Shedding light on thermal photon and dilepton production[1,2]

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[1] based on collaborations w/ Dibyendu Bala, Jessica Churchill, Charles Gale, Sangyong Jeon, Olaf Kaczmarek and Mikko Laine.

[2] supported by the DOE under grant No. DE-FG02-00ER41132

Quark-gluon plasma ...WHY?

- Thermodynamic properties *usually* hard to calculate **ab initio**
  ... Here, quantum chromodynamics (*QCD*) is the starting point

- Predictions for ultra relativistic *nucleus-nucleus* collisions

- Develop & test general methods that can be used elsewhere:

  e. g. for neutrino production in EW-plasmas (cosmology)

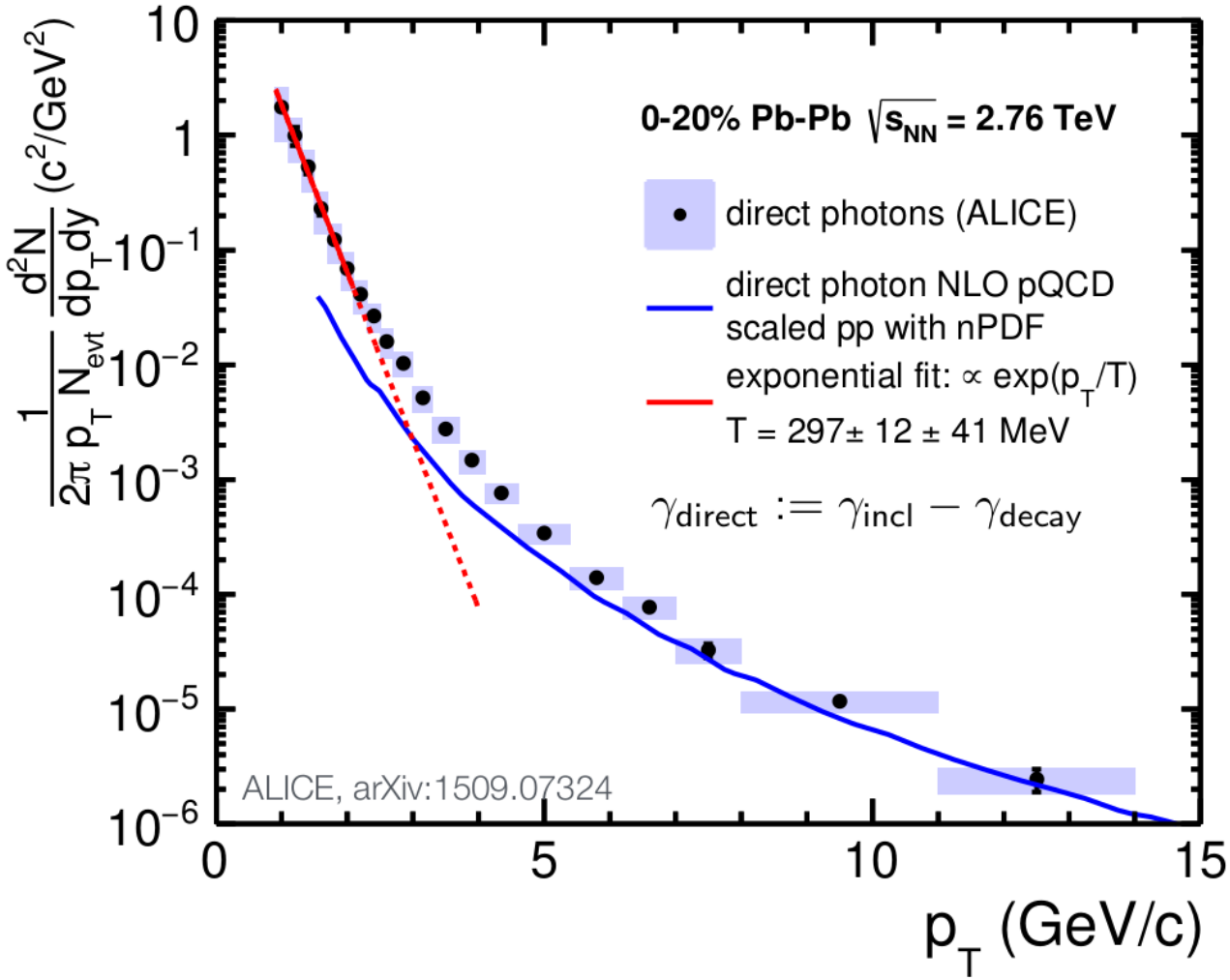
$\Rightarrow$ **In this talk:** pQCD at finite μ (new calc)

comparison w/ lattice (new data)

hydro yields (new predictions)

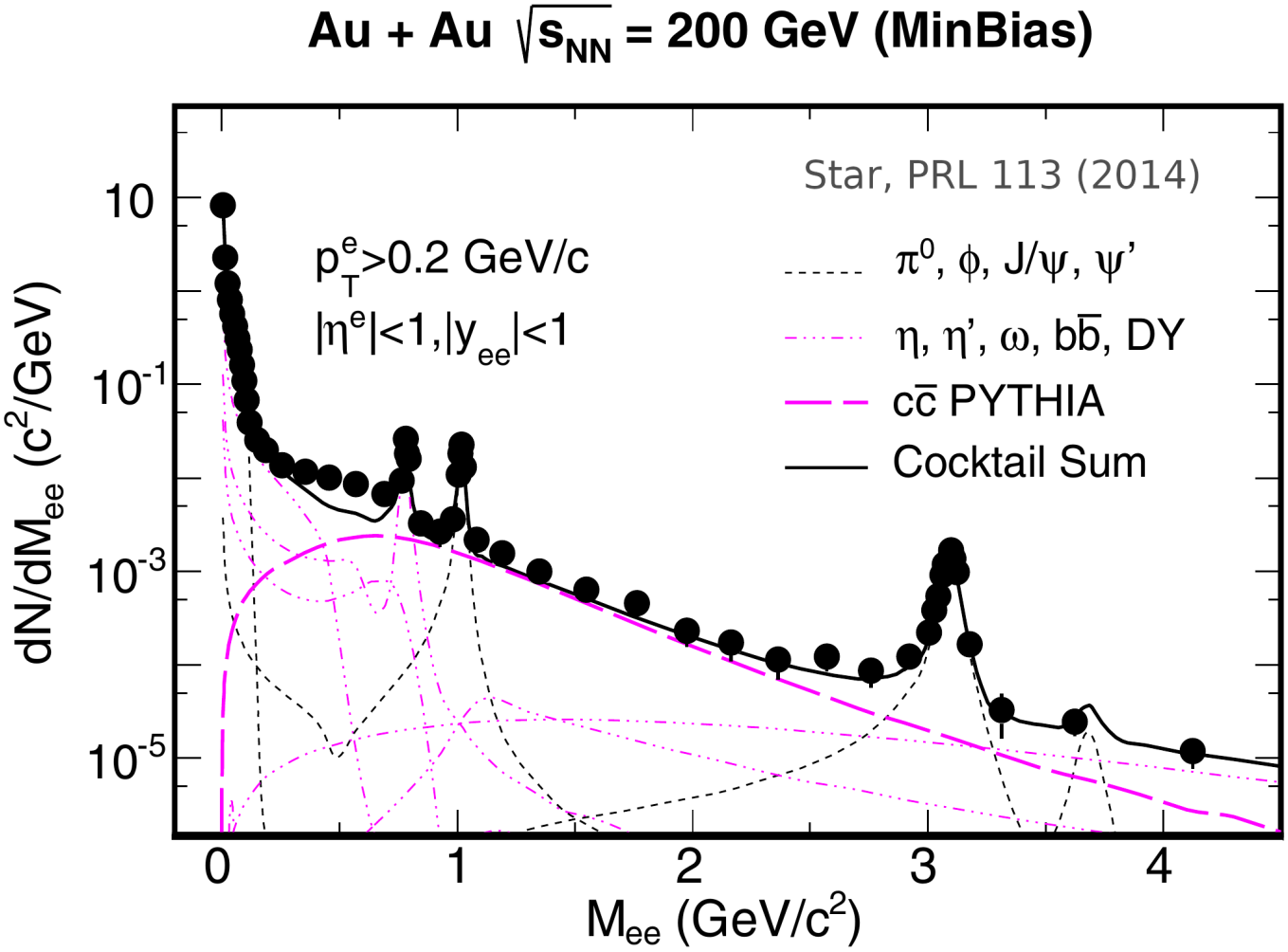
Electromagnetic probes

... photons are 'clean' messengers; they do not re-interact w/ the QGP



Electromagnetic probes

Closely related observable *dileptons pairs*, e.g. from $q\bar{q} \to \gamma^* \to e^+e^-$



Theoretical tools

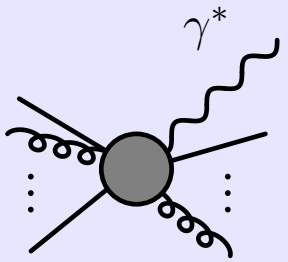
$$\mathcal{L} = -\tfrac{1}{4}F^2 + \sum_f \bar{\psi}\left(i\not{D} - m_f\right)\psi \;; \qquad F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - \underline{g f^{abc} A^b_\mu A^c_\nu}$$

perturbation theory

weak-coupling expansion:

production rates

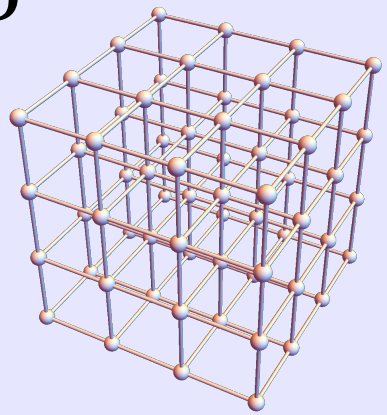
$$\omega \frac{d\Gamma}{d^3\boldsymbol{k}} = \int d\Phi \left|\right.$$



$$\left.\right|^2$$

$\times$ (thermal weight)

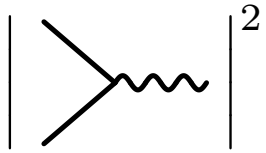
lattice QCD



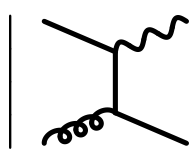
$$G_{\mu\nu}(\tau, \boldsymbol{k}) = \int_{\mathbf{x}} e^{i\,\boldsymbol{k}\mathbf{x}} \langle j_\mu(\tau, \mathbf{x}) j_\nu(0)\rangle$$

$\tau \equiv it$ 'imaginary time'

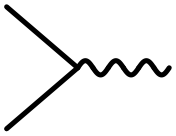
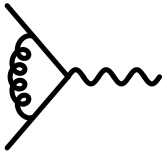
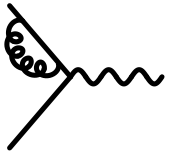
Application of perturbative QCD ($pQCD$):

$|\sum \mathcal{M}|^2 =$ 

Drell-Yan

$+$  $+ \ldots \Big|^2 + \ldots$

Compton , annihilation , ...

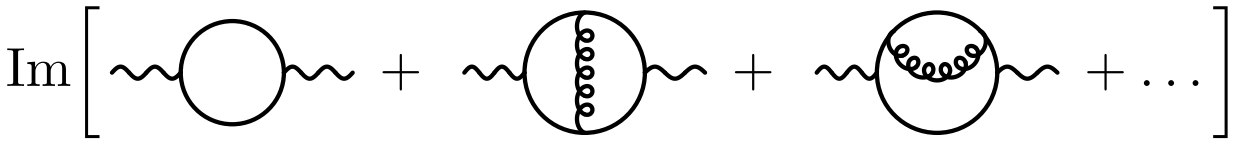
$+ \Big[$  $\Big] \Big[$  $+$  $+ \ldots \Big]^* +$ c.c.

interference

$+ \ldots$

Application of perturbative QCD ($pQCD$):

OR ...



self-energy, $\Pi_{\mu\nu}$

[Weldon (1990)] , [Bödeker, Sangel, Wörmann (2015)]

Basic relations from pert. theory

[McLerran, Toimela (1995)]

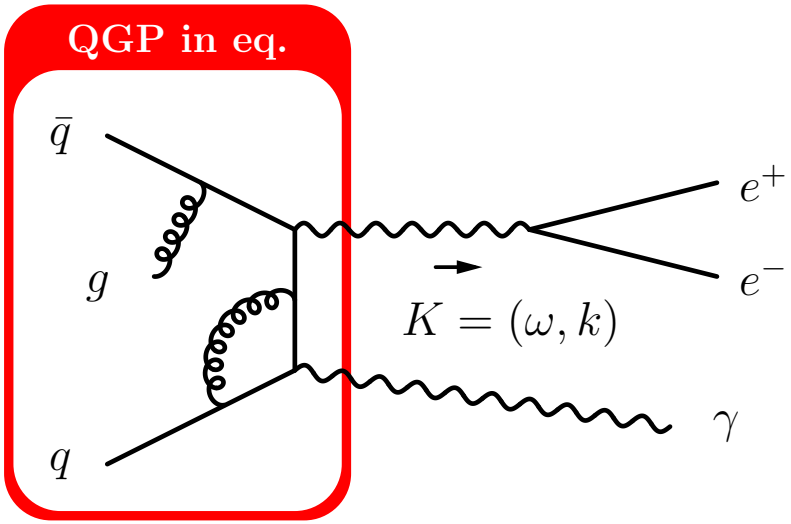
[Gale, Kapusta (1991)]

Dilepton rate:

$$\frac{d\Gamma_{e^-e^+}}{d\omega\, d^3\boldsymbol{k}} \simeq \frac{\alpha_{\rm em}^2 n_{\rm B}(\omega)}{3\pi^2 M^2} C_{\rm em}\, \rho_{\rm V}(\omega, k)$$

Photon rate:

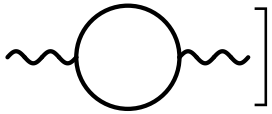
$$\frac{d\Gamma_{\gamma}}{d^3\boldsymbol{k}} \simeq \frac{\alpha_{\rm em} n_{\rm B}(k)}{2\pi^2 k} C_{\rm em}\, \rho_{\rm V}(k, k)$$

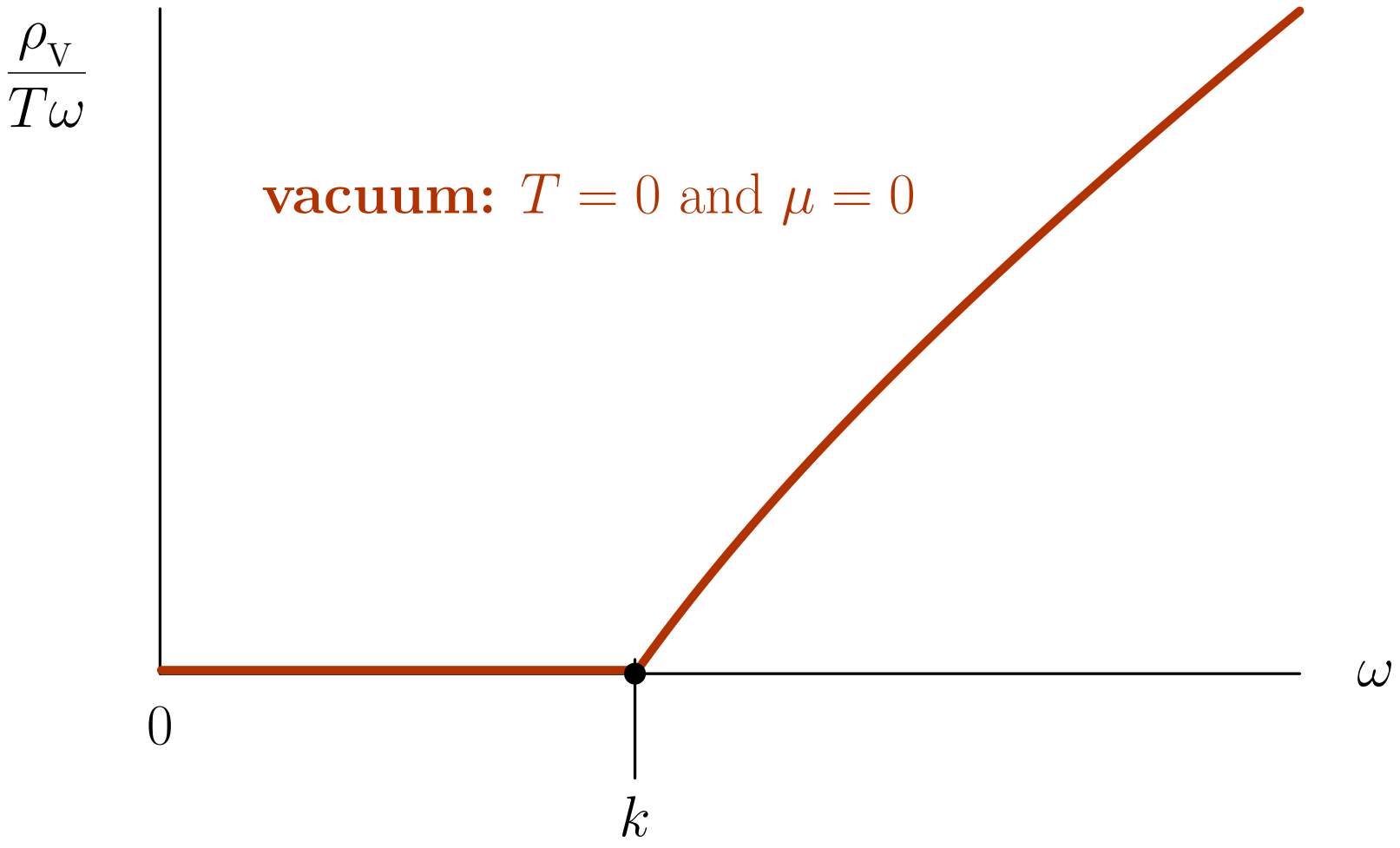


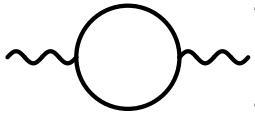
$M^2 \equiv K^2 = \omega^2 - k^2$ invariant mass, $n_{\rm B}$ is the Bose distribution

