Gravitational waves from dark confinement phase transitions

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XVth Quark Confinement and the Hadron Spectrum

Stavanger, 01. August 2022

Huang, MR, Sannino, Wang: 2012.11614
MR, Sannino, Wang, Zhang: 2109.11552

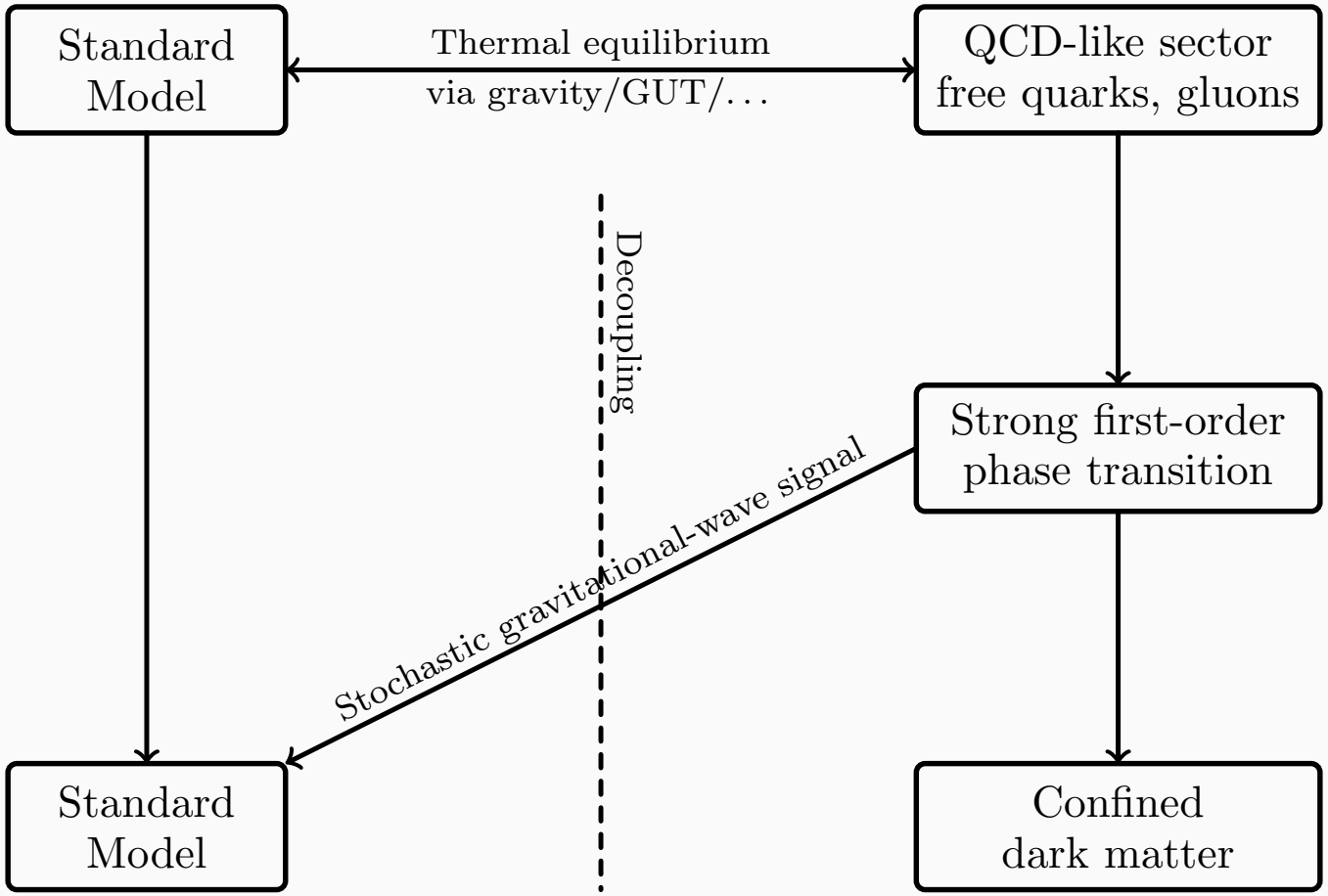
UNIVERSITY OF SUSSEX

Motivation for strongly-coupled dark sectors

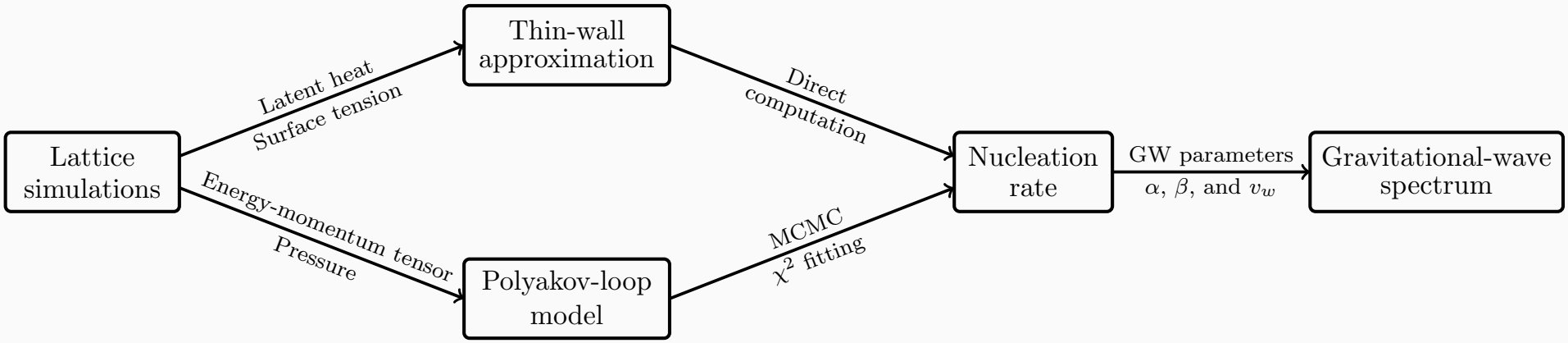
- UV complete, common SM extension
- Dark matter candidates: dark glue balls, baryons, mesons,. . .
  [Hochberg et al '14; Boddy et al '14; Appelquist et al '14; Cline et al '16; ...]
- Free parameters:
  - T_c confinement scale
  - N_c number of dark colours
  - Number N_f and representation of dark quarks

