conventions (Mandelstam variables and graviton helicities).

Graviton-graviton scattering

- The NLO graviton-graviton scattering in pure EH gravity is UV finite [G.'t Hooft and M.Veltman, Ann.Inst.H. Poincaré A**20**, 69].
- Hence, no UV scale (beyond the Planck mass M_P) is required in our expressions.
- There is a IR scale μ in our expressions regarding the NLO computation.
- The μ scale is entirely produced by low-energy massless gravitons. A detailed study of these IR divergences that appear on the one-loop graviton-graviton elastic amplitudes can be found in [F.Donoghue, T.Torma, PRD**54**, 4963; D.C.Dunbar, P.S.Norridge, Class.Quant.Grav.**14**, 351-365].

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Unitarity problem

- Unless cancellations (like those in the SM Higgs sector), NLO amplitudes rise with s^2, eventually leading to violation of unitarity at some new physics state.
- This leads to an *OVERESTIMATED* number of events due to an unphysical prediction of EFT. That is, amplitudes *cannot* grow uncontrolled.
- Two options:
  - Set up a low-energy cut-off on the theory, due to the validity limits of the EFT itself. This limit, indeed, comes from the UV completion, whose specification would require to pick up a full (renormali. and unitar.) model from the *theory zoo*.
  - **Consider the EFT a valid low-energy limit and take advantage of the analytical properties of the scattering amplitudes, encoded in the so–called unitarization procedures, to extend the validity regime of the EFT. These techniques are well known from hadron physics.**

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Unitarity and Partial Waves

- The NLO-computed EFT grows with the CM energy like $T \sim s^2$. Eventually reaching the unitarity bound, becoming non-perturbative.
- Violation of unitarity of the S-matrix. That is, an unphysical leak in the interaction probability among EW gauge bosons.
- Tool for studying this phenomena: partial waves.
- For graviton-graviton scattering,

$$a_{J\lambda_1\lambda_2\lambda_3\lambda_4}(s) = \frac{1}{64\pi}\int_{-1}^{1} d\left(\cos\theta\right)\, T_{\lambda_1\lambda_2\lambda_3\lambda_4}(s,\cos\theta)\, d^J_{\lambda,\lambda'}(\cos\theta),$$

J, total angular momentum; $\lambda = \lambda_1 - \lambda_2$; $\lambda' = \lambda_3 - \lambda_4$; λ_i, helicity state of the i-nth external gauge boson; $d^J_{\lambda,\lambda'}(\cos\theta)$, Wigner functions.

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- Unit. cond. for $S - matrix$: $SS^\dagger = \mathbb{1}$,
- plus analytical properties of matrix elements,
- plus time reversal invariance,

Unitarity condition for partial waves

$$\mathrm{Im}\, A_{IJ,p_i\to k_1}(s) = \sum_{\{a,b\}} \sqrt{1-\frac{4m_q^2}{s}}[A_{IJ,p_i\to q_{i,ab}}(s)][A_{IJ,q_{i,ab}\to k_i}(s)]^*$$

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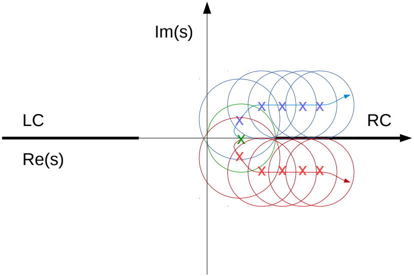

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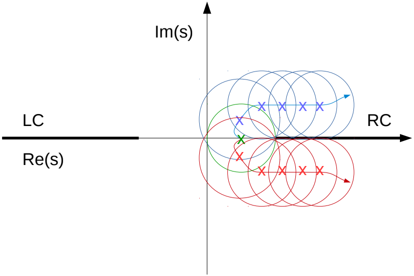

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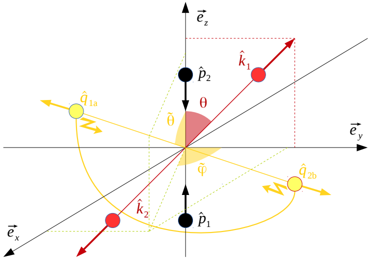

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IR cut-off

Modified amplitude $\tilde{T}^\eta(s,\theta)$: an elastic scattering unitarity relation like:

$$\mathrm{Im}\,\tilde{T}^\eta_{\lambda_1\lambda_2\lambda_3\lambda_4}(s,\theta) = \frac{1}{128\pi^2}\sum_{\lambda_a\lambda_b}\int_R d\Omega'\,\tilde{T}^\eta_{\lambda_1\lambda_2\lambda_a\lambda_b}(s,\theta')\,\tilde{T}^{\eta *}_{\lambda_a\lambda_b\lambda_3\lambda_4}(s,\theta'')$$

