Curci-Ferrari model for describing infrared correlation functions

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XVth Quark Confinement and the Hadron Spectrum
August 2022, Stavanger, Norway

Landau gauge Euclidean QCD Lagrangian

- Computation of correlation functions analytically requires gauge fixing.
- Euclidean gauge fixed Lagrangian via Faddeev-Popov in Landau gauge

$$\mathcal{L} = \frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + \sum_{i=1}^{N_f}\bar{\psi}_i(\gamma_\mu D_\mu + M_i)\psi_i + \underbrace{ih^a\partial_\mu A^a_\mu}_{\text{Landau gauge}} + \underbrace{\partial_\mu\bar{c}^a(D_\mu c)^a}_{\text{Ghosts}}.$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gf^{abc}A^b_\mu A^c_\nu,$$
$$D_\mu\psi = \partial_\mu\psi - igA^a_\mu t^a\psi$$

$$(D_\mu c)^a = \partial_\mu c^a + gf^{abc}A^b_\mu c^c.$$

Perturbation theory

- **Standard** perturbation theory:
  - Asymptotic freedom [1973, Politzer, Gross, Wilczek; Nobel prize 2004]
  - Landau pole in the infrared

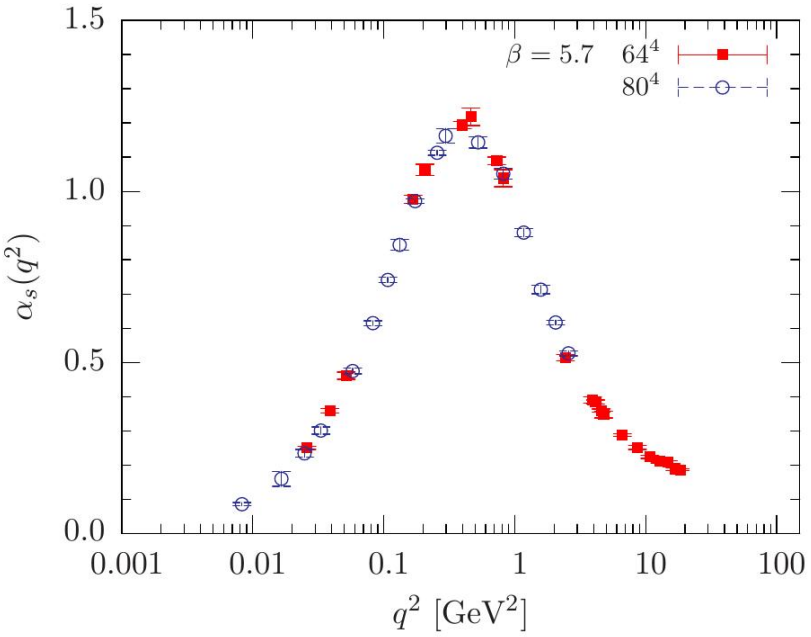
  Faddeev-Popov > Perturbation theory

➡ **Nonperturbative approaches:**

- functional renormalization group approach
- Dyson-Schwinger equations, between others
- [see Christian Fischer's talk]

Lattice simulations

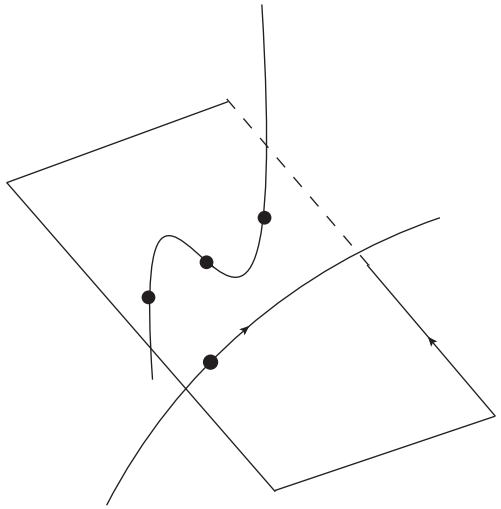
- Lattice simulations in the infrared:
  - moderate coupling constant
  - no evidence for a Landau pole
  - some kind of perturbation theory should be possible



[*Bogolubsky et al. Phys. Lett. B676 (2009)*]

Problems fixing the gauge

- Faddeev-Popov procedure is not completely justified in the infrared.
- Presence of Gribov copies