  See talk Zhi-Wei Wang, tomorrow 14:00
- Stochastic GW signal measurable at future GW observatories
- Non-perturbative: need effective models & Lattice input

Schematic picture



Procedure pure Yang-Mills theory



Related works:

- Matrix model [Halverson, Long, Maiti, Nelson, Salinas '20]
- Polyakov-loop model [Kang, Matsuzaki, Zhu, '21]
- Holography [Ares, Henriksson, Hindmarsh, Hoyos, Jokela '21]

The Polyakov loop

Polyakov loop (gauge invariant, charged under Z_{N_c} centre symmetry)

[Polyakov '78]

$$\ell(x) = \frac{1}{N_c} \mathrm{Tr}\left[\mathcal{P} \exp\left(i\, g \int_0^{1/T} A_0(x,\tau)\, \mathrm{d}\tau\right)\right]$$

Expectation value is an order parameter of the phase transition

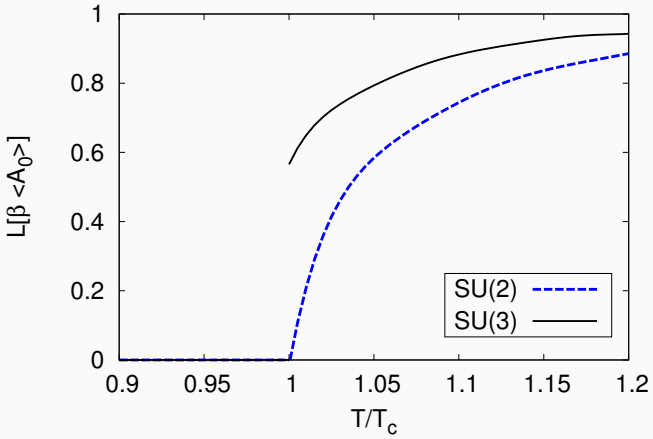
$\langle \ell \rangle_{T<T_c} = 0$

Confined

$\langle \ell \rangle_{T>T_c} > 0$

Deconfined

- Second-order for $SU(2)$
- First-order for $SU(N_c \geq 3)$



[Plot: Braun et al '07]

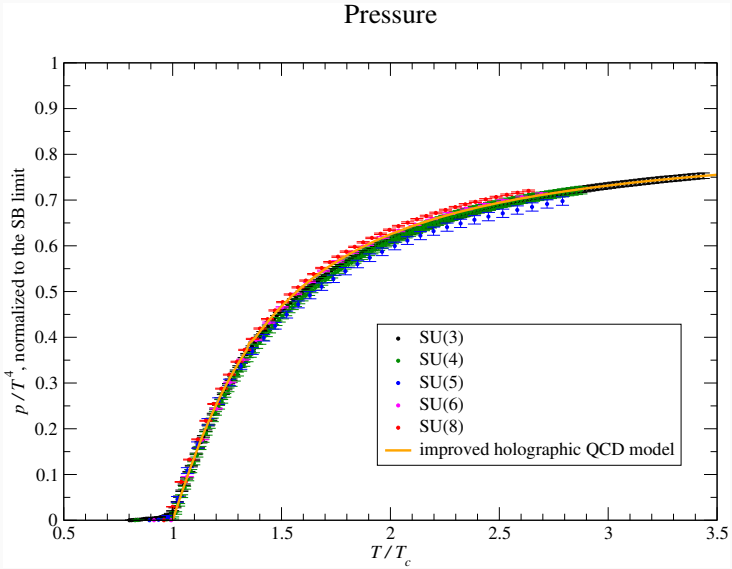
The Polyakov loop model

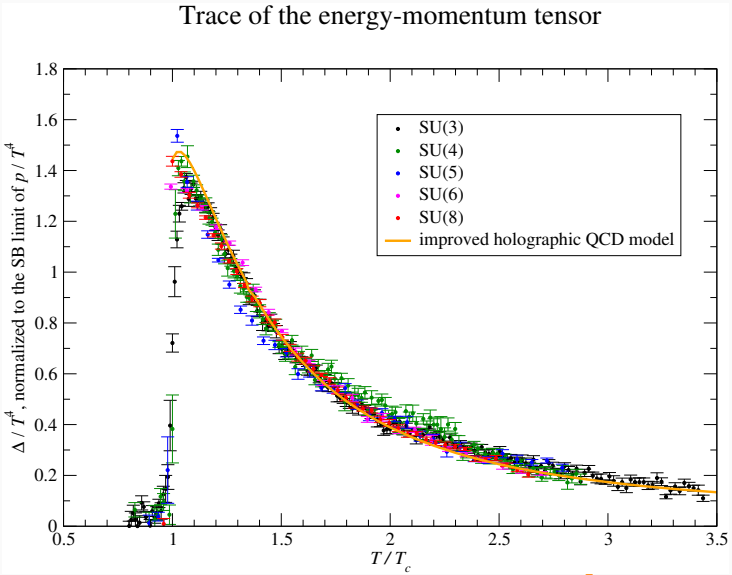
Effective Polyakov loop potential [Pisarski '00, '02]

$$V_{\text{PLM,poly}} = T^4 \left(-\frac{b_2(T)}{2}|\ell|^2 - b_N\left(\ell^{N_c} + \ell^{*N_c}\right) + b_4|\ell|^4 + \dots \right)$$

$$V_{\text{PLM,log}}^{N_c=3} = T^4 \left(-\frac{a(T)}{2}|\ell|^2 + b(T)\ln\left[1 - 6|\ell|^2 + 4(\ell^{*3} + \ell^3) - 3|\ell|^4\right] \right)$$

