Meaning of Polyakov Loop in QCD: confinement, deconfinement, and partial deconfinement

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Aug 2022 @Stavanger

Based on
MH, 2102.08982[hep-th] (PRD)
MH, Shimada, Wintergerst, 2001.10459[hep-th] (JHEP)
+ several more

[1] M. Hanada and J. Maltz, "A proposal of the gauge theory description of the small Schwarzschild black hole in $\mathrm{AdS}_5\times\mathrm{S}^5$," *JHEP* **02** (2017) 012, arXiv:1608.03276 [hep-th].

[2] M. Hanada, G. Ishiki, and H. Watanabe, "Partial Deconfinement," *JHEP* **03** (2019) 145, arXiv:1812.05494 [hep-th]. [Erratum: JHEP 10, 029 (2019)].

[3] M. Hanada, A. Jevicki, C. Peng, and N. Wintergerst, "Anatomy of Deconfinement," *JHEP* **12** (2019) 167, arXiv:1909.09118 [hep-th].

[4] M. Hanada and B. Robinson, "Partial-Symmetry-Breaking Phase Transitions," *Phys. Rev. D* **102** no. 9, (2020) 096013, arXiv:1911.06223 [hep-th].

[5] M. Hanada, H. Shimada, and N. Wintergerst, "Color confinement and Bose-Einstein condensation," *JHEP* **08** (2021) 039, arXiv:2001.10459 [hep-th].

[6] H. Watanabe, G. Bergner, N. Bodendorfer, S. Shiba Funai, M. Hanada, E. Rinaldi, A. Schäfer, and P. Vranas, "Partial deconfinement at strong coupling on the lattice," *JHEP* **02** (2021) 004, arXiv:2005.04103 [hep-th].

[7] M. Hanada, "Bulk geometry in gauge/gravity duality and color degrees of freedom," *Phys. Rev. D* **103** no. 10, (2021) 106007, arXiv:2102.08982 [hep-th].

[8] M. Hanada, J. Holden, M. Knaggs, and A. O'Bannon, "Global symmetries and partial confinement," *JHEP* **03** (2022) 118, arXiv:2112.11398 [hep-th].

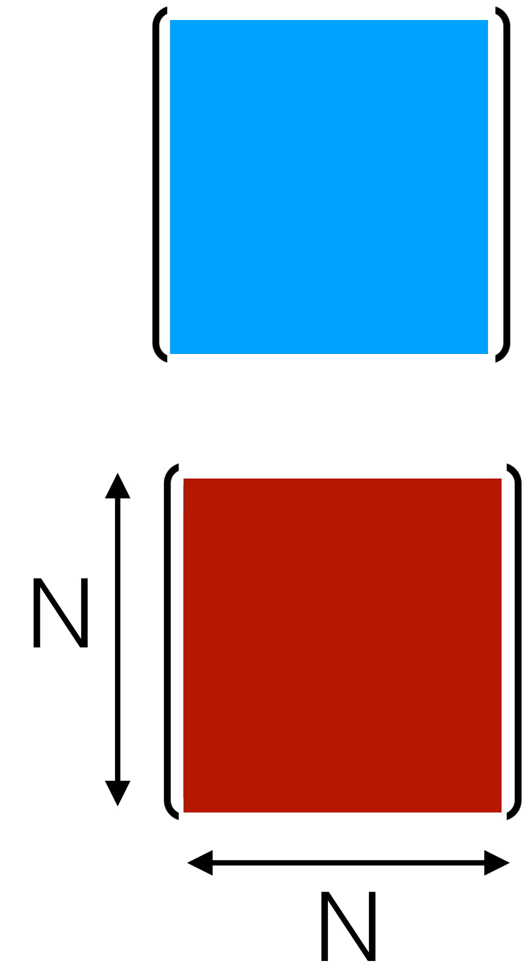
[9] Gautam, M. Hanada, Holden, Rinaldi, "Linear confinement in the partially-deconfined phase", to appear in hep-th.

- Three phases? Intermediate phase between confined and deconfined phase?
- Meaning of Polyakov loop (nothing to do with center symmetry!)
- Partial confinement, partial deconfinement
- Large-N confinement ~ BEC

Confinement ~ superfluid

Deconfinement ~ normal fluid

- Confinement phase: $E, S \sim N^0$
- Deconfinement phase: $E, S \sim N^2$



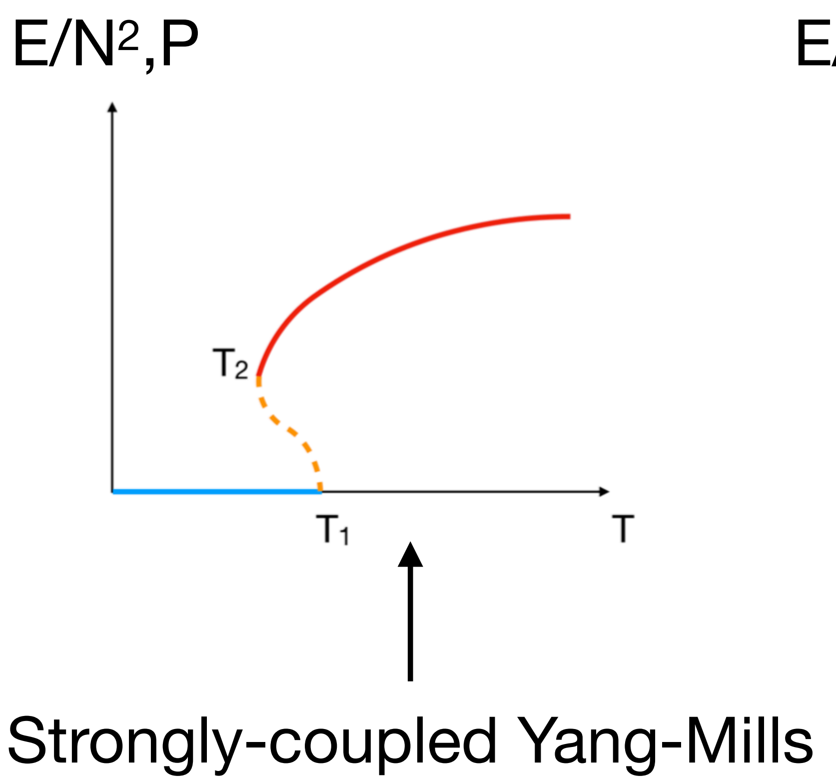
This ‘kinematical’ characterization (Witten 1998)

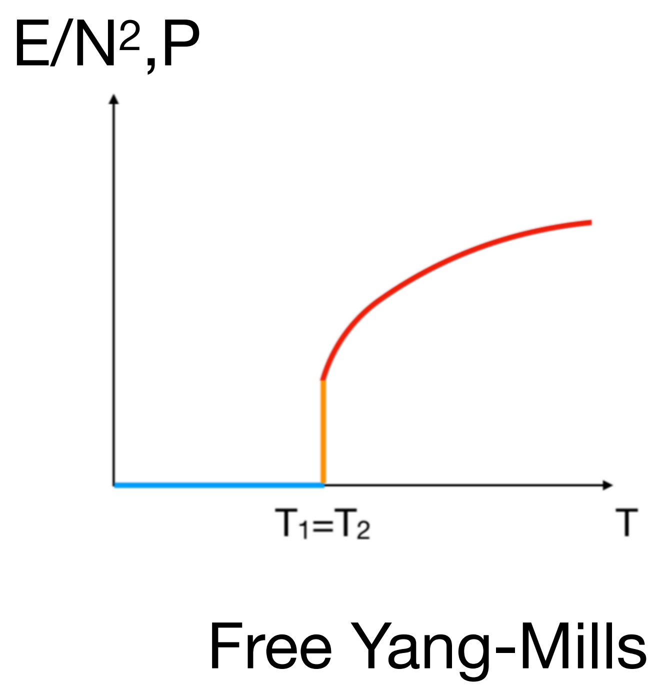
works even at weak coupling and/or small volume.