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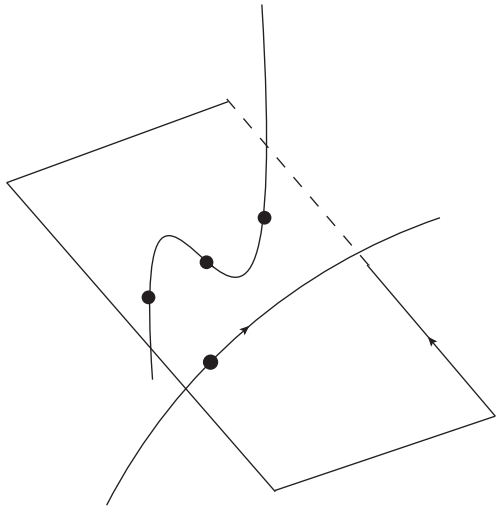


- Faddeev-Popov action needs to be extended or modified

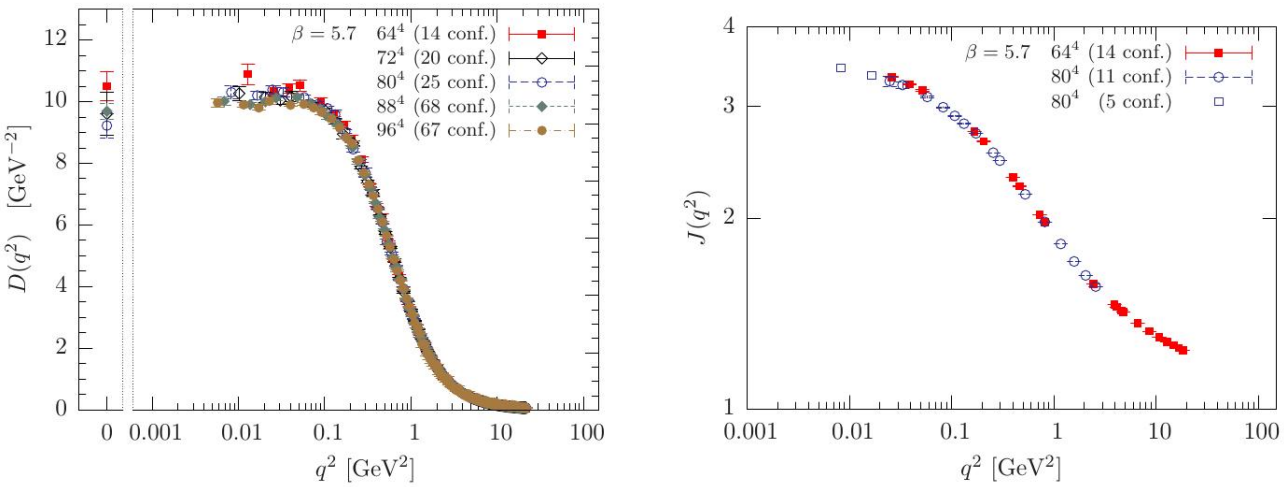
  ➡ How to find the appropriate gauge-fixed Lagrangian?

  - Studies trying to restrict the integrals to a region without Gribov copies: Gribov-Zwanziger action and refined-Gribov-Zwanziger action.

    [*Zwanziger (1989), Dudal et al (2008)*]
  - Phenomenological approach: include new operators to complete the gauge-fixing model and try to constraint their coupling

Lattice simulations

- Finite coupling constant.
- Massive gluons and massless ghosts.



[*I. L. Bogolubsky, E. M. Ilgenfritz, M. Müller-Preussker, A. Sternbeck, Phys. Lett. B676 (2009)*]

The model: Massive gluons (Curci-Ferrari)

What is the simplest Lagrangian that allows us to do perturbation theory reproducing lattice results?

- Let's try just adding a gluon mass term:

$$\mathcal{L} = \mathcal{L}_{\text{inv}} + ih^a \partial_\mu A^a_\mu + \partial_\mu \bar{c}^a (D_\mu c)^a + \frac{m^2}{2} A^a_\mu A^a_\mu$$

[*Curci-Ferrari (1975)*]

- **This term breaks BRST symmetry**. [*Becchi, Rouet, Stora (1975) and Tyutin (1975)*] But it still has a modified-BRST symmetry which allows to prove renormalizability.
- It violates positivity ... but also lattice simulations do

[Cucchieri, Mendes,Taurines Phys.Rev.D71 (2005)].

