Field, current, and charge distribution in a pure gauge SU(3) flux tube

M. Baker, V. Chelnokov, L. Cosmai, F. Cuteri, A. Papa

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Outline

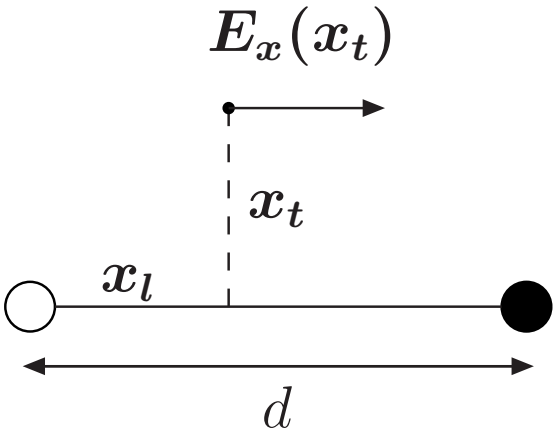
Flux tubes on the lattice

- The chromoelectric field between a static quark and an antiquark forms tube-like structures connecting them. This creates a linear confining potential.
- The field distributions can be extracted from the lattice simulations, and used to visualize the flux tube, and study its structure.

We work at zero temperature in a quenched theory without dynamic quarks and a standard Wilson gauge action.

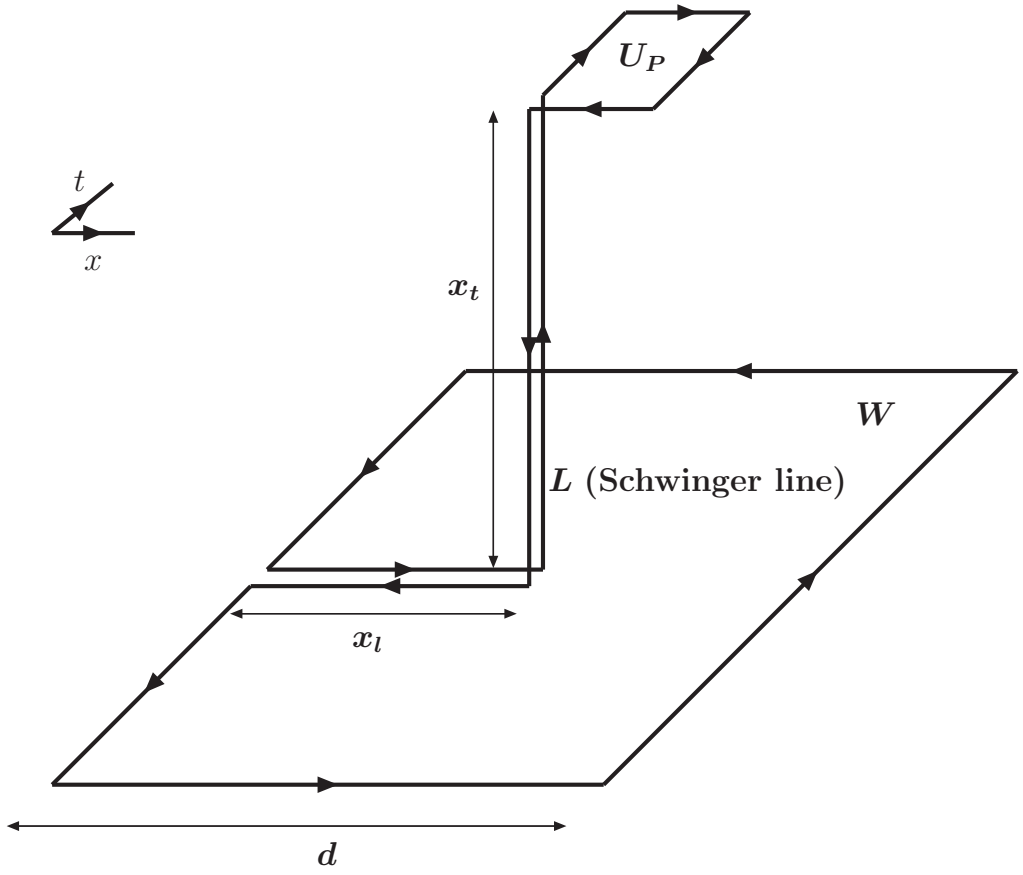
L	$\beta = 6/g^2$	a [fm]	d
48	6.240	0.0639	$8a = 0.511$ fm
48	6.544	0.0426	$12a = 0.511$ fm
48	6.769	0.0320	$16a = 0.511$ fm

Geometry



The system has cylindrical symmetry, so we can limit ourselves to studying just the (x_l, x_t) plane.

Fields in SU(3) gauge theory



$$\rho^{\text{conn}}_{W,\mu\nu} = \frac{\langle \mathrm{Tr}(W L U_P L^*) \rangle}{\langle \mathrm{Tr}(W) \rangle} - \frac{1}{N} \frac{\langle \mathrm{Tr}(U_P)\, \mathrm{Tr}(W) \rangle}{\langle \mathrm{Tr}(W) \rangle} \equiv a^2 g\, F_{\mu\nu}$$

Simulation details

Simulations are done with a modified version of the MILC code.

