Warped dimensions: Green's functions and continuous spectra

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Other references: A. Falkowski, M. Pérez-Victoria PRD79 (2009); E.M., G.Panico, O.Pujolàs, M.Quirós, JHEP1609(2016) 118.

Issues

1. The extradimensional model
   - General formalism
   - The soft-wall model
2. Gapped continuous spectra: Gauge bosons
   - Green's functions for gauge bosons
   - Resonances
   - Spectral functions
   - Regularized continuum model
3. Gapped continuous spectra: Graviton and radion
   - The linear dilaton model
   - Graviton
   - Radion
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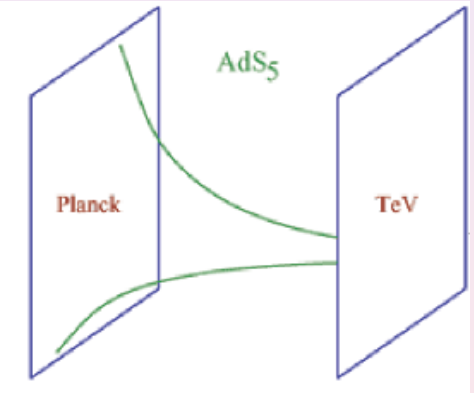
Introduction

Randall-Sundrum model in warped extra-dimension

- Proposed in 1999 by Randall and Sundrum (RS) **[PRL83, 3370 '99]**
- It was based on a 5D space-time with line element

$$ds^2 = e^{-2A(y)}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2\,, \qquad A(y) = ky\,,$$

- and two branes:



$$\mathrm{TeV} = e^{-ky_1} M_{\mathrm{Planck}}\,, \qquad ky_1 \sim 35\,.$$

Higgs boson profile: $h(y) \propto e^{aky}$, $a > 2$.

AdS $\Leftrightarrow$ CFT correspondence

- Heavy (light) fermions are mainly localized at the IR (UV) brane: composite (elementary).
- KK modes: $m_{\mathrm{KK}} \sim \mathrm{TeV} \ll M_{\mathrm{Planck}}$ $\longrightarrow$ Solve the hierarchy problem.
- Brane distance stabilized by a bulk scalar field ϕ with bulk/brane potentials fixing its VEVs **[W. Goldberger, M. Wise, PRD60, 107505 '99]**.

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The Model

[Cabrer, Gersdorff, Quirós, (2010)]

- Scalar-gravity system with UV and IR branes:

$$S = \int d^5x \sqrt{|\det g_{MN}|}\left[-\frac{1}{2\kappa^2}R + \frac{1}{2}g^{MN}(\partial_M\phi)(\partial_N\phi) - V(\phi)\right] - \sum_\alpha \int_{B_\alpha} d^4x \sqrt{|\det \bar{g}_{\mu\nu}|}\lambda_\alpha(\phi) + S_{\rm GHY}$$

- Metric: $ds^2 = g_{MN}dx^M dx^N \equiv \underbrace{e^{-2A(y)}\eta_{\mu\nu}}_{\bar{g}_{\mu\nu}} dx^\mu dx^\nu - dy^2$.
- $V(\phi)$ bulk potential.
- λ_α $(\alpha = 0, 1) \equiv$ UV, IR 4-dim brane potentials at $(y(\phi_0), y(\phi_1))$.
- $S_{\rm GHY} :=$ Gibbons-Hawking-York boundary term.
- Solve the hierarchy problem $\longrightarrow$ Brane dynamics should fix (ϕ_0, ϕ_1) to get $A(\phi_1) - A(\phi_0) \approx \mathcal{O}(35) \Longrightarrow M_{\rm Planck} \simeq 10^{15} M_{\rm TeV}$.

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The soft-wall model

[Cabrer et al. NJP '10], [C. Csaki et al. '19], [E.M., M. Quirós, JHEP '19 & JHEP '21]

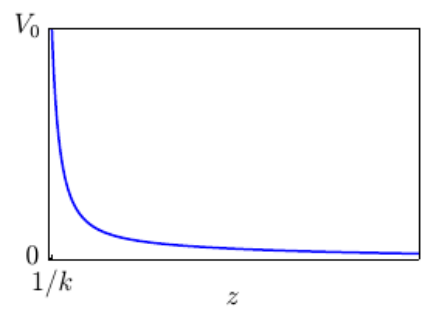
Superpotential: $$W(\phi) = \frac{6k}{\kappa^2}\left(1 + e^{\nu\phi}\right).$$

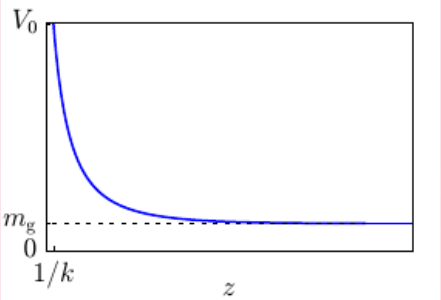
- Critical case $\nu = \nu_c \equiv \kappa/\sqrt{3}$ ➡ Gapped continuous KK spectra.

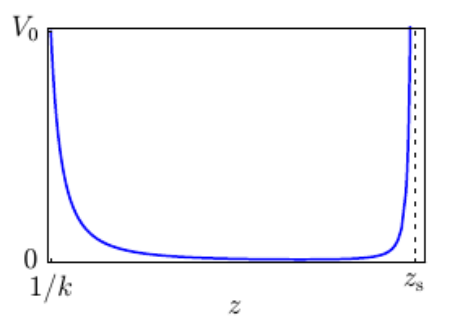
$$A(z) \simeq \begin{cases} \log(kz) & 1/k \leq z \leq z_1 \quad \leftarrow RS \\ \log(kz_1) + \rho(z - z_1) & z_1 < z < \infty \quad \leftarrow LDM \end{cases}.$$

- For $\nu > \nu_c$ ➡ discrete KK spectra with TeV spacing.
- For $\nu < \nu_c$ ➡ ungapped continuous KK spectra.