We would like to check

... if the perturbative analysis reproduces the lattice data

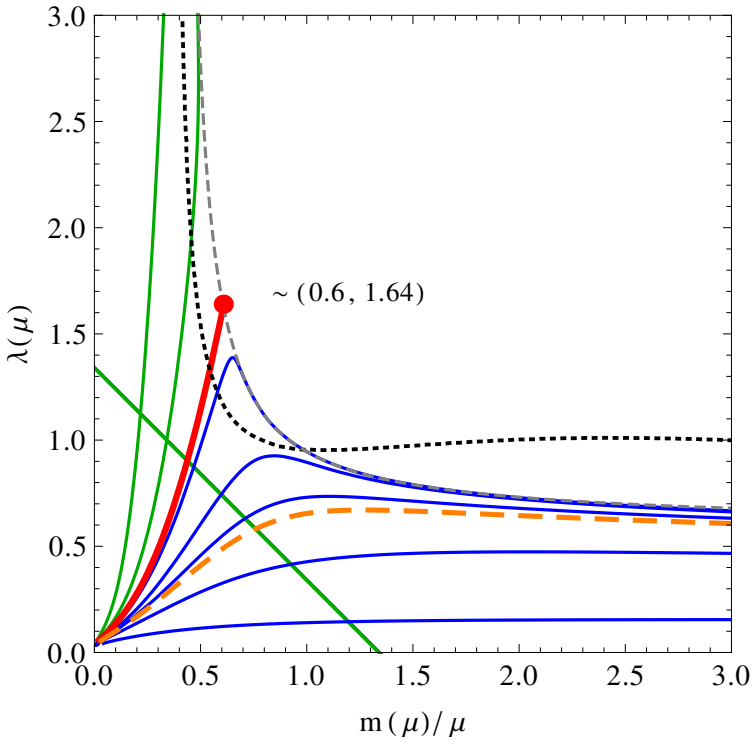
Renormalization Scheme

Infrared safe scheme:

$$\Gamma^{(2)}_{AA}(p=\mu,\mu) = \mu^2 + m^2(\mu),$$
$$\Gamma^{(2)}_{C\bar{C}}(p=\mu,\mu) = \mu^2,$$
$$Z_g\sqrt{Z_A}Z_c = 1,$$
$$Z_{m^2}Z_A Z_c = 1$$

$$\beta_g(g,m^2) = \mu\frac{dg}{d\mu}\Big|_{g_0,m_0^2},$$
$$\beta_{m^2}(g,m^2) = \mu\frac{dm^2}{d\mu}\Big|_{g_0,m_0^2},$$
$$\gamma_A(g,m^2) = \mu\frac{d\log Z_A}{d\mu}\Big|_{g_0,m_0^2},$$
$$\gamma_c(g,m^2) = \mu\frac{d\log Z_c}{d\mu}\Big|_{g_0,m_0^2}.$$

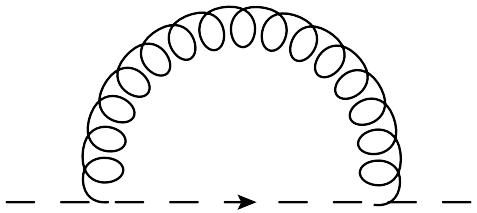
The model admits trajectories without Landau pole ($\lambda(\mu) = \frac{Ng(\mu)^2}{16\pi^2}$):



One-loop contribution

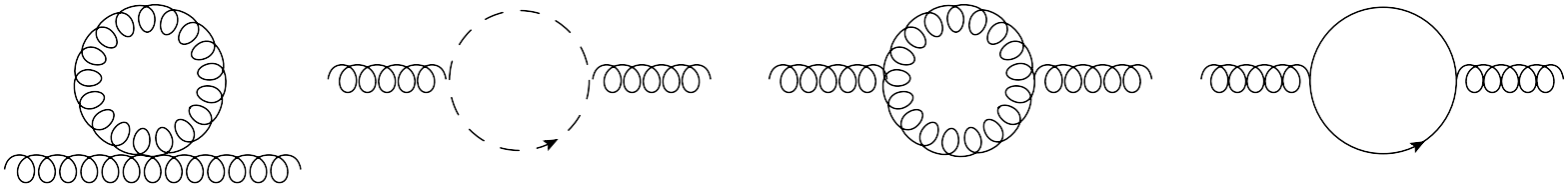
- One loop diagrams for the propagators

Ghost propagator



(e)