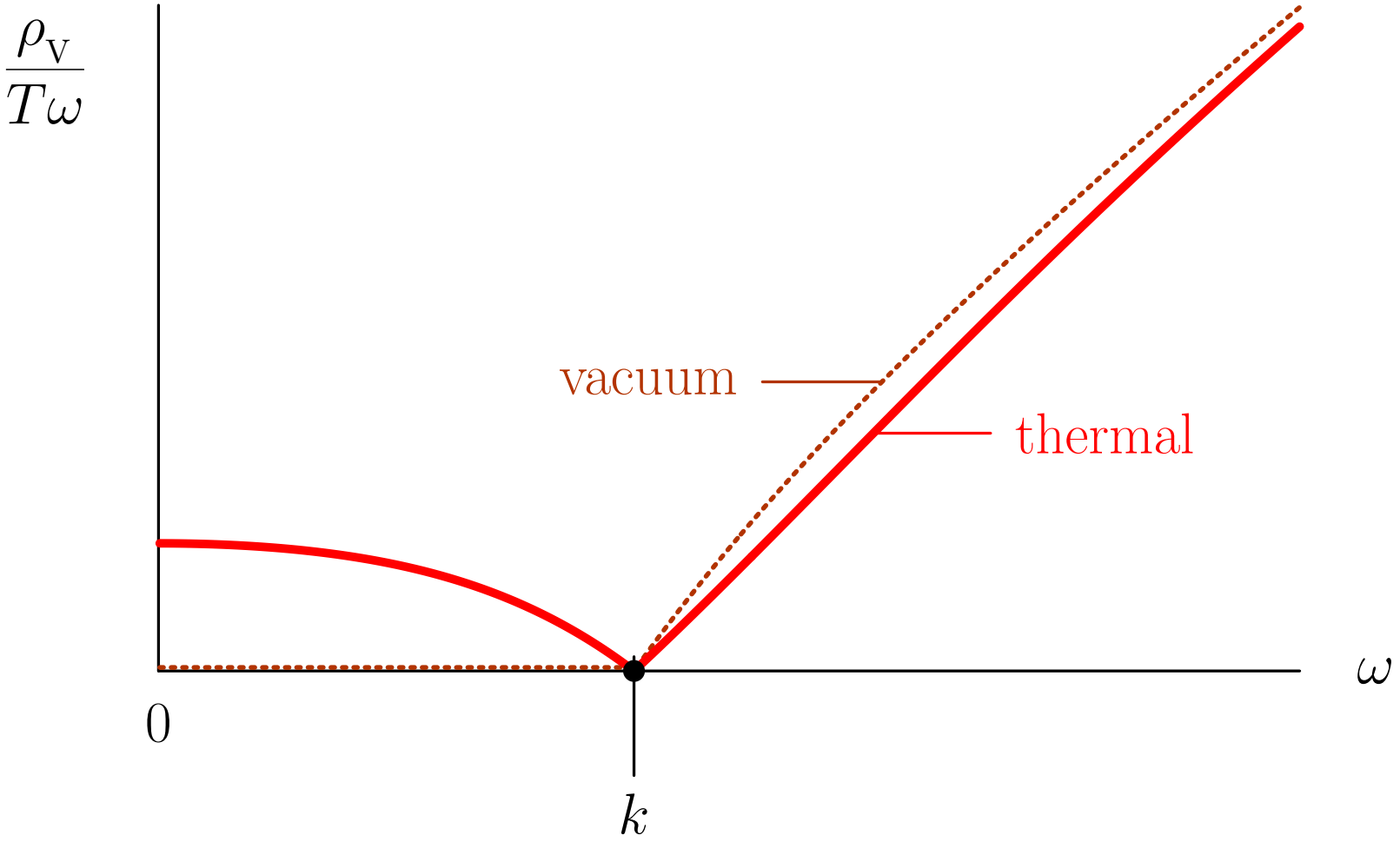
$$\rho_{\mu\nu}(\omega, k) = \mathrm{Im}\Big[\, \Pi_{\mu\nu}(\omega + i0^+, k)\, \Big]$$

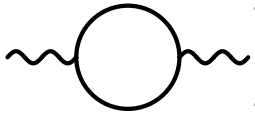
Vector channel spectral function $\rho_{\rm V} \equiv \rho_\mu^{\ \mu} = 2\rho_{\rm T} + \rho_{\rm L}$

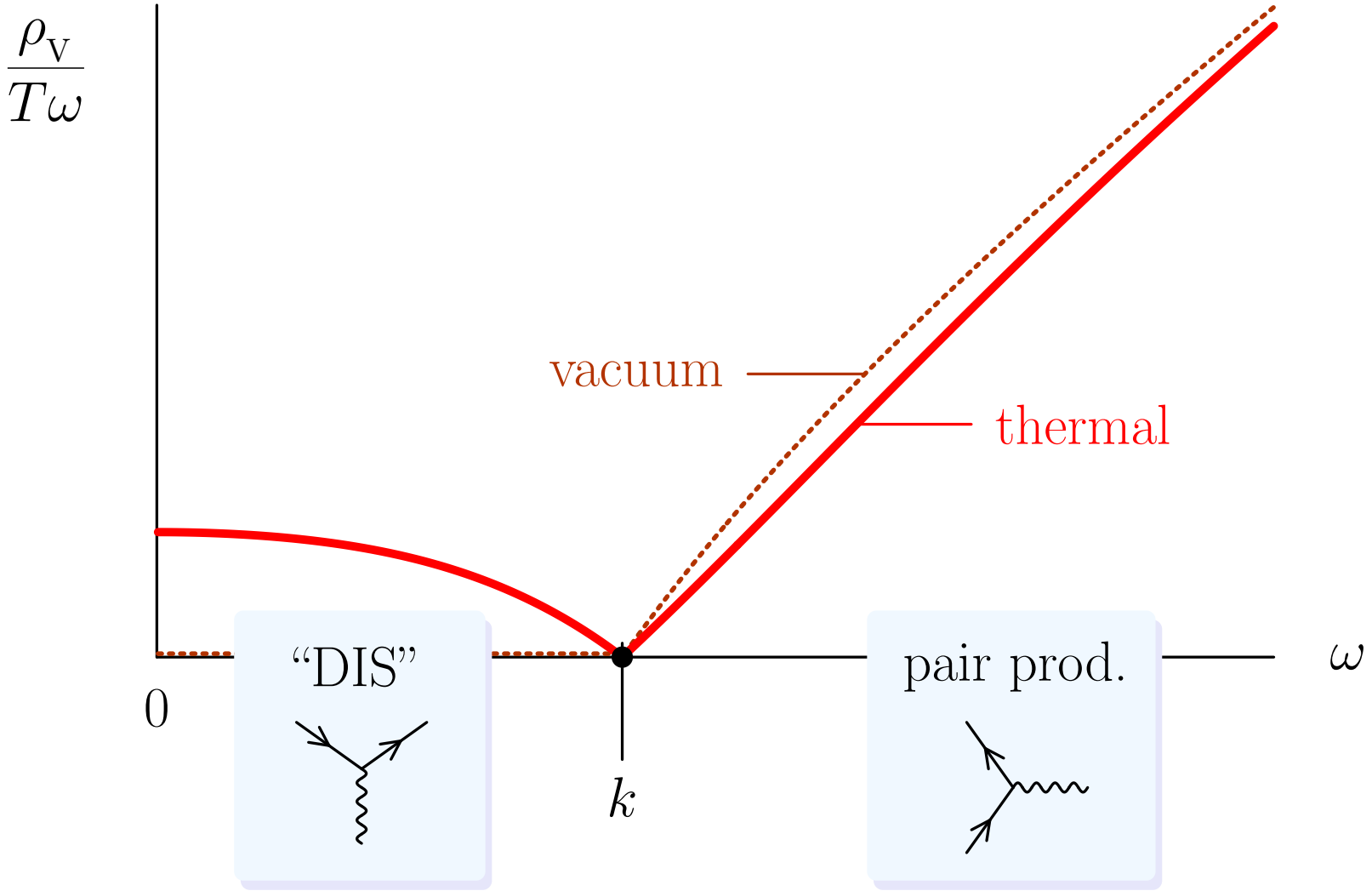
$\mathrm{Im}\Big[$  $\Big] \xrightarrow{T=0} \dfrac{N_\mathrm{c}\, K^2}{4\pi}\,\Theta(K^2)$

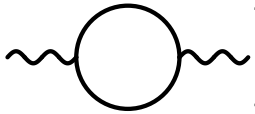


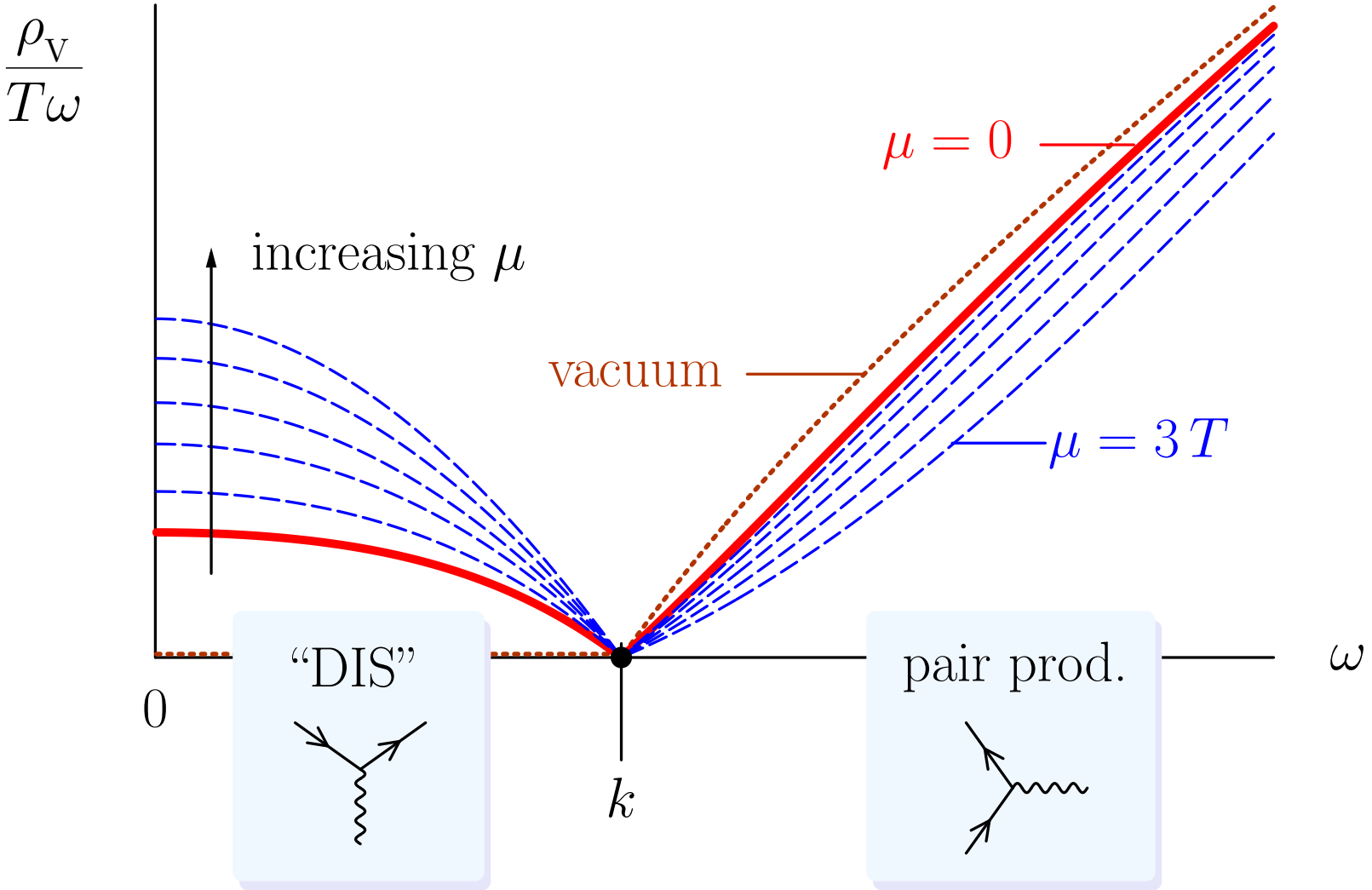
$$\mathrm{Im}\Big[$$  $$\Big] \xrightarrow{T>0} \frac{N_\mathrm{c}\,K^2}{4\pi}\left\{\frac{2\,T}{k}\log\left[\frac{1+e^{-\frac{1}{2}(\omega+k)/T}}{1+e^{-\frac{1}{2}|\omega-k|/T}}\right]+\Theta(K^2)\right\}$$



$$\mathrm{Im}\Big[$$  $$\Big] \xrightarrow[\mu=0]{T>0} \frac{N_\mathrm{c}\,K^2}{4\pi}\left\{\frac{2\,T}{k}\log\left[\frac{1+e^{-\frac{1}{2}(\omega+k)/T}}{1+e^{-\frac{1}{2}|\omega-k|/T}}\right]+\Theta(K^2)\right\}$$

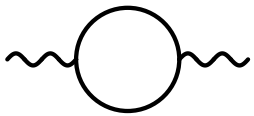


$$\mathrm{Im}\Big[$$  $$\Big] = \frac{N_c K^2}{4\pi}\left\{\frac{T}{k}\sum_{\nu=\pm\mu}\log\left[\frac{1+e^{(\nu-\frac{1}{2}(\omega+k))/T}}{1+e^{(\nu-\frac{1}{2}|\omega-k|)/T}}\right]+\Theta(K^2)\right\}$$



Fixed-order calculation

$$\Pi^{\mu\nu} \;=\; \Big[\, \sum_{l=0}^{\infty} g_{\rm s}^{2l}\, \Pi^{\mu\nu}_{(l)} \,\Big] \;+\; O(e^2)\;; \qquad \alpha_{\rm s} \;=\; \frac{g_{\rm s}^2}{4\pi}$$

$=$  $+\;\ldots$

Result for each polarisation: $\big(\,\text{abbrev. } k_\pm \equiv \frac{1}{2}(\omega \pm k)\,\big)$

$$\rho_{\rm V} \;=\; \ldots$$

$$\rho_{00} \;=\; \frac{N_{\rm c}}{12\pi k}\Big\{ 12\,T^3 \sum_{\nu\,=\,\pm\mu} \Big[\, l_{3{\rm f}}\big(k_+ - \nu\big) - l_{3{\rm f}}\big(|k_-| - \nu\big) \,\Big]$$

$$+\;\; 6kT^2 \sum_{\nu\,=\,\pm\mu} \Big[l_{2{\rm f}}\big(k_+ - \nu\big) + {\rm sign}(k_-)\, l_{2{\rm f}}\big(|k_-| - \nu\big) \Big] + k^3\,\Theta(k_-) \Big\}\;,$$

$$\text{where} \quad l_{2{\rm f}}(x) \;\equiv\; {\rm Li}_2\Big(-\,e^{-x/\,T}\Big)\;, \qquad l_{3{\rm f}}(x) \;\equiv\; {\rm Li}_3\Big(-\,e^{-x/\,T}\Big)\;.$$

to finally give ... $\Pi_{\rm L} = \dfrac{K^2}{\boldsymbol{k}^2}\,\Pi_{00}\;, \quad \Pi_{\rm T} = -\,\dfrac{1}{2}\Big(\,\Pi_{\rm V} + \dfrac{K^2}{\boldsymbol{k}^2}\Pi_{00}\,\Big)$

