Topological objects and the local Polyakov loop

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Overview

- Objective
  - Understand the connection between the local Polyakov loop and topological objects
  - We will look for the correlation between these on $32^3 \times 8$ finite temperature lattices
- Polyakov loop
- Topological object
  - How do we observe Topological objects $\rightarrow$ Zero-modes of overlap Dirac operator
- Results
  - 3d plots of overlap between zero-modes and local polyakov loop
  - Correlation between zero-modes and local polyakov loop

The Polyakov loop

- Polyakov loop is a wilson line that goes all the way around the complex time direction
- Is an order parameter for center symmetry

$$P(x) \quad = \quad Path(\exp(i \int_0^{1/T} A_4(x,t)dt)) \tag{1}$$

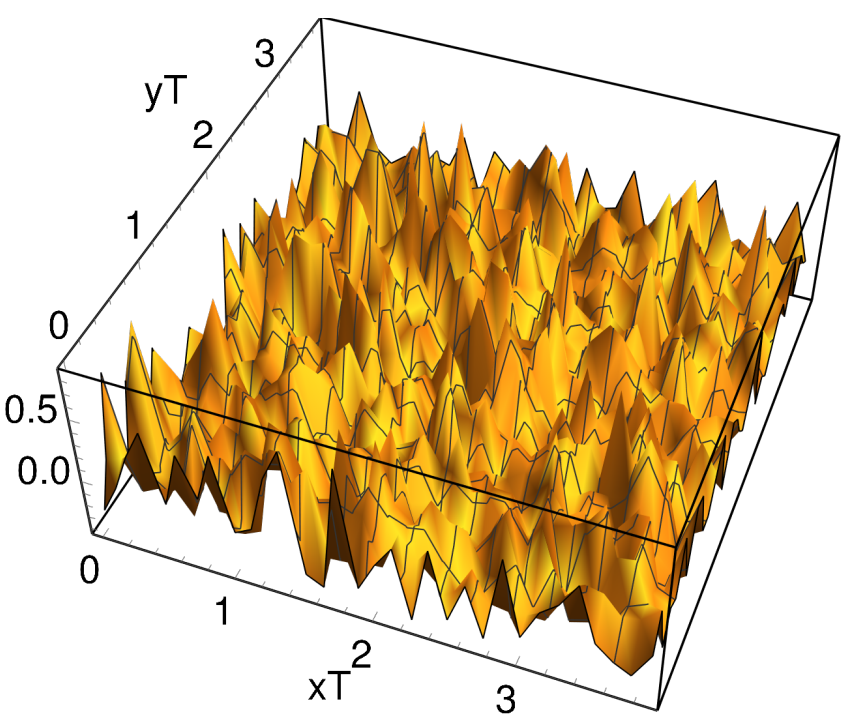
- On the lattice

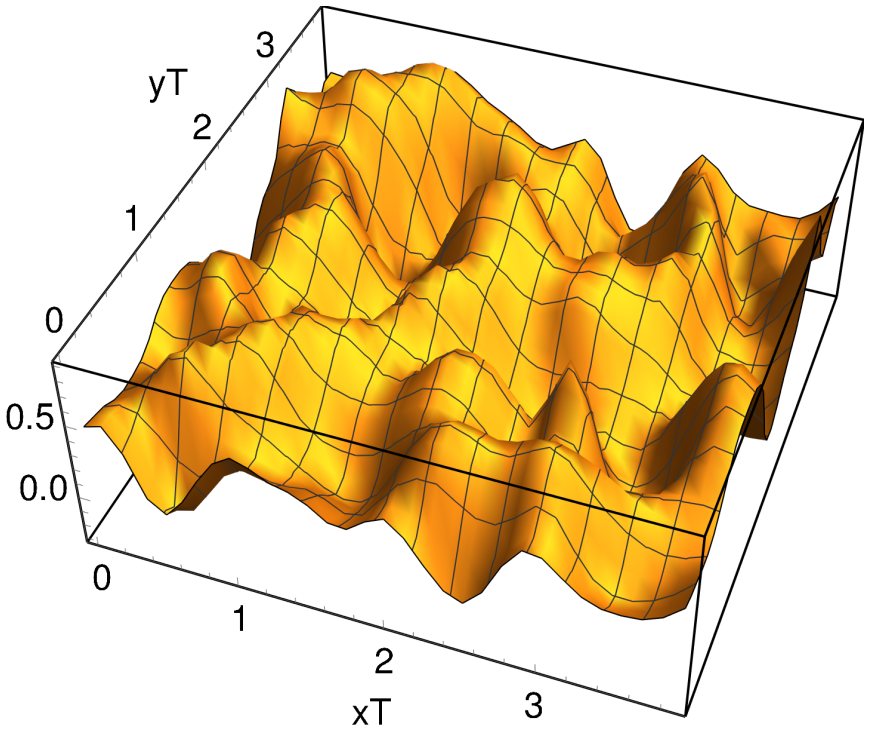
$$P(x) = \prod_{t=1}^{N_t} U_4(x,t) \tag{2}$$