Effective potential for gauge bosons:







$\nu < \nu_c$ $\qquad$ $\nu = \nu_c$ $\qquad$ $\nu > \nu_c$

- $\rho \equiv k\, e^{-ky_1} \sim \text{TeV}$ ➡ $A_1 \simeq 35.$

Green's functions: gapped continuous spectra

- Up to now, searches of new physics at the LHC ⟶ detection of bumps in the invariant mass of final states.
- An exploring possibility to cope with the elusiveness of signals is a continuum of Kaluza-Klein (KK) states beyond a mass gap m_g.
- ⟶ New physics associated with an excess in the measured cross section with respect to the SM prediction.
- The Green's functions generalize the particle propagators

$$\frac{1}{p^2 - m^2 + i\epsilon} = \mathcal{P}\frac{1}{p^2 - m^2} - i\pi\delta(p^2 - m^2)$$

- ... to Green's functions with an isolated pole (the zero mode) and a continuum of states with a mass gap m_g

$$G(p^2, m_g^2) = \mathrm{Re}\, G(p^2, m_g^2) + i\left[c_0\delta(p^2) + \eta(p^2, m_g^2)\Theta(p^2 - m_g^2)\right] .$$

- Same behavior as gapped unparticles **[M. Pérez-Victoria et al, PRD '09 & JHEP '09]**. Here $m_g \sim$TeV is linked to the solution of the *hierarchy problem*.

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Massless gauge bosons

- Lagrangian for massless gauge bosons (in the gauge $A_5 = 0$):

$$\mathcal{L} = \int_0^{y_s} dy \left[-\frac{1}{4} \mathrm{tr} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} e^{-2A} \mathrm{tr} A'_\mu A'_\mu \right] ,$$

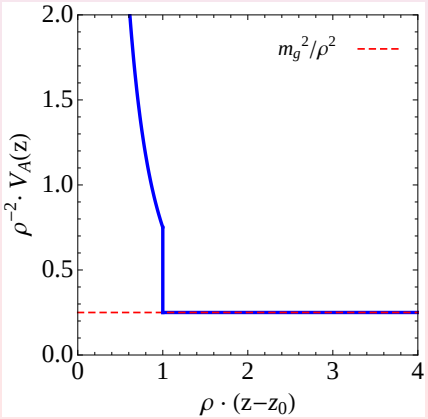
$$A_\mu(p, y) = f_A(y) A_\mu(p) / \sqrt{y_s} .$$

- Schrödinger like form of the EoM of the fluctuations

$$-\tilde{f}''_A(z) + V_A(z) \tilde{f}_A(z) = p^2 \tilde{f}_A(z) , \qquad [f_A(z) = e^{A(z)/2} \tilde{f}_A(z)] ,$$

with potential

$$V_A(z) = \tfrac{1}{4} A'(z)^2 - \tfrac{1}{2} A''(z) , \qquad V_A(z) \underset{z > z_1}{\longrightarrow} m_g^2 = \left(\tfrac{\rho}{2}\right)^2 .$$



➡ Existence of a mass gap.
➡ Continuum of states above the mass gap.

Massless gauge bosons

- The *Green's function* is given by

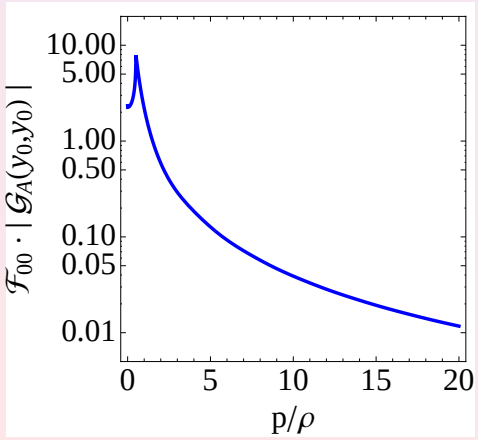
$$G_A^{\mu\nu}(y,y';p) = [\eta^{\mu\nu} - (1-\xi)p^\mu p^\nu/p^2]G_A(y,y';p)\,.$$

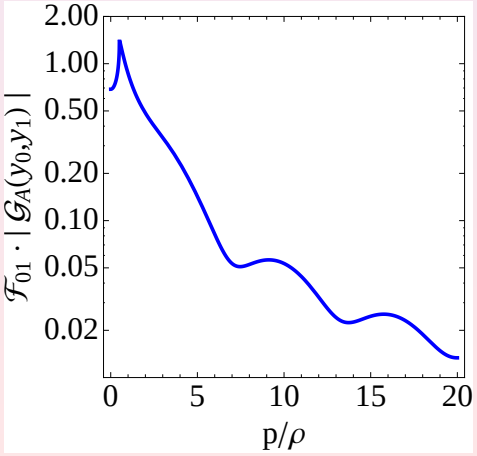
- EoM for the Green's function:

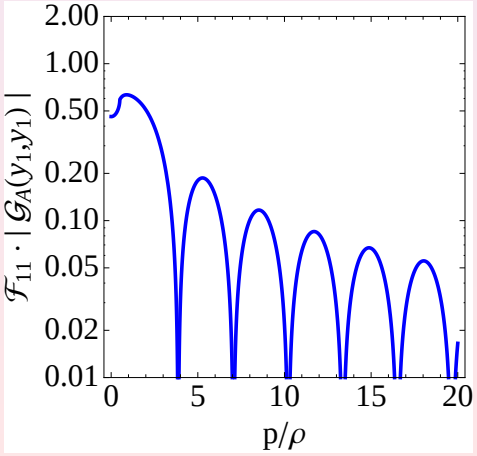
$$p^2 G_A(y,y';p) + \partial_y\left(e^{-2A(y)}\partial_y G_A(y,y';p)\right) = \delta(y-y')\,.$$