Fixed-order calculation

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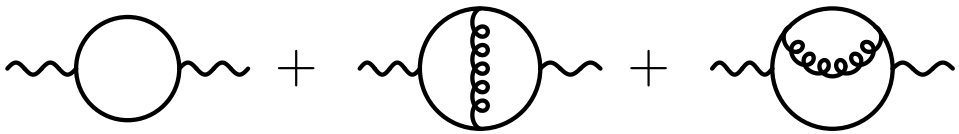
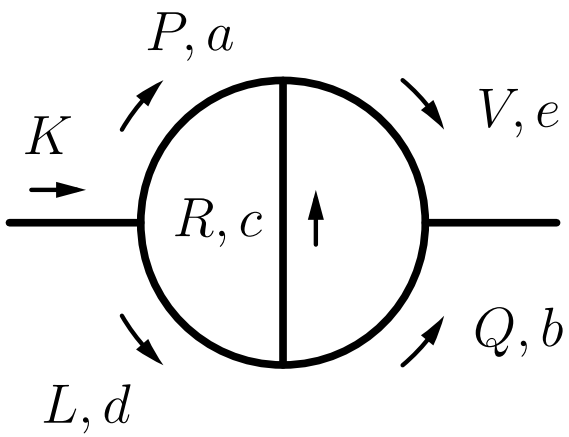
$=$  $+ \ldots$

Project 2-loop result onto 'basis' of **master diagrams** and evaluate:



$$L = K - P$$

$$V = K - Q$$

$$R = K - P - Q$$

$$\boxed{\rho^{(m,n)}_{abcde}(\omega, \boldsymbol{k}) \;\equiv\; \mathrm{Im} \sum\!\!\!\!\!\!\int_{P,Q} \frac{p_0^m\, q_0^n}{P^{2a} Q^{2b} (K-P-Q)^{2c} (K-P)^{2d} (K-Q)^{2e}}}$$

Fixed-order calculation

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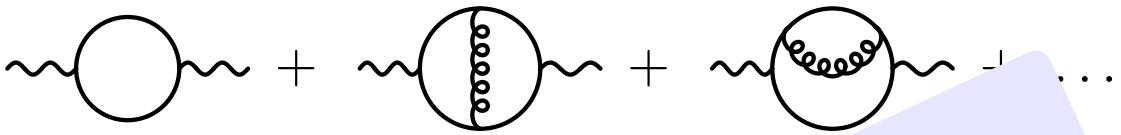
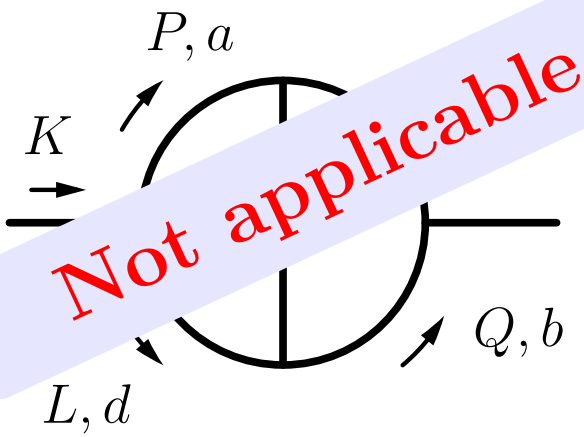
$=$ 

Project 2-loop result onto 'basis' of **master** [illegible] and evaluate:

Not applicable for $K^2 \to 0$!



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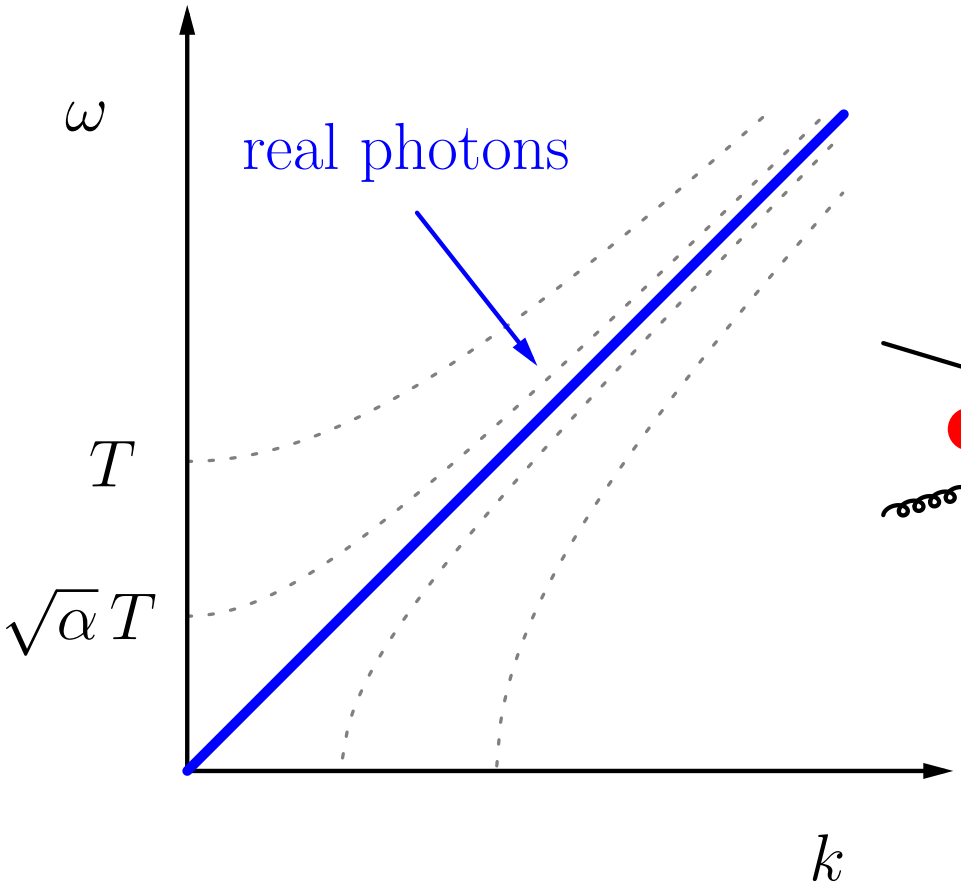
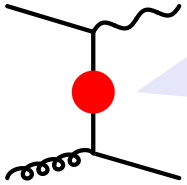


Thermal Screening

$$\frac{\mathrm{d}\sigma}{\mathrm{d}t} = \frac{-\pi\,\alpha_{\mathrm{em}}\alpha_{\mathrm{s}}}{3s^2}\,\frac{t^2+s^2}{t\,s}$$



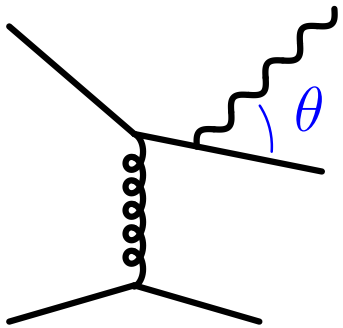
$\sim \alpha_s T^2$ *Hard Thermal Loops*

[Kapusta, Lichard, Seibert (1991)]

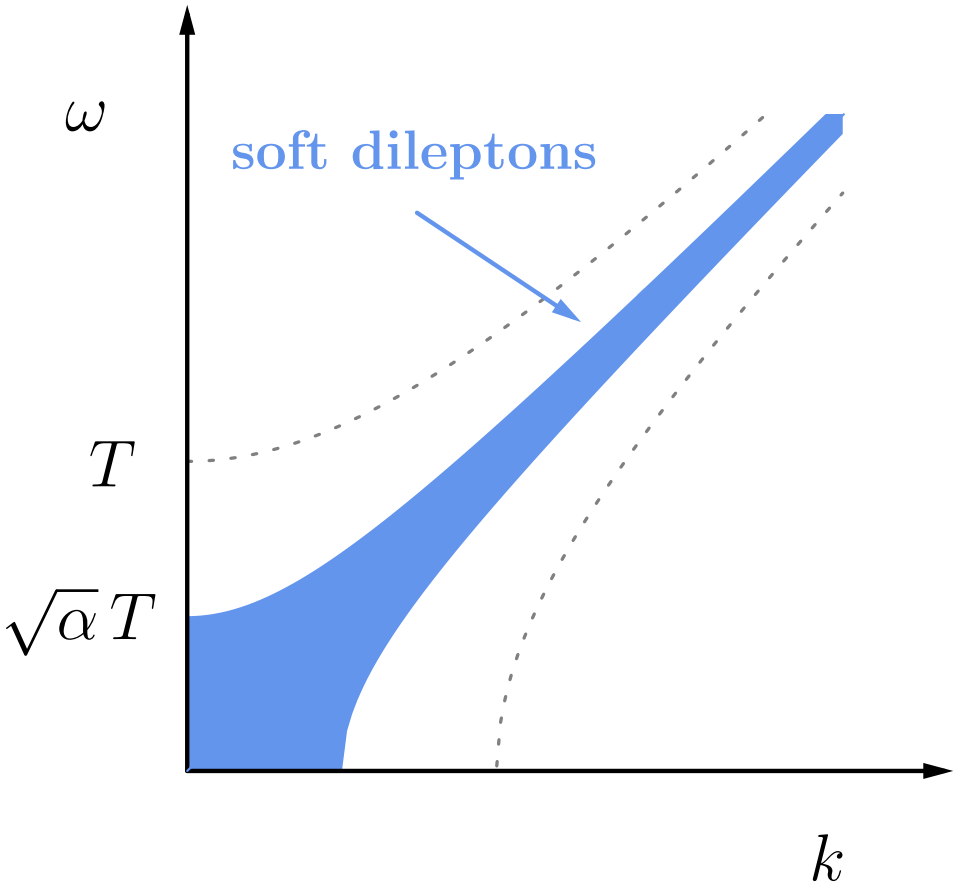
[Baier, *et al* (1992)]

Landau-Pomeranchuk-Migdal (LPM)

$$\int \frac{\mathrm{d}\cos\theta}{E(1-\cos\theta)} = \infty$$



LO: [Arnold, Moore, Yaffe (2001)] , NLO: [Ghiglieri, *et al* (2013)]



LO: [Aurenche, *et al* (2002)]

NLO: [Ghiglieri, Moore (2014)]

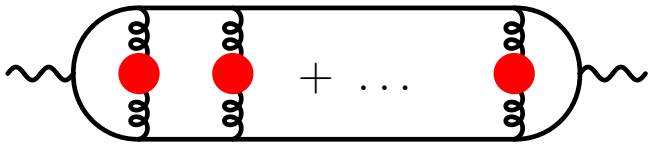
light-like correlator

$\Downarrow$ [Caron-Huot (2009)]

Effective Field Theory

'ladder diagrams' for $M^2 \ll T^2 \rightarrow$ LPM effect + *Hard Thermal Loops*

$$\rho_{\mu\nu}(\omega, \boldsymbol{k}) \quad = \quad \mathrm{Im}\Big[\mu$$

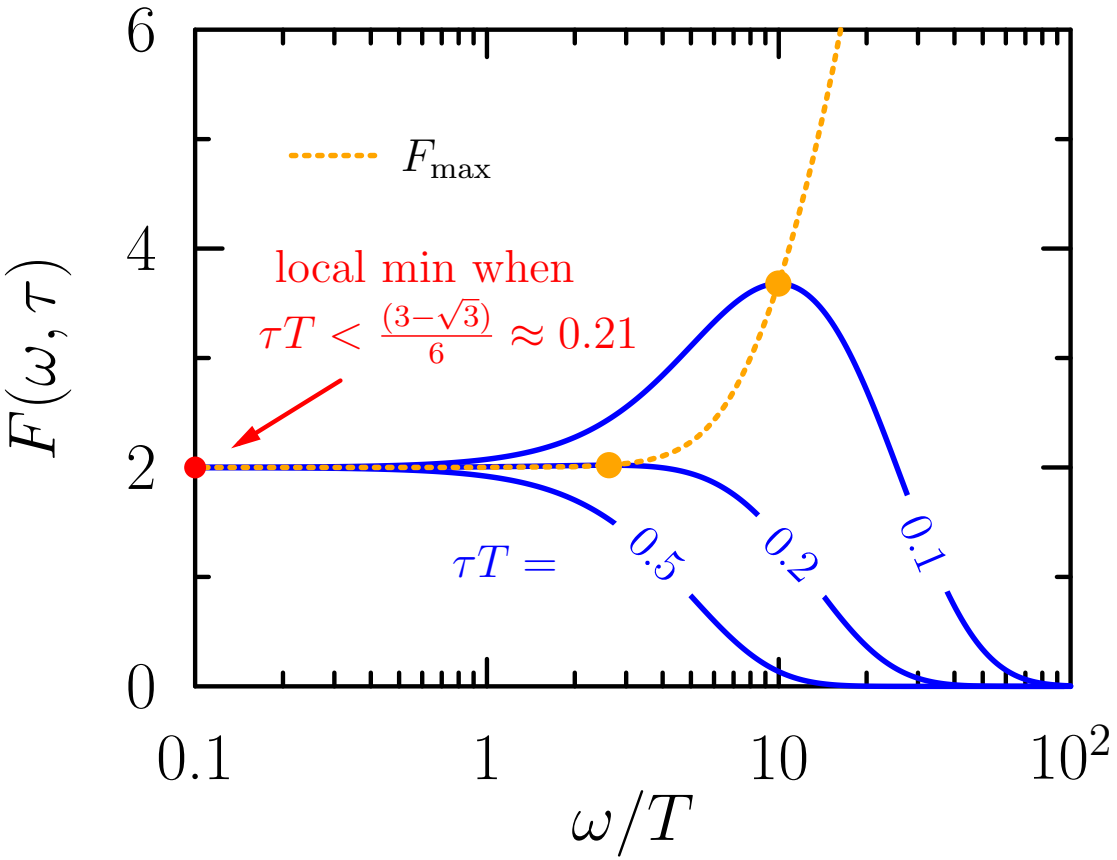


$$\nu\Big]$$

Information from lattice ($\mu_{\rm B} = 0$)

... can 'measure' imaginary time Euclidean current-current correlator:

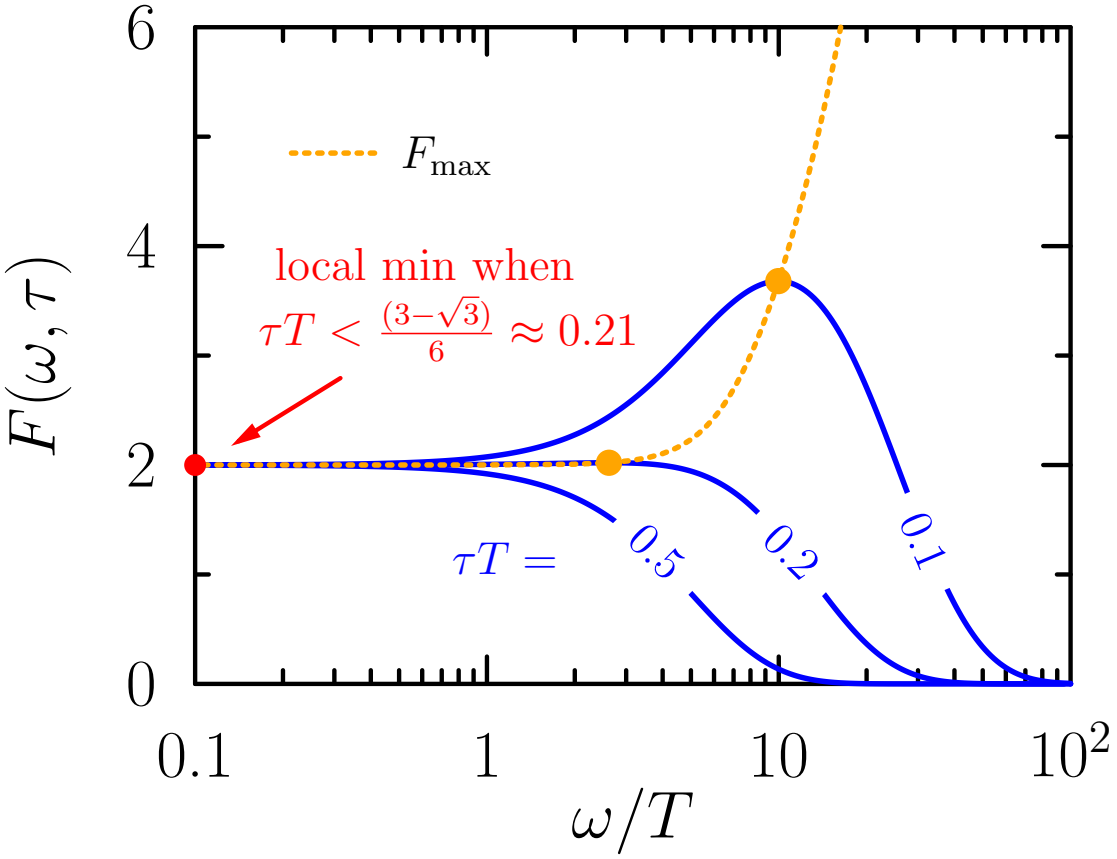
$$\frac{G(\tau,k)}{T^3} = \int_0^\infty \frac{d\omega}{2\pi T} \frac{\rho(\omega,k)}{\omega T} F(\omega,\tau) \quad \text{with} \quad F \equiv \frac{\omega}{T} \frac{\cosh[(\frac{1}{2}\beta - \tau)\omega]}{\sinh[\frac{1}{2}\beta\omega]}$$