- With fermions, the connections to confinement is not as clear, since fermions break center symmetry
- Normally average over x is performed
- We will look at correlation between $Tr[P(x)]/3$ and topological objects

Smeared Polyakov Loop

- Polyakov loop dominated by High Energy Noise
- Apply 10 steps of HYP smearing to smooth out, without destroying too much
- (Left) No smearing, (right) 10 steps of HYP smearing





- Real part of trace

How do we see topological objects on the lattice

- Indirect Method:
  - Find fermionic zero-modes
  - Localized around topological object
- How to find exact zero modes:
  - Use overlap Dirac operator
- Extra:
  - Calorons have $N_c = 3$ independent degrees of positions
  - Where zero-mode is localized depend on boundary conditions $\psi(\tau + 1/T) = \psi(\tau)e^{i\phi}$:
  - Will explore 3 boundary conditions $\phi = \pi$, $\pi/3$, $-\pi/3$

The overlap Dirac operator

- Needs exact zero-modes $->$ overlap Dirac operator

$$D_{ov} = 1 - \gamma_5 sign(H_W) \ , H_W = \gamma_5 (M - aD_W) \tag{3}$$

- D_W is the massless Wilson-Dirac operator
- Obey the Ginsparg-Wilson relation within numerical precision (10^{-9})

$$\gamma_5 D_{ov}^{-1} + D_{ov}^{-1} \gamma_5 \quad = \quad \gamma_5 \tag{4}$$

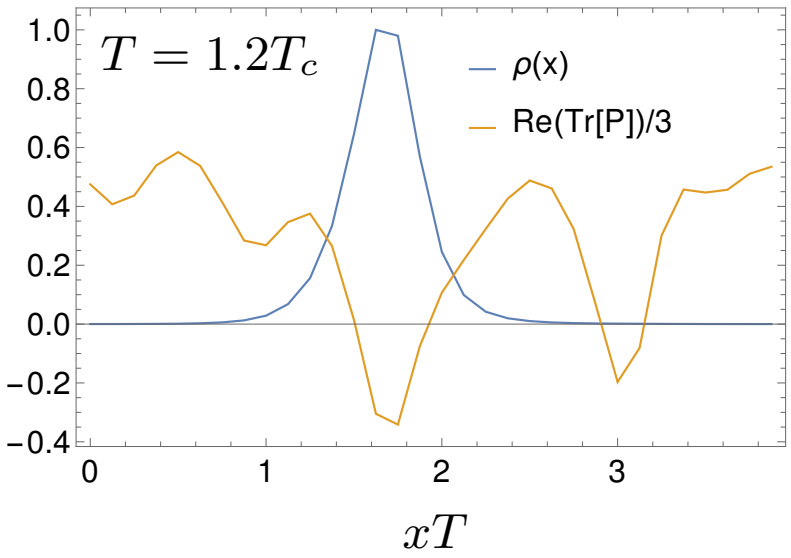
- Operator is not local
- Expensive due to having to calculate the sign
- Can make boundary condition anything we want