- All Green's functions include the zero-mode contribution:

$$G_A^0 = \frac{1}{y_s p^2} = \lim_{p\to 0} G_A(y,y';p) \quad \longrightarrow \quad \text{Define:} \quad \mathcal{G}_A(y,y';p) = G_A(y,y';p) - G_A^0\,.$$







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Green's functions in the complex plane: resonances

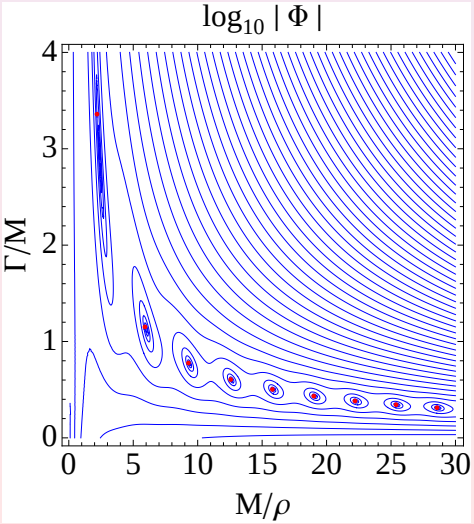
- Complex plane ($s \equiv p^2$):

$$s = M^2 - iM\Gamma = M^2(1 - ir), \qquad (r \equiv \Gamma/M)\,.$$

- Riemann sheets:

$$\begin{cases} \text{1st Riemann sheet} & \delta_A^{\rm I} = +\sqrt{1 - s/m_g^2} \\ \text{2nd Riemann sheet} & \delta_A^{\rm II} = -\sqrt{1 - s/m_g^2} \end{cases}.$$

- $G_A(y, y'; p) \propto \frac{1}{\Phi(p)}$ and $\Phi(p)|_{\text{2nd Riemann}} \propto \left[e^{i2p/\rho} - 8i(p/\rho)^2\right]$



→ Zeros of $\Phi(p) \equiv$ Poles of $G_A(y, y'; p)$:

$$\frac{s}{\rho^2} \simeq -\mathcal{W}_n\left[\pm\frac{1}{4}(1+i)\right]^2, \quad n = -1, -2, \cdots$$

$\mathcal{W}_n \equiv$ n-th branch of the Lambert function

Poles interpreted as resonances!!!

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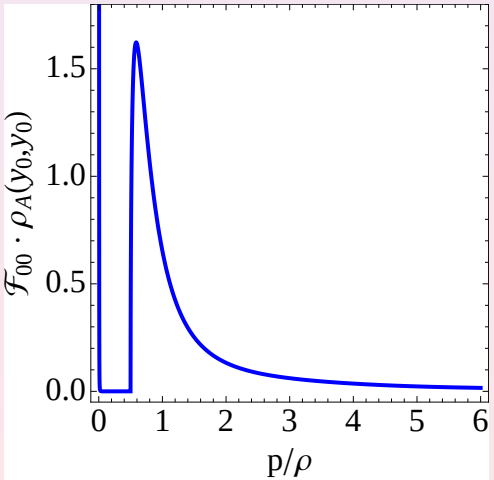
Spectral function (soft-wall model)

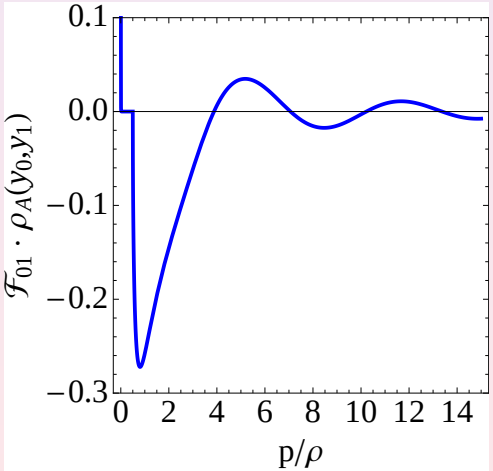
- Spectral function:

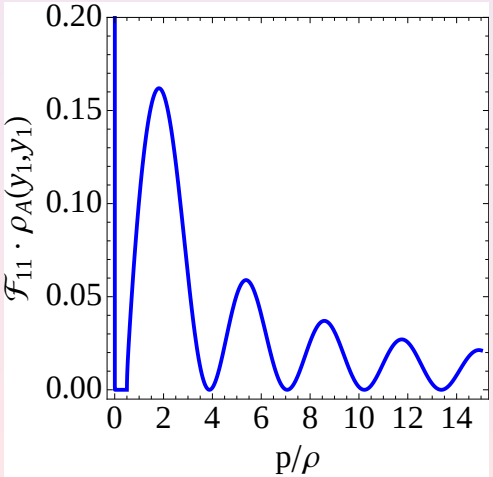
$$\rho_A(y,y';s) = -\frac{1}{\pi}\mathrm{Im}\, G_A(y,y';s+i\epsilon)\,, \qquad s \equiv p^2\,.$$

- Zero mode + continuum:

$$\rho_A(y,y';s) = \frac{1}{y_s}\,\delta(s) + \eta_A(s)\Theta(s-m_g^2)\,.$$







- Kramers-Kronig relations: $\mathrm{Re}G_A \Longleftrightarrow \mathrm{Im}G_A$ ➡

➡ All the information of G_A is in ρ_A.