Information from lattice ($\mu_{\rm B} = 0$)

Extract $\rho(\omega, k)$ for *real* frequencies?

simple inversion is ill-posed : sensitive to input (overdetermined)



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simple inversion is ill-posed : sensitive to input (overdetermined)

Study UV-finite corr. $G_{\rm H} = 2(G_{\rm T} - G_{\rm L})$ [Brandt, *et al* (2018)]

Properties:

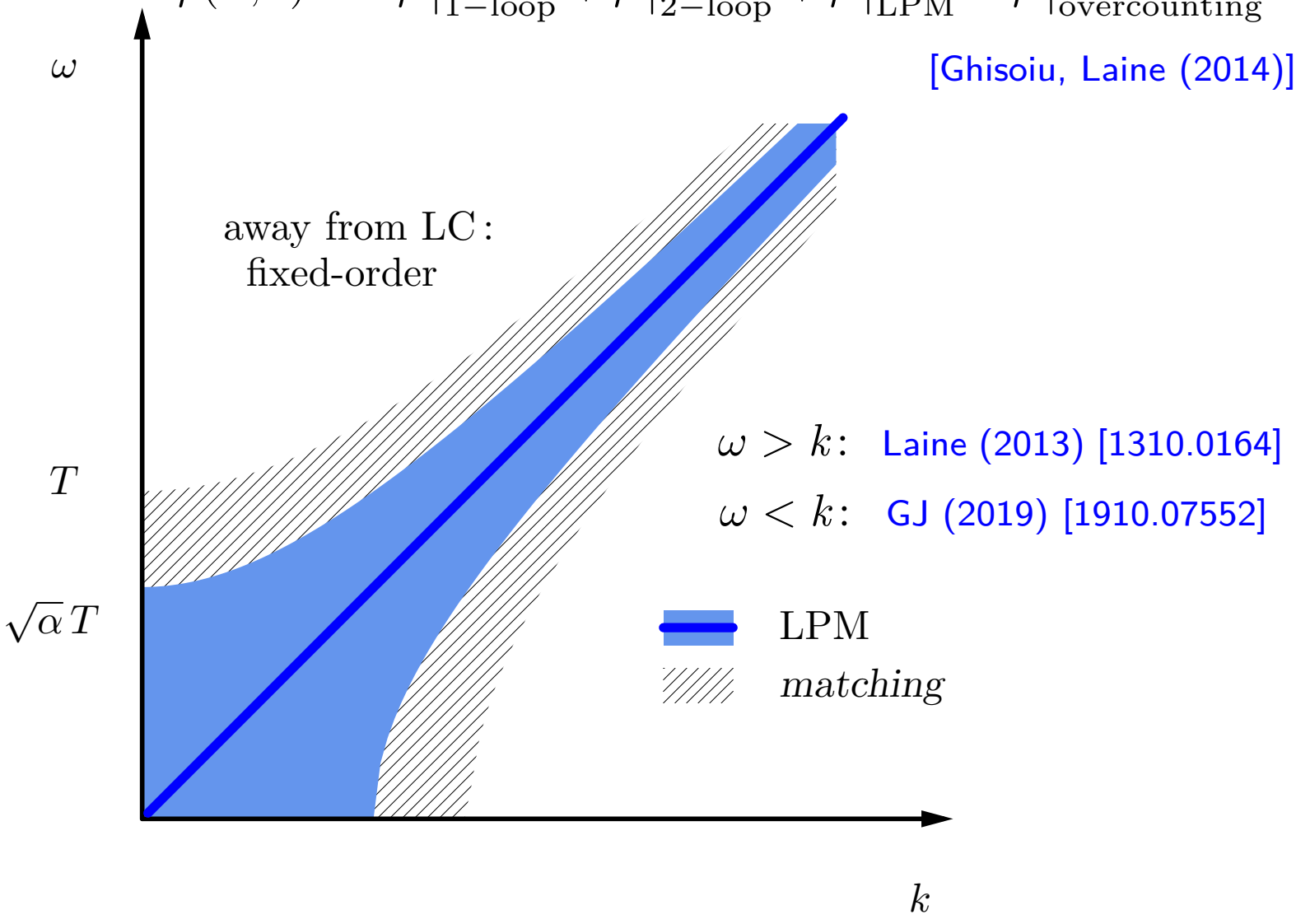
- no vacuum part, $\lim_{T\to 0} \rho_{\rm H} = 0$
- expansion, $\rho_{\rm H} = \alpha_{\rm s}\, 64\pi\, k^2 \int_{\boldsymbol{p}} \frac{p}{\pi} \frac{4(4n_{\rm F} - n_{\rm B})}{9M^4} + O\left(\frac{T^6}{M^4}\right)$
- sum rule, $\int_0^\infty d\omega\, \omega\, \rho_{\rm H}(\omega, k) = 0$ [Caron-Huot (2009)]

$\Rightarrow$ Improved control over systematic uncertainties!

What we do

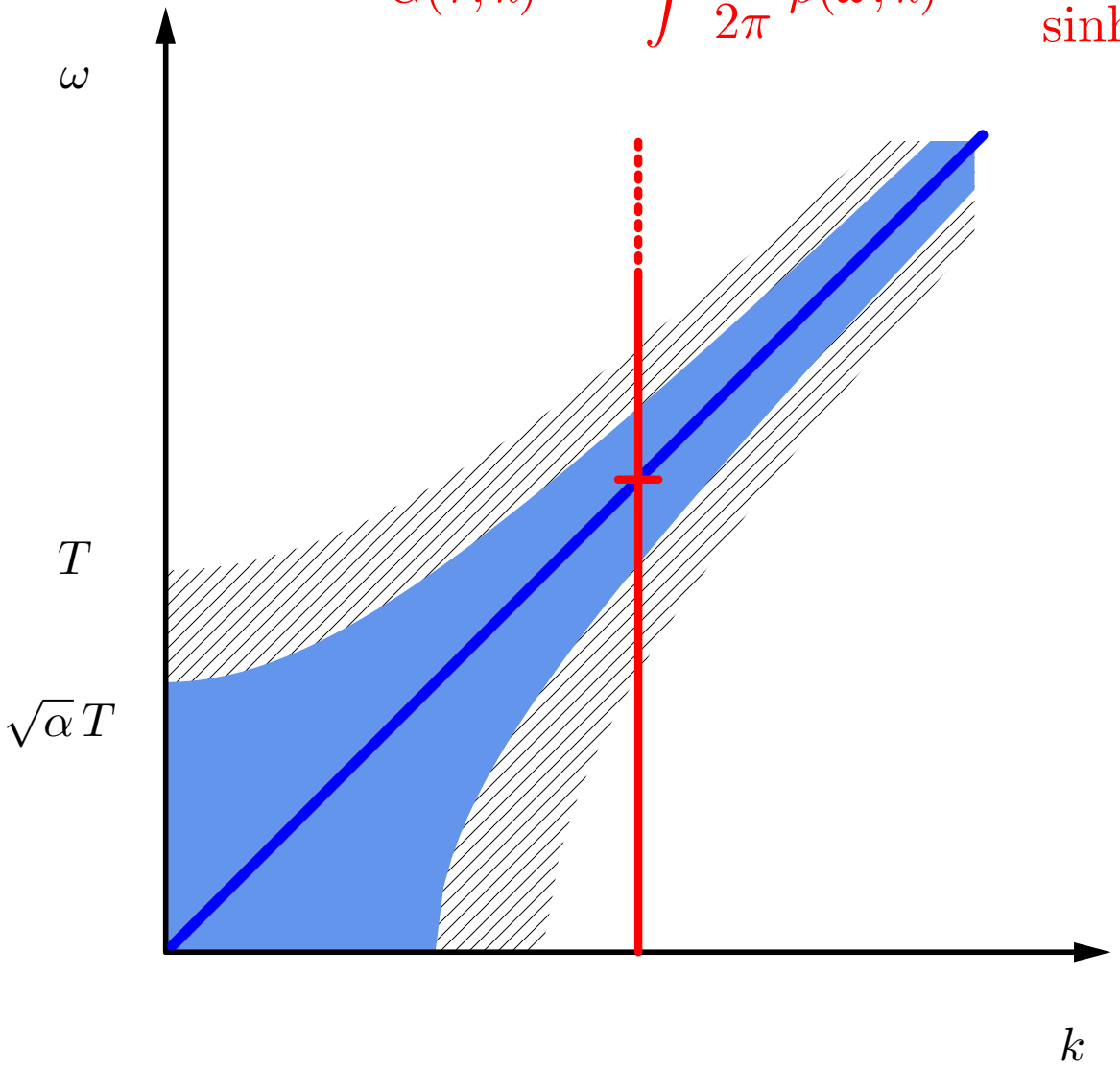
$$\rho(\omega,k) \;\simeq\; \rho\,\Big|^{\text{strict}}_{1-\text{loop}} + \rho\,\Big|^{\text{strict}}_{2-\text{loop}} + \rho\,\Big|^{\text{full}}_{\text{LPM}} - \rho\,\Big|_{\text{overcounting}}$$

[Ghisoiu, Laine (2014)]



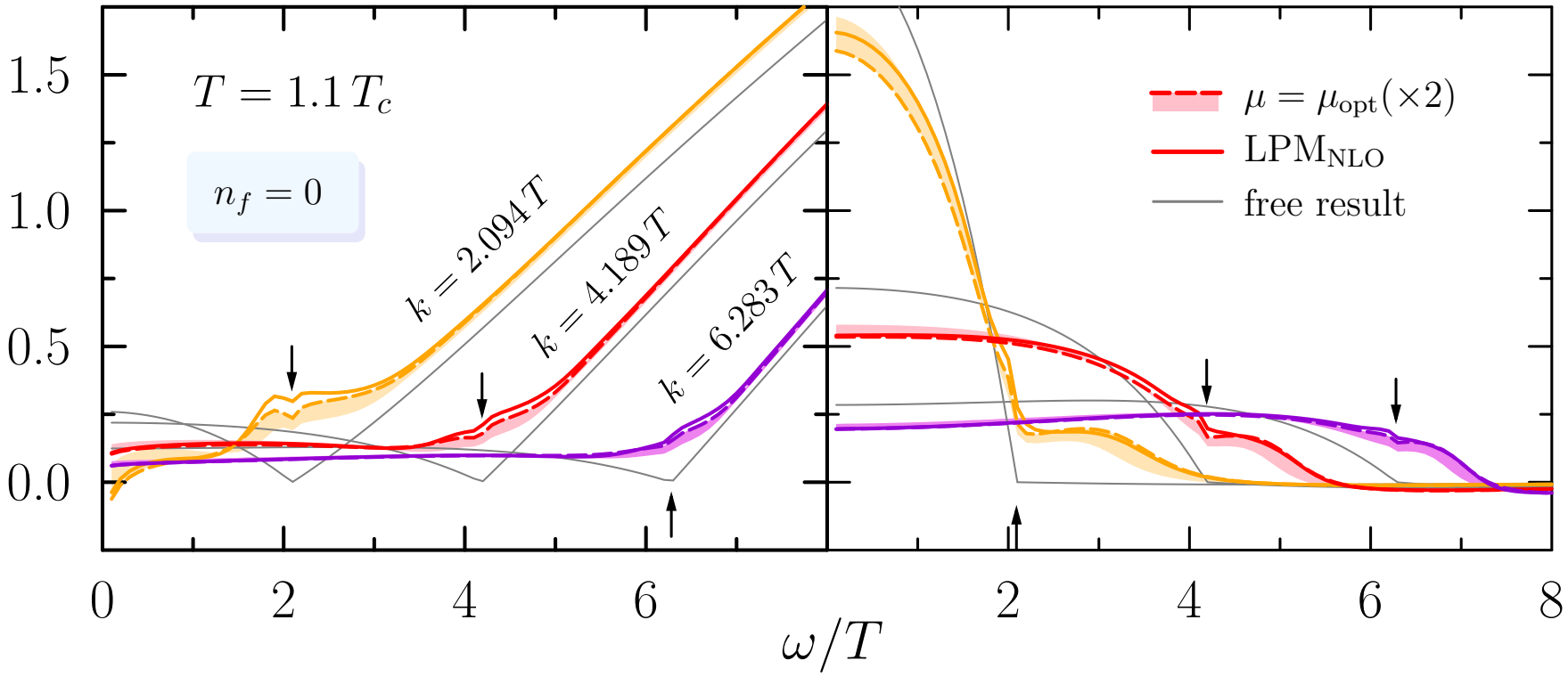
What we do

$$G(\tau, k) \;=\; \int \frac{d\omega}{2\pi}\, \rho(\omega, k)\, \frac{\cosh[(\frac{1}{2}\beta - \tau)\omega]}{\sinh[\frac{1}{2}\beta]}$$



What we find

$G(\tau)$ requires knowing $\rho(\omega, k)$, for *ALL* frequencies:

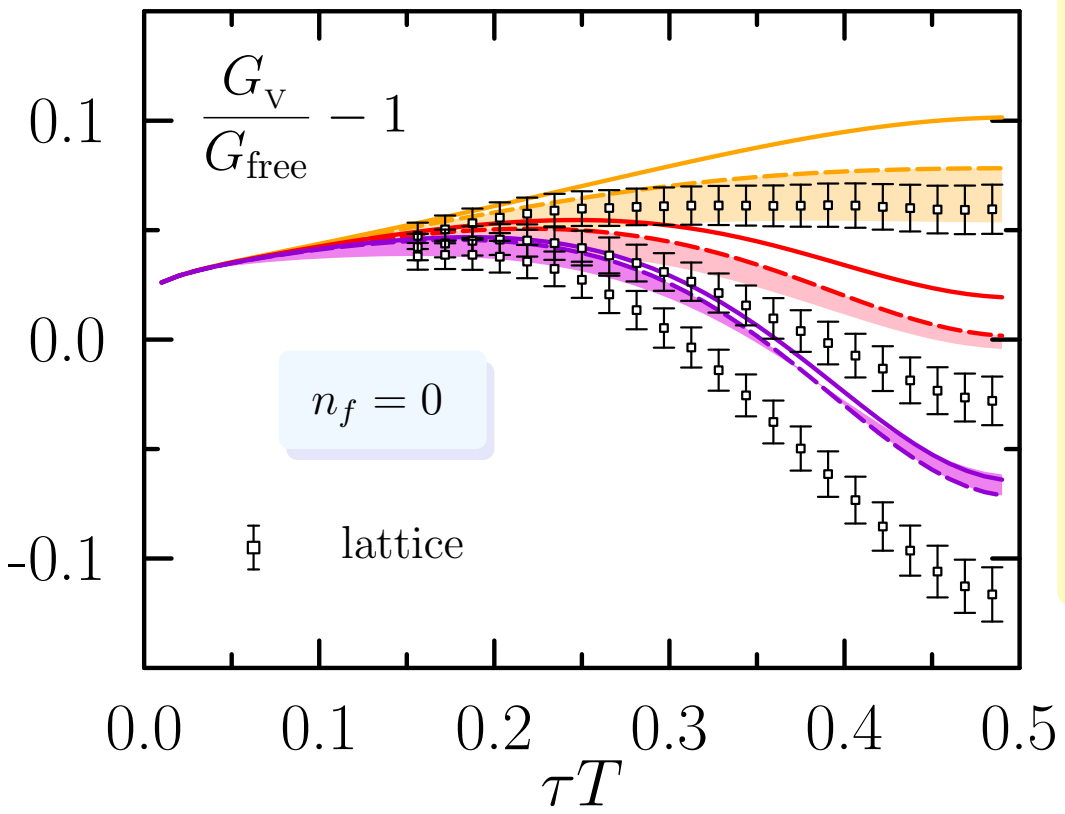


Spectral functions,

$$\text{Left: } \rho_{\rm V}/(\omega T) = (2\rho_{\rm T} + \rho_{\rm L})/(\omega T)$$
$$\text{Right: } \rho_{\rm H}/(\omega T) = 2(\rho_{\rm T} - \rho_{\rm L})/(\omega T)$$