To improve the signal-to-noise ratio a smearing procedure was applied:

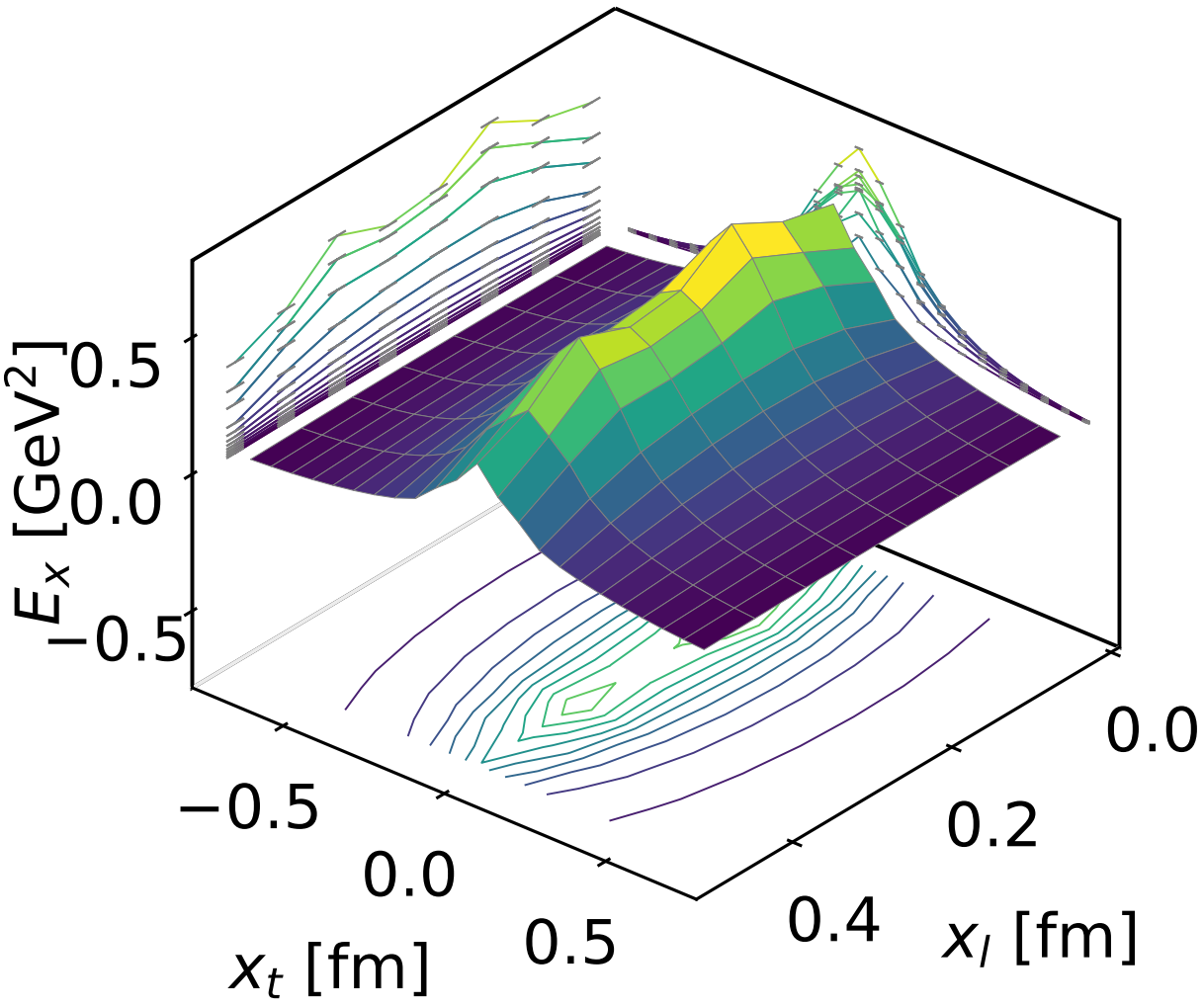
- one step of 4-dimensional hypercubic smearing on the temporal links (HYPt)
- N_{HYP3d} steps of hypercubic smearing restricted to the three spatial directions (HYP3d)

Results:

- Chromomagnetic field is compatible with zero
- Longitudinal chromoelectric field shows a tube-like structure
- Transverse chromoelectric field is smaller than longitudinal but nonzero