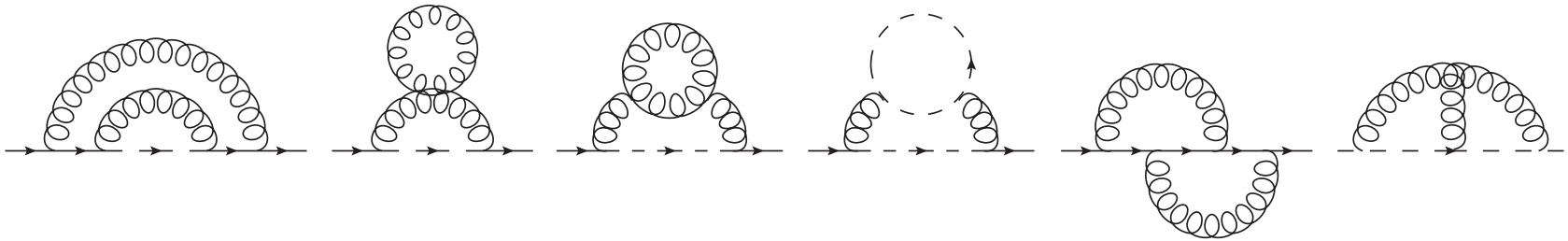
Gluon propagator



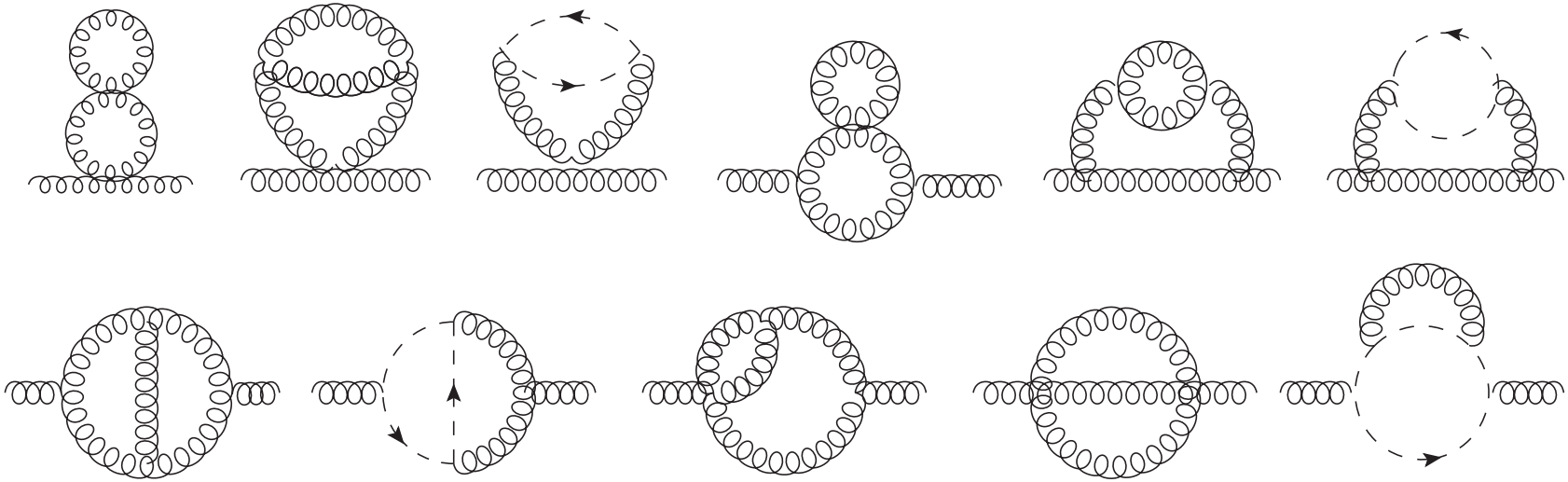
(a) (b) (c) (d)

Two-loop contribution

- Two loop diagrams for the ghost propagator



- Two loop diagrams for the gluon propagator

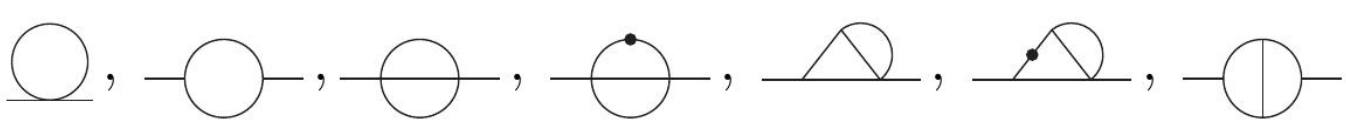


[JaxoDraw: A Graphical user interface for drawing Feynman diagrams, D. Binosi, L. Theussi]

