Disentangling the gravity dual of Yang-Mills theory from the lattice

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XVth Quark Confinement and the Hadron Spectrum

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Motivation

- Wish to understand confinement

  $\boxed{\textit{conformal symmetry must be broken}}$

- Non-perturbative: defied analytic attempts. Numerical progress.
- Use non-AdS/non-CFT
- Tools at disposal:
  - Wilson etc loops
  - Entanglement entropy
  - C-functions
- Holography is never going to solve confinement, but

  $\boxed{\textit{we may learn valuable lessons}}$

- Holography score card by comparison to YM simulations
- We need to gain understanding of entanglement entropy in gauge theories $\leftrightarrow$ this is our first goal

Entanglement is sensitive to confinement transition

- Entanglement roughly probes confinement
  [Nishioka-Takayanagi hep-th/0611035,Klebanov-Kutasov-Murugan 0709.2140,NJ-Subils 2010.09392]
- Effective degrees of freedom:
  - deconfining phase: colorful (e.g. gluons) $\sim \mathcal{O}(N_c^2)$
  - confining phase: color singlets (e.g. glueballs) $\sim \mathcal{O}(1)$
- Derived quantities of EE capture the number of dofs
- Key idea: C-function constructed from EE acts as an order parameter for deconfinement at $N_c \sim \infty$

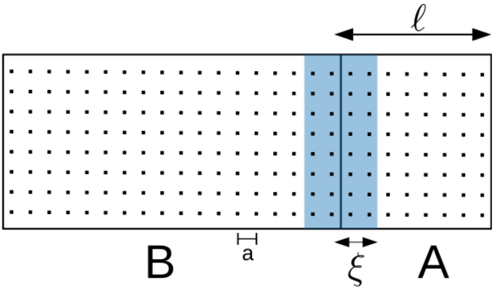
Entanglement entropy itself motivates

- Quantum information theory (related entanglement measures)
- Universal order parameter for quantum phase transitions
  [Kitaev-Preskill hep-th/0510092]
- "Measured" in cold atom systems
  [Islam et al. 1509.01160]
- $N_c \ll \infty$: Computable from the lattice
  [Buividovich-Polikarpov 0802.4247, ...]
- $N_c = \infty$: Holography
  - simple prescription via minimal surfaces
    [Ryu-Takayanagi hep-th/0603001,Hubeny-Rangamani-Takayanagi 0705.0016]
  - relationship to black hole thermo $\rightarrow$ get thermo for QFT

Definition of EE

- Take (scalar) QFT in $\mathbb{R}^{d+1}$ in vacuum state $|0\rangle$ and split

$$A = \mathbb{R}^d \times I_\ell \qquad , \qquad B = \mathbb{R}^d \times (\mathbb{R} - I_\ell)$$



- Quantum entanglement (von Neumann) entropy

$$S_A = -\mathrm{Tr}\rho_\mathrm{A} \log \rho_\mathrm{A} \;, \text{ reduced density matrix } \rho_\mathrm{A} = \mathrm{Tr}_\mathrm{B}|0\rangle\langle 0|$$

- EE for region A is entropy seen by an observer unable to access dofs in B

Holographic EE and $q\bar{q}$ potential

- Consider SU($N_c \sim \infty$) in 4d and 3d
  - $\rightsquigarrow$ rough duals from stacks of D3- and D2-branes
- Compute potentials for quarks separated by L and entanglement entropies for slabs of width ℓ using holography.
  [Rey-Yee hep-th/9803001,Maldacena hep-th/9803002,Ryu-Takayanagi hep-th/0603001]
- Holographic results for 4d SYM

$$V(L) \propto 1/L$$
$$S_{RT} \propto 1/\ell^2$$

- Holographic results for 3d SYM

$$V(L) \propto 1/L^{2/3}$$
$$S_{RT} \propto 1/\ell^{4/3}$$

Existing results in the literature

EE for slabs from the lattice in

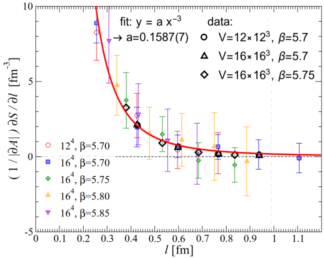
- Relation of EE to free energy / path integral
  [Calabrese-Cardy'04&'05]
- Prescription for free energy measurements
  [Fodor'07,Endrödi-Fodor-Katz-Szabo'07]
- Pure glue SU(2) in 4d
  [Buividovich-Polikarpov'08]
- Pure glue SU(3) in 4d
  [Nakagawa-Nakamura-Motoki-Zakharov'09,Itou-Nagata-Nakagawa-Nakamura-Zakharov'11&'15]
- Pure glue SU(2,3,4) in 4d
  [Rabenstein-Bodendorfer-Buividovich-Schäfer'18]

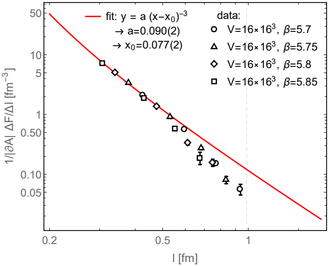
Our work, pure glue SU(N_c)

- in 4d for $N_c = 3$ and 5 at $T = 0$
- in 3d for $N_c = 2$ (upto $N_c = 7$ running) at $T \gtrsim T_c$

$SU(N_c = 3)$, 4d

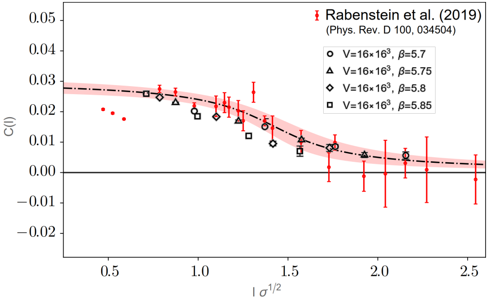
- Compares well with previous results in the literature
  [here shown Nakagawa-Nakamura-Motoki-Zakharov 1104.1011]
- Our statistical errors are ∼invisible to eye (black markers).