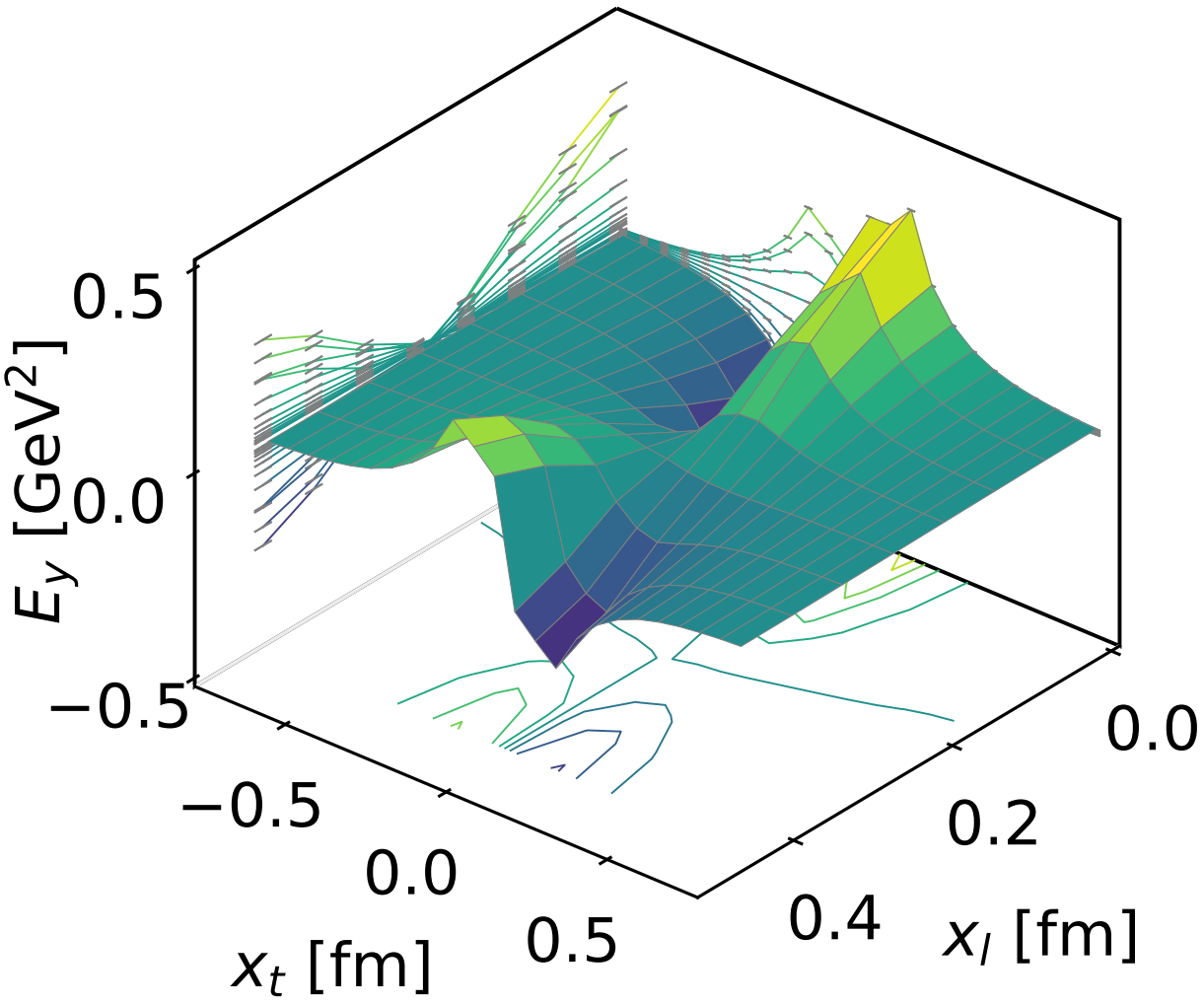
Simulation results: longitudinal field

$$\beta = 6.240,\ d = 8a = 0.511\ \text{fm}$$



Simulation results: transverse field

$$\beta = 6.240,\ d = 8a = 0.511\ \text{fm}$$



Subtraction procedure

We expect that the full field is a sum of two parts: perturbative part which behaves like an electrostatic field, and nonperturbative part that forms the flux tube

$$\vec{E} = \vec{E}^{\mathrm{C}} + \vec{E}^{\mathrm{NP}}$$

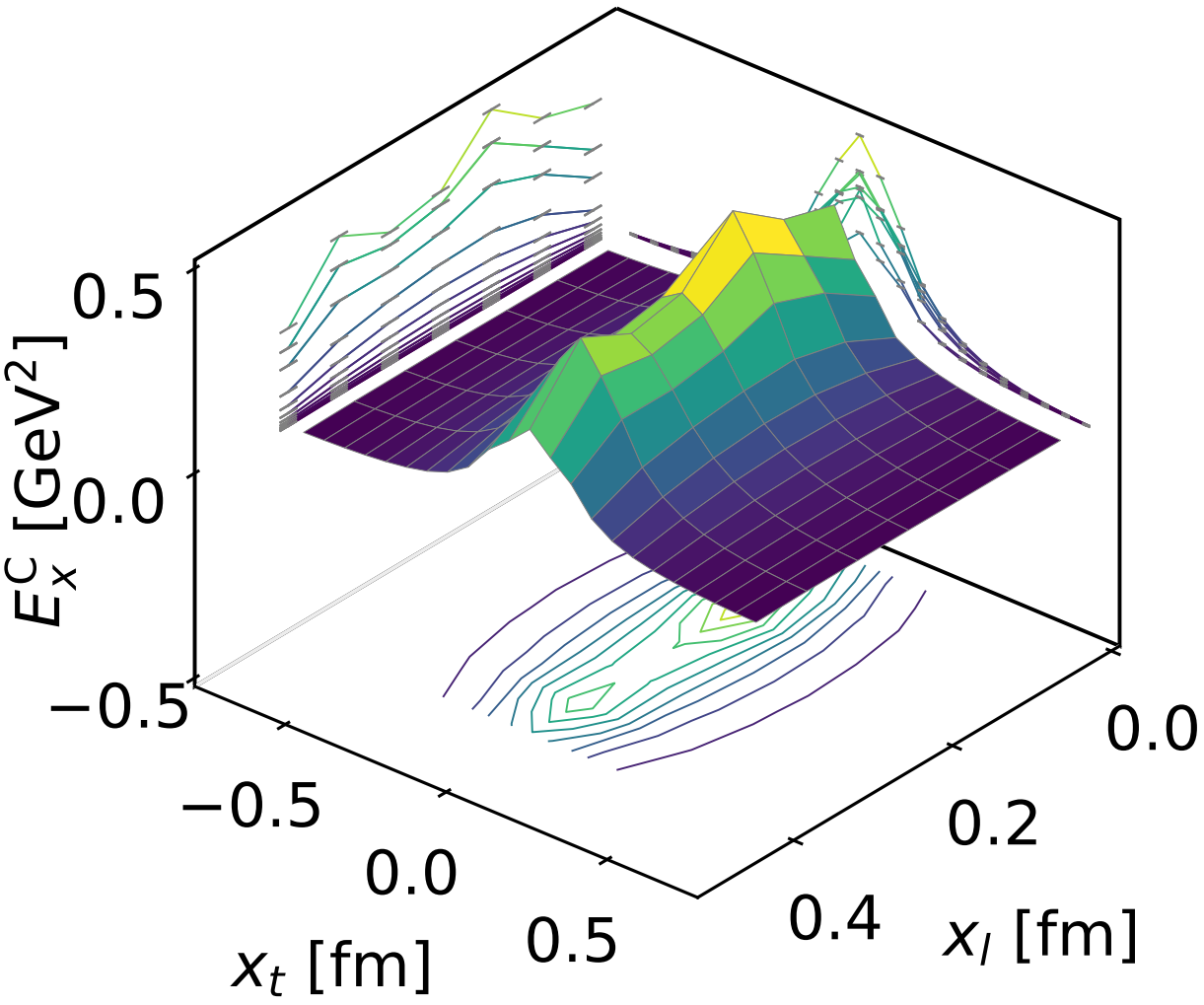
To separate these fields following assumptions are made:

- $\vec{E}^{\mathrm{C}}$ is a potential field ($\vec{\nabla} \times \vec{E}^{\mathrm{C}} = 0$),
- $\vec{E}^{\mathrm{NP}}$ is purely longitudinal ($E_y^{\mathrm{NP}} = 0$),
- both $\vec{E}^{\mathrm{C}}$ and $\vec{E}^{\mathrm{NP}}$ are zero at large transverse separations from the quark-antiquark axis.

These assumptions, together with the calculated values of $\vec{E}$, uniquely determine the field parts.

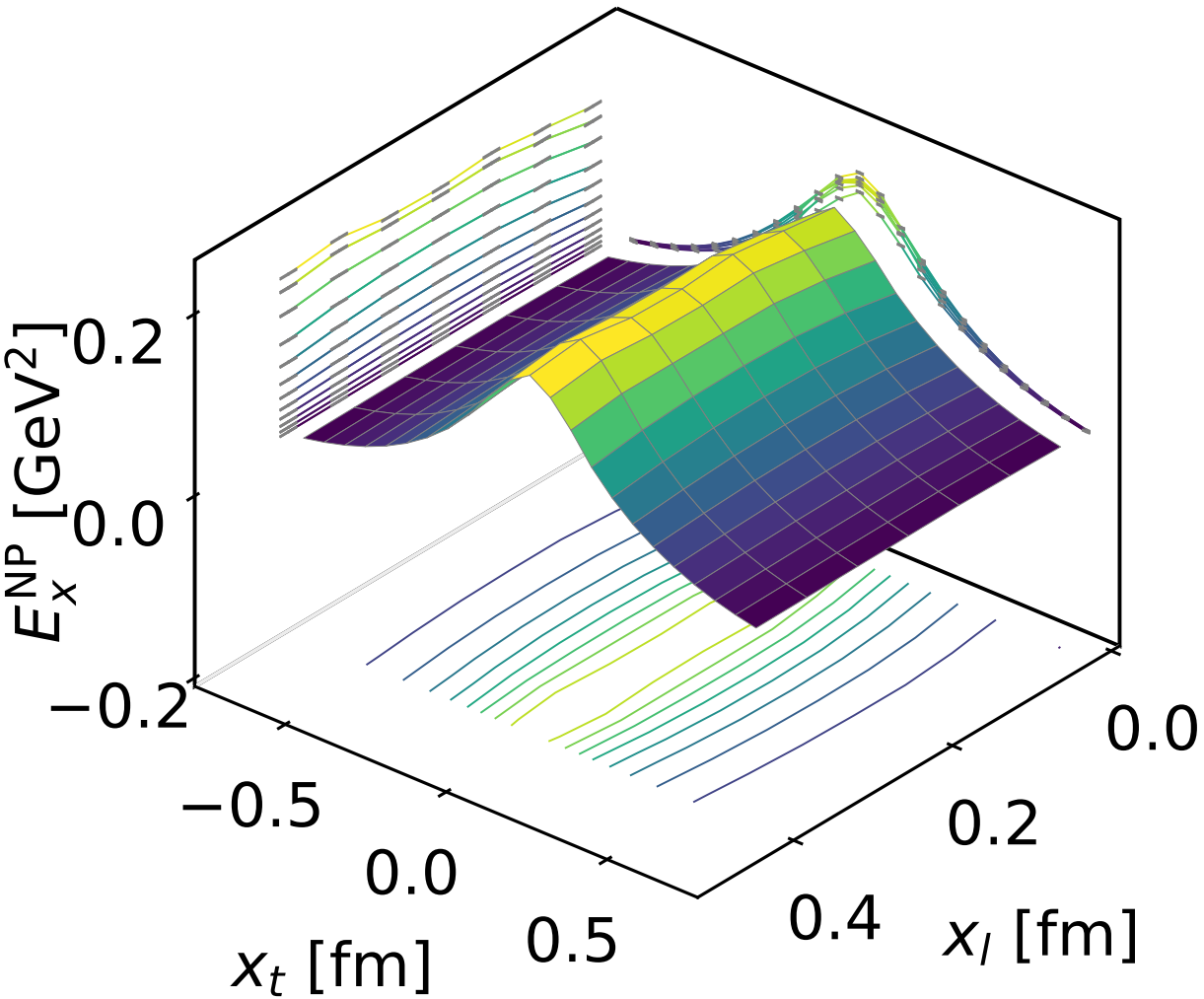
Subtraction procedure: perturbative part