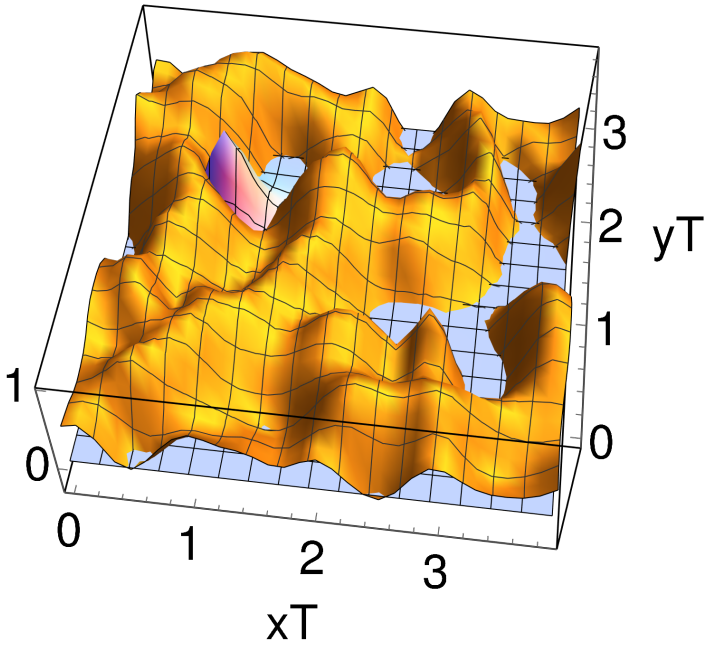
Setup

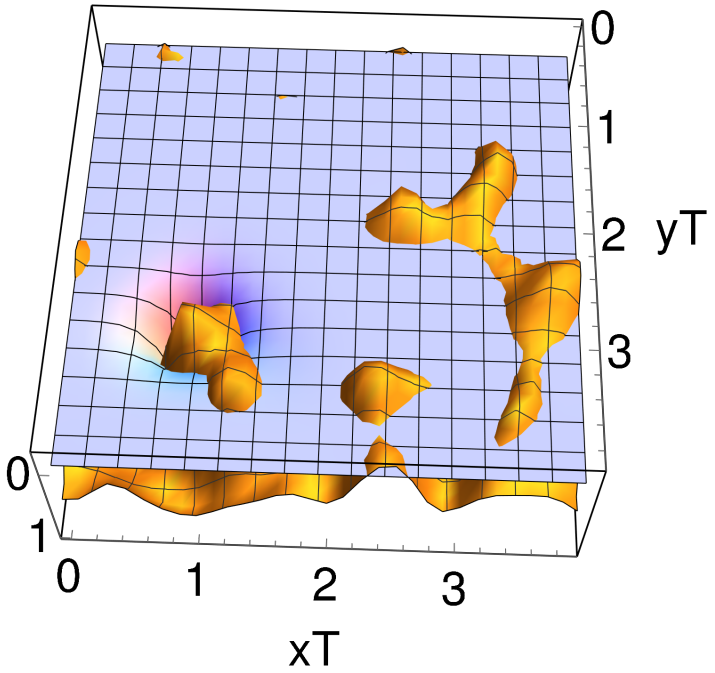
- We explore the range $T = 1.1 - 1.2T_c$
- Configurations was generated with Physical masses using domain wall fermions
- Size: $N_s = 32$ and $N_\tau = 8$
- We find the zero-modes using the overlap operator with zero fermion mass
- near zero-modes (λ around 10^{-6}) appears in pairs
- $N_c = 3$

Polyakov Loop and Zero modes

- (blue) zero-mode density and (Yellow) $Re[Tr(P(x))]/3$
- (Left) $1.1T_c$, anti periodic boundary conditions
- (right) Same plot from below, local valley seen at position of zero mode