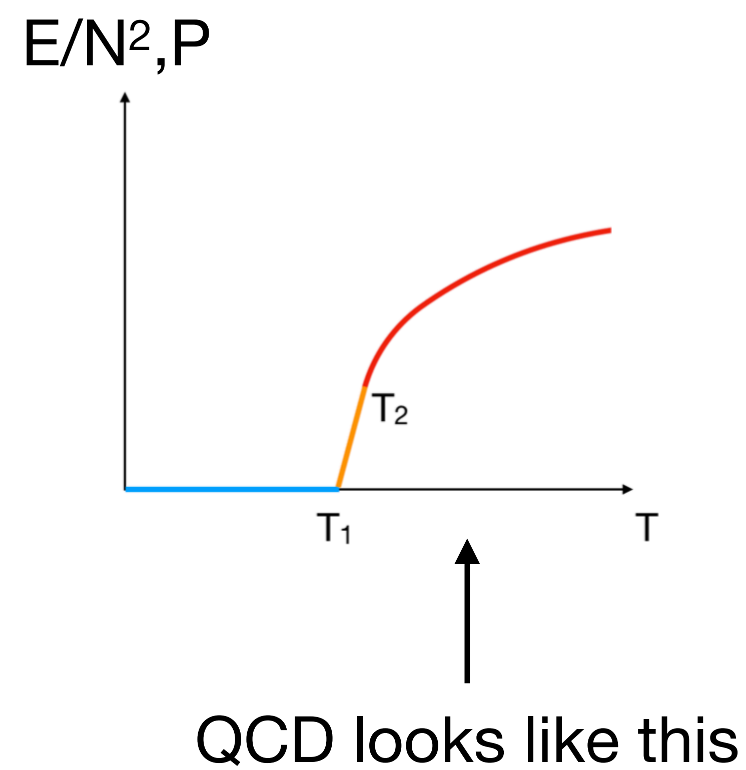
(Sundborg 1998; Aharony et al 2003)

We consider the <u>large-N</u> limit.

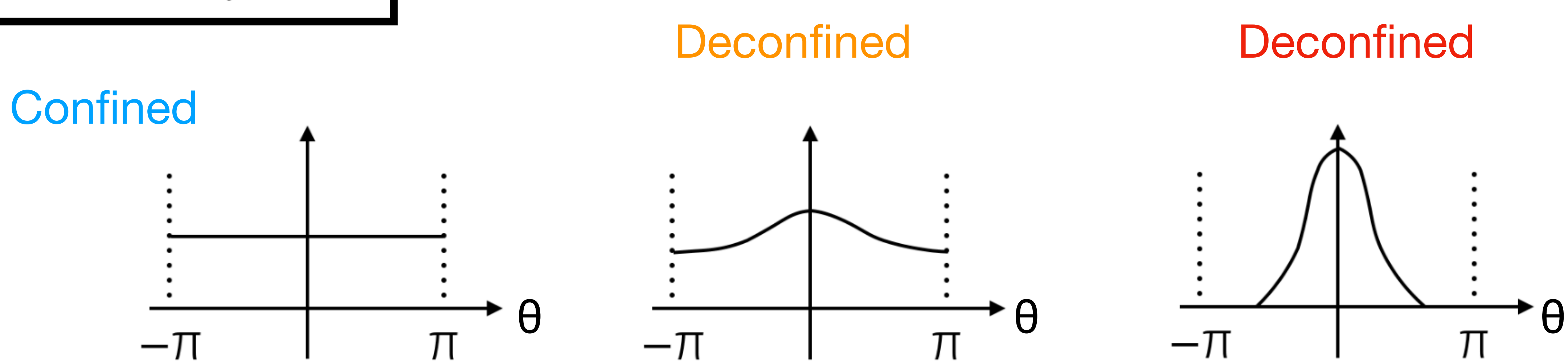
Three patterns of the phase diagram in gauge theory (Aharony et al, 2003)



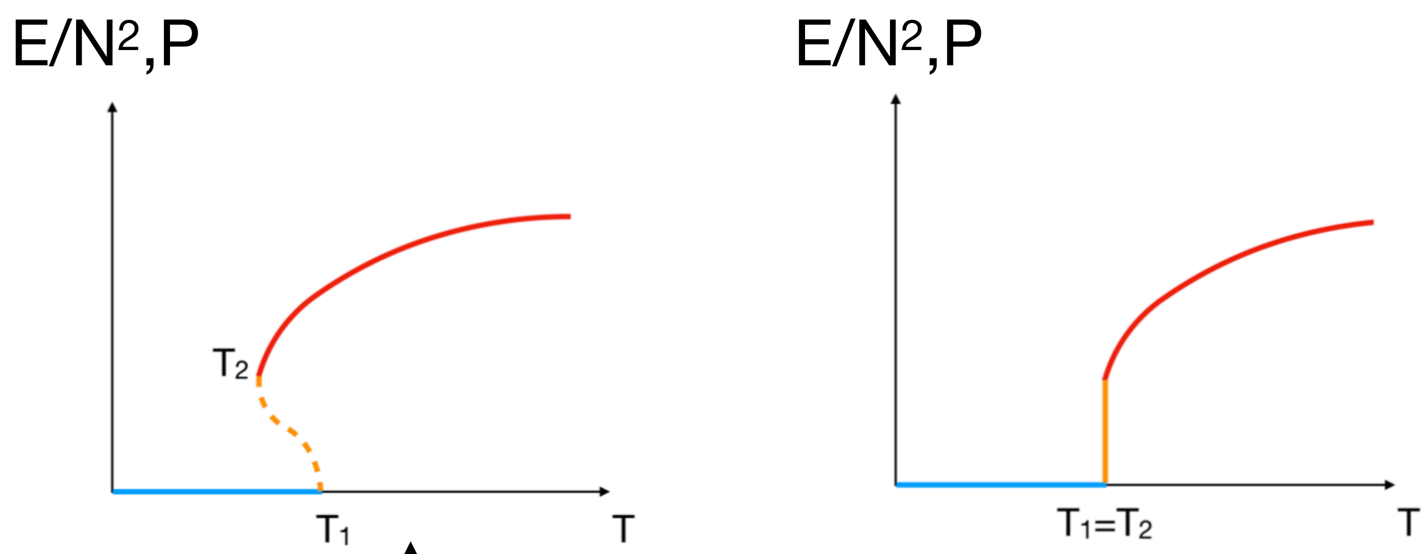




Polyakov loop $P = \frac{1}{N}\sum_{j=1}^{N} e^{i\theta_j}$



Three patterns of the phase diagram in gauge theory (Aharony et al, 2003)