Spectral functions (RS model)

- Randall-Sundrum model: $A(y) = ky\,, \qquad 0 \leq y \leq y_1\,.$
- Green's function:

$$G_{A,\mathrm{RS}}(y,y';p) = \frac{1}{y_1}\sum_n \frac{f_A^{(n)}(y) f_A^{(n)}(y')}{p^2 - m_n^2 + i\epsilon}\,.$$

- Spectral function:

$$\rho_{A,\mathrm{RS}}(y,y';p) = \frac{1}{y_1}\sum_n f_A^{(n)}(y) f_A^{(n)}(y')\,\delta(p^2 - m_n^2)\,.$$

- KK modes $\longrightarrow$ $m_n/\rho \simeq$ zero mode + zeros of $J_0(p/\rho)$:

$$m_n/\rho \simeq 0,\ 2.45,\ 5.57,\ 8.70,\ 11.84,\ 14.98, \cdots\,.$$

- $f_A^{(n)}(y)$ has <u>n nodes</u> $\longrightarrow$ n changes of sign from $y = 0$ to $y = y_1$.

$$\longrightarrow \begin{cases} \rho_{A,\mathrm{RS}}(y,y) \geq 0 & \forall y \\ \rho_{A,\mathrm{RS}}(y,y') = \text{can be negative} & y \neq y' \end{cases}\,.$$

- IR-brane-to-IR-brane:

$$\rho_{A,\mathrm{RS}}(y_1,y_1;p) \simeq \frac{1}{y_1}\delta(p^2) + 2k\sum_{n=1}^{\infty}\delta(p^2 - m_n^2) \geq 0\,.$$

Spectral function: positivity

- The function $\rho_A(y, y')$ can be understood as a matrix element of a spectral operator **[L.L. Salcedo, Private communication]**

$$\hat{\rho}_A = -\frac{1}{\pi}\mathrm{Im}\ \hat{G}_A\,, \qquad \text{where} \qquad \mathrm{Im}\ \hat{G}_A = \frac{1}{2i}\left(\hat{G}_A - \hat{G}_A^\dagger\right),$$

$$\longrightarrow \qquad \rho_A(y, y') = \langle y|\hat{\rho}_A|y'\rangle \equiv \rho_{yy'}\,.$$

- The matrix element is factorizable:

$$\rho_{yy'} = \rho_y \rho_{y'}\,.$$

- Then, the infinitely dimensional matrix $\hat{\rho}_A$, with elements $(\hat{\rho}_A)_{yy'} = \langle y|\hat{\rho}_A|y'\rangle$ has the following properties:
  1. $\det \hat{\rho}_A = 0$.
  2. All the eigenvalues of $\hat{\rho}_A$ are vanishing, except one of them:

$$\boxed{\lambda = \mathrm{tr}\,\hat{\rho}_A = \int_0^{y_s} dy\,\langle y|\hat{\rho}_A|y\rangle = \int_0^{y_s} dy\,\rho_y^2 \geq 0\,.}$$

- This proves that $\hat{\rho}_A$ is positive semidefinite.
- All the information of $\hat{\rho}_A$ is in λ.

Spectral function: positivity (RS model)

- In the RS model

$$\rho_{A,\mathrm{RS}}(y,y;p) = \frac{1}{y_1}\sum_n f_A^{(n)2}(y)\delta(p^2-m_n^2) \quad \text{and} \quad \int_0^{y_1} dy\, f_A^{(n)2}(y) = y_1\,,$$

so that

$$\lambda_{A,\mathrm{RS}}(p) = \int_0^{y_1} dy\, \rho_{A,\mathrm{RS}}(y,y;p) = \sum_n \delta(p^2-m_n^2)\,,$$

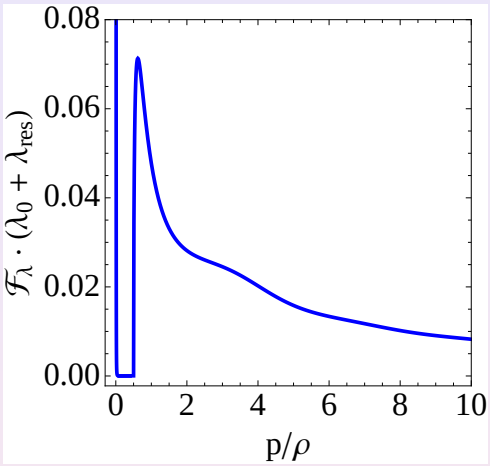
is interpreted as the *density of states*.

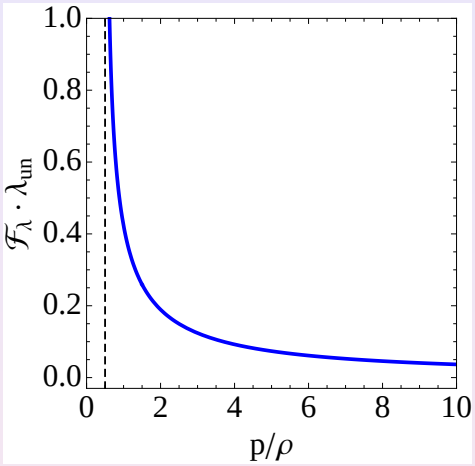
- The integral of $\lambda_{A,\mathrm{RS}}(s)$:

$$\int_0^\infty ds\, \lambda_{A,\mathrm{RS}}(s) = \int_0^\infty ds \sum_n \delta(s-m_n^2) = N_{\mathrm{states}} \to \infty\,,$$

is the *number of states*.

Spectral function: positivity (soft-wall model)





- $\lambda(p)$ has several contributions:

$$\boxed{\lambda(p) = \lambda_0(p) + \lambda_{\text{resonant}}(p) + \lambda_{\text{unparticles}}(p)\,,}$$

where

$$\begin{aligned}
\lambda_0(p) &= \delta(p^2)\,,\\
\lambda_{\text{resonant}}(p) &= \xi_A(p)\Theta(p^2 - m_g^2)\,,\\
\lambda_{\text{unparticles}}(p) &= -\frac{\log(\rho\epsilon)}{2\pi\rho\sqrt{p^2 - m_g^2}}\Theta(p^2 - m_g^2)\,.
\end{aligned}$$

See e.g. [A.Delgado, J.R. Espinosa, J.No, M.Quirós, PRD79 '09] for unparticles.