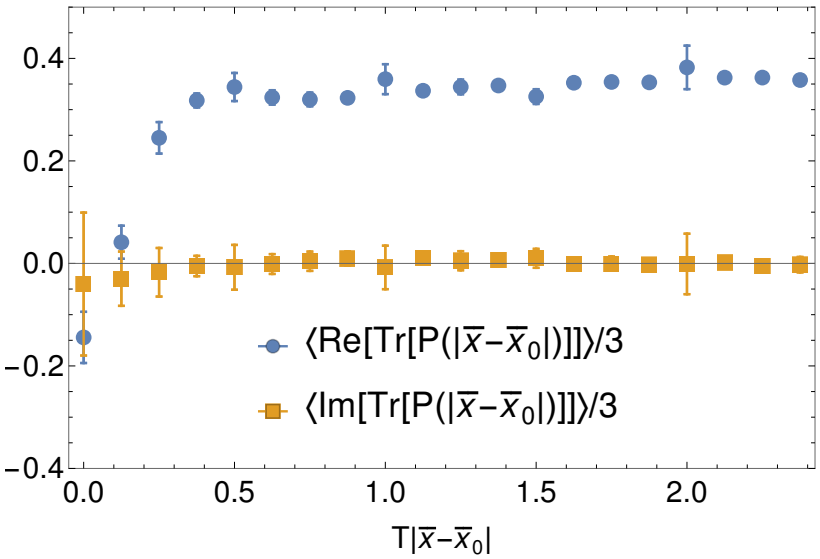


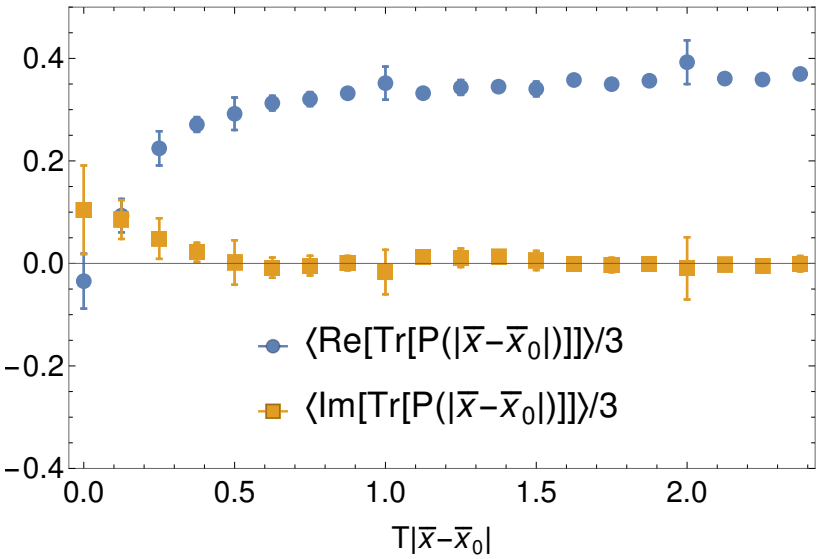




Correlating P and zero-mode

- Picture consistent between many different configurations, some part of a larger valley
- Look for average over "many" configurations
- Idea 1: Measure $< Tr(P) >$ as function of distance to zero mode



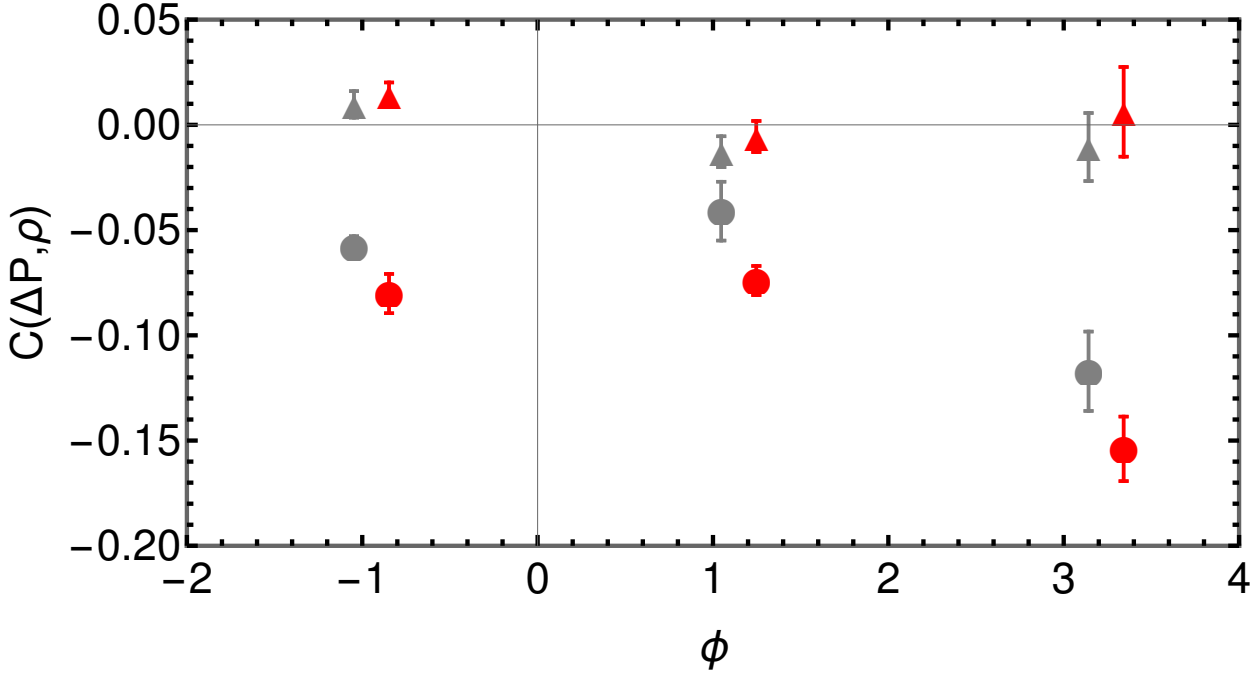


- (Left) $\phi = \pi$, (right) $\phi = -\pi/3$, $T = 1.2T_c$

Zero-mode Overlap on Polyakov loop

- Idea 2: Measure the zero mode density overlap with the local polyakov loop, compared to average P

$$C(\Delta P, \rho) = \int d^3\vec{x}\ \rho(\vec{x})\ \frac{1}{3}\left[\mathrm{Tr}[P(\vec{x})] - \langle \mathrm{Tr}[P(\vec{x})]\rangle\right]\ . \tag{5}$$



- (Grey) $T = 1.1T_c$
- (Red) $T = 1.2T_c$
- (Circle) Real part
- (Triangle) Imag part

Summary

- We have seen that the local polyakov loop is related to the position of the zero modes of the Dirac operator
- We see an anti-correlation between the local polyakov loop and the zero modes
- We hope to expand this to lower temperatures