R: the phase space region cut off in the IR (only states where $s = 4E^2_{CM} > \mu^2$). This must be related in some way with the η parameter introduced in the modified amplitude. Crossing requires $t < -\mu^2$ and the same for t' and t''. By trading $x = \cos\theta$ by t,

$$\int_{-1+\eta}^{1-\eta} dx = \frac{s}{2}\int_{t_{min}}^{t_{max}} dt = \frac{s}{2}\int_{-s+\mu^2}^{-\mu^2} dt \Leftrightarrow \mu^2 = \eta\frac{s}{2}$$

Furthermore, if we give a small mass μ to the graviton, as a way of introducing an IR cut-off, the integral over the θ angle becomes automatically regulated and the relation $\eta = 2\mu^2/s$ is automatically recovered at leading order in μ^2.

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Unitarity with the IR cut-off

- We need to see how this IR cut-off works.
- For the T_{++++} case, at all orders in η: . . . . $a_0^{(0)}(s,\eta) = \frac{s}{2M_P^2} \log \frac{2-\eta}{\eta}$
- The NLO: . . . . $\mathrm{Im}\, a_1^{(0)}(s,\mu) = \frac{s^2}{4M_P^4} \log \frac{2-\eta}{\eta} \left[\log \frac{\eta(\eta-2)}{4} + 2 \log \frac{s}{\mu^2} \right]$
- Defining η as $\eta = 2\mu^2/s$, at all orders in η,

$$\mathrm{Im}\, a_0^{(1)}(s,\eta) = \left(a_0^{(0)}(s,\eta) \right)^2$$

- Note that **one should only expect $\boldsymbol{\eta = 2\mu^2/s}$ to work for small $\boldsymbol{\mu}$**.
- For $J \neq 0$ this only works up to order η, $\mathrm{Im}\, a_J^{(1)} = \left(a_J^{(0)} \right)^2 + O(\eta)$
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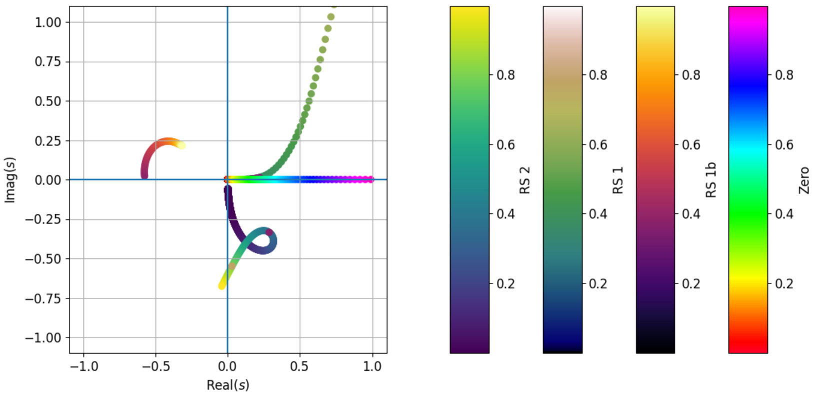

arXiv:2207.06070 [hep-th]