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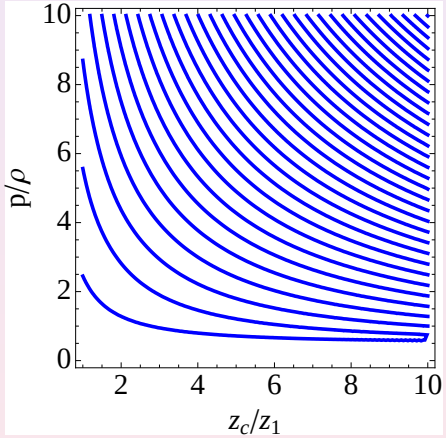
Regularized continuum model

[E.M., M. Pérez-Victoria, M. Quirós, to appear '22]

- Spectrum: $z \in [z_0, z_c] \longrightarrow$ Discrete. $z \in [z_0, \infty] \longrightarrow$ Continuum.
- *'Discrete'* $\longrightarrow$ *'Continuum'* by considering $z \in [z_0, z_c]$ with

$$z_0 \quad < \quad z_1 \quad < \quad z_c \qquad \text{and} \qquad z_c \to \infty \, .$$



$$\frac{p_n^2}{\rho^2} \simeq \frac{1}{4} + \left[\frac{z_1}{z_c}\pi\left(n - \frac{1}{4}\right)\right]^2 , n = 1, 2, 3, \cdots$$

(Eigenvalues tighten as z_c increases).

- <u>*Green's function*</u> 'G_A' and <u>*spectral density*</u> 'σ':

$$G_A(z, z'; s) \quad = \quad \sum_n \frac{1}{||f_n||^2} \frac{f_n(z) f_n^*(z')}{s - p_n^2 + i\epsilon} \quad \underset{z_c \to \infty}{\longrightarrow} \quad \int_0^\infty dp^2 \sigma(p^2) \frac{f_{p^2}(z) f_{p^2}^*(z')}{s - p^2 + i\epsilon} \, .$$

Gapped model in flat space-time: 1-loop corrections

[E.M., M. Pérez-Victoria, M. Quirós, to appear '22]

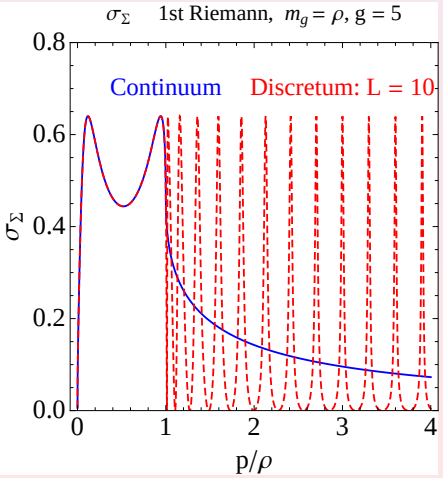
- Scalar field A (unparticle) coupled to massless scalar field φ:

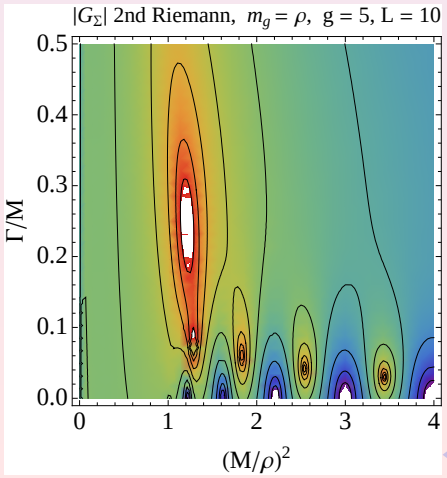
$$\mathcal{L} = -\frac{1}{2} A \Pi(-\partial^2) A + \partial_\mu \varphi^\dagger \partial^\mu \varphi + g A \varphi^\dagger \varphi \,.$$

- UV-UV Green's function for A ($g = 0$):

$$G_A^{(0)}(z_0, z_0; p) = \frac{1}{\sqrt{p^2 - m_g^2}\tan\left(\sqrt{p^2 - m_g^2}\, L\right)} \underset{L\to\infty}{\longrightarrow} -\frac{1}{\sqrt{m_g^2 - p^2}} \,.$$

- Including 1-loop corrections: $G_{A,\Sigma}(z_0, z_0; p) = \dfrac{1}{\left[G_A^{(0)}(p^2)\right]^{-1} + \Sigma(p^2)} \,.$





$$p_n^2 = m_g^2 + \frac{\pi^2}{L^2} n^2, \;\; n = 0, 1, 2, \cdots$$

$$p^2 = M^2(1 - i\Gamma/M)$$

Gapped model in flat space-time: Time evolution & Unitarity

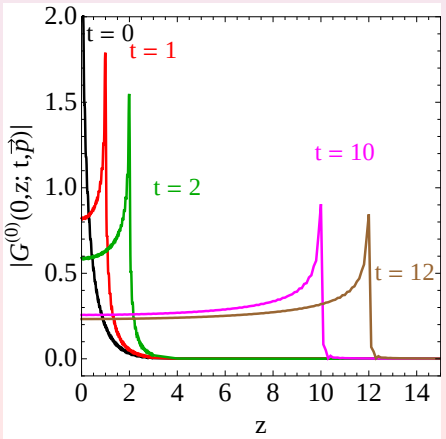
[E.M., M. Pérez-Victoria, M. Quirós, to appear '22]