Fitted to lattice data of pressure and latent heat





[Panero '09]

$p/(N_c^2 - 1)$ and $\Delta/(N_c^2 - 1)$ are almost N_c independent

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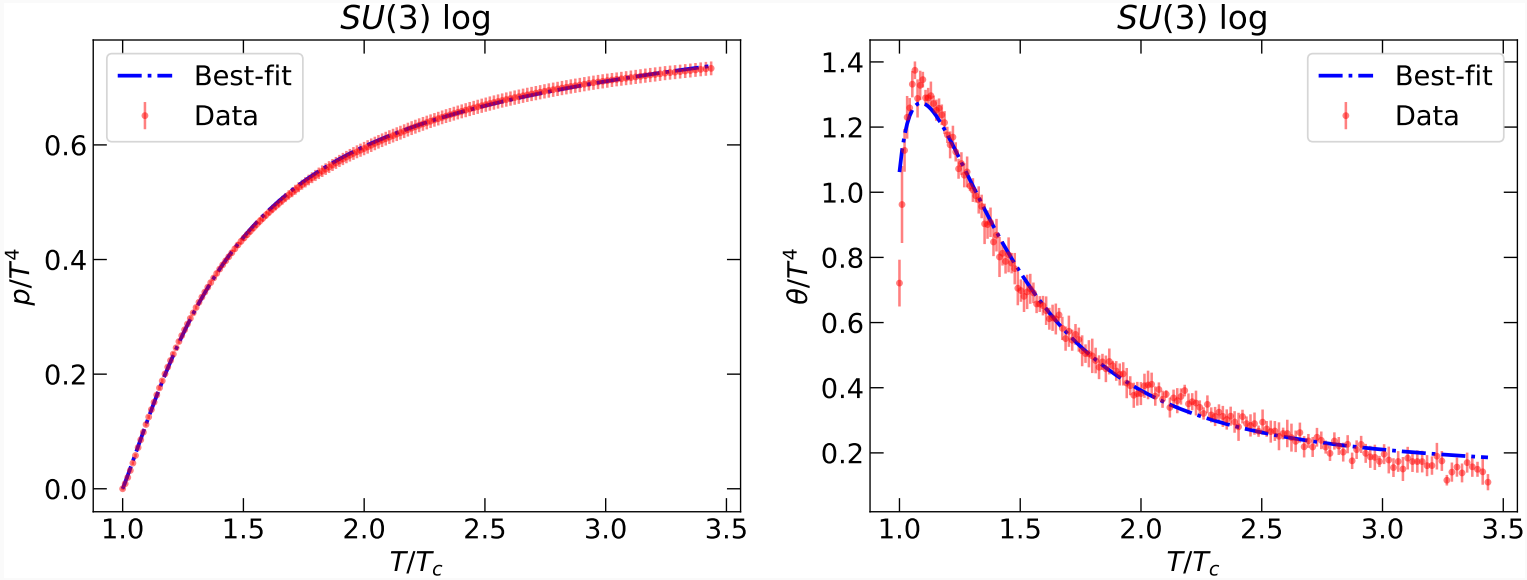

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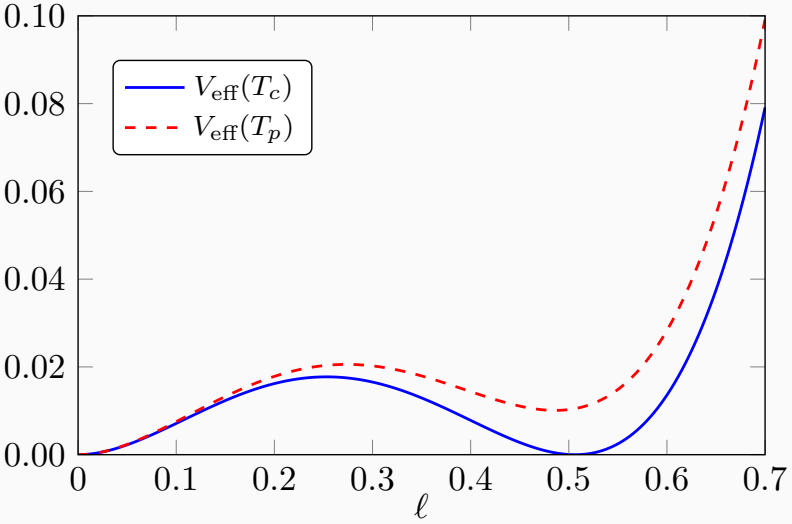
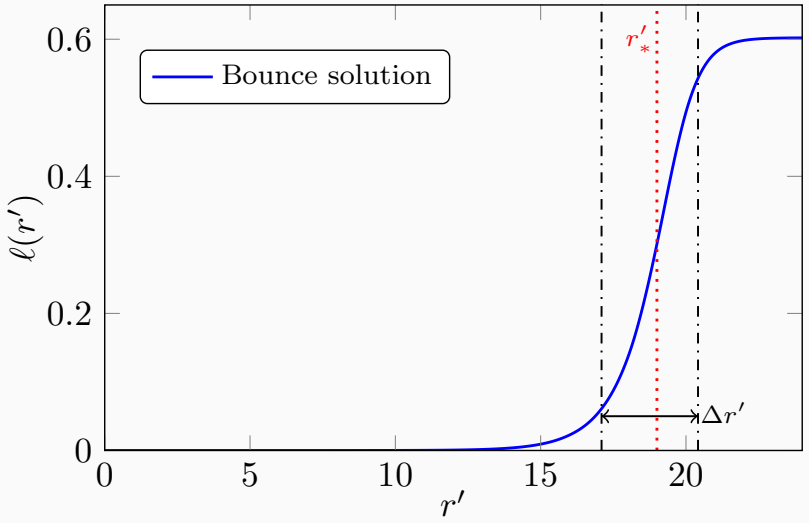
Bubble nucleation