Strongly-coupled Yang-Mills, black hole

Free Yang-Mills

QCD looks like this

Polyakov loop $P = \frac{1}{N}\sum_{j=1}^{N} e^{i\theta_j}$

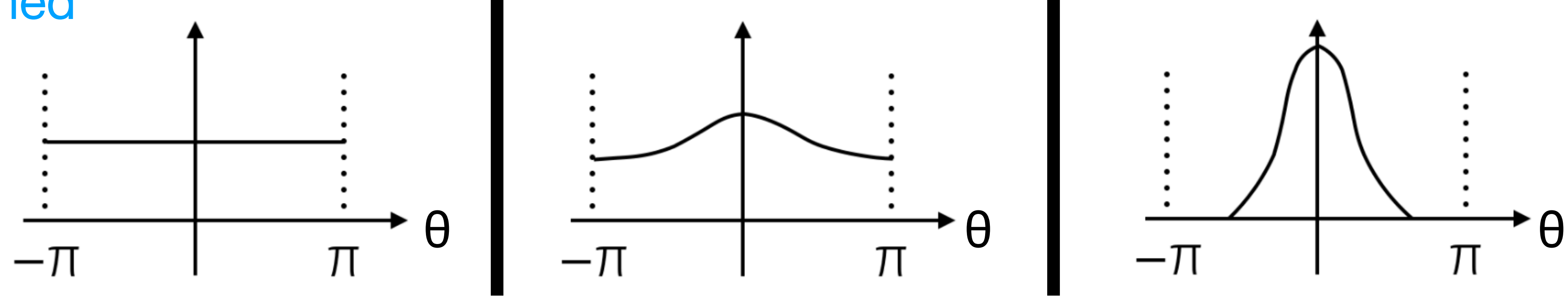
Hagedorn

Gross-Witten-Wadia

Confined

Deconfined

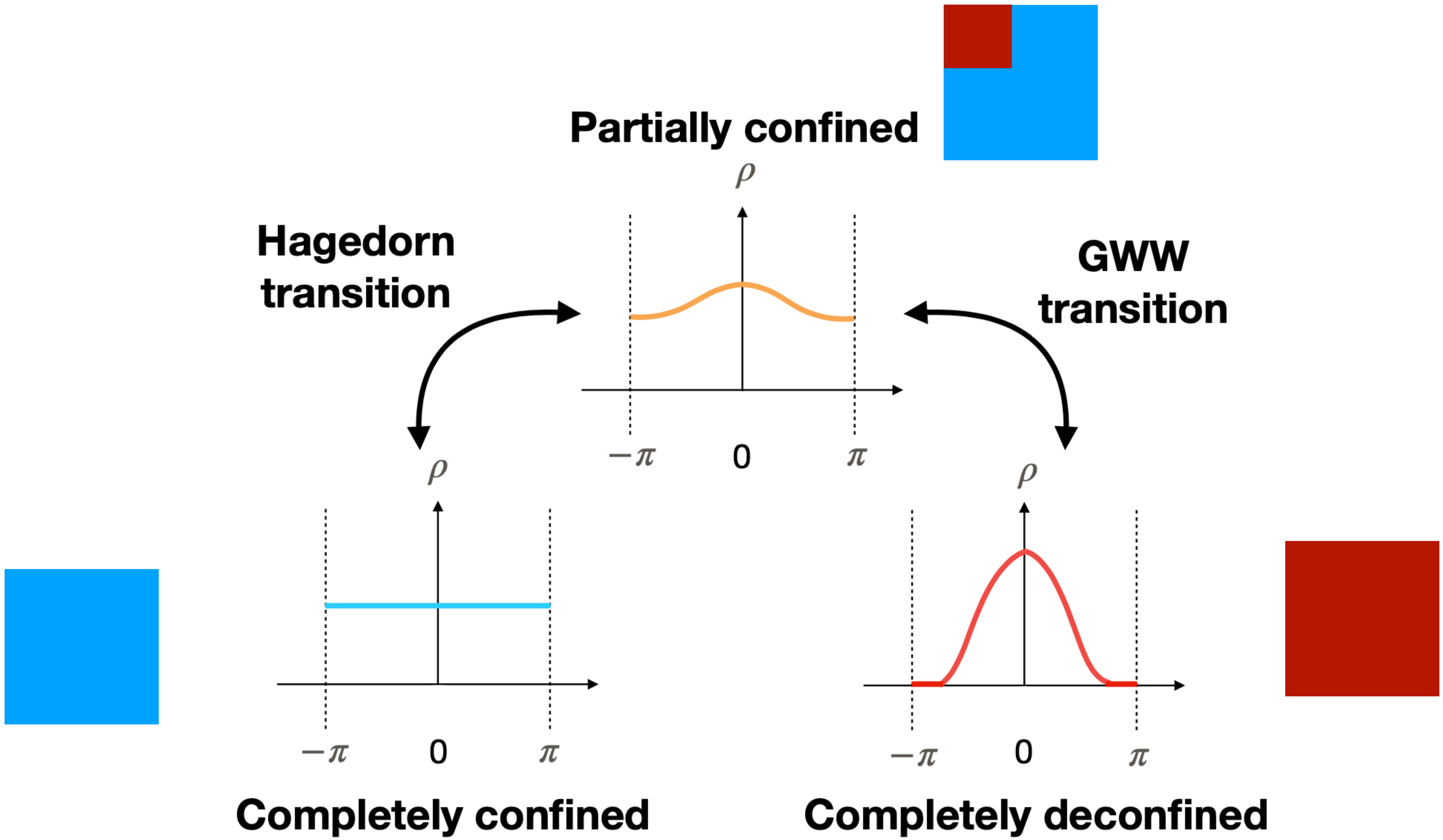
Deconfined



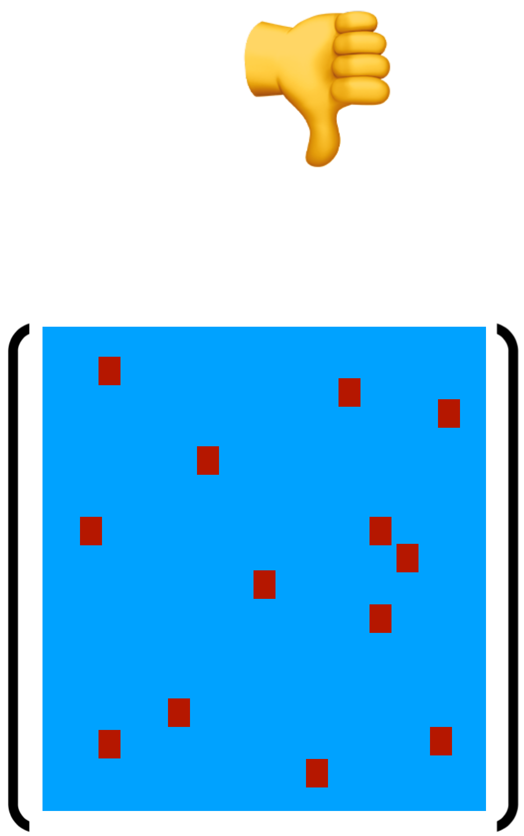
Physical meaning?

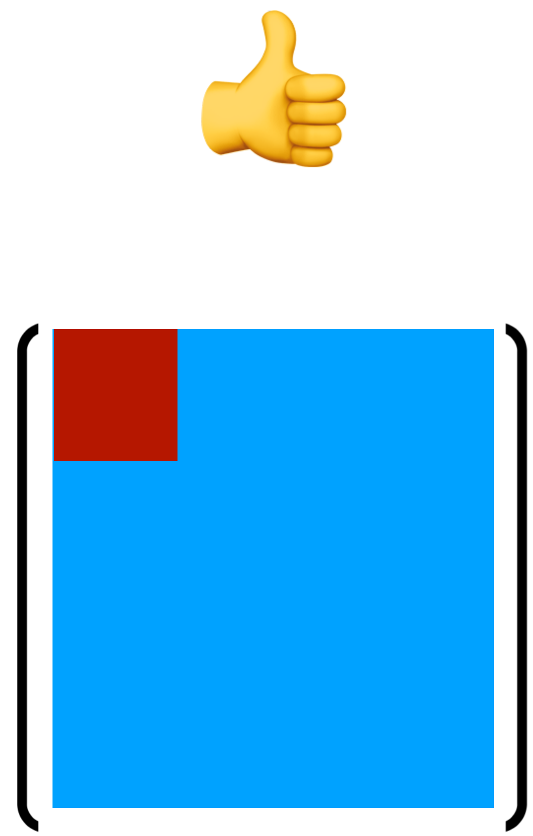
Partial confinement (MH-Maltz, 2016)

Polyakov Loop $$P = \frac{1}{N}\sum_{j=1}^{N} e^{i\theta_j}$$

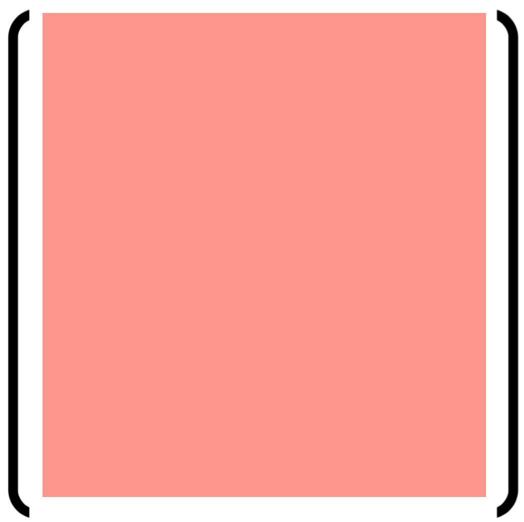


MH-Maltz, 2016 (JHEP)
MH-Ishiki-Watanabe, 2018 (JHEP)
MH-Shimada-Wintergerst, 2020 (JHEP)





no symmetry

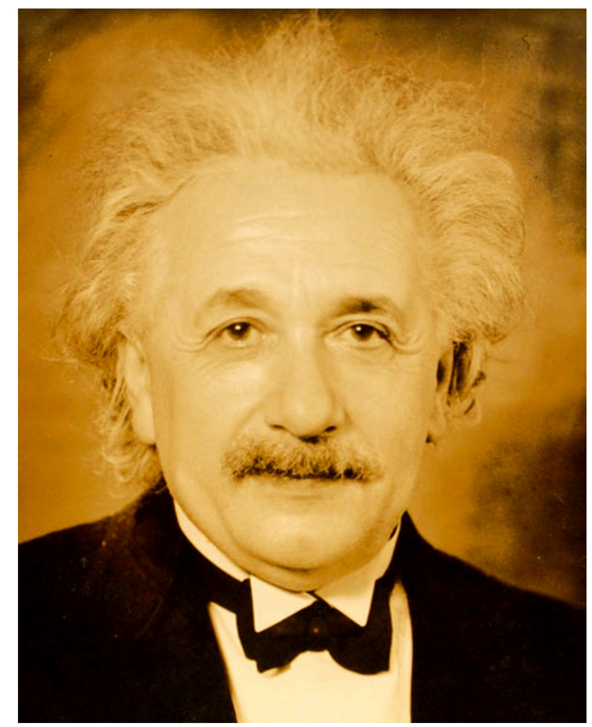


The mechanism is the same as Bose-Einstein condensation

(MH-Shimada-Wintergerst, 2020)

Historically the first example of non-abelian gauge theory

Bose



Einstein

$\boxed{\text{N indistinguishable bosons}}$

Summation over singlet states $Z(T) = \mathrm{Tr}_{\mathcal{H}_{\mathrm{inv}}}(e^{-\hat{H}/T})$

Summation over all states & projection to singlet states

$$Z(T) = \frac{1}{\mathrm{vol}(G)} \int_G dg \mathrm{Tr}_{\mathcal{H}_{\mathrm{ext}}}(\hat{g} e^{-\hat{H}/T})$$