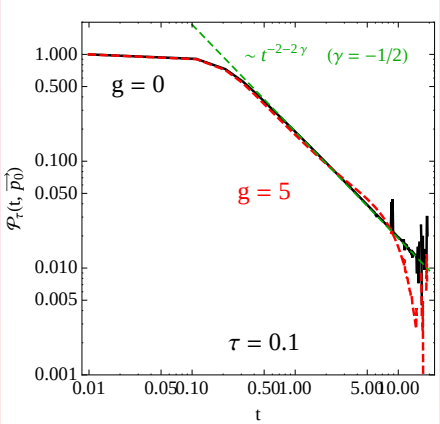
- Time evolution can be obtained by the inverse Fourier transform:

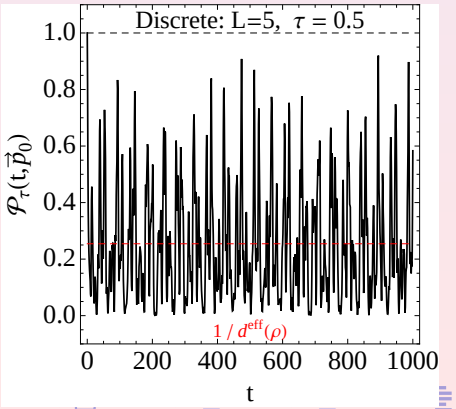
$$G(0,z;t,\vec{p}) = i\int_{-\infty}^{\infty}\frac{dp^0}{2\pi}e^{-ip^0 t}G(0,z;p^0,\vec{p})$$

- Free theory ($g=0$): $G^{(0)}(0,0;t,\vec{p}) = -i\sqrt{\frac{2}{\pi}}K_0\left(\sqrt{-\vec{p}^2-m_g^2+i\epsilon}|t|\right)$.
- Survival probability ➞ Probability of finding the system in the initial state after a time t:

$$\mathcal{P}(t) \simeq \left|\frac{G(t,\vec{p})}{G(0,\vec{p})}\right|^2_{z=0}.$$







Issues

1. The extradimensional model
   - General formalism
   - The soft-wall model
2. Gapped continuous spectra: Gauge bosons
   - Green's functions for gauge bosons
   - Resonances
   - Spectral functions
   - Regularized continuum model
3. Gapped continuous spectra: Graviton and radion
   - The linear dilaton model
   - Graviton
   - Radion
4. Conclusions

The linear dilaton model

$$W(\phi) = \frac{6k}{\kappa^2} e^{\nu\phi} .$$

- Simplified model with no AdS_5 in the UV.
- Scalar field and warp factor:

$$\bar{\phi}(y) \equiv \nu_c \, \phi(y) = -\log\left[k(y_s - y)\right] , \quad A(y) = -\log\left[1 - \frac{y}{y_s}\right] .$$

- In conformal coordinates:

$$\bar{\phi}(z) = A(z) + \bar{v}_0 , \qquad A(z) = \rho \cdot (z - z_0) , \qquad z_0 \leq z < \infty .$$

- $\rho = \pm 1/y_s$.
- Dual to Little String Theories **[I.Antoniadis et al, PRL '12], [P. Cox, T. Gherghetta, JHEP '12].**
- Gapped continuous spectrum **[E.M., M. Quirós, APP B52 '21].**

$$m^2 \geq m_g^2 \quad \left[= \lim_{A(z_1)\to\infty} \left(m_g^2 + \frac{\pi^2 n^2}{A(z_1)^2} \rho^2 \right) \right] .$$

The linear dilaton model

[E.M., M. Quirós, Acta Phys. Polon. B52 (2021)]

- Within the linear dilaton model, propagation of gauge bosons in the bulk produces a violation of EW precision observables.
  → Consider:
  - SM fields located in the IR brane.
  - Propagation in the bulk of *graviton* and *radion*.

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Graviton

- Transverse traceless fluctuations of the metric:

$$ds^2 = e^{-2A(y)}(\eta_{\mu\nu} + 2\kappa h_{\mu\nu}(x,y))dx^\mu dx^\nu - dy^2 \,.$$

- Lagrangian

$$\mathcal{L} = -\frac{1}{2}\int_0^{y_s} dy\, e^{-2A}\left[\partial_\rho h_{\mu\nu}\partial^\rho h^{\mu\nu} + e^{-2A} h'_{\mu\nu} h'^{\mu\nu}\right] \,.$$

- EoM of fluctuations $\quad [h_{\mu\nu}(x,y) = \mathfrak{h}(y) h_{\mu\nu}(x); \quad \mathfrak{h}(z) = e^{3A(z)/2}\tilde{\mathfrak{h}}(z)]$

$$-\tilde{\mathfrak{h}}''(z) + V_{\mathfrak{h}}(z)\tilde{\mathfrak{h}}(z) = p^2\tilde{\mathfrak{h}}(z) \quad \text{with} \quad V_{\mathfrak{h}}(z) \underset{z\to\infty}{\longrightarrow} m_g^2 = (3\rho/2)^2 \,.$$

- Interaction with matter

$$\mathcal{L}_{4D} = -\frac{1}{\sqrt{2}M_5^{3/2}} h^{\mu\nu}(x,y_1) T_{\mu\nu}(x,y_1) \,.$$

Graviton

- Green's function:

$$D_{\mu\nu,\rho\sigma} = \frac{1}{2}\left(\eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho} - \frac{2}{3}\eta_{\mu\nu}\eta_{\rho\sigma}\right) G_{\mathfrak{h}}(y, y'; p)\,.$$

- EoM for the Green's function:

$$p^2 G_{\mathfrak{h}}(y, y') + e^{2A(y)} \frac{d}{dy}\left(e^{-4A(y)} G'_{\mathfrak{h}}(y, y')\right) = e^{2A(y')} \cdot \delta(y - y')\,.$$