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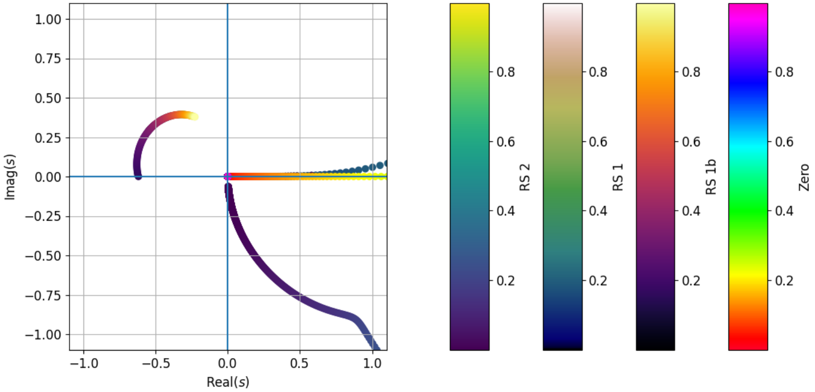
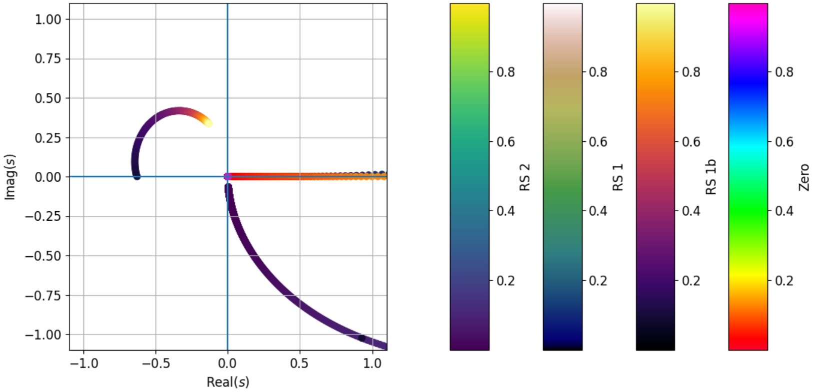
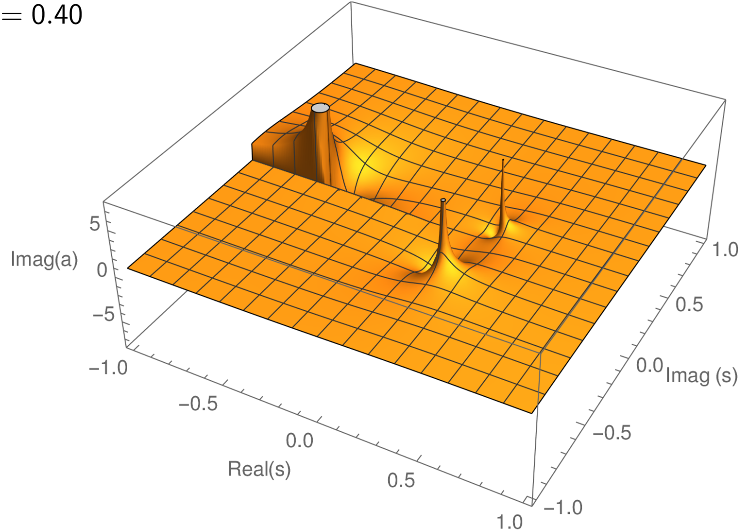