$G = \mathrm{SU}(N)$ + adjoint fields → Yang-Mills, matrix model

$G = \mathrm{S}_N$ + fundamental fields → N indistinguishable bosons

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Polyakov loop

$G = \mathrm{SU}(N)$ + adjoint fields → Yang-Mills, matrix model

$G = \mathrm{S}_N$ + fundamental fields → N indistinguishable bosons

N bosons in 3d harmonic trap

$$\hat{H} = \sum_{i=1}^{N} \left(\frac{\hat{\vec{p}}_i^{\,2}}{2m} + \frac{m\omega^2}{2} \hat{\vec{x}}_i^{\,2} \right)$$

$$\hat{\vec{x}}_i = (\hat{x}_i, \hat{y}_i, \hat{z}_i)$$

$$\hat{\vec{p}}_i = (\hat{p}_{x,i}, \hat{p}_{y,i}, \hat{p}_{z,i})$$

Fock states $|\vec{n}_1, \vec{n}_2, \cdots, \vec{n}_N\rangle \equiv \prod_{i=1}^{3} \frac{\hat{a}_{i1}^{\dagger n_{i1}}}{\sqrt{n_{i1}!}} \frac{\hat{a}_{i2}^{\dagger n_{i2}}}{\sqrt{n_{i2}!}} \cdots \frac{\hat{a}_{iN}^{\dagger n_{iN}}}{\sqrt{n_{iN}!}} |0\rangle$

$$\begin{aligned} Z(T) &= \frac{1}{N!} \sum_{\sigma \in \mathrm{S}_N} \sum_{\vec{n}_1, \cdots, \vec{n}_N} \langle \vec{n}_1, \cdots, \vec{n}_N | \hat{\sigma} e^{-\hat{H}/T} | \vec{n}_1, \cdots, \vec{n}_N \rangle \\ &= \frac{1}{N!} \sum_{\vec{n}_1, \cdots, \vec{n}_N} e^{-(E_{\vec{n}_1} + \cdots + E_{\vec{n}_N})/T} \underline{\left(\sum_{\sigma \in \mathrm{S}_N} \langle \vec{n}_1, \cdots, \vec{n}_N | \vec{n}_{\sigma(1)}, \cdots, \vec{n}_{\sigma(N)} \rangle \right)} \end{aligned}$$

measures the amount of redundancy

京都　三十三間堂

Sanjusangendo, Kyoto

N=1001

(Einstein visited Kyoto in 1922)

$$Z(T) = \frac{1}{N!}\sum_{\sigma\in \mathrm{S}_N}\sum_{\vec{n}_1,\cdots,\vec{n}_N}\langle \vec{n}_1,\cdots,\vec{n}_N|\hat{\sigma}e^{-\hat{H}/T}|\vec{n}_1,\cdots,\vec{n}_N\rangle$$

$$= \frac{1}{N!}\sum_{\vec{n}_1,\cdots,\vec{n}_N} e^{-(E_{\vec{n}_1}+\cdots+E_{\vec{n}_N})/T}\underline{\left(\sum_{\sigma\in \mathrm{S}_N}\langle \vec{n}_1,\cdots,\vec{n}_N|\vec{n}_{\sigma(1)},\cdots,\vec{n}_{\sigma(N)}\rangle\right)}$$

$|\vec{0},\vec{0},\cdots,\vec{0}\rangle$ $\quad N!$

$|\vec{n}_1,\cdots,\vec{n}_N\rangle$ $\quad 1$

(all of them are different)

$|\vec{n}_1,\cdots,\vec{n}_M,\vec{0},\cdots,\vec{0}\rangle$ $\quad (N-M)!$

$\boxed{\text{This enhancement triggers BEC.}}$

Einstein, 1924

Polyakov loop

$$Z(T) = \frac{1}{N!}\sum_{\sigma\in \mathrm{S}_N}\sum_{\vec{n}_1,\cdots,\vec{n}_N}\langle \vec{n}_1,\cdots,\vec{n}_N|\hat{\sigma}e^{-\hat{H}/T}|\vec{n}_1,\cdots,\vec{n}_N\rangle$$

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$|\vec{0},\vec{0},\cdots,\vec{0}\rangle$ $\quad N!$

$$G_{|\Phi\rangle} = \{\hat{\sigma}\in G = \mathrm{S}_N \text{ s.t. } \hat{\sigma}|\Phi\rangle = |\Phi\rangle\}$$

$$\mathrm{Vol}G_{|\Phi\rangle}$$

$|\vec{n}_1,\cdots,\vec{n}_M,\vec{0},\cdots,\vec{0}\rangle$ $\quad (N-M)!$

$\boxed{\text{This enhancement triggers BEC.}}$

Einstein, 1924

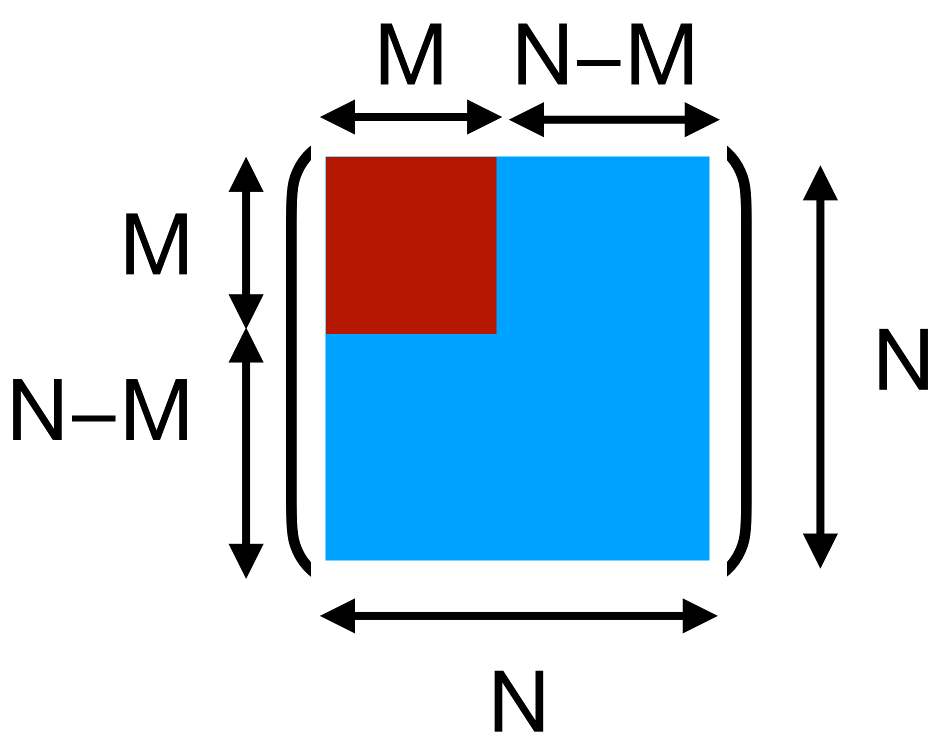
(MH-Shimada-Wintergerst, 2020)

Partially-BEC state

$$|\vec{n}_1, \cdots, \vec{n}_M, \vec{0}, \cdots, \vec{0}\rangle \qquad (N-M)!$$

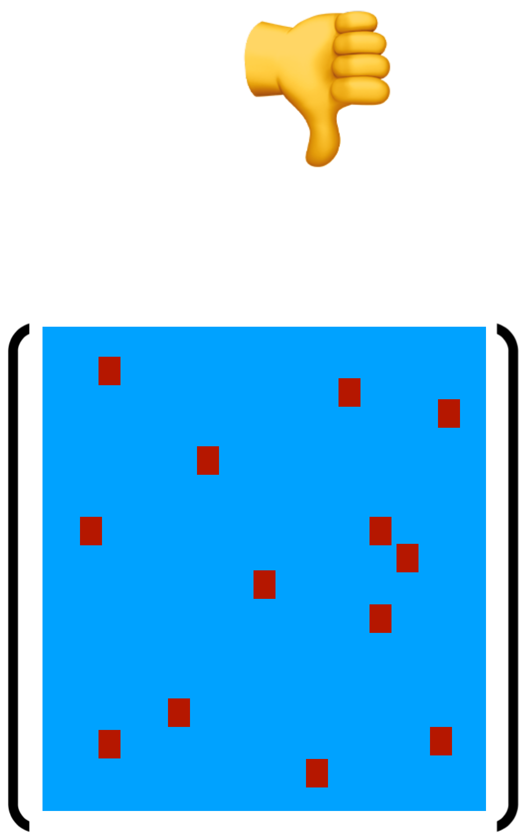
Partially-confined state

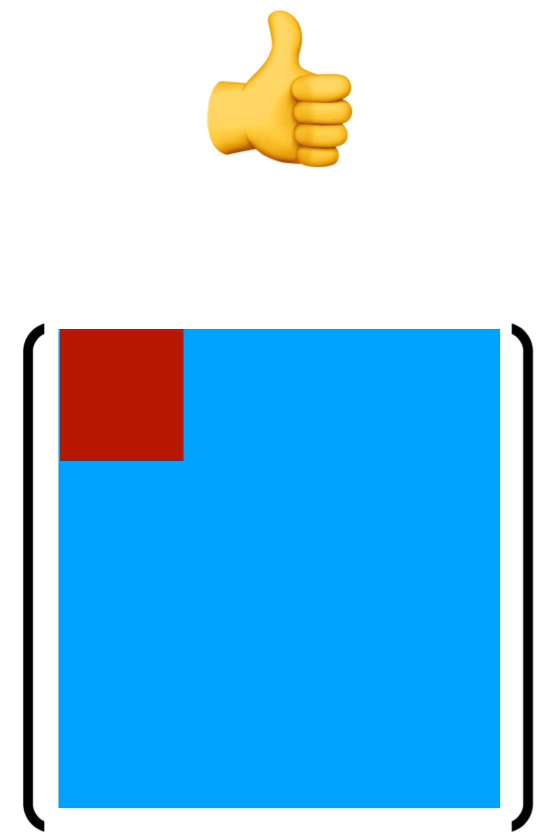
(MH-Maltz, 2016; Berenstein, 2018;
MH-Ishiki-Watanabe, 2018;
MH-Jevicki-Peng-Wintergerst, 2019;
Watanabe et al, 2020)