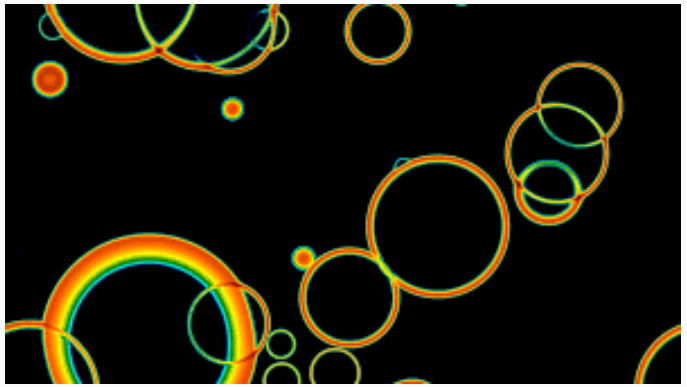


Bubble nucleation rate

[Coleman '77; Linde '81, '83]

$$\Gamma = T^4 \left(\frac{S_3(T)}{2\pi T} \right)^{3/2} e^{-S_3(T)/T}$$

$$S_3 = 4\pi T \int_{r'} r'^2 \left[\frac{1}{2} \left(\frac{\mathrm{d}\ell}{\mathrm{d}r'} \right)^2 + \tilde{V}_{\rm eff}(\ell, T) \right]$$

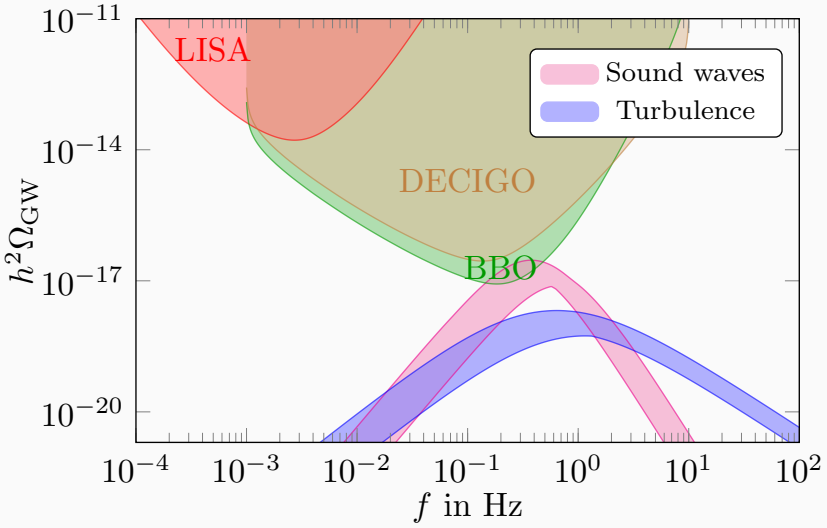


[Weir '17]

Gravitational-wave spectrum

- Collision of bubble walls
  [Kosowsky, Turner, Watkins '92; Kamionkowski, Kosowsky, Turner '93; ...]
- Sound waves in the plasma after the collision
  [Hindmarsh, Huber, Rummukainen, Weir '13; Giblin, Mertens '13; ...]
- Magnetohydrodynamic turbulence in the plasma
  [Kosowsky Mack, Kahniashvili '01; Dolgov, Grasso, Nicolis '02; ...]

$$h^2\Omega_{\mathrm{GW}}^{\text{sound-wave, peak}} \sim \left(\frac{v_w}{\tilde{\beta}}\right)\left(\frac{\kappa\,\alpha}{1+\alpha}\right)^2\left(\frac{100}{g_*}\right)^{\frac{1}{3}}$$



Gravitational-wave parameters

- Percolation temperature T_p or nucleation temperature T_n
  - T_n: One bubble nucleation per Hubble volume $\Gamma(T_n) \approx H(T_n)^4$
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- Strength parameter α (energy budget)
  - Jump in energy & pressure across boundary weighted by enthalpy

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- Wall velocity v_w
  - Depends on non-perturbative $1 \to n$ friction processes across wall [Bodeker, Moore '09; ...]
  - Assume v_w is large than speed of sound $v_w \geq c_s = 1/\sqrt{3}$

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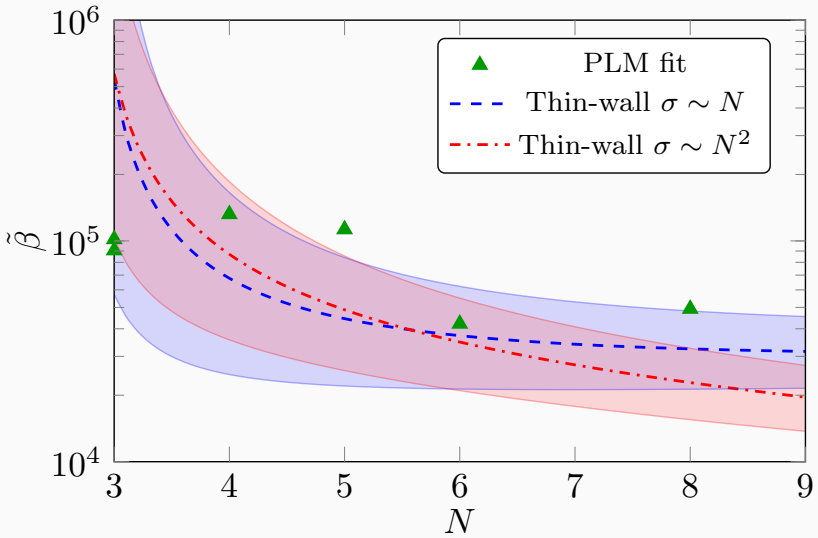
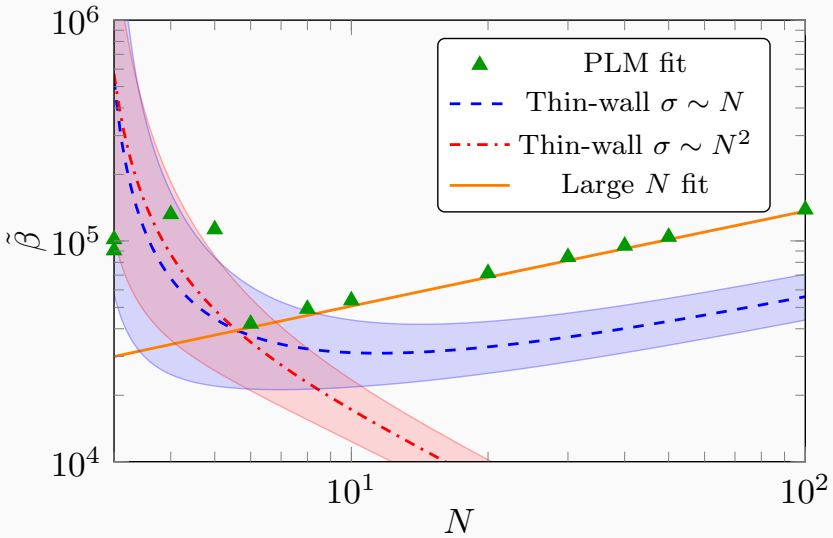
- Inverse duration time β or mean bubble separation R