$\beta = 6.240,\ d = 8a = 0.511\ \mathrm{fm}$



Subtraction procedure: nonperturbative part

$$\beta = 6.240,\ d = 8a = 0.511\ \text{fm}$$



Field derivatives

Considering that, in our case, the fields are time-independent, and $\vec{B} = 0$, nonzero derivatives of the fields

$$\rho_{\mathrm{el}} = \vec{\nabla} \cdot \vec{E} \ ,$$
$$\vec{J}_{\mathrm{mag}} = \vec{\nabla} \times \vec{E} \ ,$$

allow one to write the force density $\vec{f}$ as

$$\vec{f} = \rho_{\mathrm{el}} \cdot \vec{E} + \vec{J}_{\mathrm{mag}} \times \vec{E} \ .$$

Field derivatives

$$\rho_{\mathrm{el}} = \vec{\nabla} \cdot \vec{E} = \vec{\nabla} \cdot \vec{E}^{\mathrm{C}} + \vec{\nabla} \cdot \vec{E}^{\mathrm{NP}} = \rho_{\mathrm{el}}^{\mathrm{C}} + \rho_{\mathrm{el}}^{\mathrm{NP}}$$

Since the nonperturbative field is purely longitudinal

$$\vec{\nabla} \cdot \vec{E}^{\mathrm{NP}} = \frac{\partial}{\partial x} E_x^{\mathrm{NP}} \ ,$$

We expect the flux tube to be constant in the longitudinal direction, so $\rho_{\mathrm{el}}^{\mathrm{NP}}$ should be close to zero.

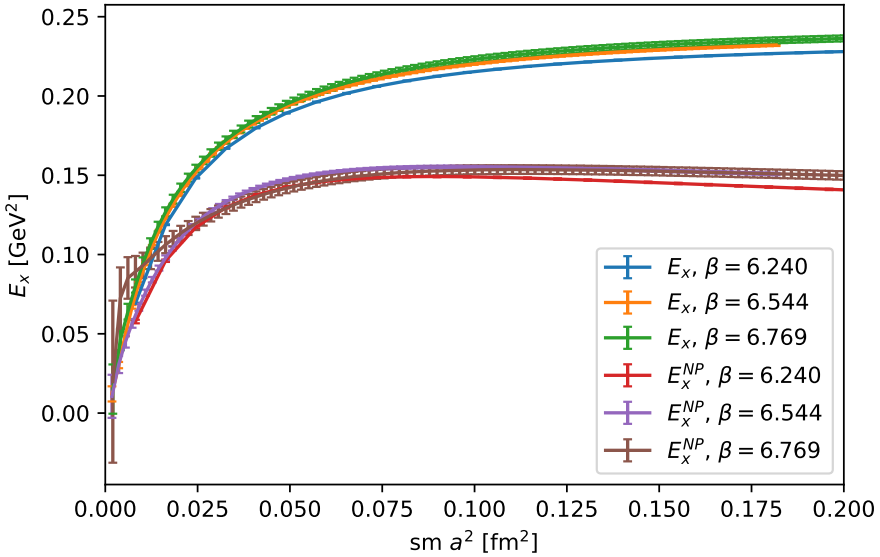
$$\vec{J}_{\mathrm{mag}} = \vec{\nabla} \times \vec{E} = \vec{\nabla} \times \vec{E}^{\mathrm{C}} + \vec{\nabla} \times \vec{E}^{\mathrm{NP}} = \vec{J}_{\mathrm{mag}}^{\mathrm{NP}}$$

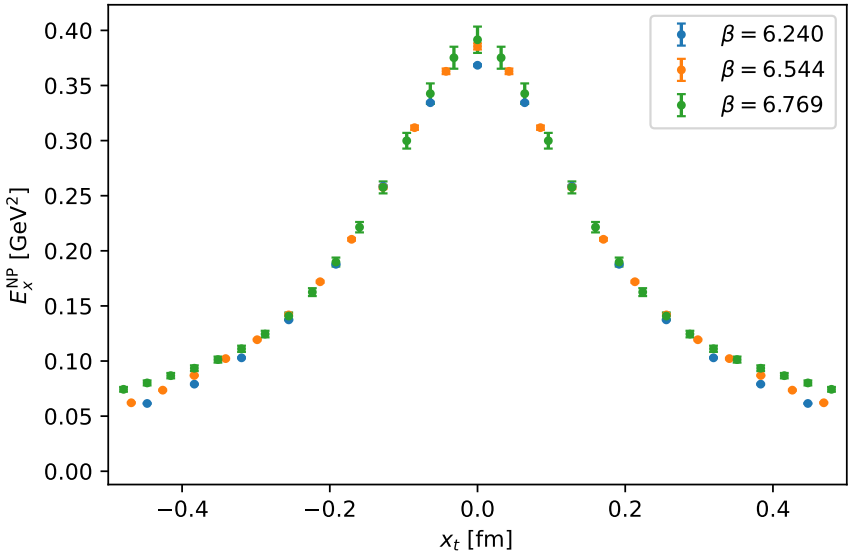
Due to the rotational symmetry, in our case, the only nonzero component of $\vec{J}_{\mathrm{mag}}$ is $(J_{\mathrm{mag}})_z$ winding around the quark-antiquark axis.

Smearing as an effective renormalization

The connected field operator that we use undergoes a nontrivial renormalization , depending on both x_l and x_t, which has to be taken into account if we want to reach the continuum limit. Battelli, Bonati (2019)

Our smearing procedure effectively works as a renormalization, restoring the continuum scaling. To check it we perform simulations on three lattices corresponding to the same physical quark-antiquark distance $d = 0.512$ fm, but having different lattice steps a.