What we find

Testing pQCD: [Ghiglieri, Kaczmarek, Laine, Meyer (2016)] [GJ, Laine (2019)]



Simulations @ $T = 1.1\,T_c$:

$N_s^3 \times N_\tau$	confs
$96^3 \times 32$	314
$144^3 \times 48$	358
$192^3 \times 64$	242

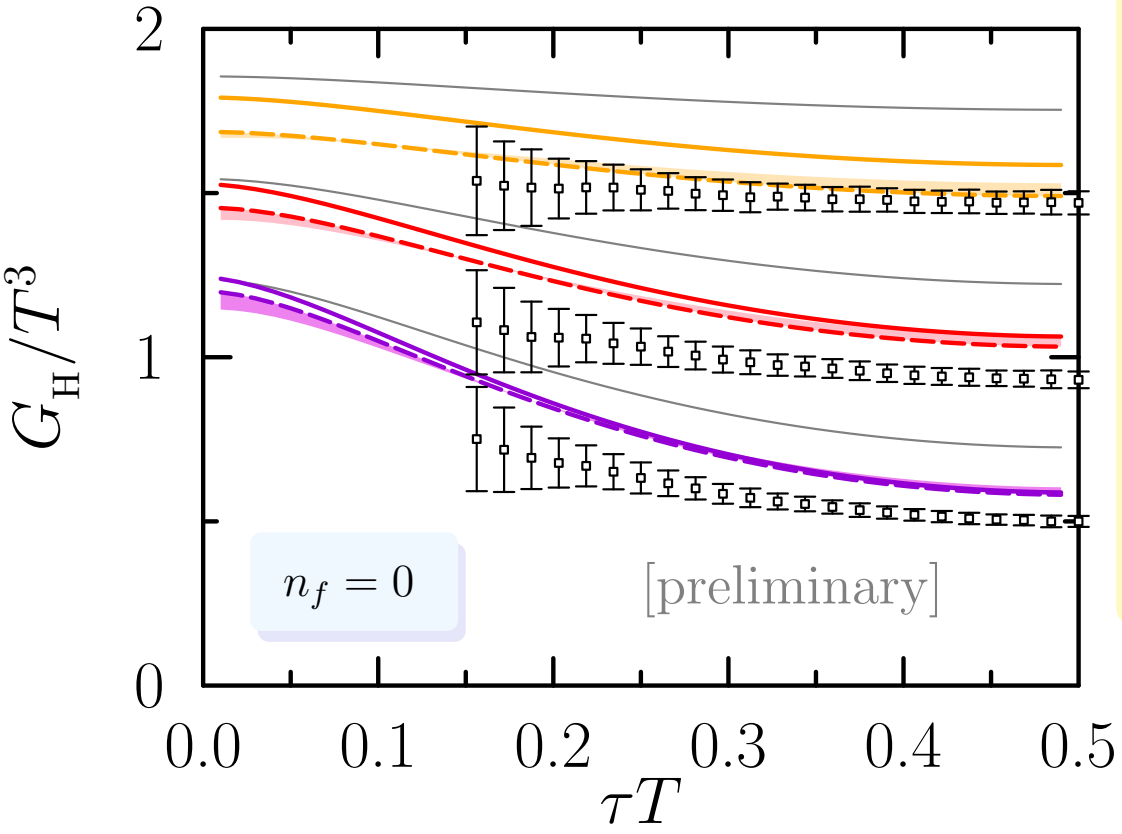
... finite-size box, $k = \dfrac{2\pi n}{aN_s}$

where $n = \{1, 2, 3\}$

Normalisation: $G_{\rm free} \to$ no QCD corrections, $\alpha_s = 0$

What we find

Testing pQCD: (and more!) work in progress w/ Bala, Kaczmarek



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... finite-size box, $k = \dfrac{2\pi n}{aN_s}$

where $n = \{1, 2, 3\}$

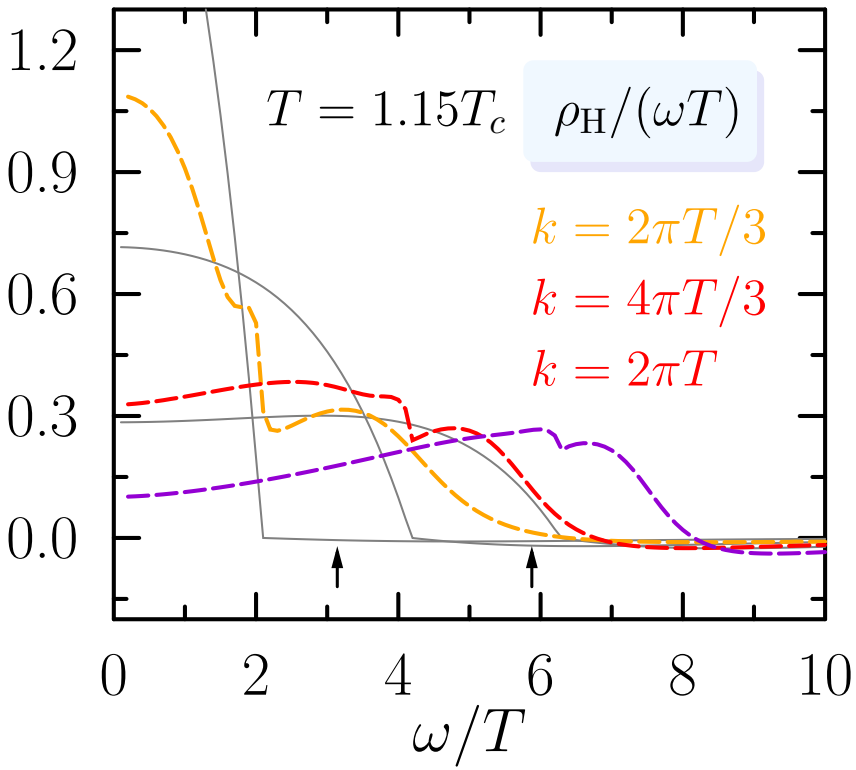
$n_f = 0$ data for $T = \{1.3, 1.5\}\,T_c$

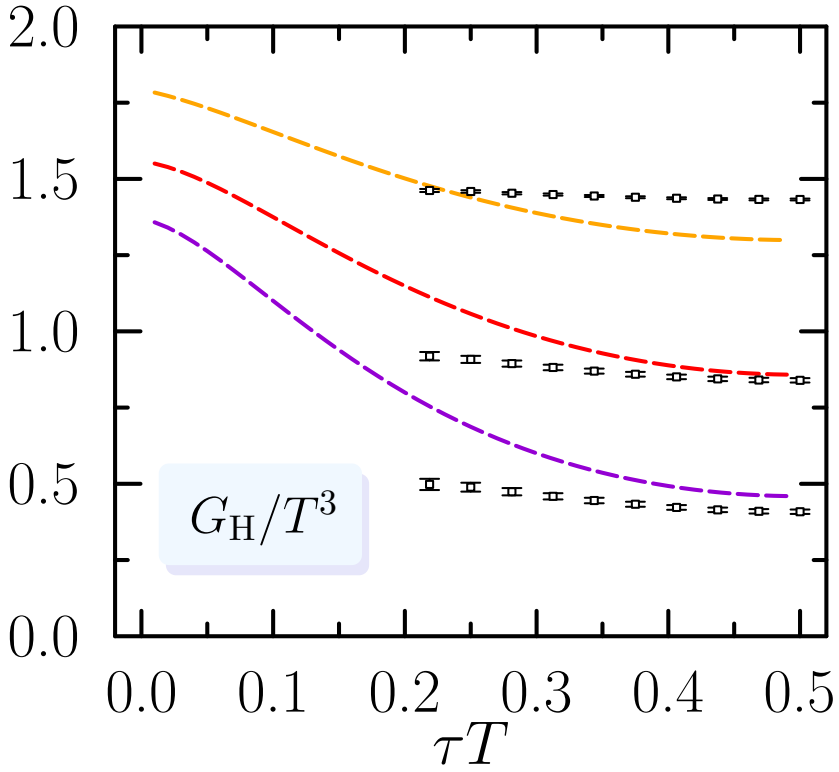
$n_f = 2$ data for $T = 1.2\,T_c$

What we find

AND ... $n_f = 3$ data for $T = \{1.15, 1.3\}\, T_c$ work in progress w/ Bala, Kaczmarek

clover-improved Wilson fermions, HISQ config. w/ $m_l = m_s/5$





NB: not yet continuum extrapolated!!

What we find

AND ... $n_f = 3$ data for $T = \{1.15, 1.3\}\, T_c$ work in progress w/ Bala, Kaczmarek

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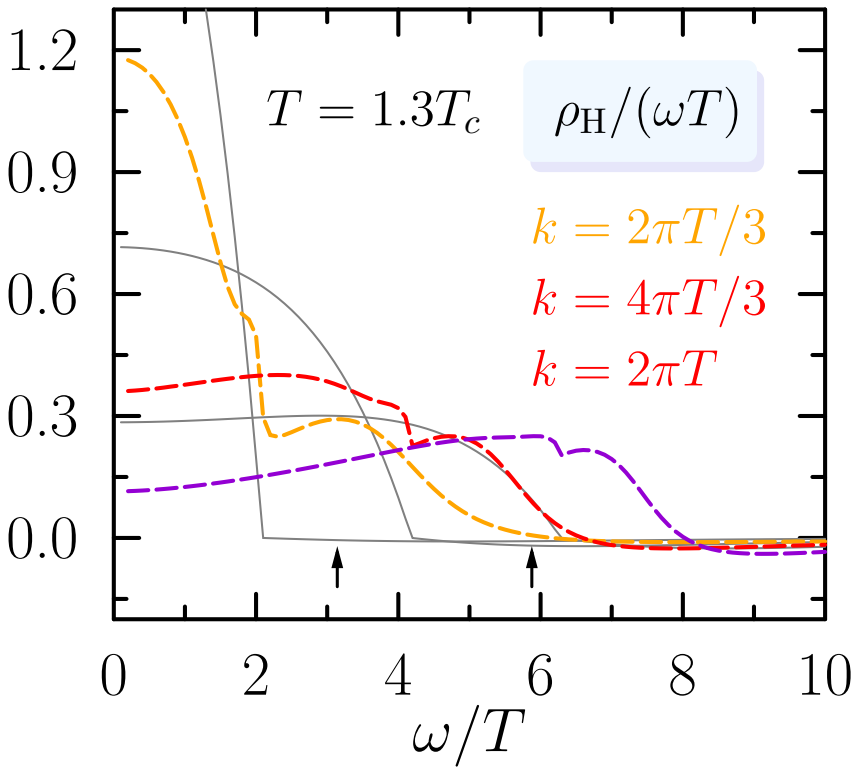
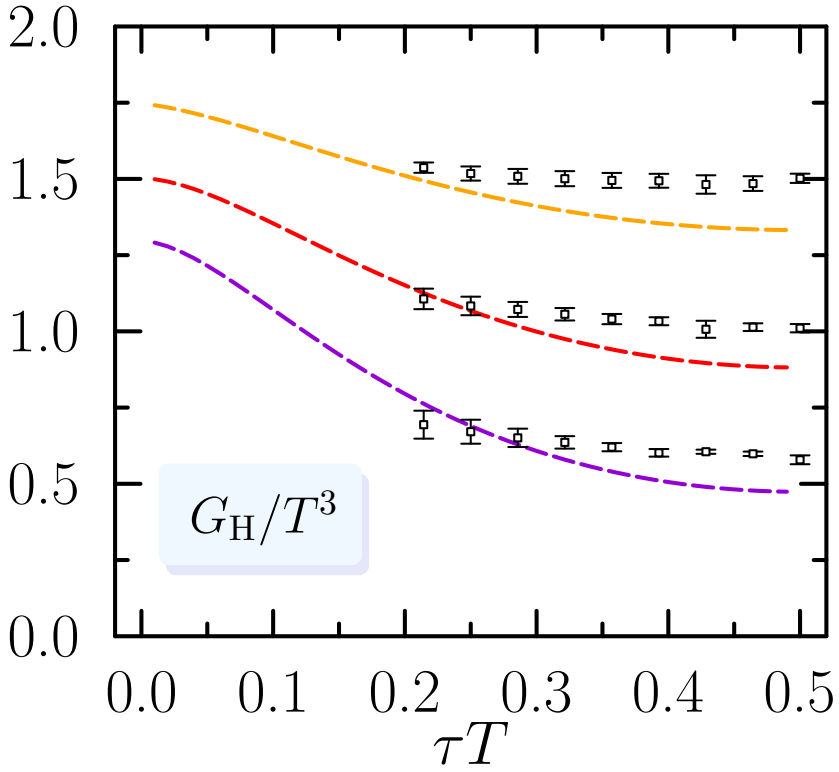


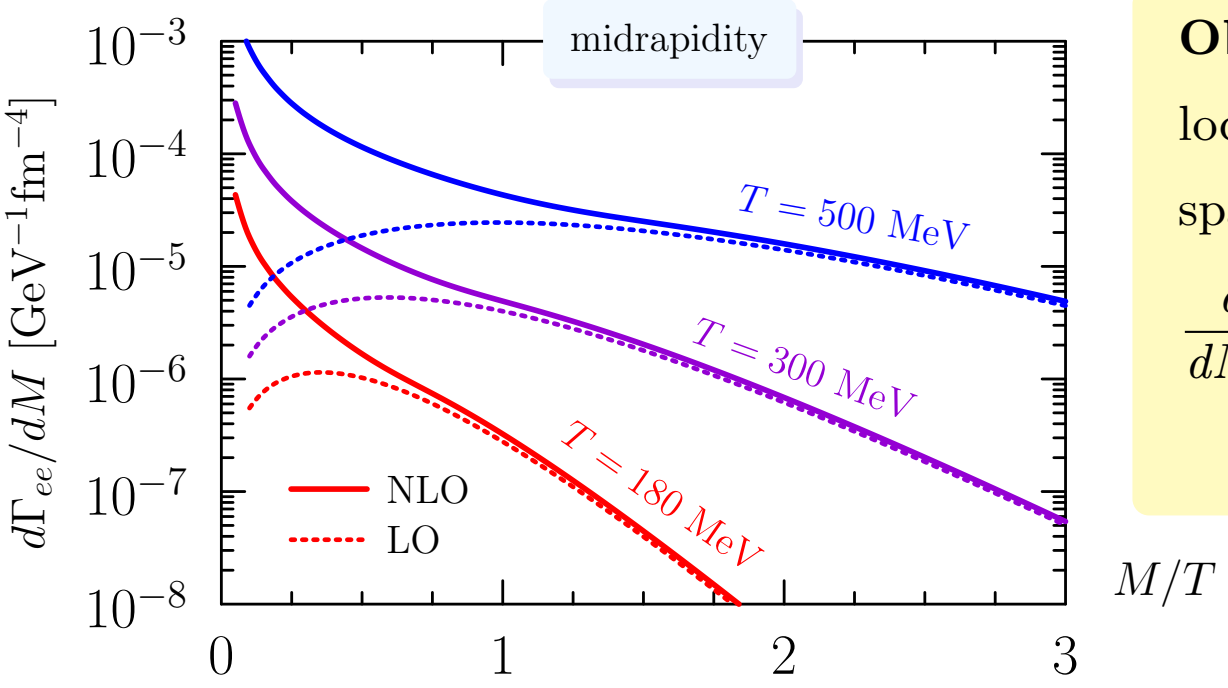


NB: not yet continuum extrapolated!!