- We use Laporta algorithm to decompose the two-loop two-point functions into master integrals

$$\Gamma^{(2)}_{AA}(p) = p^2 + m^2 + \sum_{D\in\mathcal{M}} \mathcal{R}_{AA}(D)\mathcal{I}(D)$$
$$\Gamma^{(2)}_{C\bar{C}}(p) = p^2 + \sum_{D\in\mathcal{M}} \mathcal{R}_{C\bar{C}}(D)\mathcal{I}(D)$$

where $\mathcal{R}_{AA}(D)$ and $\mathcal{R}_{C\bar{C}}(D)$ are rational functions of p^2 and m^2, and $\mathcal{I}(D)$ is a master Feynman integral, with D among

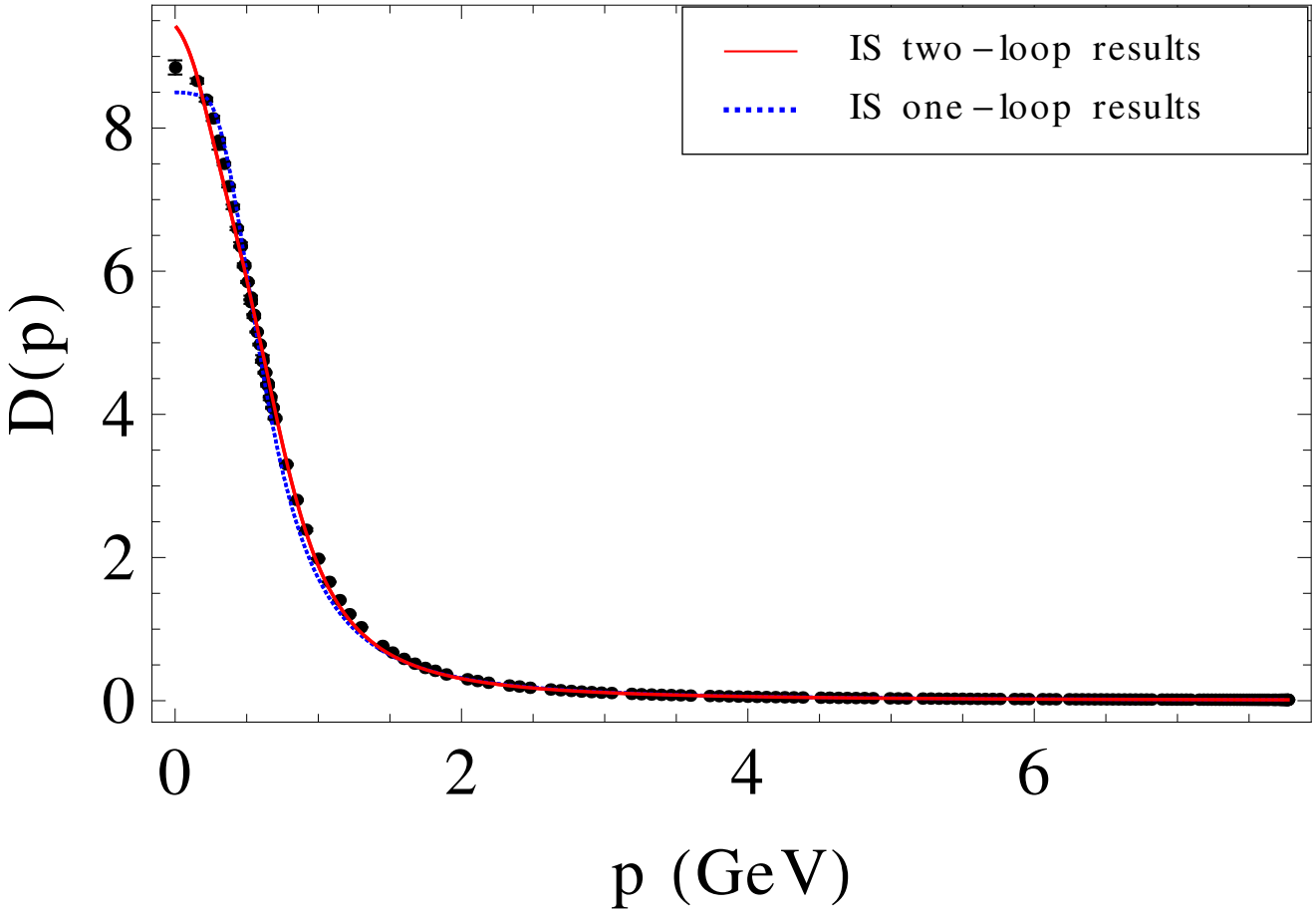