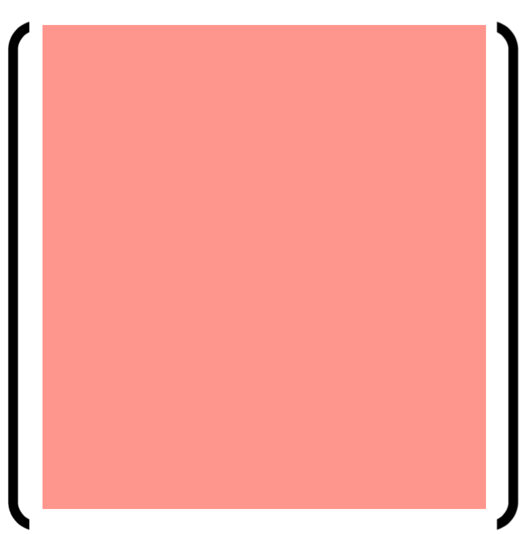
$$\mathrm{Vol}(\mathrm{SU}(N-M)) \sim e^{(N-M)^2}$$

deconfined sector = extended bound state of strings and D-brane
= black hole





no symmetry



Larger enhancement factor (volume of SU(N-M))

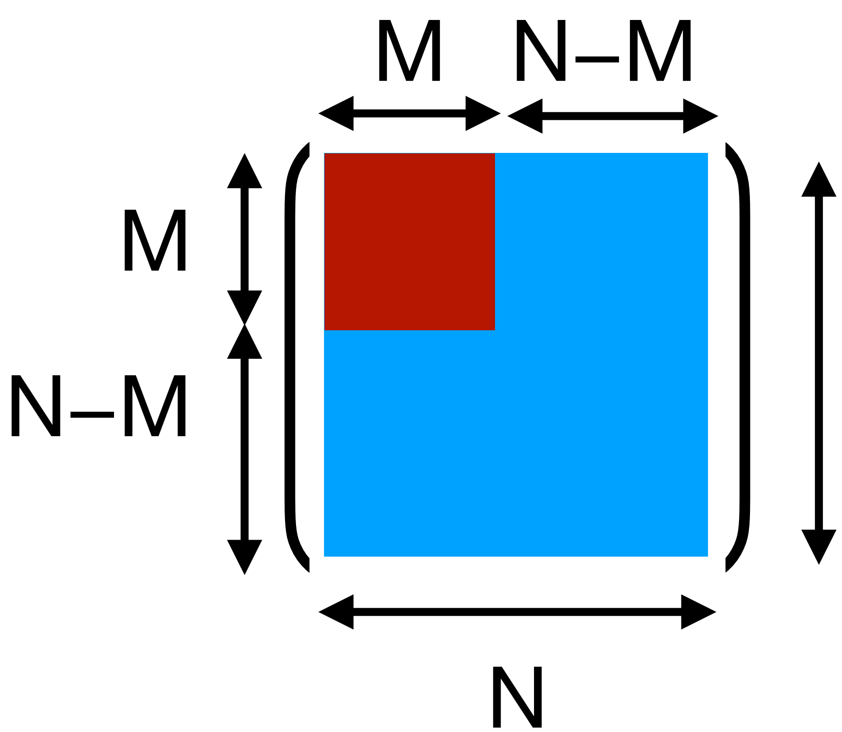
MH, Shimada, Wintergerst, 2001.10459[hep-th] (JHEP)
MH, 2102.08982[hep-th] (PRD)

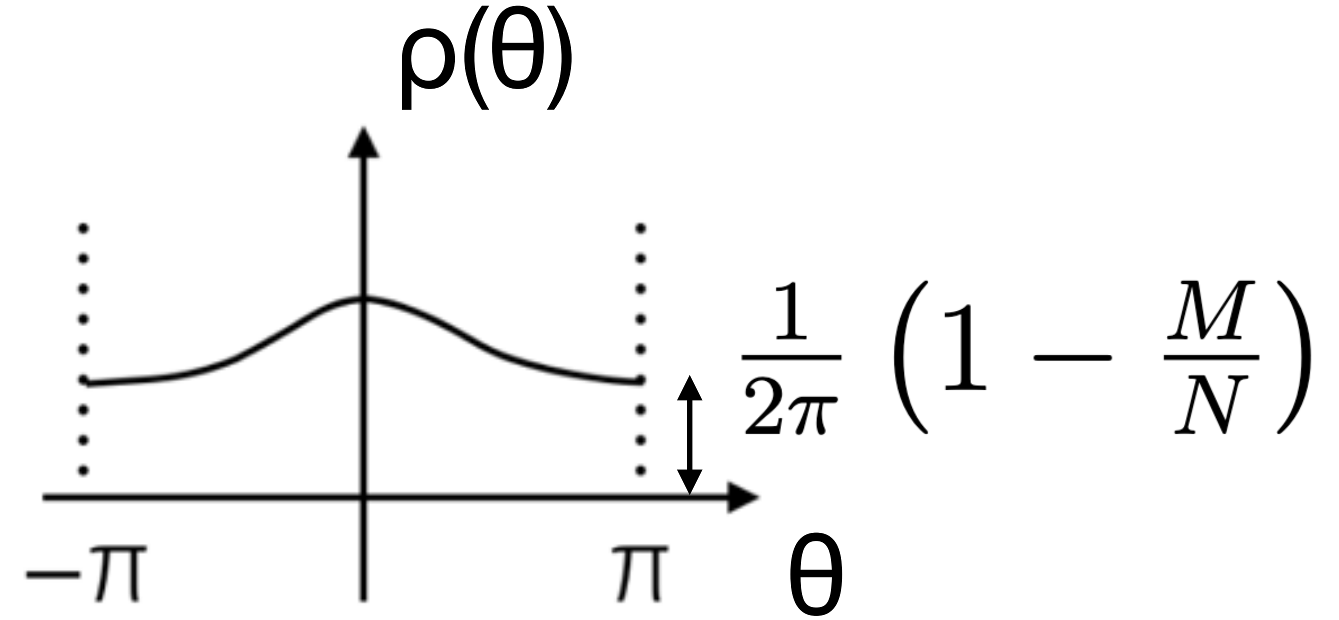
$$Z(T) = \frac{1}{\mathrm{vol}G}\int_G dg \mathrm{Tr}_{\mathcal{H}_{\mathrm{ext}}}\left(\hat{g}e^{-\hat{H}/T}\right)$$

$$\sim \frac{1}{\mathrm{vol}G} e^{-E_{\mathrm{typical}}/T}\int_G dg \langle \mathrm{typical}|\hat{g}|\mathrm{typical}\rangle$$

Polyakov loop

Typical $\hat{g}$'s which leave $|\mathrm{typical}\rangle$ unchanged dominate the phase distribution





$$|\vec{n}_1, \cdots, \vec{n}_M, \vec{0}, \cdots, \vec{0}\rangle$$

MH-Shimada-Wintergerst, 2020
(Similar to Feynman, 1953)