Impact on yield ($\mu_{\rm B} = 0$)

Embed pQCD rates into hydro work in progress w/ Churchill, Gale, Jeon

$$\left.\frac{d\Gamma_{\ell\bar{\ell}}}{dM\,dy}\right|_{y=0} = 2\pi\, M \int_0^\infty dk_\perp \cdot k_\perp \, \frac{d\Gamma_{\ell\bar{\ell}}}{d\omega\, d^3k}\left(\omega = M^2 + k_\perp^2, k = k_\perp\right)$$



Observable yields:

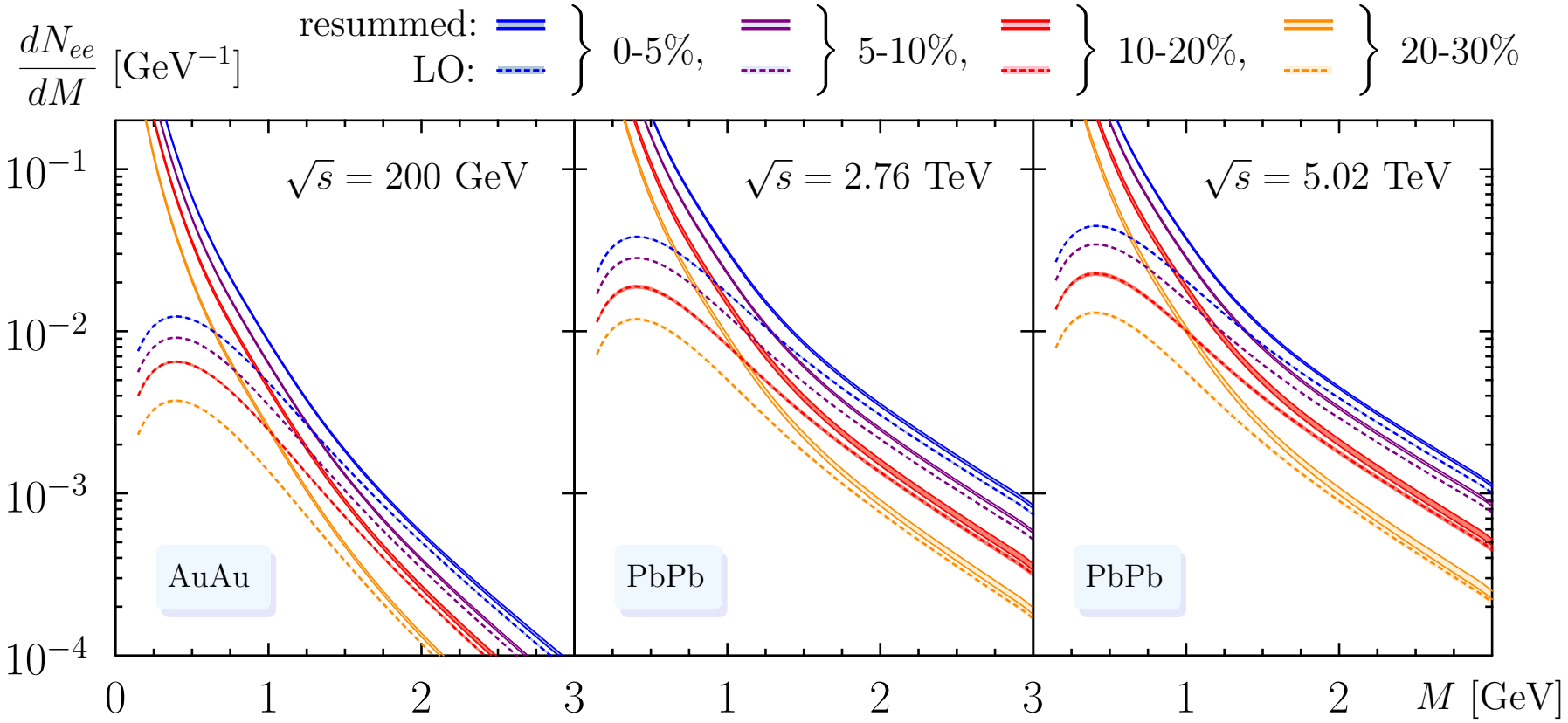
local equillib. $T(t, \mathbf{x})$, ...

spacetime evolution

$$\frac{dN}{dM\,dy} = \int dt \int_{\mathbf{x}} \frac{d\Gamma}{dM\,dy}$$

$\Rightarrow$ hydro simulation

even for $1 \lesssim M \lesssim 3$ GeV, NLO corrections give at least 10% enhancement!



classical YM action in 2D

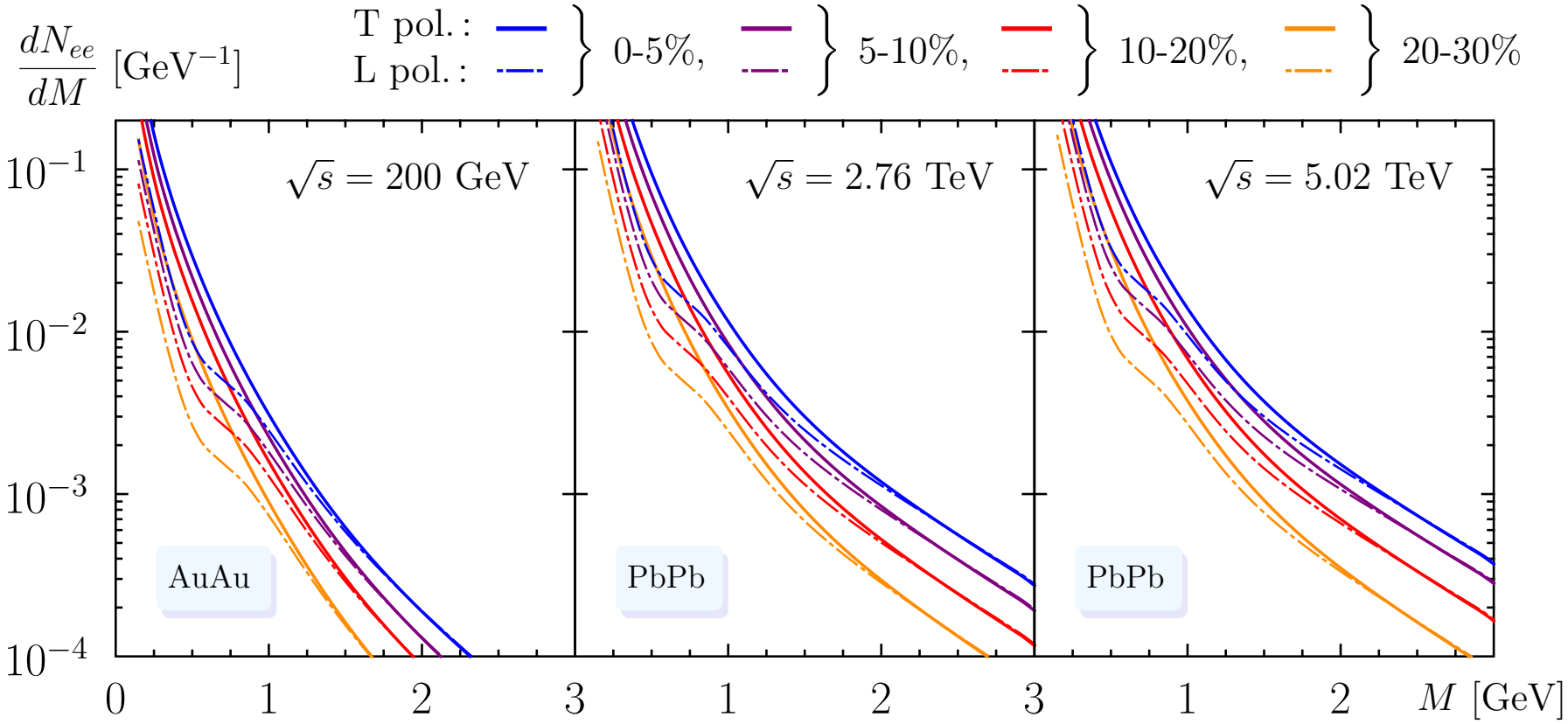
→ saturation scale Q_s

IP-Glasma w/ $b = 0...20$ fm

thermal dileptons during *local* equilib

→ specified by viscous EM tensor...

$$T^{\mu\nu} = \varepsilon u^\mu u^\nu - (g^{\mu\nu} - u^\mu u^\nu)(p + \Pi) + \pi^{\mu\nu}$$



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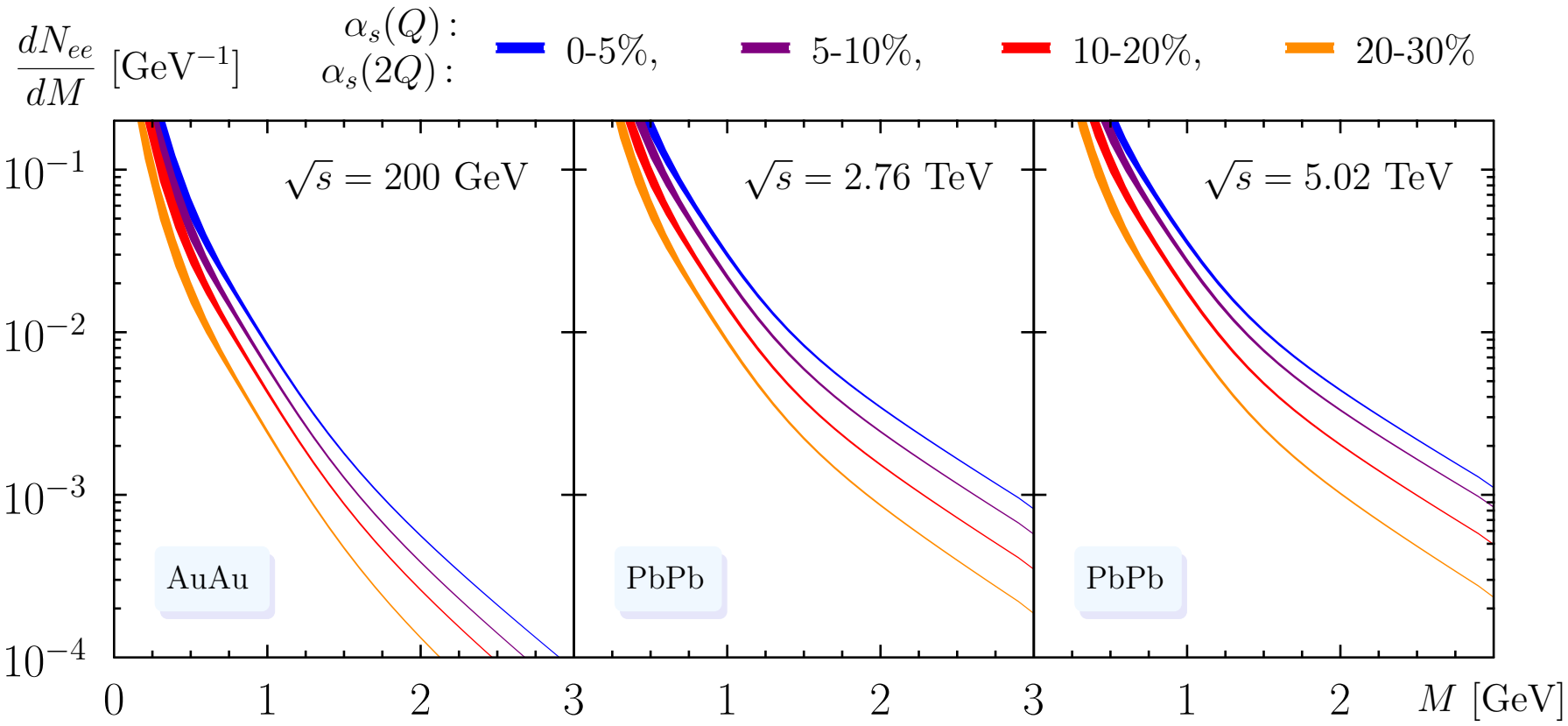
CGC — **hydro:** MUSIC $2+1$D — Cooper-Frye (iS3D) → UrQMD

$\tau_0 \simeq 0.4$ fm

$\tau_{\text{freezeout}}$ @ $T = 145$ MeV



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CGC

$\tau_0 \simeq 0.4$ fm

hydro: MUSIC $2+1$D

$\tau_{\text{freezeout}}$ @ $T = 145$ MeV

Cooper-Frye (iS3D) → UrQMD

Considerations for non-zero $\mu_{\rm B}$

- chemical equilibrium $\Rightarrow \mu \equiv \mu_{\rm q} = \frac{1}{3}\mu_{\rm B}$

- Debye mass m_D and the 'asymptotic' quark mass m_∞

$$
\begin{aligned}
m_D^2 &\equiv g^2\left[\left(\tfrac{1}{2}\,n_f + N_{\rm c}\right)\frac{T^2}{3} + n_f\,\frac{\mu^2}{2\pi^2}\right] \\
m_\infty^2 &\equiv g^2\,\frac{C_{\rm F}}{4}\left(T^2 + \frac{\mu^2}{\pi^2}\right)
\end{aligned}
$$

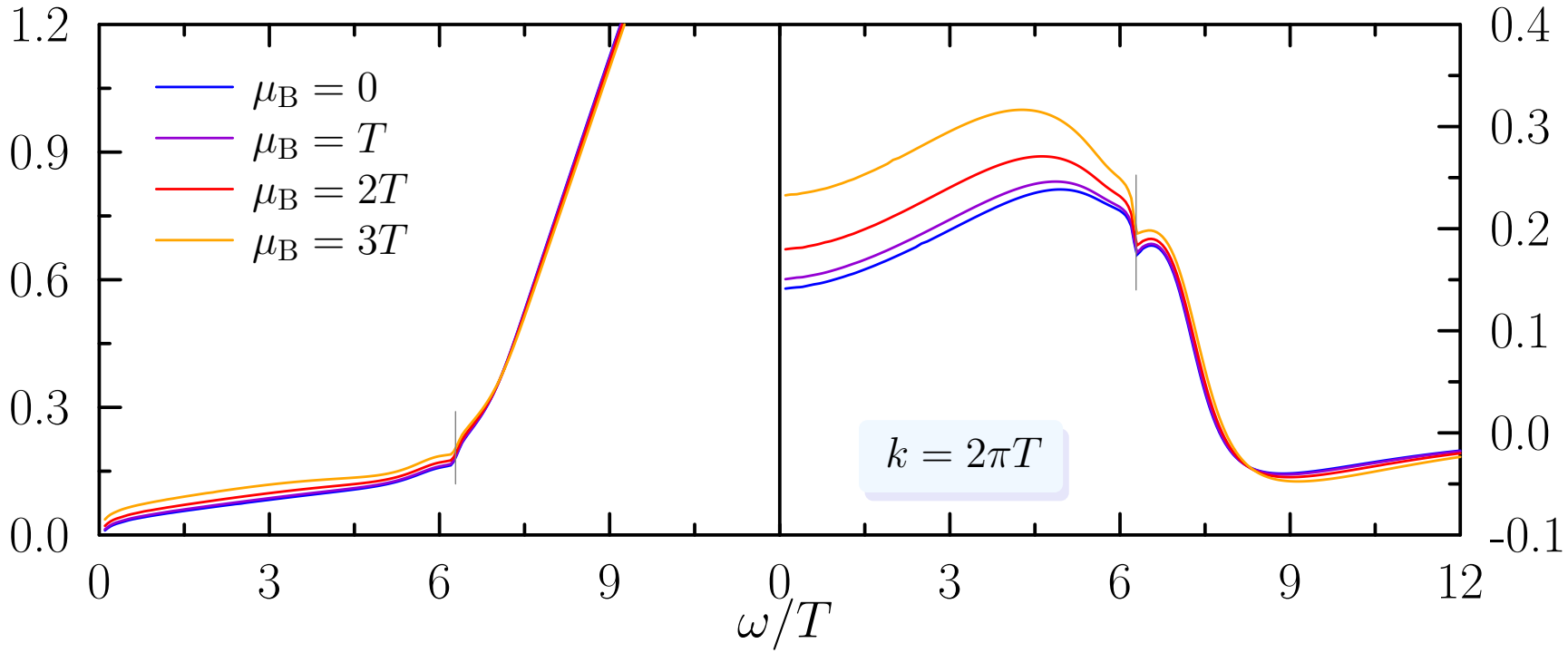
- large frequency limit: enhancement ↘

$$
\rho_{\rm V} \simeq \frac{N_{\rm c} M^2}{4\pi} + 4g^2 C_{\rm F} N_{\rm c}\left\{\frac{3M^2}{4(4\pi)^3} + \frac{\pi\left(\omega^2 + \frac{k^2}{3}\right)}{36M^4}\left(T^4 + \frac{6}{\pi^2}T^2\mu^2 + \frac{3}{\pi^4}\mu^4\right)\right\}
$$

NEW RESULTS: the **full** effect of $\mu_{\rm B}$ on $\rho(\omega,k)\,\big|^{\rm NLO}_{\rm resummed}$ ...