$D \in \mathcal{M} = \{$  $\}$

- We then evaluate each of the master integrals using the TsiL package. [https://www.niu.edu/spmartin/TSIL/]

- From the propagators we can compute $\beta-$functions.
- In the infrared-safe scheme, the gluon and the ghost propagators are given explicitely in terms of the running parameters.

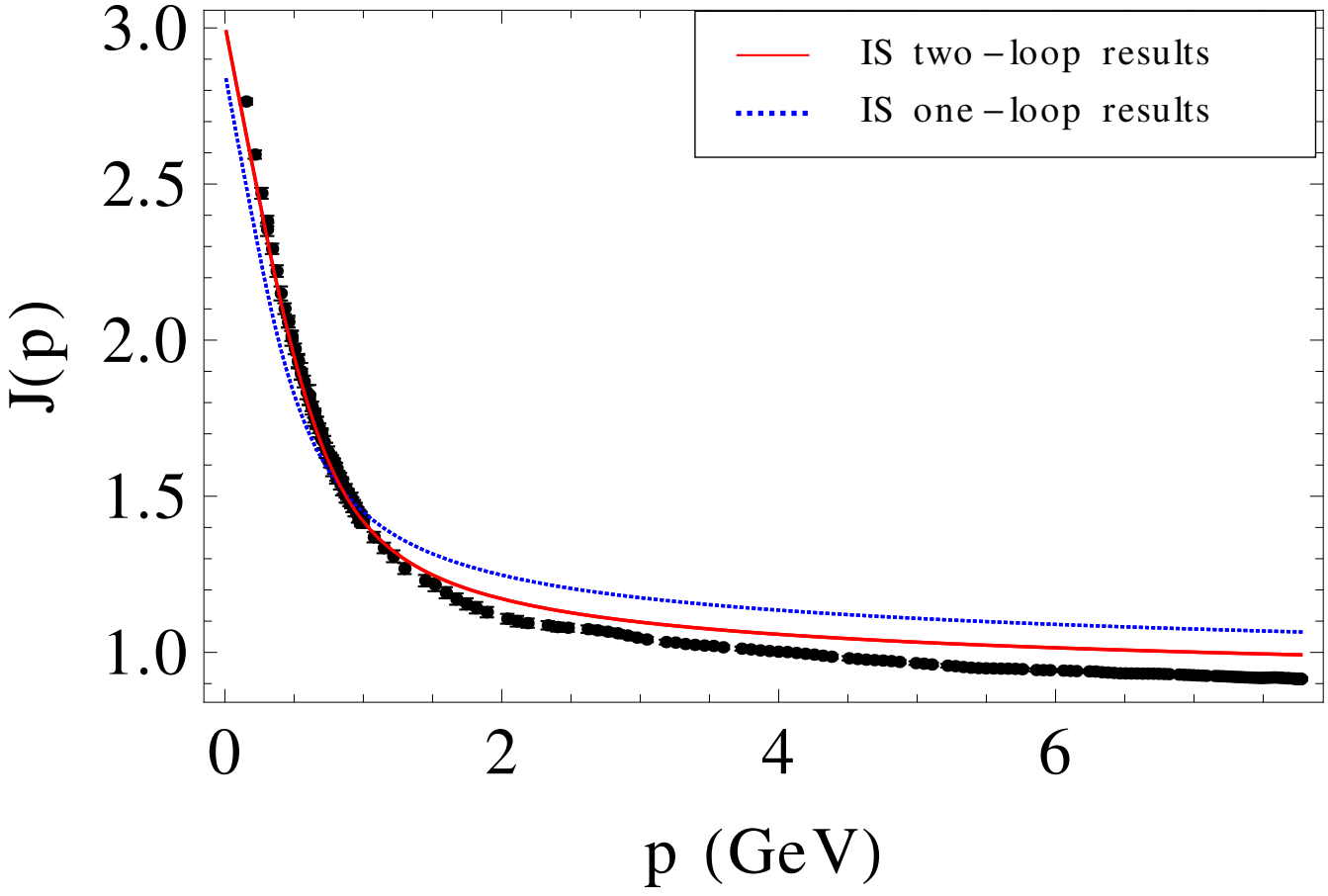
$$D(p) = \frac{g^2(\mu_0)}{m^4(\mu_0)} \frac{m^4(p)}{g^2(p)} \frac{1}{p^2 + m^2(p)}, \qquad J(p) = \frac{m^2(\mu_0)}{g^2(\mu_0)} \frac{g^2(p)}{m^2(p)}$$