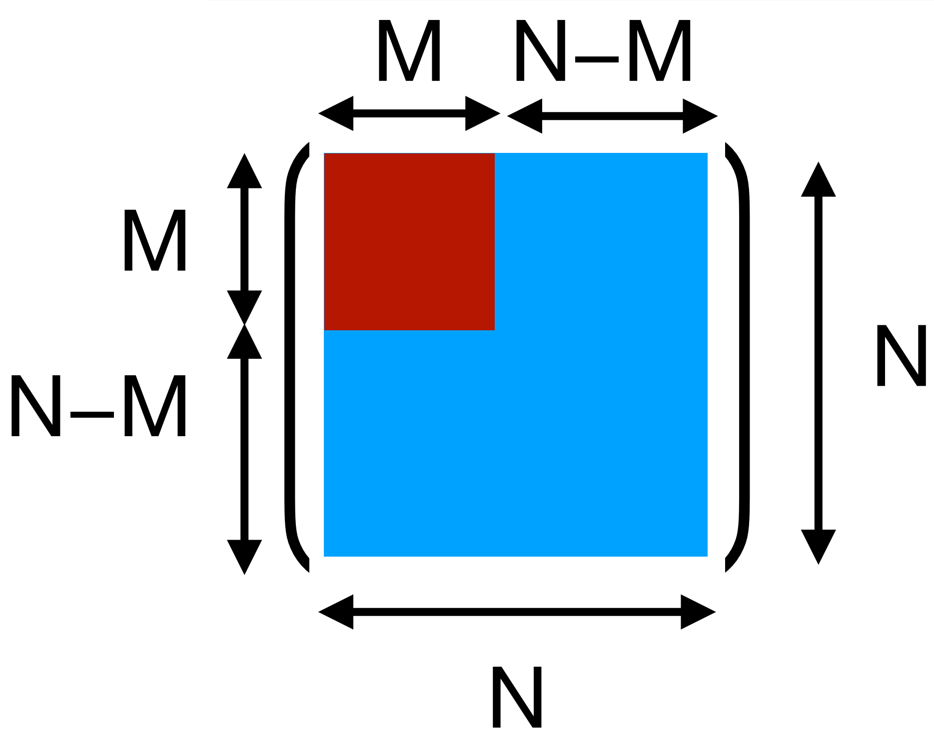
$$Z(T) = \frac{1}{\mathrm{vol}G}\int_G dg\,\mathrm{Tr}_{\mathcal{H}_{\mathrm{ext}}}\left(\hat{g}e^{-\hat{H}/T}\right)$$

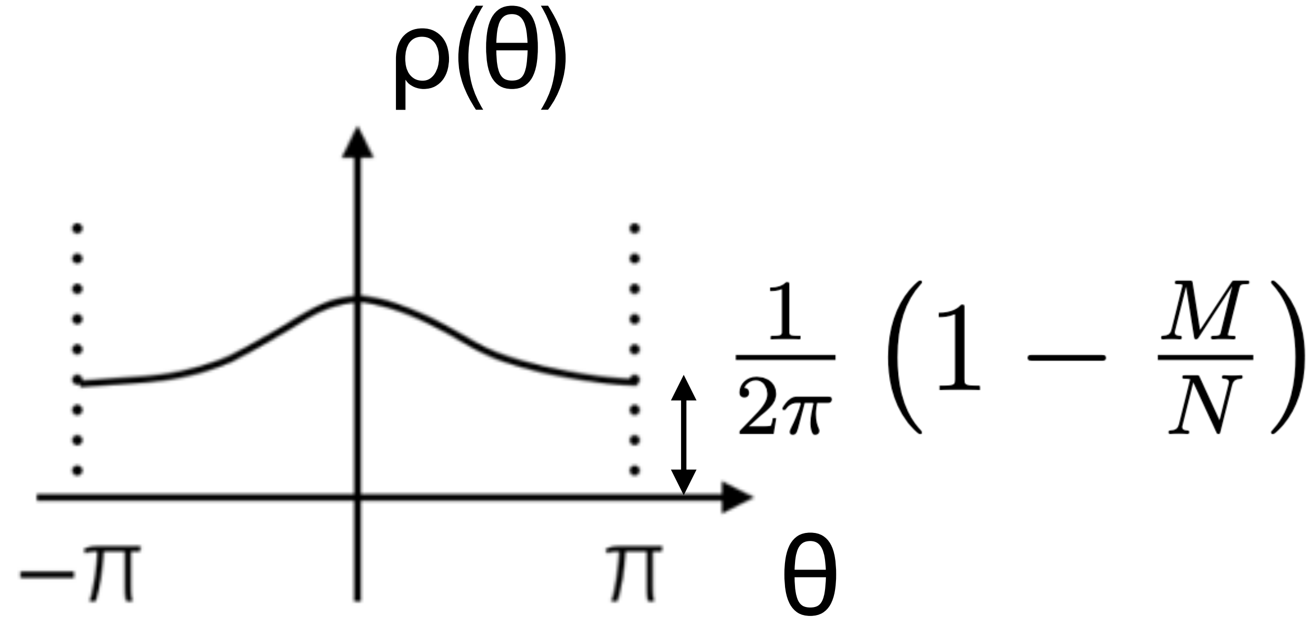
$$\sim \frac{1}{\mathrm{vol}G}e^{-E_{\mathrm{typical}}/T}\int dg\langle\mathrm{typical}|\hat{g}|\mathrm{typical}\rangle$$

Typ

Feynman's characterization of BEC (1953)