- Boundary and matching conditions $[G_{\mathfrak{h}}(y) \equiv G_{\mathfrak{h}}(y, y'),\ 0 < y, y' < y_s]$:

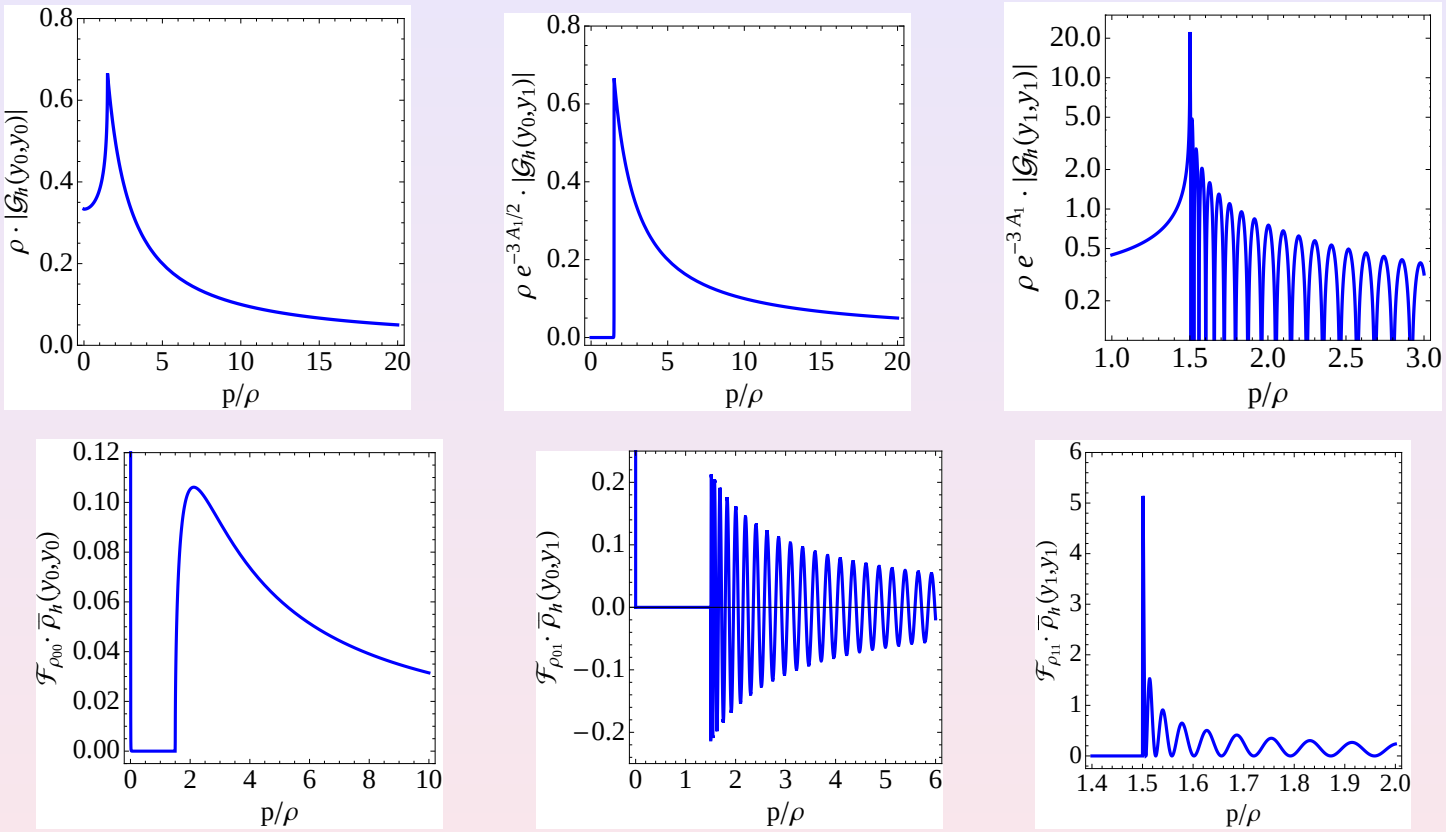
$$G'_{\mathfrak{h}}(0) = 0\,, \qquad \Delta G'_{\mathfrak{h}}(y') = e^{4A(y')}\,, \qquad G_{\mathfrak{h}}(y_s) = \text{regular}\,.$$

- Solution $[\Delta_{\mathfrak{h}} = \Delta_A\,, \quad \delta_{\mathfrak{h}} = \delta_A]$:

$$\boxed{G_{\mathfrak{h}}(y, y') = \frac{1}{3\rho}\frac{1}{\delta_{\mathfrak{h}}}(1 - \bar{y}_\uparrow)^{\frac{3}{2}\Delta_{\mathfrak{h}}^+}\left(-(1 - \bar{y}_\downarrow)^{\frac{3}{2}\Delta_{\mathfrak{h}}^-} + \frac{\Delta_{\mathfrak{h}}^-}{\Delta_{\mathfrak{h}}^+}(1 - \bar{y}_\downarrow)^{\frac{3}{2}\Delta_{\mathfrak{h}}^+}\right)}\,,$$

where $\quad y_\downarrow = \min(y, y')\,, \quad y_\uparrow = \max(y, y')$ and $\bar{y} \equiv \rho\, y$.