Inverse duration of the phase transition β

Ansatz for nucleation rate $\Gamma \approx \Gamma(t_p) e^{\beta(t-t_p)}$

$$\tilde{\beta} = \frac{\beta}{H(T_p)} = T \frac{\mathrm{d}}{\mathrm{d}T} \frac{S_3(T)}{T} \bigg|_{T=T_p}$$





Very fast phase transition $\tilde{\beta} = \mathcal{O}(10^5)$

Agreement with thin-wall approximation for $\sigma / T_c^3 = 0.065 N_c$

Thin-wall approximation

For thin bubble walls, the nucleation rate can be approximated in term of Latent heat L and surface tension σ [Linde '83; Fuller, Mathews, Alcock '88]

$$S_3(T) \approx \frac{16\pi}{3} \frac{\sigma(T_c)^3}{L(T_c)^2} \frac{T_c^2}{(T_c - T)^2}$$

Lattice data [Lucini, Teper, Wenger '05]

$$\frac{L}{N_c^2} = \left(0.766(40) - \frac{0.34(1.60)}{N_c^2}\right)^4 T_c^4$$

$$\sigma = \begin{cases} T_c^3\,(0.118(3)N_c - 0.333(9)) \\ T_c^3\,(0.0138(3)N_c^2 - 0.104(3)) \end{cases}$$

Unclear Lattice data for σ for large N_c (improvement in progress)
[MR, Rinaldi, Sannino, Schaich, Springer, Wang]

Gravitational-wave spectrum from sound waves

GW spectrum [Hindmarsh, Huber, Rummukainen, Weir '13; Giblin, Mertens '13; ...]

$$h^2\Omega_{\rm GW}(f) = h^2\Omega_{\rm GW}^{\rm peak}\left(\frac{f}{f_{\rm peak}}\right)^3\left[\frac{4}{7}+\frac{3}{7}\left(\frac{f}{f_{\rm peak}}\right)^2\right]^{-\frac{7}{2}}$$

Peak frequency

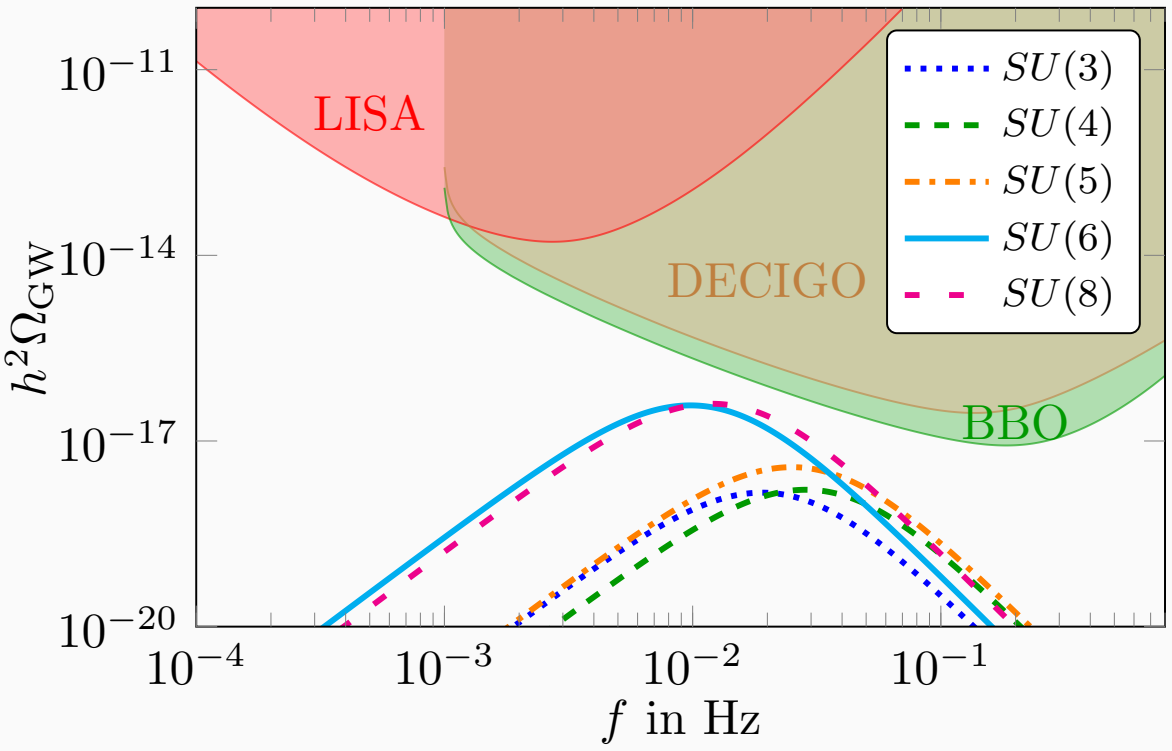
$$f_{\rm peak} \simeq 1.9\cdot 10^{-5}\,{\rm Hz}\left(\frac{g_*}{100}\right)^{\frac{1}{6}}\left(\frac{T_p}{100\,{\rm GeV}}\right)\left(\frac{\tilde{\beta}}{v_w}\right)$$