= Gross-Witten-Wadia transition

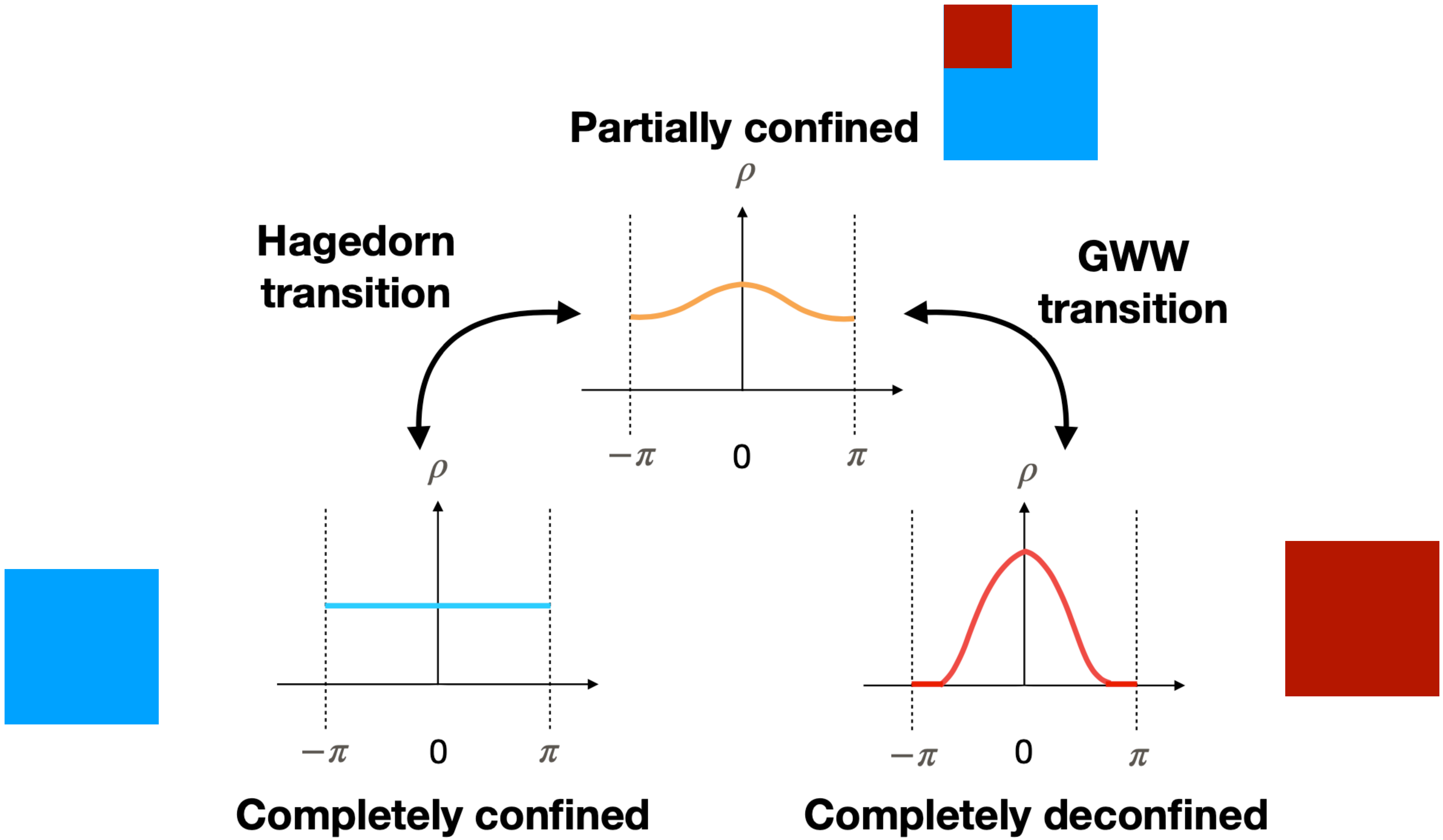




$$|\vec{n}_1,\cdots,\vec{n}_M,\vec{0},\cdots,\vec{0}\rangle$$

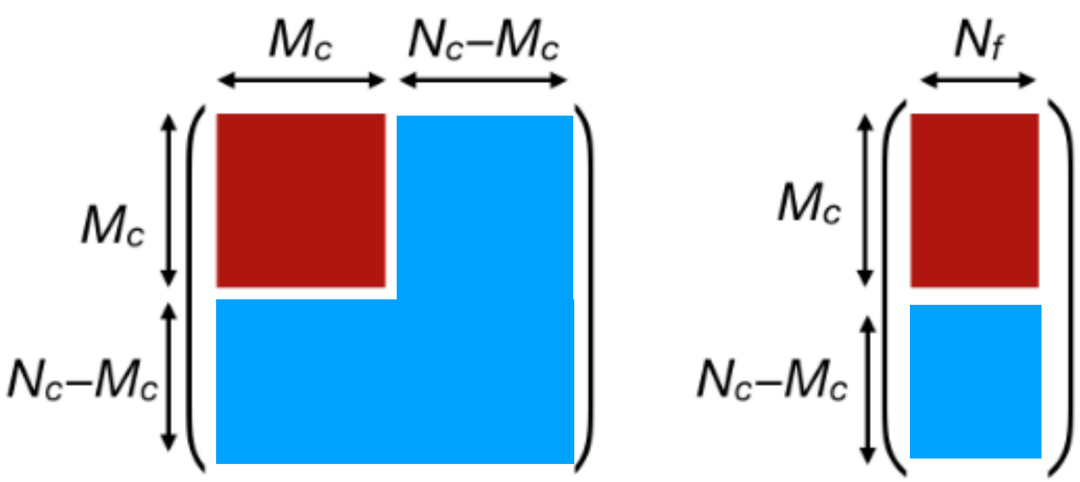
MH-Shimada-Wintergerst, 2020
(Similar to Feynman, 1953)

Polyakov Loop $P = \frac{1}{N}\sum_{j=1}^{N} e^{i\theta_j}$

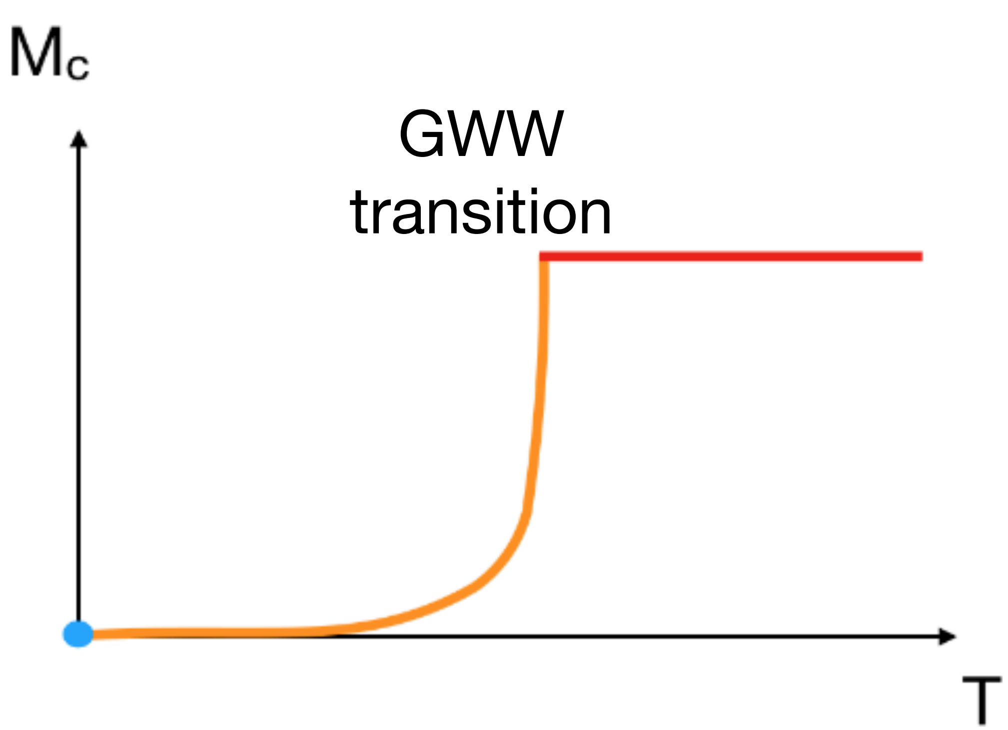


MH-Maltz, 2016 (JHEP)
MH-Ishiki-Watanabe, 2018 (JHEP)
MH-Shimada-Wintergerst, 2020 (JHEP)

QCD on S^3 at weak coupling



MH-Robinson, 1911.06223

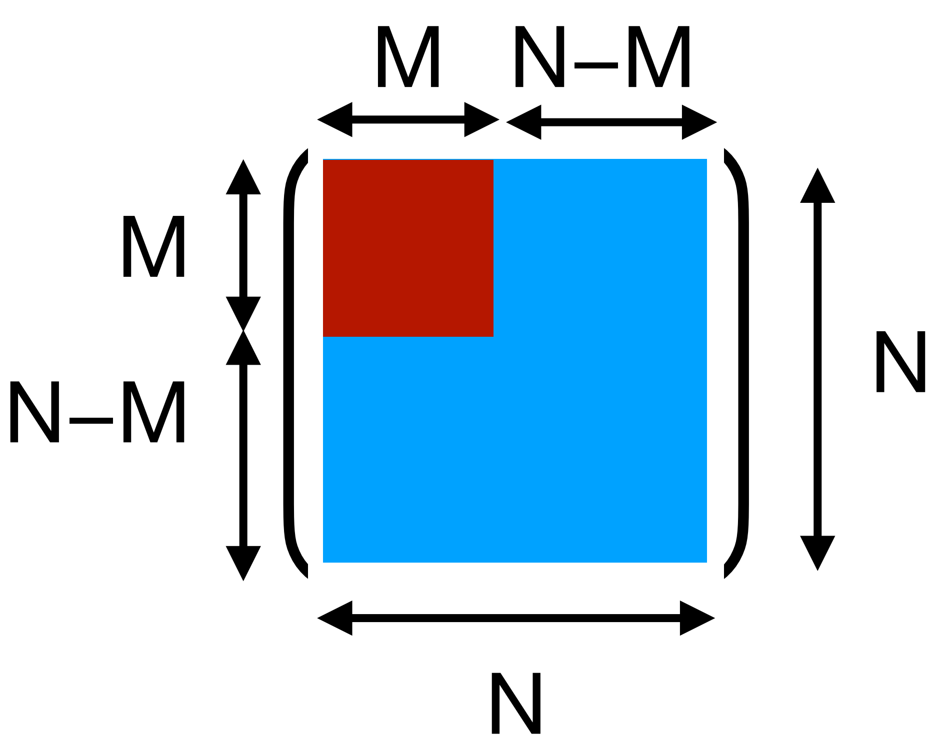


Re-interpretation of Schnitzer, hep-th/0402219

Flux tube in pure YM

Gautam-MH-Holden-Rinaldi, 2022

$$\mathrm{Pol.} = \mathrm{diag}(\underbrace{e^{i\theta_1}, \cdots, e^{i\theta_M}}_{\text{Deconfined Polyakov loop}}, \underbrace{e^{i\theta_{M+1}}, \cdots, e^{i\theta_N}}_{\text{Confined Polyakov loop}})$$



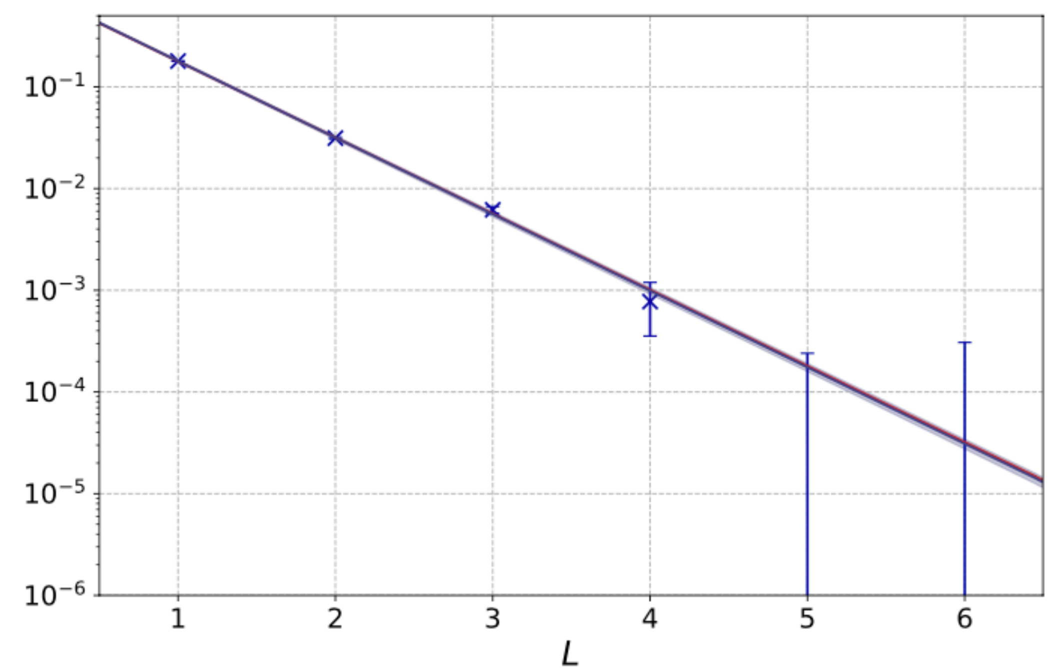
- Strong coupling limit → Analytic prediction