What we find

pQCD spectral functions $T = 280$ MeV and $n_f = 3$:



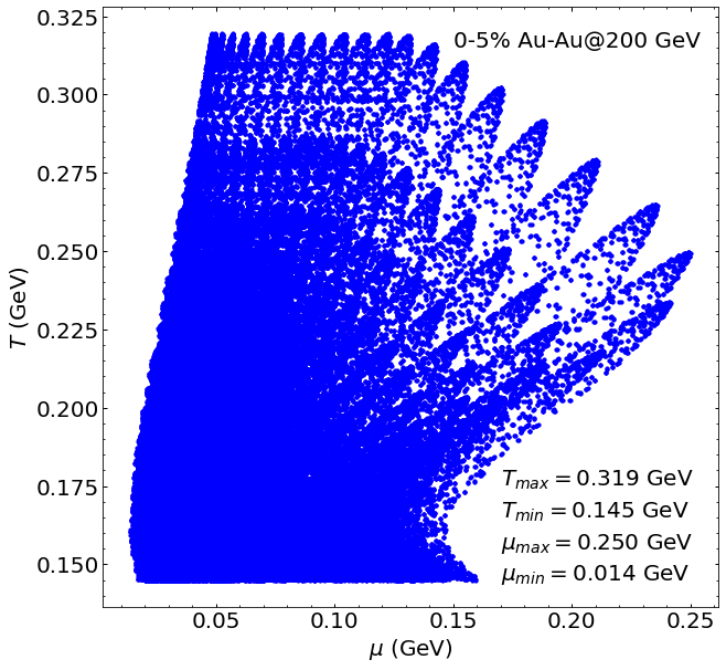
Spectral functions,

$$\text{Left: } \rho_V/(\omega T) = (2\rho_T + \rho_L)/(\omega T)$$
$$\text{Right: } \rho_H/(\omega T) = 2(\rho_T - \rho_L)/(\omega T)$$

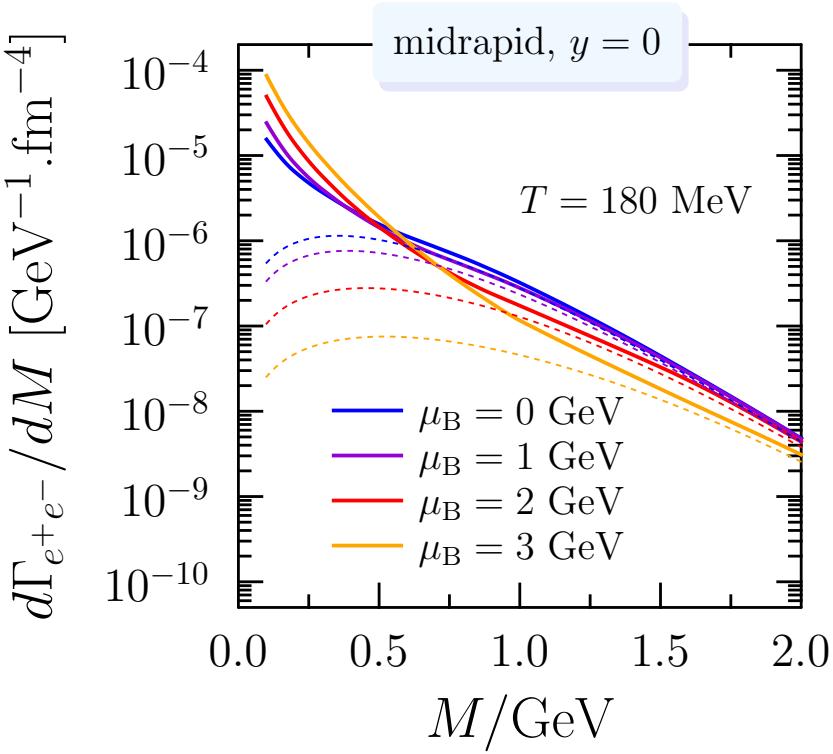
Impact on yield (non-zero μ_B)

BES $\Rightarrow$ probe baryon rich region work in progress w/ Churchill, Gale, Jeon

MUSIC: [Schenke, Jeon, Gale (2010)]



(↑ figure by Lipei Du)

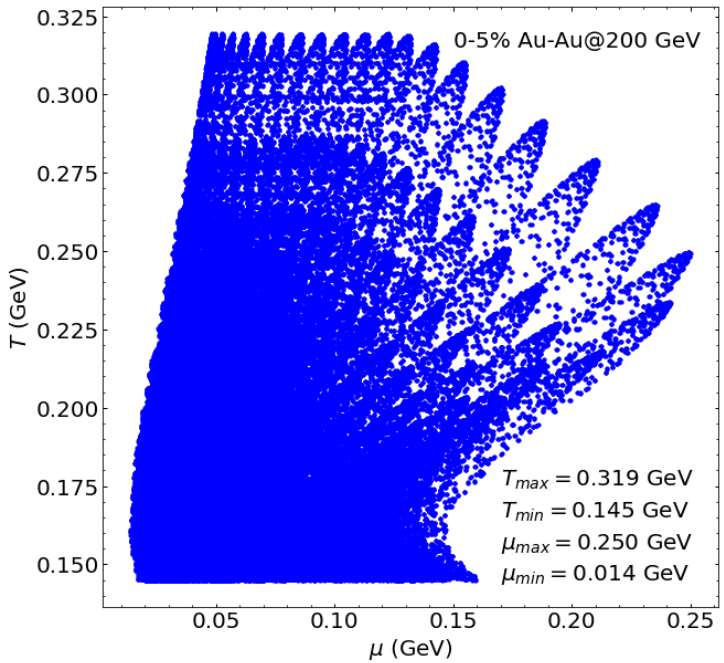


$\Rightarrow$ compensation of LO suppression & NLO enhancement! ...

Impact on yield (non-zero μ_B)

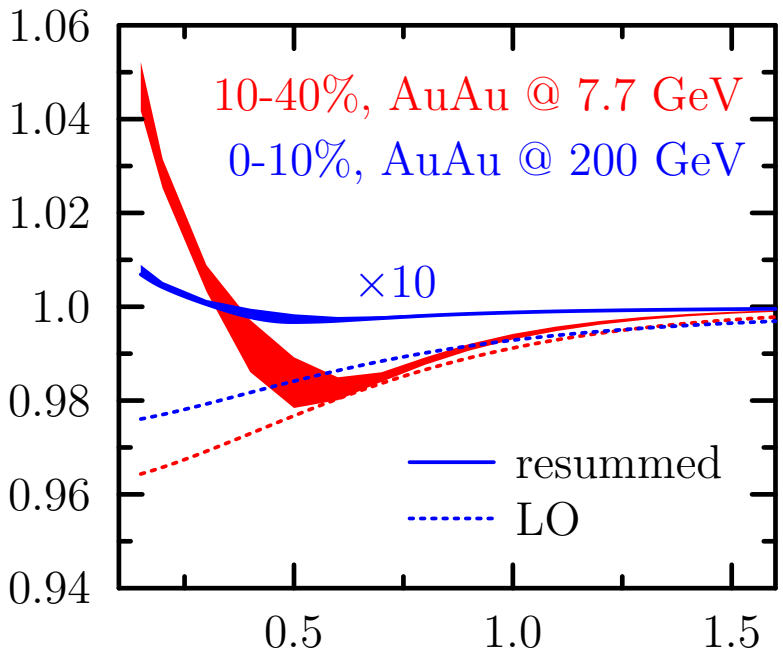
BES $\Rightarrow$ probe baryon rich region work in progress w/ Churchill, Gale, Jeon

MUSIC: [Schenke, Jeon, Gale (2010)]



(↑ figure by Lipei Du)

ratio: $\dfrac{dN_{e^+e^-}/dM|_{\mu>0}}{dN_{e^+e^-}/dM|_{\mu=0}}$



smooth MC-Glauber initial conditions + baryon diffusion

Summary

- extended pQCD calculation to finite μ_{B}
- tested spectral fncs. with **lattice** data for $G_{\mathrm{T}} - G_{\mathrm{L}}$
- many new hydrodynamic predictions:
  dileptons a good *thermometer*, poor *baryometer*!

$$\boxed{\rho^{(m,n)}_{abcde}(K) \equiv \mathrm{Im} \sum\!\!\!\!\!\!\int_{P,Q} \frac{p_0^m\, q_0^n}{P^{2a}\, Q^{2b} (K-P-Q)^{2c} (K-P)^{2d} (K-Q)^{2e}}}$$

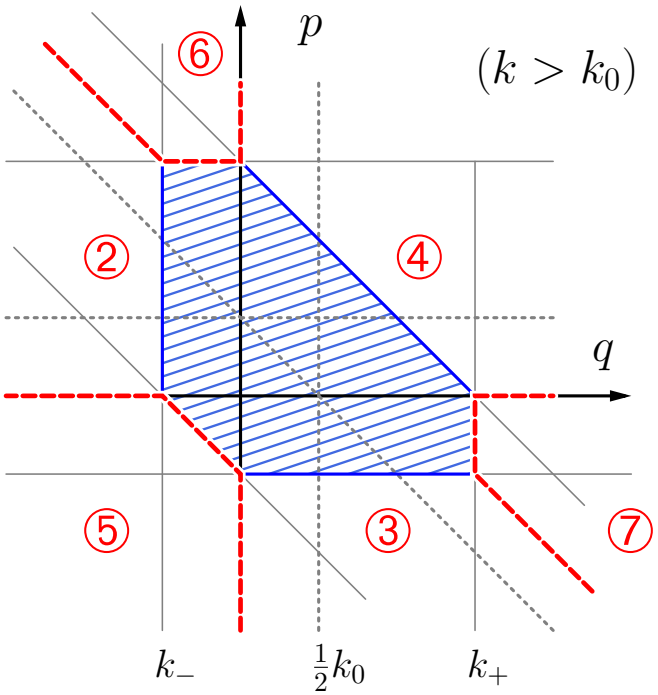
$$\begin{aligned}
\rho_{\mathrm{V}}|^{(g^2)}_{\mathrm{NLO}} &= 8(1-\epsilon) g^2\, C_{\mathrm{F}}\, N_{\mathrm{c}} \Big\{ (1-\epsilon) K^2 \Big(\rho^{(0,0)}_{11020} + \rho^{(0,0)}_{11002} - \rho^{(0,0)}_{10120} - \rho^{(0,0)}_{01102} \Big) \\
&+ \quad \rho^{(0,0)}_{11010} + \rho^{(0,0)}_{11001} + 2\epsilon\, \rho^{(0,0)}_{11100} - \tfrac{1}{2}(3+2\epsilon) K^2 \rho^{(0,0)}_{11011} \\
&- \quad (1-\epsilon) \Big(\rho^{(0,0)}_{1111(-1)} + \rho^{(0,0)}_{111(-1)1} \Big) + 2K^2 \Big(\rho^{(0,0)}_{11110} + \rho^{(0,0)}_{11101} \Big) - K^4 \rho^{(0,0)}_{11111} \Big\} \,, \\
\rho_{00}|^{(g^2)}_{\mathrm{NLO}} &= 4 g^2\, C_{\mathrm{F}}\, N_{\mathrm{c}} \Big\{ (1-\epsilon) \Big(\rho^{(0,0)}_{10110} + \rho^{(0,0)}_{01101} \Big) + 2\epsilon \rho^{(0,0)}_{11100} + (1+\epsilon) k^2\, \rho^{(0,0)}_{11011} \\
&- \quad (1-\epsilon) \Big(\rho^{(0,0)}_{1111(-1)} + \rho^{(0,0)}_{111(-1)1} \Big) + 2 \big[(1-2\epsilon)\omega^2 - k^2 \big] \Big(\rho^{(0,0)}_{11110} + \rho^{(0,0)}_{11110} \Big) \\
&+ \quad 8\epsilon\, \omega \Big(\rho^{(1,0)}_{11110} + \rho^{(0,1)}_{11101} \Big) - 8(1-\epsilon)\omega \Big(\rho^{(0,1)}_{11110} + \rho^{(1,0)}_{11101} \Big) + 4\epsilon\, K^2 \rho^{(1,1)}_{11111} \\
&+ \quad \big[(1-2\epsilon)\omega^2 + k^2 \big]\, K^2 \rho^{(0,0)}_{11111} - 2(1-\epsilon) K^2 \Big(\rho^{(2,0)}_{11111} + \rho^{(0,2)}_{11111} \Big) \Big\} \,.
\end{aligned}$$

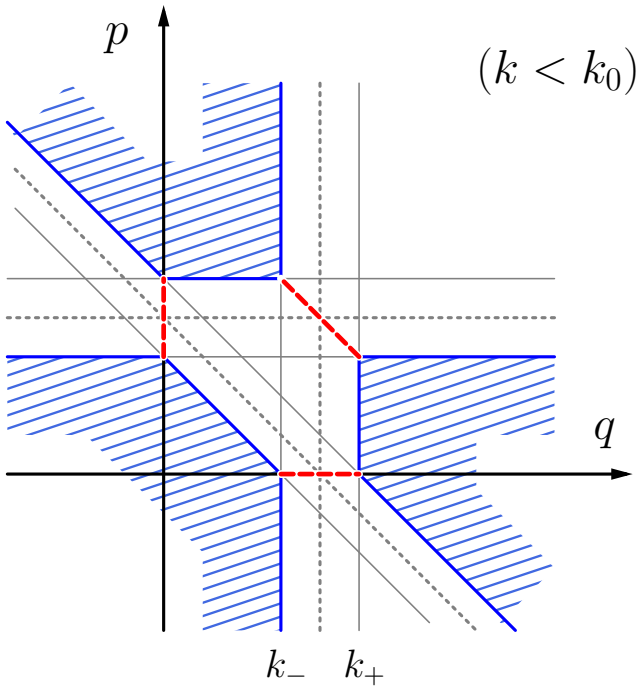
Apply general 'cutting' rules to each **master diagram** ... [Jeon (1993)]

Numerical evalution of master diags.

$$\rho^{(0,0)}_{11100} = \frac{n_0^{-1}}{(4\pi)^3} \int dp\, dq\; W(p,q)\; n_1(p-\mu_1)\, n_2(q-\mu_2)\, n_3(k_0-p-q-\mu_3)$$

$$W(p,q) = \frac{1}{2k} \Big\{ \; |p-k_+| + |q-k_+| - |p+q-k_+| - |p-k_-| - |q-k_-| + |p+q-k_-| - \min[k_0,k] \; \Big\},$$





Approach to the LPM regime

The spectral functions can be calculated from

$$\rho_{\mathrm{T,L}} = e^2\ K^2 \frac{4N}{\pi^2\, k_0}(e^{k_0/T} - 1) \int dp\, d\ell\ n_F(p) n_F(\ell) \delta(k_0 - p - \ell)\, \chi_{\mathrm{T,L}}(p, \ell)$$

$$\chi_{\mathrm{L}} = \frac{p\ell}{2k_0^2} \int_0^\infty dx\, \mathrm{Im}\, \frac{1}{[u_0^{\mathrm{reg.}}(x)]^2}$$

$$\chi_{\mathrm{T}} = \frac{p^2 + \ell^2}{2p\ell\, K^2} \int_0^\infty dx\, \mathrm{Im}\, \frac{m_D^2}{[u_1^{\mathrm{reg.}}(x)]^2}$$

$u_0^{\mathrm{reg.}}$ and $u_1^{\mathrm{reg.}}$ are the S- and P-wave channel solutions to a Schrödinger equation ...

$$\kappa = \frac{M_{\mathrm{eff}}^2}{m_D^2}\,, \quad \lambda = 8\frac{p\ell}{k_0\, T}\frac{m_\infty^2}{m_D^2}$$

$$\left[-\partial_x^2 + \frac{\alpha^2 - \frac{1}{4}}{x^2} + \kappa + \frac{i\lambda}{2\pi}\left(\log\frac{x}{2} + \gamma + K_0(x) \right) \right] u_\alpha(x) = 0$$

Boundary conditions: $\quad u_\alpha^{\mathrm{reg.}}(x) \sim x^{\frac{1}{2}+|\alpha|}\,,\ (x \to 0) \quad$ where $\alpha = \{0, 1\}$.