Peak amplitude

$$h^2\Omega_{\rm GW}^{\rm peak} \simeq 2.65\cdot 10^{-6}\left(\frac{v_w}{\tilde{\beta}}\right)\left(\frac{\kappa\,\alpha}{1+\alpha}\right)^2\left(\frac{100}{g_*}\right)^{\frac{1}{3}}\Omega^2_{SU(N_c)}$$

- $\Omega^2_{SU(N)}$ accounts for the dilution due to non-participating SM dofs
- Larger $\tilde{\beta}$ $\longrightarrow$ larger $f_{\rm peak}$ & smaller $\Omega_{\rm GW}^{\rm peak}$
- Larger α $\longrightarrow$ larger $\Omega_{\rm GW}^{\rm peak}$

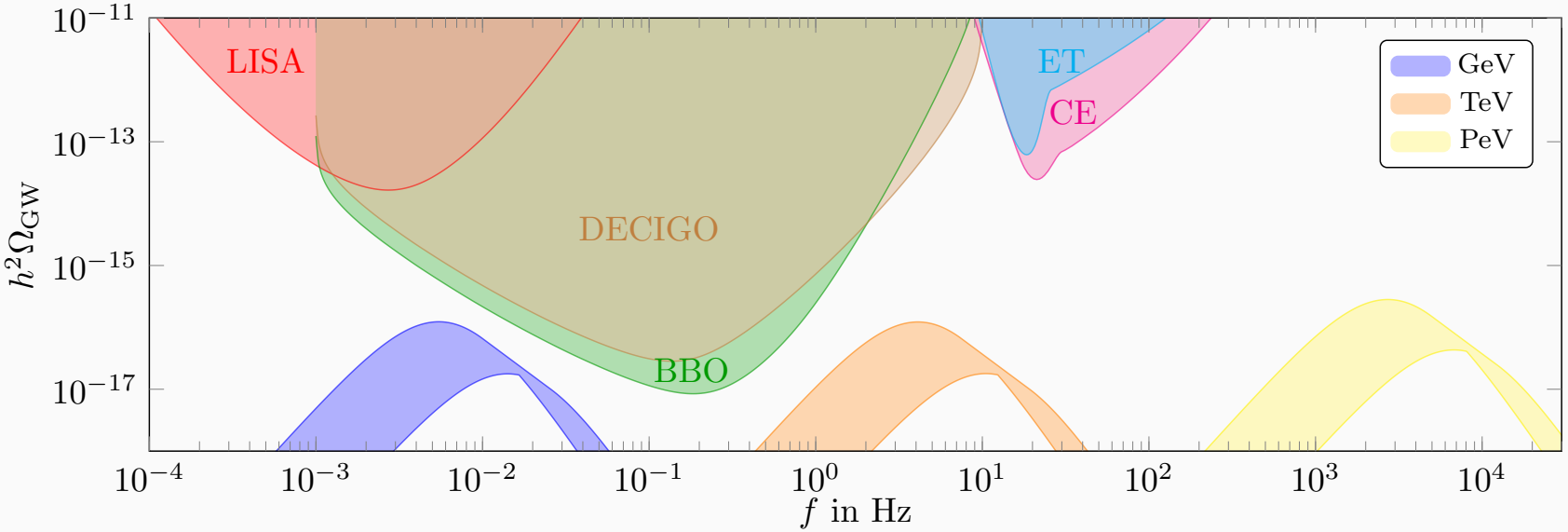
Dependence on the number of colours N_c



- Almost independent for $N_c \geq 6$
- Dilution with SM dofs suppresses GW signal for small N_c

$$\Omega_{SU(N_c)} = \frac{g_{*,SU(N_c)}}{g_{*,SU(N_c)} + g_{*,SM}}$$

Dependence on confinement scale T_c (for $N_c = 6$ and $v = v_J$)