  (Assuming the formation of flux tube)
- Check it numerically.
- Use large-N volume reduction technique

Flux tube in pure YM

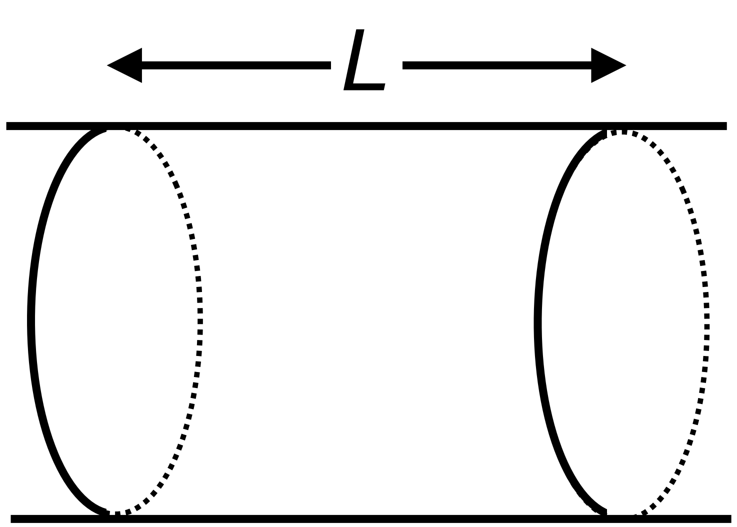
Polyakov loop correlators

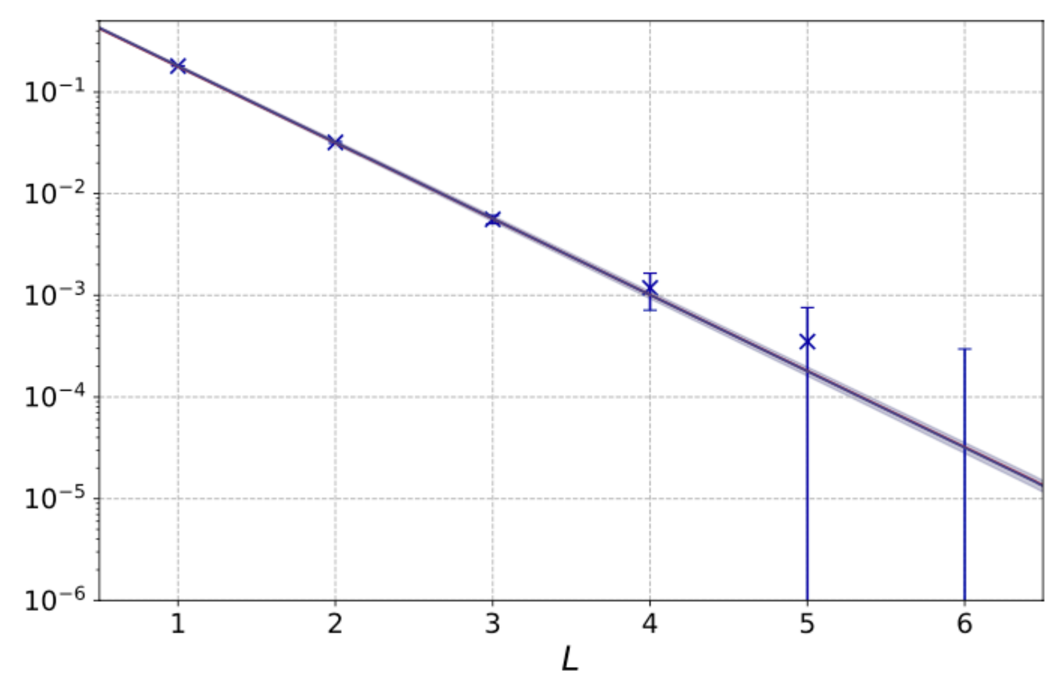
Gautam-MH-Holden-Rinaldi, 2022



M/N=0.5

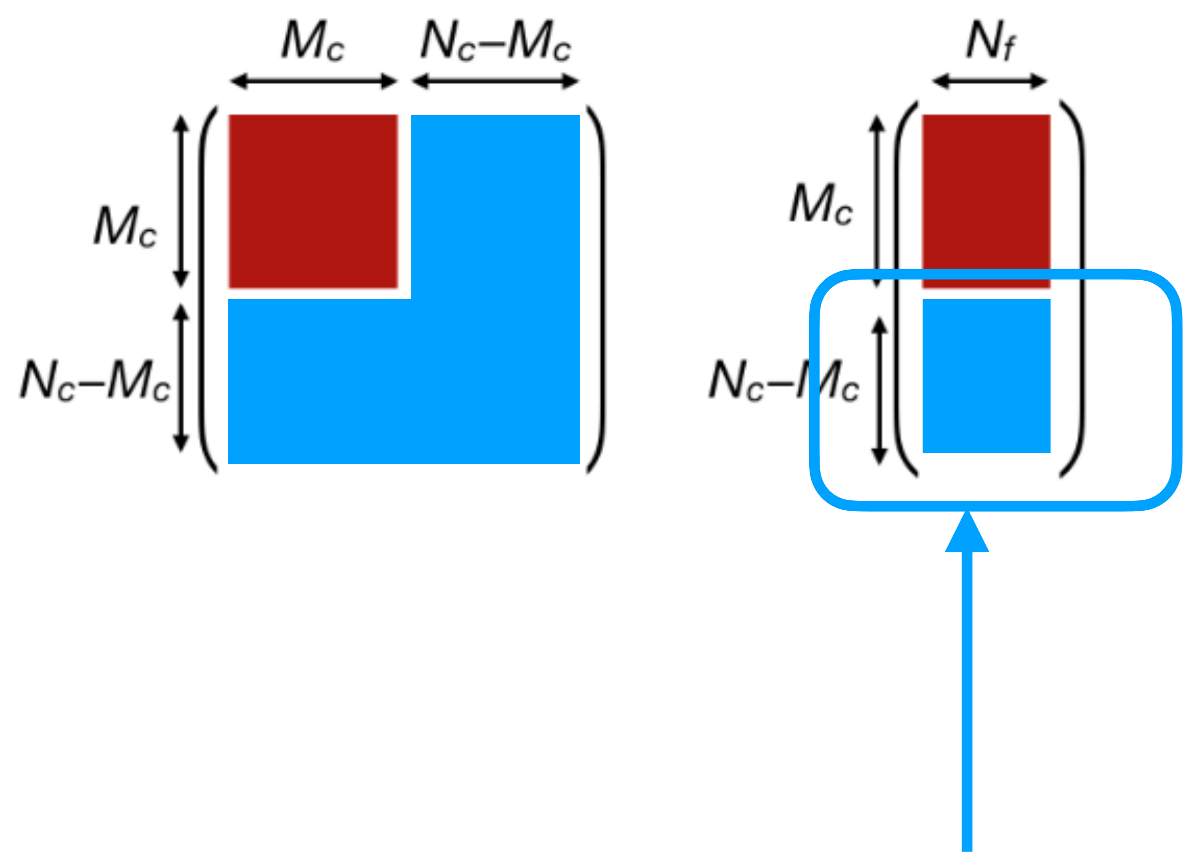
String-coupling
Lattice Gauge Theory





M/N=0.63

Chiral symmetry



't Hooft anomaly matching

→ Chiral symmetry breaking in this sector

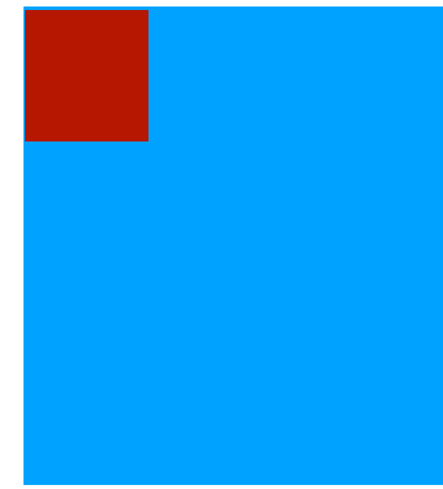
$\boxed{\text{Chiral symmetry breaking point = GWW point}}$

MH-Robinson, 1911.06223

We can check it numerically.

MH-Holden-Knaggs-O'Bannon, 2112.11398

Summary & Outlook



- Partial (de)confinement @ large N
  - Confined sector + deconfined sector
  - Only gauge symmetry is needed.
- Generic feature of large-N gauge theory
- Triggers chiral symmetry breaking (also CP breaking @$\theta=\pi$)
- Finite-N?

"Partial phase = center breaking, chiral breaking phase"

(in some concrete examples with global symmetries)