What we do

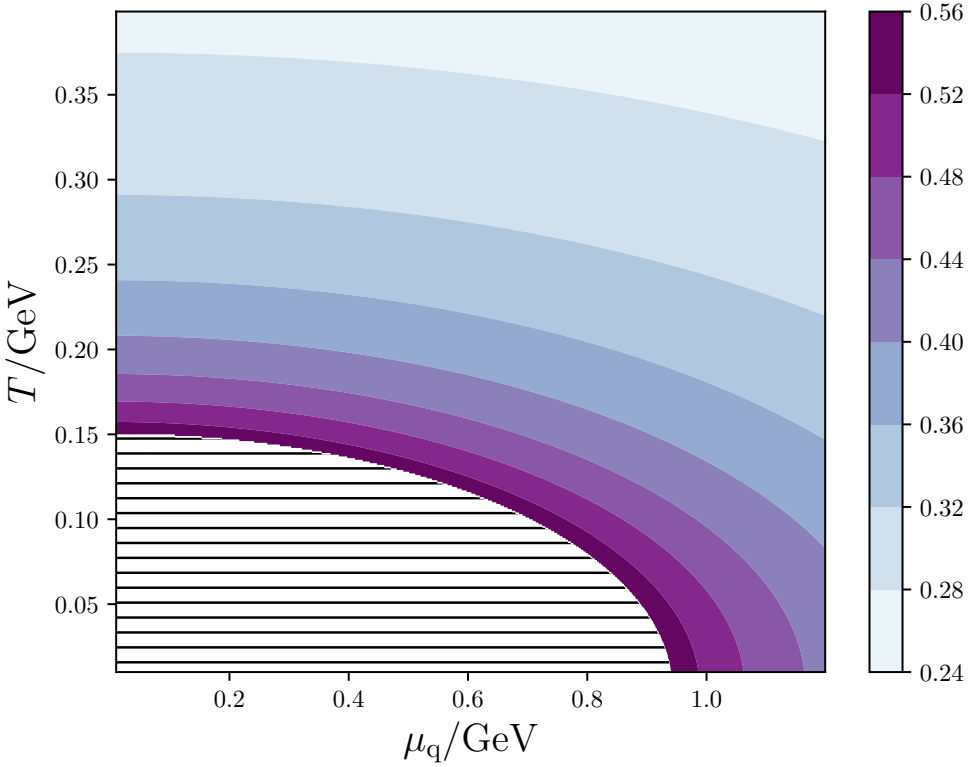
$G(\tau)$ requires knowing $\rho(\omega, k)$, for ALL frequencies:

5-loop $\alpha_{\rm s}(\mu)$ at 'optimal' scale

$$Q_{\rm opt} = \sqrt{M^2 + (\xi \pi T)^2 + \mu^2}$$

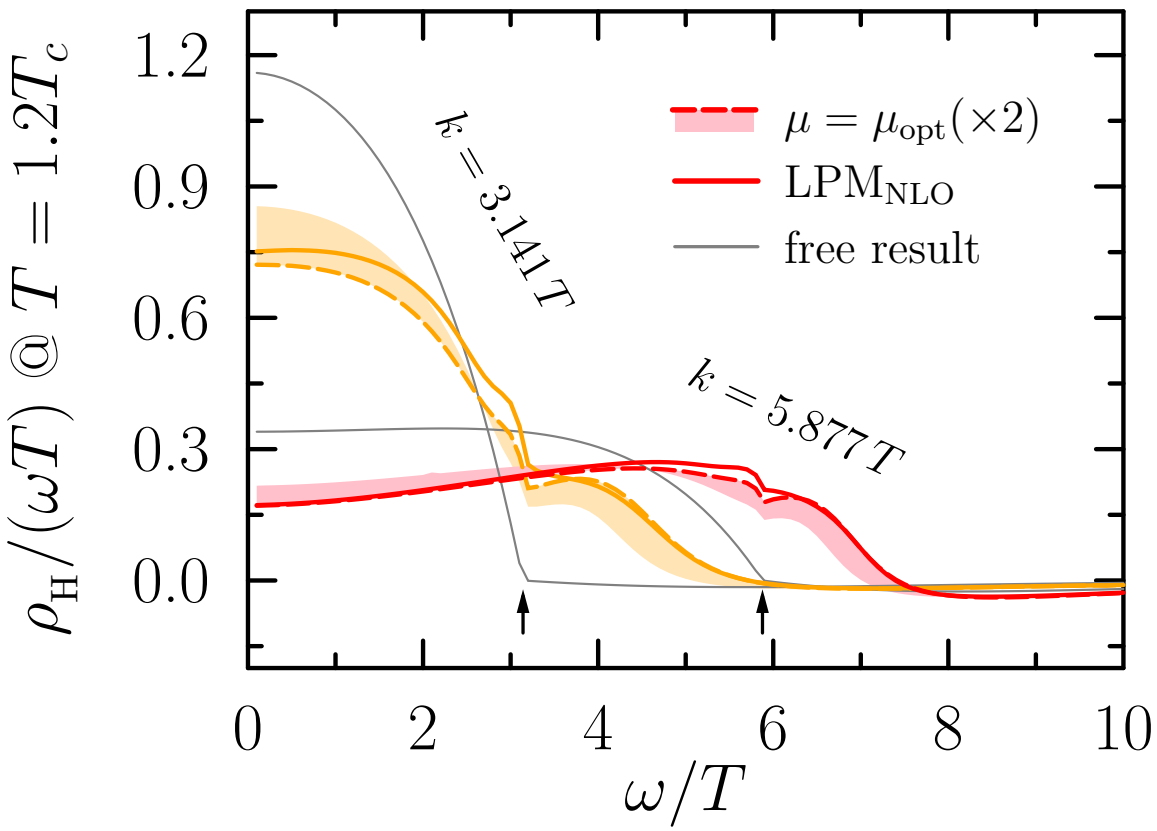
where $\xi = 1\,(2)$ for $n_f = 0\,(2,3)$

Near the light cone, $Q_{\rm opt} \sim \ldots$



What we find

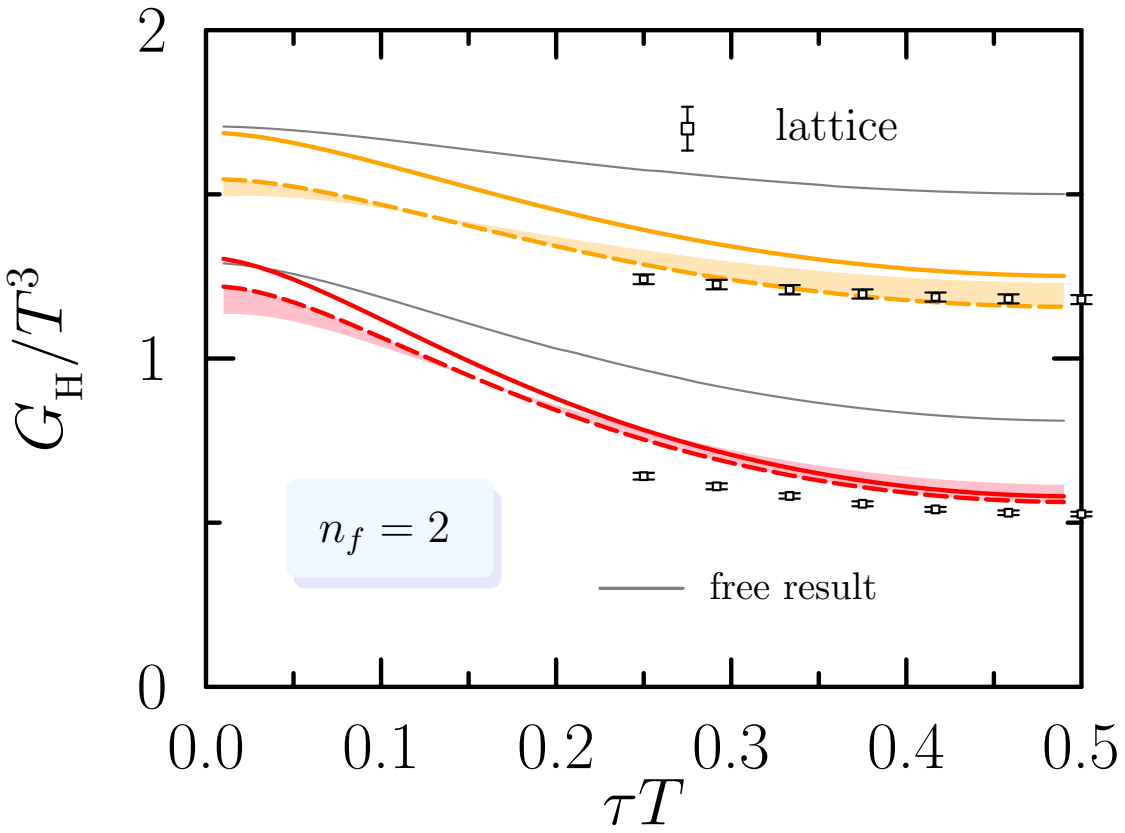
UV-finite correlator $G_{\rm H} = 2(G_{\rm T} - G_{\rm L})$



Spectral function depicted for 2-flavour QCD ($n_f = 2$)

What we find

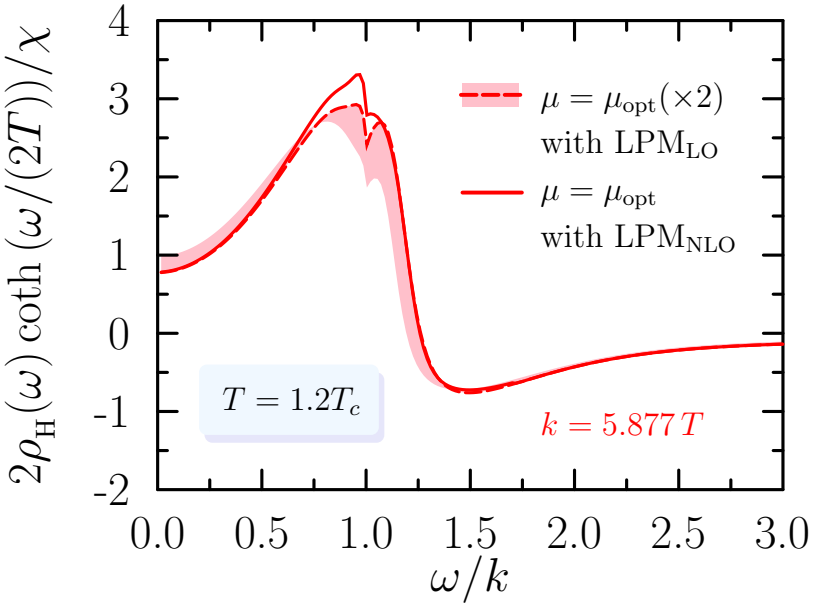
Comparing with the lattice [Cè, *et al* (2020)]



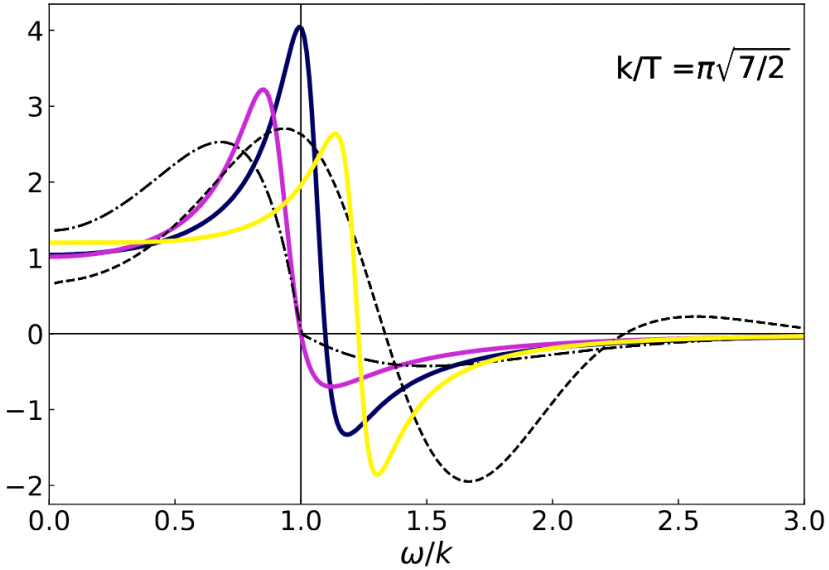
values of $\dfrac{k}{T}$ @ $T = 1.2\,T_c$: π, (3.14159) $\sqrt{7/2}\,\pi$ (5.87738)

Comparing with the lattice, $\rho_{\rm H}$ for $n_f = 2$ [Cè, *et al* (2020)]

perturbation theory



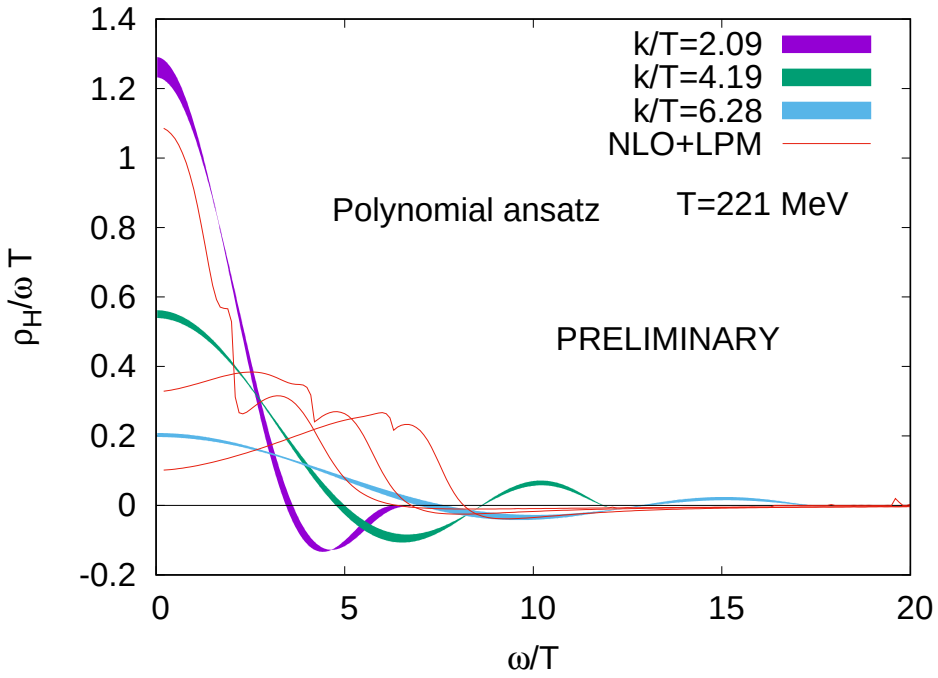
lattice reconstruction

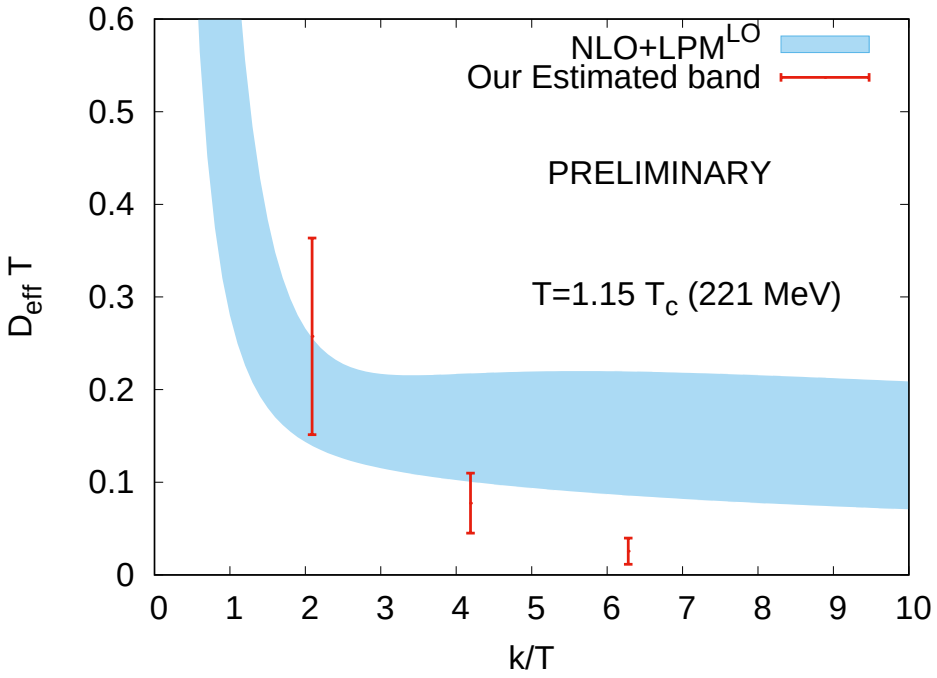


Comparing with the lattice, $\rho_{\rm H}$ for $n_f = 3$ work in progress w/ Bala, Kaczmarek

spectral fncs.

$$D_{\rm eff}(k) \equiv \rho_{\rm H}(k, \boldsymbol{k})/(2k\chi_{\rm q})$$





Should ρ_{V} be negative in the very IR?

For $\omega, k \ll T$, the **hydrodynamic** prediction gives:

$$\frac{\rho_{\mathrm{T}}}{\omega} = -\chi_{\mathrm{q}} D$$

$$\frac{\rho_{\mathrm{L}}}{\omega} = -\chi_{\mathrm{q}} D \frac{K^2}{\omega^2 + D^2 k^4}$$

[Hong, Teaney (2010)]

$D =$ diffusion coefficient

$\chi_{\mathrm{q}} =$ charge susceptibility

Therefore $\lim_{\omega \to 0} \rho_{\mathrm{V}}/\omega$ crosses zero at $k = 1/(\sqrt{2} D)$