Pole position, $J = 4$



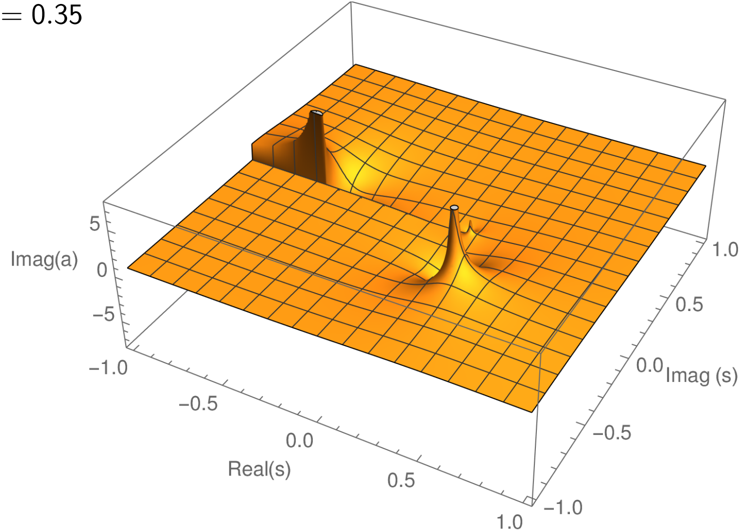
Unitarized partial waves, $J = 0$

$\mu/M_P = 0.40$



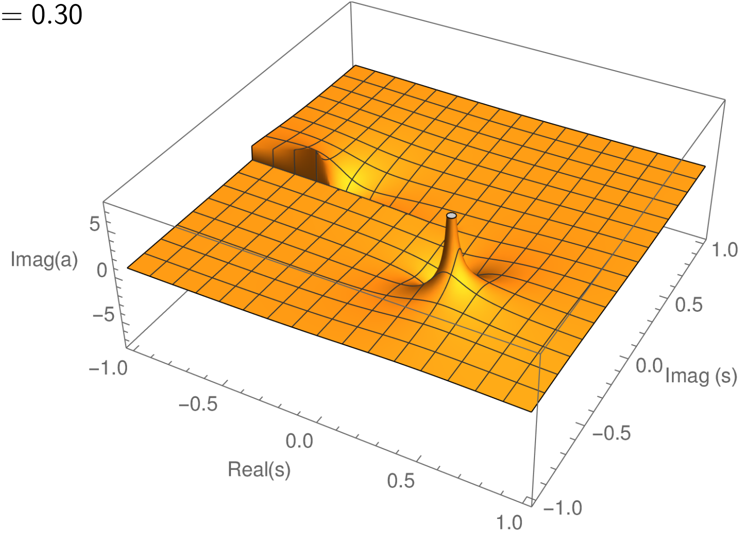
Unitarized partial waves, $J = 0$

$\mu/M_P = 0.35$



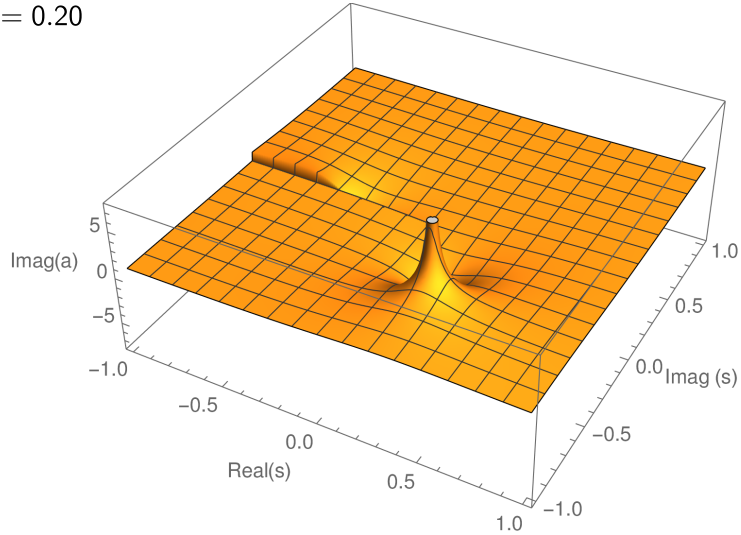
Unitarized partial waves, $J = 0$

$\mu/M_P = 0.30$



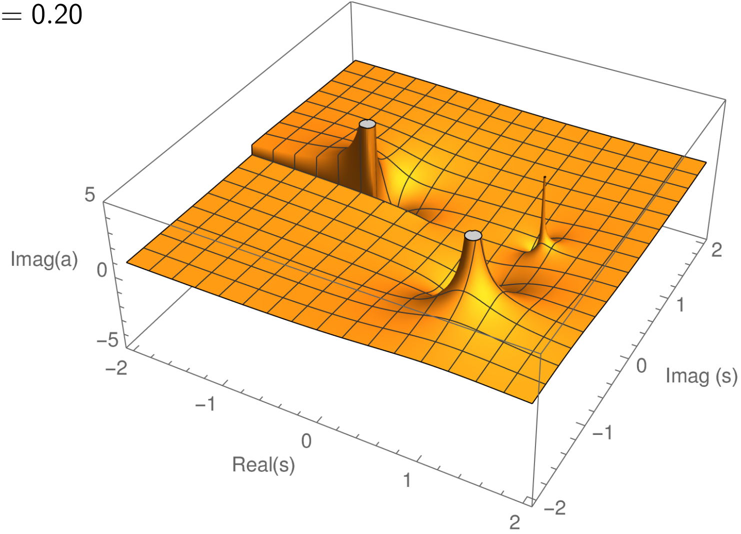
Unitarized partial waves, $J = 0$

$\mu/M_P = 0.20$



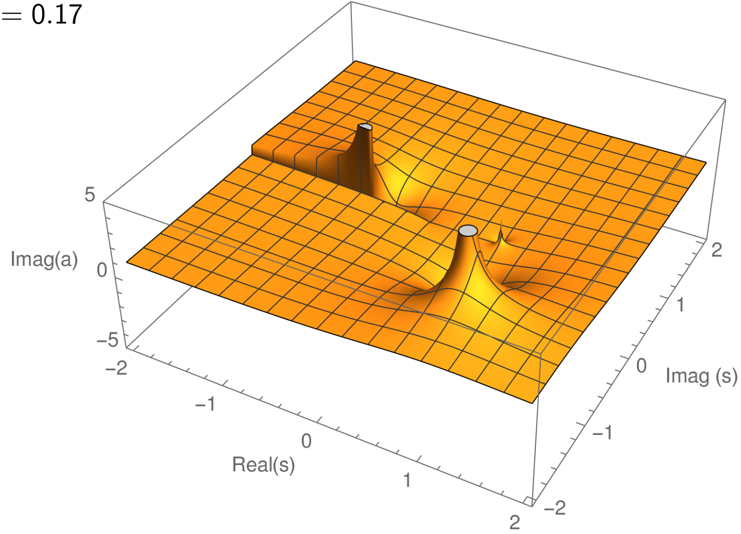
Unitarized partial waves, $J = 2$

$\mu/M_P = 0.20$



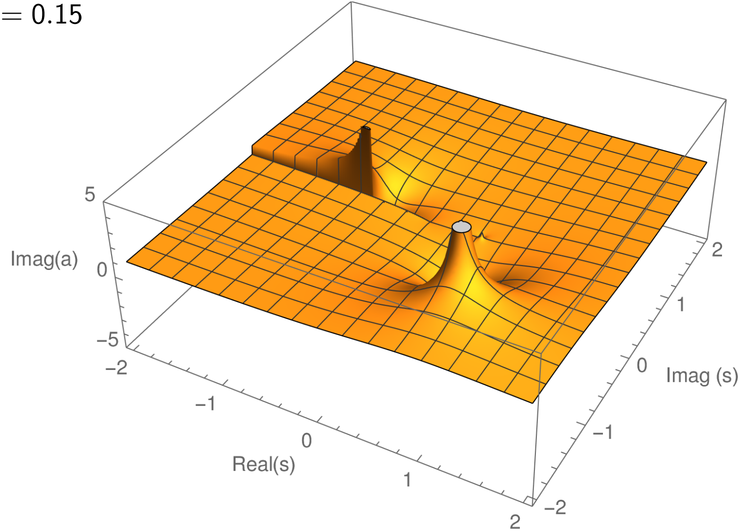
Unitarized partial waves, $J = 2$

$\mu/M_P = 0.17$



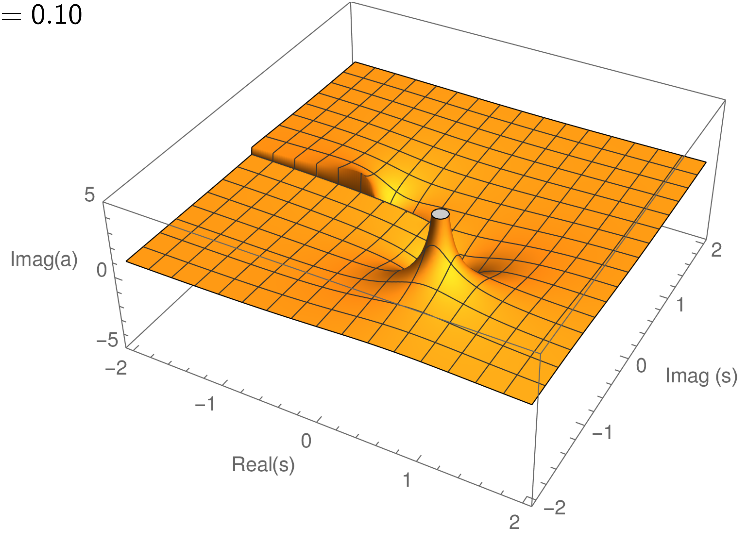
Unitarized partial waves, $J = 2$

$\mu/M_P = 0.15$

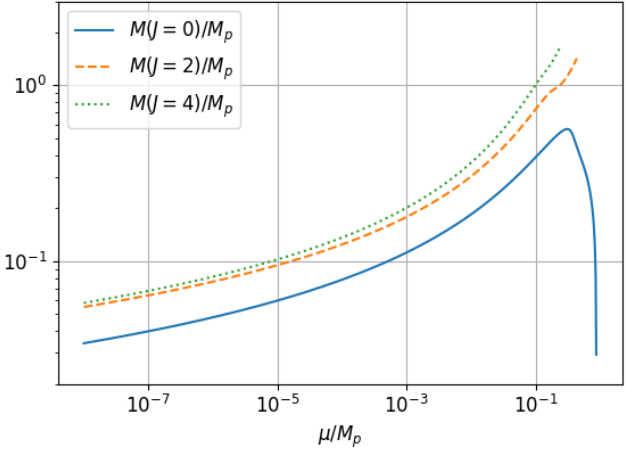


Unitarized partial waves, $J = 2$

$\mu/M_P = 0.10$



Mass



Conclusions (1/2)

- We have studied the NLO graviton-graviton elastic scattering in Einstein-Hilbert (pure) gravity by means of dispersion relations via the Inverse Amplitude Method.
- We do not need an UV cutoff, thanks to the theory being UV finite.
- We do need an infrared cut-off, that can be identified with the experimental resolution of existing graviton mass determinantions ($\mu \ll M_P$).
- We do not find causality breaking poles on the first Riemann sheet for low values of $\mu/M_P < 0.092$.

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- Such an analysis, on the QCD pion Lagrangian, was able to recover resonances (for instance, vector bosons) that are experimentally found.
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