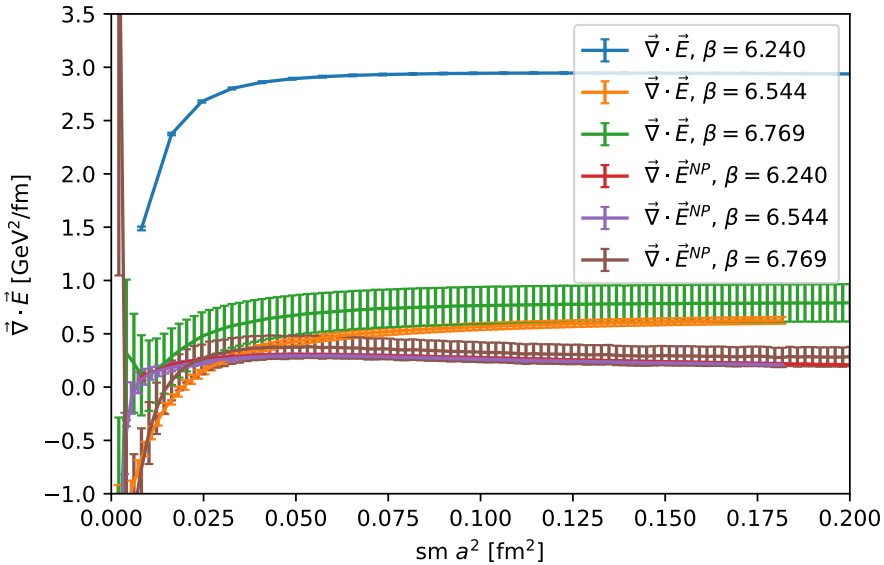


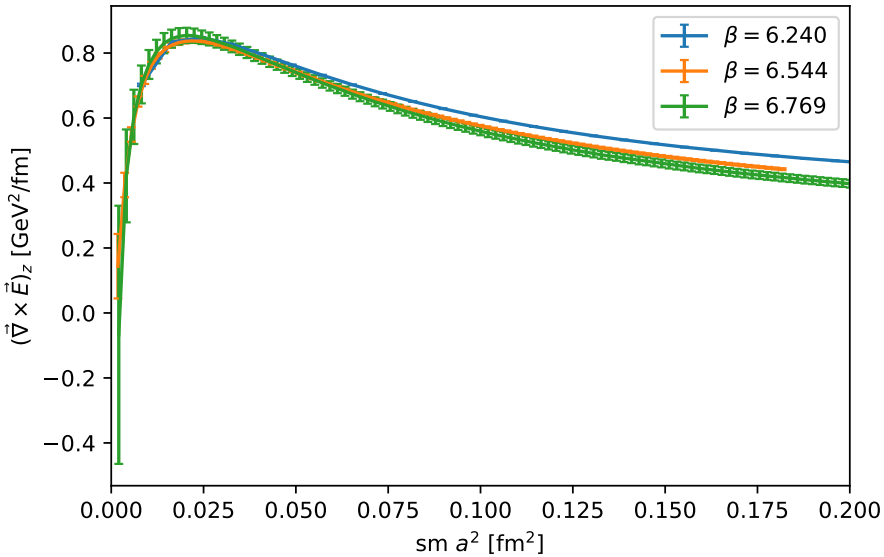
Optimal number of smearing steps

The optimal number of smearing steps depends on

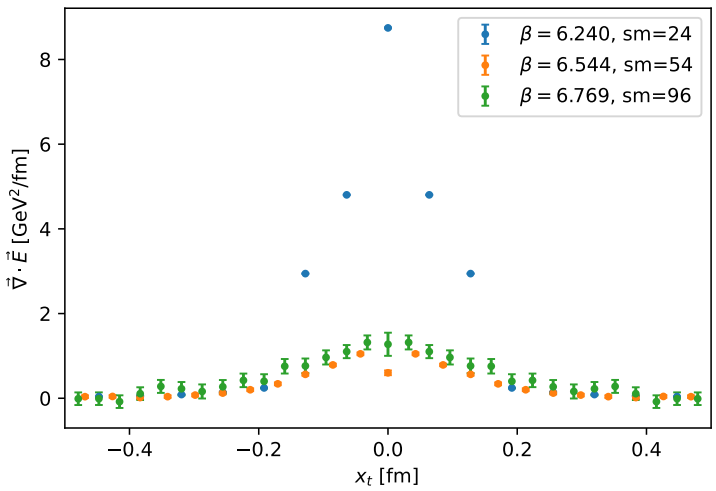
- Observable that we are interested in
- Coordinates x_l and x_t (large coordinates require more smearing steps)

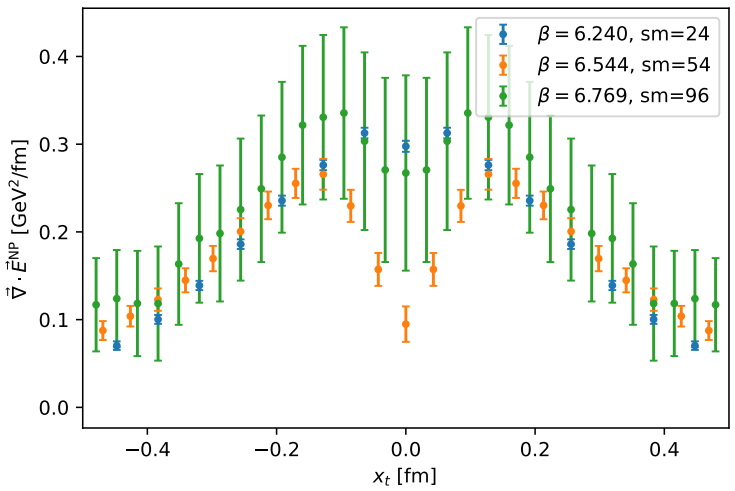
In general, scaling seems to be better at the maximum of the observable.