Two loop results: Gluon propagator



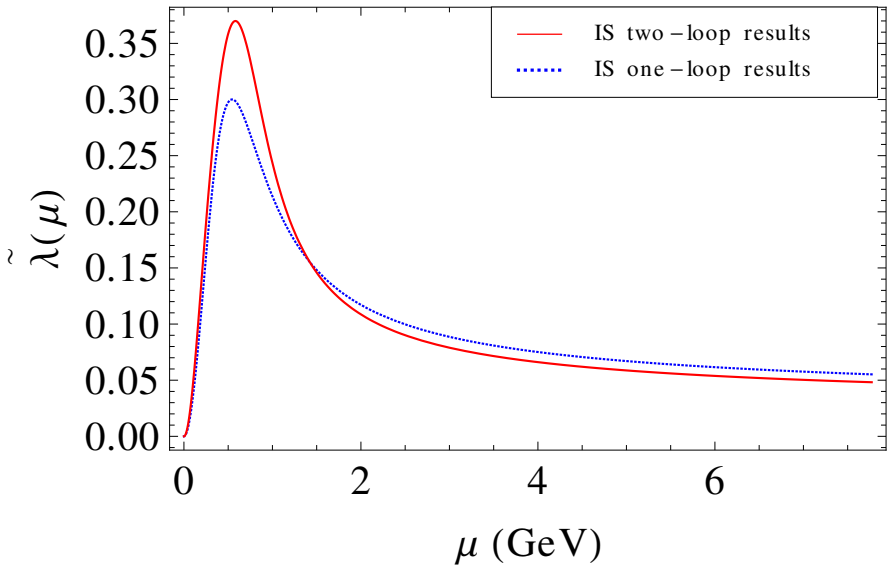
[J.A. Gracey et al. Phys.Rev.D 100 (2019) 3, 034023]

Two loop results: Ghost dressing function



[J.A. Gracey et al. Phys.Rev.D 100 (2019) 3, 034023]

Two loop results: Coupling constant



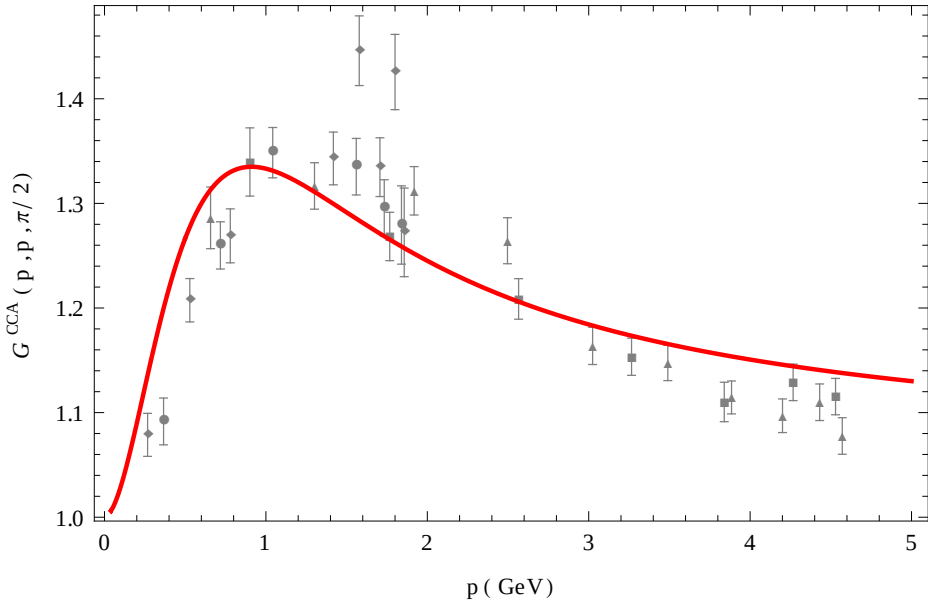
[J.A. Gracey et al. Phys.Rev.D 100 (2019) 3, 034023]