Graviton



- All Green's functions include: zero mode + continuum + unparticles

$$\begin{cases} \mathcal{G}_{\mathfrak{h}}(y,y') \equiv G_{\mathfrak{h}}(y,y') - G^0_{\mathfrak{h}}\,, \\ \rho_{\mathfrak{h}}(y,y';s) = 3\rho\,\delta(s) + \eta_{\mathfrak{h}}(s)\Theta(s-m_g^2)\,. \end{cases} \qquad G^0_{\mathfrak{h}} = \frac{3\rho}{p^2}\,.$$

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Radion

- Scalar perturbation of the metric **[C.Csaki et al. PRD63 '01]**

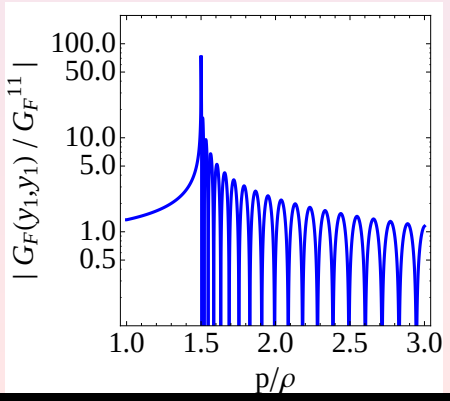
$$ds^2 = -[1+2F(x,y)]^2 dy^2 + e^{-2[A(y)+F(x,y)]}\eta_{\mu\nu}dx^\mu dx^\nu \,,$$
$$\phi(x,y) = \phi(y) + \varphi(x,y)\,, \qquad \text{with} \qquad F(x,y) = F(y)\mathcal{R}(x)\,..$$

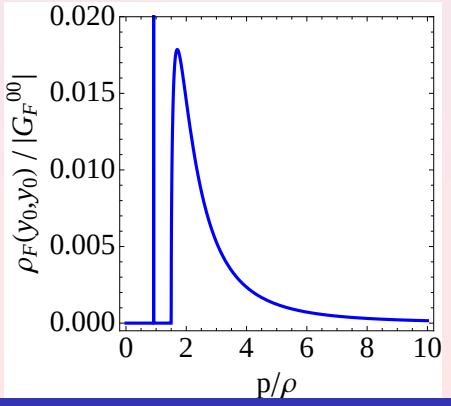
- Interaction with matter

$$\mathcal{L}_{4D} = -\frac{1}{\sqrt{2}M_5^{3/2}}F(x,y_1)\mathcal{T}\,, \qquad \mathcal{T} \equiv \mathrm{tr}\,T^\mu_\nu(x,y_1)\,.$$

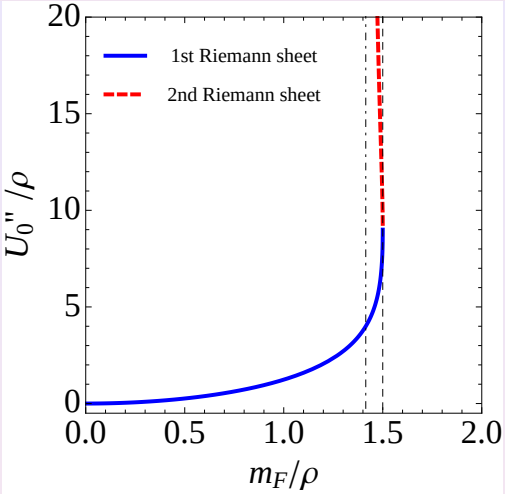
- B.C.: $G_F'(0) = \left(\frac{1}{3}\kappa^2 W(\phi(y)) - \frac{2p^2 e^{2A(y)}}{U_0''(\phi(y))}\right) G_F(y)\Big|$.

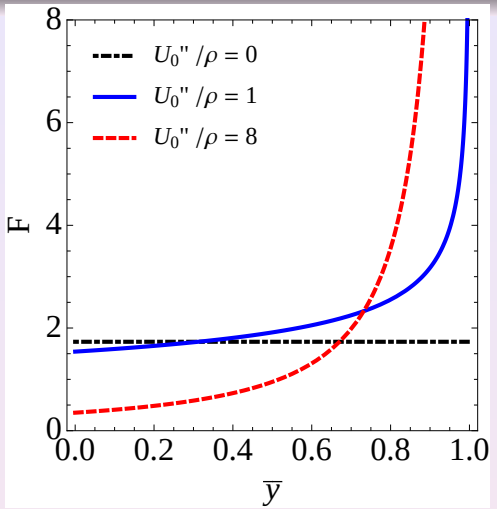
$$G_F(y,y') \propto \frac{1}{\Phi_F(p)} \qquad \text{where} \qquad \Phi_F(p) = \frac{p^2}{\rho^2} - \frac{U_0''}{4\rho}(1+3\delta_F)\,.$$





Radion





- Green's func. include 1 isolated massive mode ➡ light radion

$$G_F(y, y'; p) \propto \frac{1}{\Phi_F(p)} .$$

- Radion spectrum: $\Phi_F(m_F) = 0$

$$\boxed{\frac{m_F^2}{\rho^2} = \frac{1}{8}\frac{U_0''}{\rho}\left[-\left(\frac{U_0''}{\rho} - 2\right) + \sqrt{\left(\frac{U_0''}{\rho} - 2\right)^2 + 32}\right] , \quad 0 \leq U_0''/\rho \leq 9 .,}$$

- Wave function of the radion mostly localized toward the IR brane.

Conclusions

- We have studied 5D warped models solving the hierarchy problem:
  - AdS_5 near the UV brane.
  - Linear dilaton theory near the IR singularity.
- The KK spectra of all particles (gauge bosons, fermions, graviton, radion, Higgs boson) are continua of states with a mass gap.
- We have computed the Green's functions $G(y, y'; p)$ and spectral functions $\rho(y, y'; p)$:
  - Gauge bosons $\Longrightarrow$ resonance effects at tree level.
  - Positivity of the spectral function.
  - Interacting theory and unitarity.
  - Connection with unparticles.
- Phenomenological consequences:
  - Colliders: Increase in the cross section $\sigma(pp \to Q\bar{Q})$.
  - Cosmology: Continuum dark matter [Csaki et al. '22].
- Other phenomenological applications should be inspired on *unparticle phenomenology*.

Thank You!