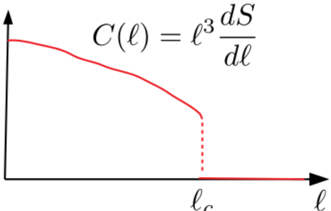
$SU(N_c = 3)$, 4d

Comparing the C-function



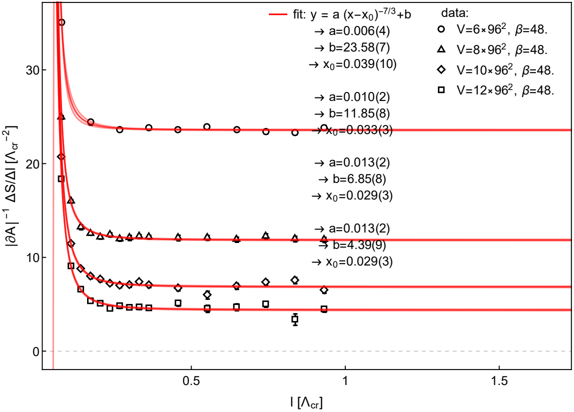
vs. holography $S_A = \frac{N_c^2}{\epsilon^2} - \frac{N_c^2}{\ell^2} + \dots$

[Ryu-Takayanagi hep-th/0605073]



$SU(N_c = 2)$, 3d

- New results for $SU(N_c)$ in 3d at non-zero temperature



- We confirm predictions from D2-brane background
  - at UV (small ℓ) $dS/d\ell \sim \ell^{-7/3}$
  - at IR (large ℓ) $dS/d\ell \sim \text{const.} = S_{BH} \propto T^{7/3}$

Bulk reconstruction

- This is applied AdS/CFT in reverse
- Idea: take boundary measurements and attempt to "guess" what is bulk
- Need either lattice simulations or experimental measurements
  - they are *noisy*

  $\Rightarrow$ one cannot find precise dual geometry
- How to deal with inherently noisy data?

[NJ-Pönni 2007.00010]

Bulk reconstruction from EE

- Make as little assumptions on bulk as possible: use RT
  - can only reconstruct some components of the bulk metric
- Can be shown, for a given metric that:

$$\frac{dS_{RT}}{d\ell} = \mathrm{function}(r_*)\ ,\ \ r_* = \text{tip of RT surface}$$

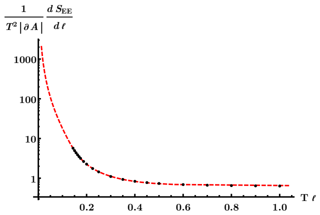
- Use Hamiltonian Monte Carlo to reconstruct the metric (and errors!)

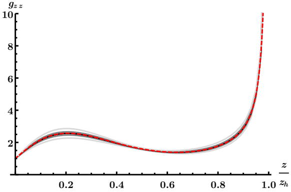
$$\left\{ \left(\frac{1}{T^2 V}\frac{dS}{d\ell}\right)_i, \ell_i, \sigma_i \right\}\ ,\ \ i \in 1, \ldots, N_{\text{data}}$$

$$ds^2 = \left(\frac{r_p}{z}\right)^{\frac{5}{2}}\left(-\frac{1-(z/z_h)^5}{a(z)^2}dt^2 + d\vec{x}^2\right) + \left(\frac{z}{r_p}\right)^{\frac{5}{2}}\frac{a(z)^2}{1-(z/z_h)^5}dz^2 + r_p^{\frac{3}{2}}\sqrt{z}\,d\Omega_6^2$$

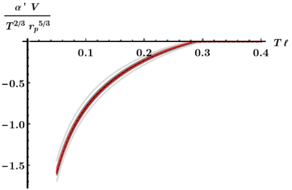
$$a(z) = 1 + \sum_{i=1}^{N_{\text{basis}}} a_i f_i(z)$$

- Bulk metric (posteriors a_i, Newton's constant) reconstructed from 3d lattice data



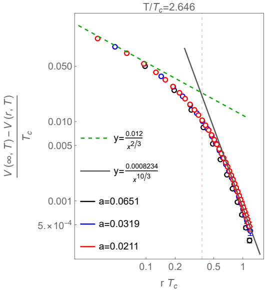


- predictions w/ AdS/CFT w/ confidence intervals!
- static $q\bar{q}$ potential using D2-brane Ansatz for the dilaton to switch frames

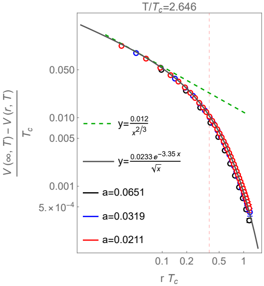


Quark-anti-quark potentials

We also tested D2-background further by computing the $q\bar{q}$ potential.



Complex or Debye screened?



- If $ReV \sim L^{-10/3}$ then $ImV \sim 1/L$
- Can test following the method [Burnier-Kaczmarek-Rothkopf 1410.2546]

Summary

- Lattice data for EE or Wilson loops can be generated $N_c \ll \infty$
- This data can be used to reconstruct the dual geometry
- Allows new predictions in same QFT w/ confidence intervals
- New era has begun

$$\boxed{\textit{Precision holography!}}$$

Thank you!