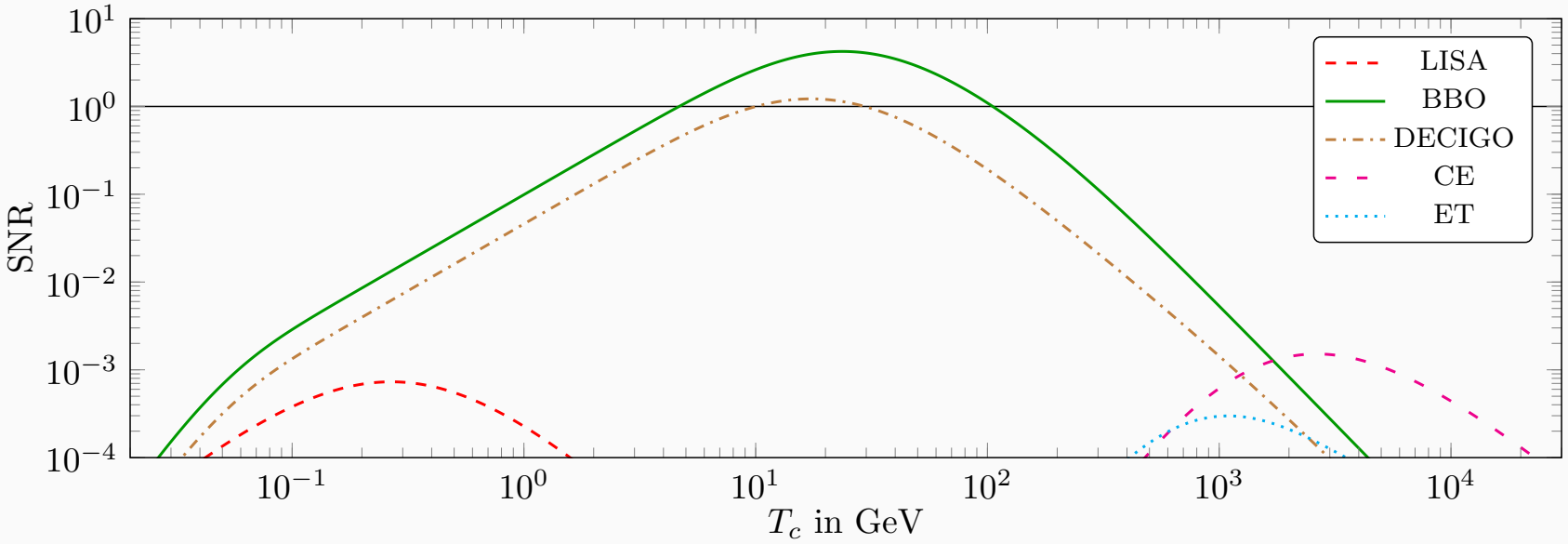


$f_{\rm peak}$ shifts with T_c

$\Omega_{\rm GW}^{\rm peak}$ almost independent on T_c

Signal-to-noise ratio (for $N_c = 6$ and $v = v_J$)

$$\mathrm{SNR} = \sqrt{\frac{3\,\mathrm{years}}{\mathrm{s}} \int_{f_{\mathrm{min}}}^{f_{\mathrm{max}}} \mathrm{d}f \left(\frac{h^2\Omega_{\mathrm{GW}}}{h^2\Omega_{\mathrm{det}}}\right)^2}$$



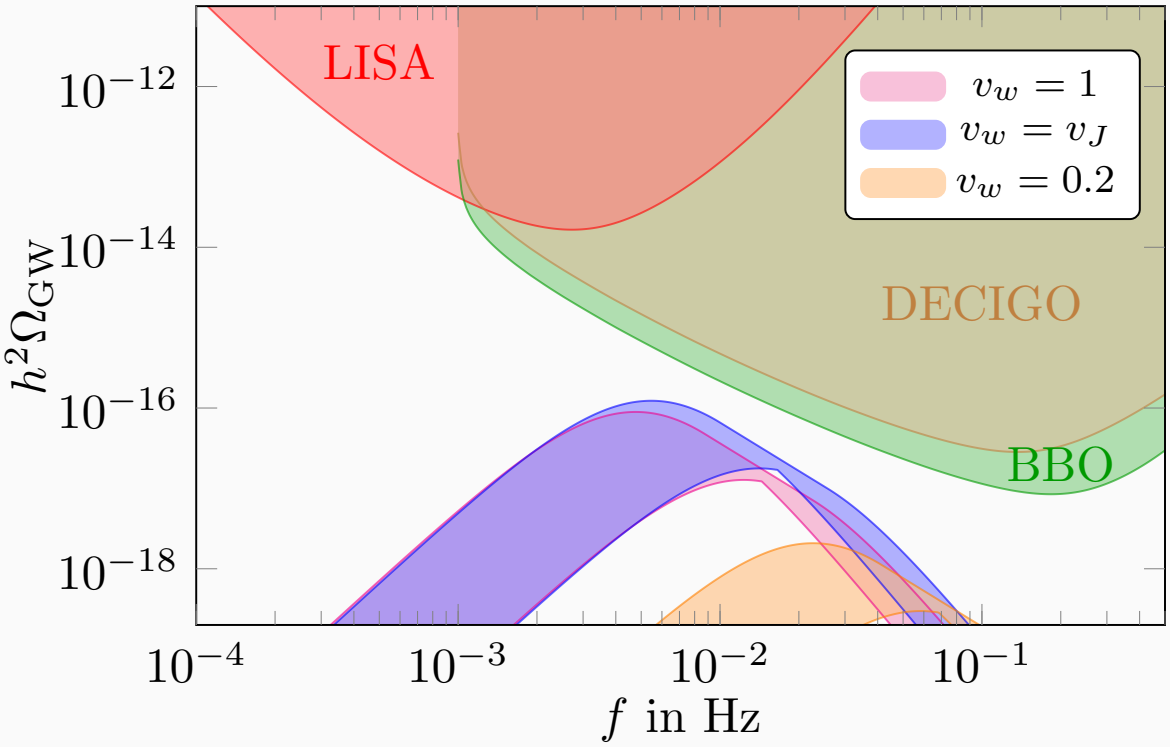
- BBO and DECIGO will test $\mathcal{O}(1\,\mathrm{GeV}) < T_c < \mathcal{O}(100\,\mathrm{GeV})$ range
- Multiple copies of $SU(N_c)$ dilute the signal

Summary

- Mapped Lattice data into effective models to study gravitational waves from confinement phase transitions
- Testable signals for BBO and DECIGO for $\mathcal{O}(1\,\mathrm{GeV}) < T_c < \mathcal{O}(100\,\mathrm{GeV})$
- Small N_c dependence
- How does the picture change if we add quarks?