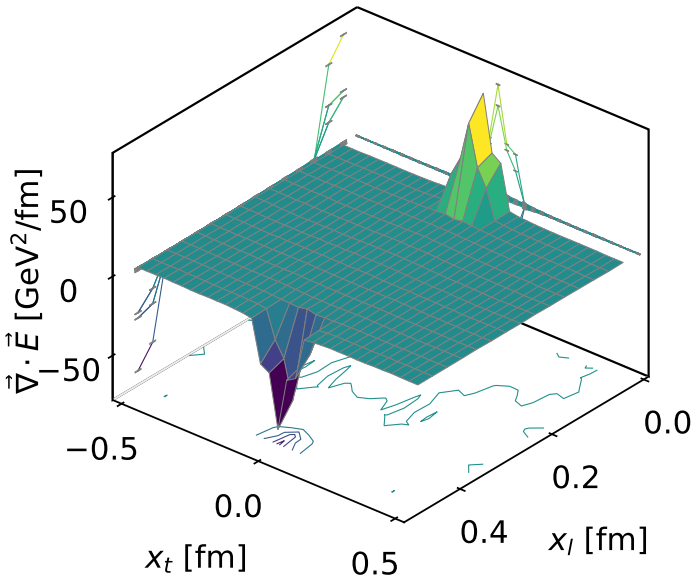


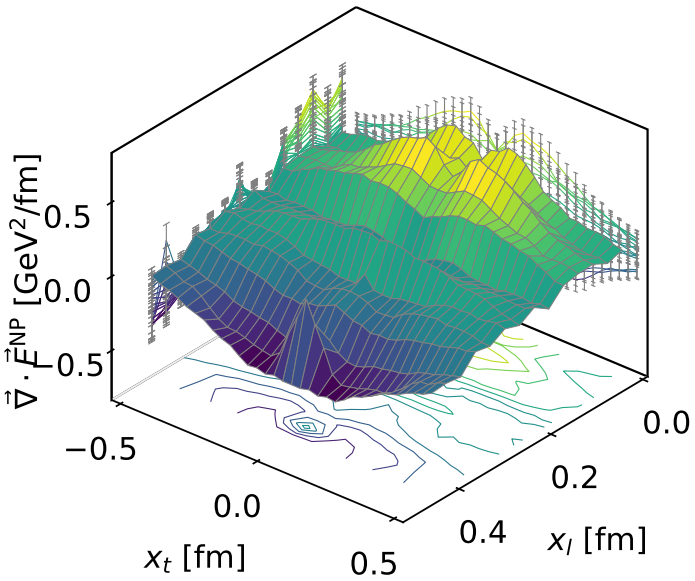


Electric charge distribution

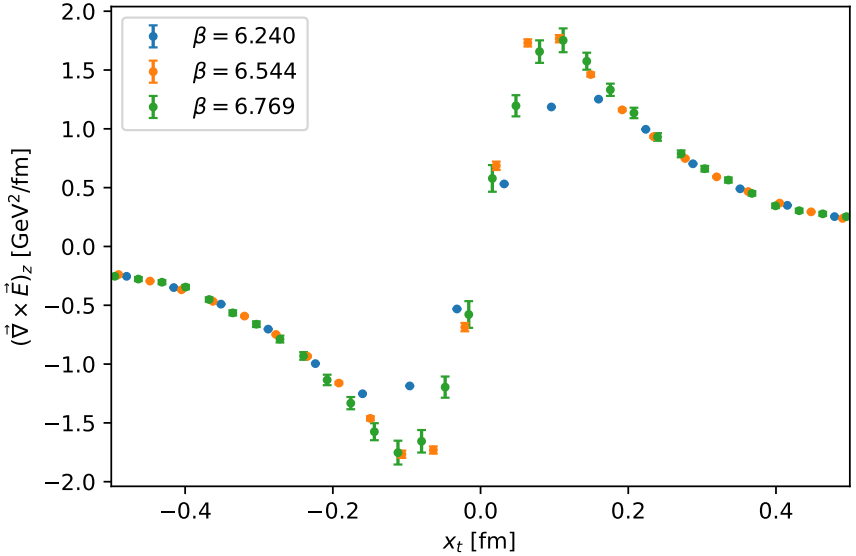


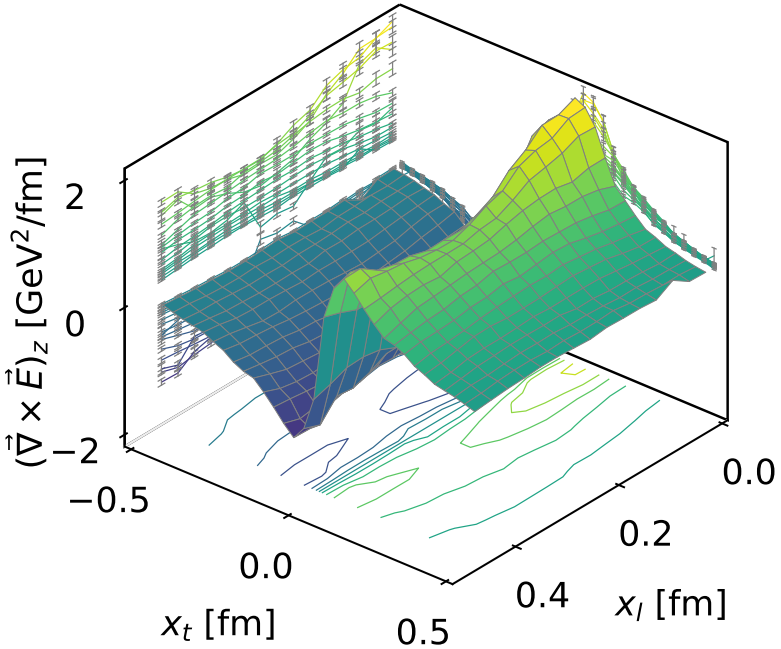






Magnetic current distribution





Confining force

Let us cut our space in two by a plane $y = 0$, that contains the quark-antiquark line.