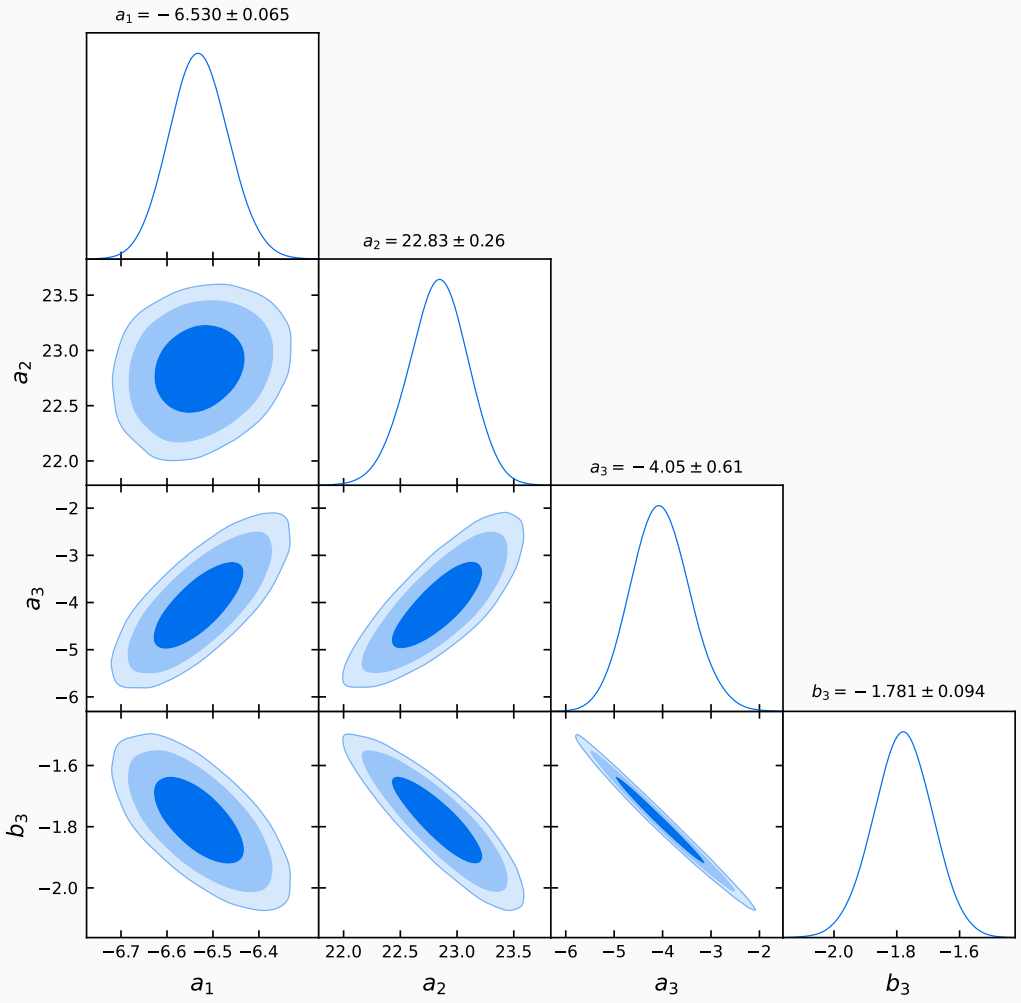
Thank you for your attention!

Dependence on wall velocity v_w



- Largest amplitude for Chapman-Jouguet detonation velocity $v_J(\alpha = 1/3) \approx 0.866$
- Error from Lattice enhanced to account for systematic errors

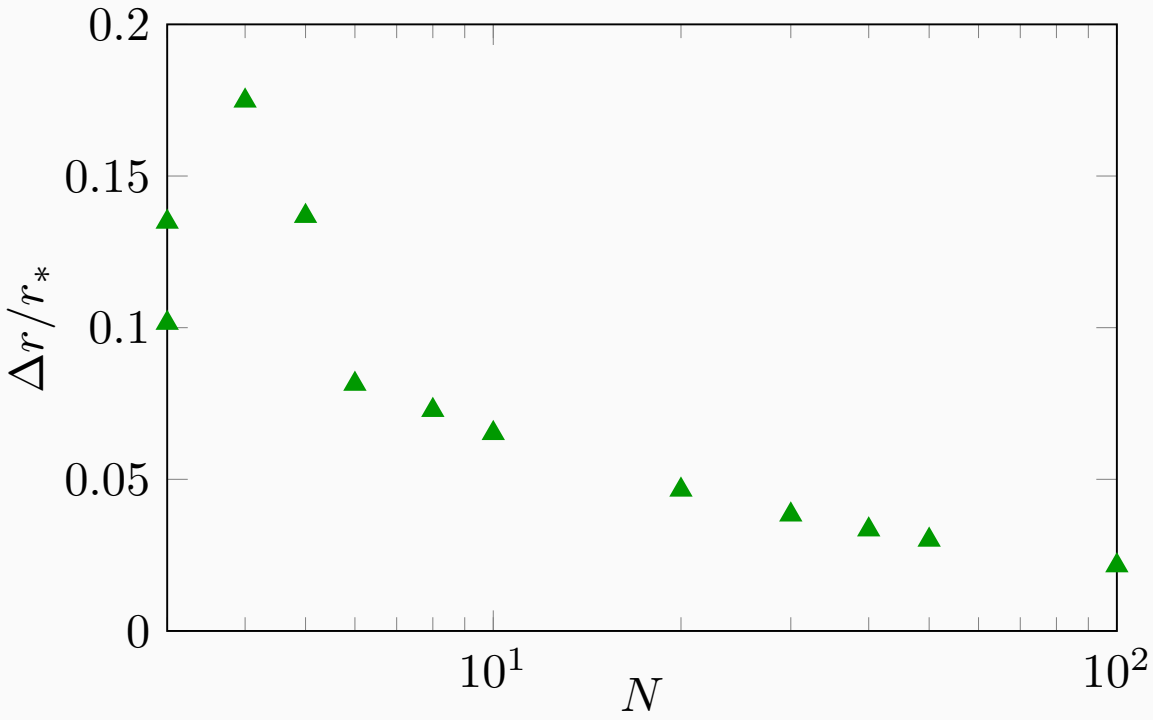
χ^2 fit


$a_1 = -6.530 \pm 0.065$
$a_2 = 22.83 \pm 0.26$
$a_3 = -4.05 \pm 0.61$
$b_3 = -1.781 \pm 0.094$
a_2
a_3
b_3
a_1
a_2
a_3
b_3

Quality of thin-wall approximation

$r_* =$ Bubble radius

$\Delta r =$ Thickness of bubble wall



- Thickest wall for $N_c = 3, 4, 5$
- Decays for large N_c