$$\vec{F} = \int_{y>0} d^3\vec{r}\, \vec{J}_{\mathrm{mag}} \times \vec{E}^{\mathrm{NP}} =$$
$$= -2\hat{e}_y \int_0^d \mathrm{d}x_l \int_0^\infty \mathrm{d}x_t\, x_t f^{\mathrm{NP}}(x_l, x_t) \equiv -\hat{e}_y F$$

Force $\vec{F}$ acts perpendicular to the cut plane "squeezing" the flux tube. We estimate this force and compare it with different estimations of the string tension.

String tension

String tension σ can be estimated from the integration of the energy of the nonperturbative field $\vec{E}^{\mathrm{NP}}$ over the transverse cross-section going through the midpoint of the flux tube

$$\sigma_{\mathrm{int}} = \int \mathrm{d}^2 x_t \frac{(E_x^{\mathrm{NP}}(d/2, x_t))^2}{2} = \\ = \pi \int \mathrm{d}x_t \, x_t \, (E_x^{\mathrm{NP}}(d/2, x_t))^2 \, .$$

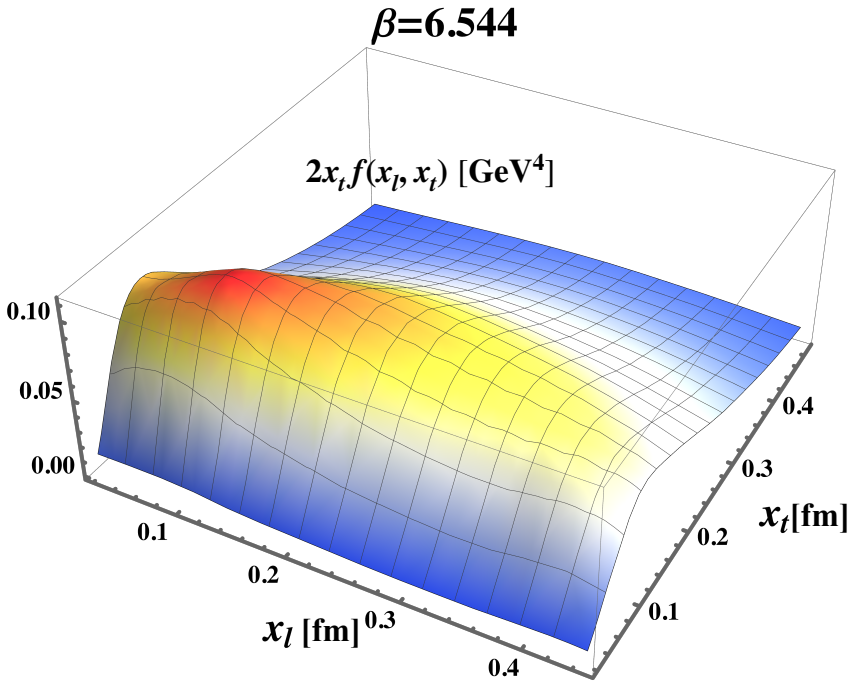
An alternative approach would be to estimate the string tension from the nonperturbative field at the position of the quark

$$\sigma_0 = g E_x(0) \, .$$

Finally, one can compare these results with $\sqrt{\sigma_{\mathrm{NS}}} = 0.464 \ \mathrm{GeV}$ used in setting the physical scale for our simulations Necco, Sommer (2002)

Confining force

β	$\sqrt{F}$ [GeV]	$\sqrt{\sigma_{\rm int}}$ [GeV]	$\sqrt{\sigma_0}$ [GeV]	$\sqrt{\sigma_{\rm NS}}$ [GeV]
6.240	$0.4859(4)^{+645}$	0.4742(12)	0.56353(81)	
6.544	$0.5165(8)^{+611}_{-214}$	0.4692(16)	0.5962(38)	0.464
6.769	$0.5297(22)^{+547}_{-322}$	0.4672(49)	0.617(16)	



Conclusions

- The chromoelectric field created by a static quark-antiquark pair can be separated into zero-curl perturbative and longitudinal nonperturbative parts, where the nonperturbative part results in long-range linear behavior of the quark-antiquark potential and can be identified as the flux tube.
- Anisotropic smearing can be used to both improve the signal-to-noise ratio and to act as an effective renormalization. The optimal amount of smearing depends on the observable and the position at which the field is measured.
- The perturbative chromoelectric charge density is concentrated around the sources, while the nonperturbative part is close to zero (assumed to go to zero in the continuum limit).
- The chromomagnetic current density has a nonzero continuum limit. The Lorentz force arising from the interaction of the current with the chromoelelectric field creates a force that acts towards the axis of the tube. Integrating this force over a half-space gives a measure of confinement of the tube in the transverse direction, that is numerically compatible with the string tension.