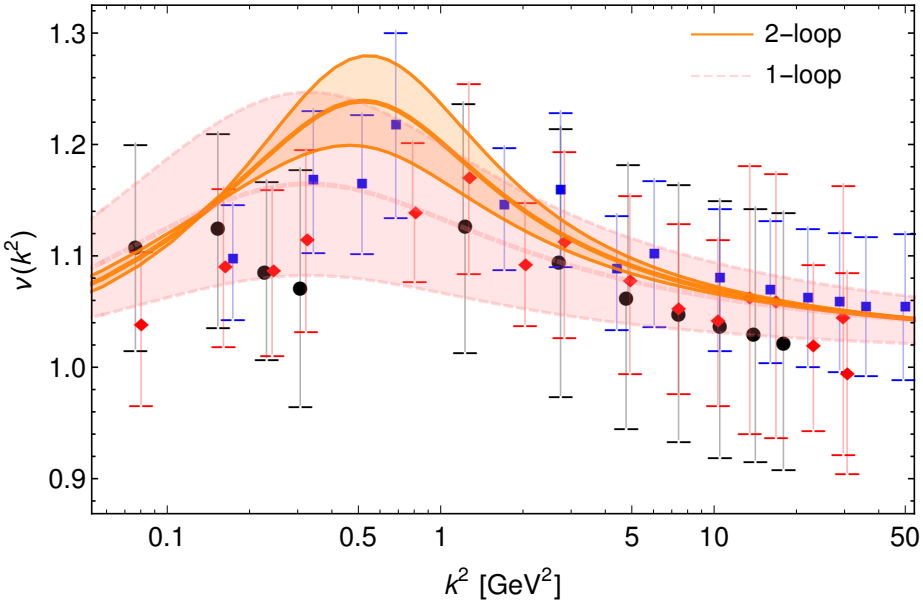
order	λ_0	m_0 (MeV)
1-loop	0.30	350
2-loop	0.27	320

Ghost-gluon vertex

- This vertex is a pure prediction of the model, since free parameters were already fixed by propagators.

One loop ghost-gluon vertex & Two-loops ghost-gluon vertex





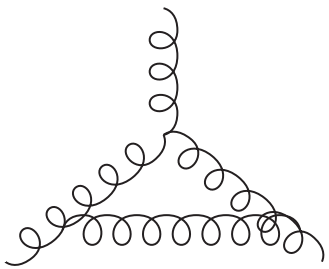
kinematics:Two orthogonal and equal norm momenta
data from: [Cucchieri et al. Phys.Rev.D 77 (2008) 094510]

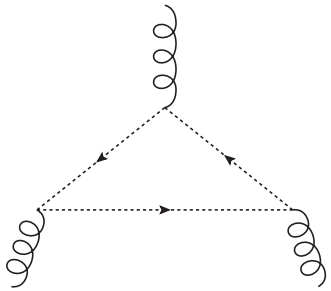
kinematics: Gluon vanishing momentum
data from: [E. Ilgenfritz et al. Braz. J. Phys. 37 (2007)]

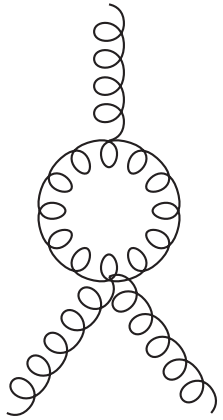
figure from [Barrios, Peláez, Reinosa, Wschebor Phys.Rev.D 102 (2020) 114016]

Three-gluon vertex: One-loop

$$\Gamma_{\mu\nu\rho}(p,k,r) = A(p^2,k^2,r^2)\delta_{\mu\nu}(p-k)_\rho + B(p^2,k^2,r^2)\delta_{\mu\nu}(p+k)_\rho$$
$$- C(p^2,k^2,r^2)(\delta_{\mu\nu}p.k - p_\nu k_\mu)(p-k)_\rho + \frac{1}{3}S(p^2,k^2,r^2)(p_\rho k_\mu r_\nu + p_\nu k_\rho r_\mu)$$
$$+ F(p^2,k^2,r^2)(\delta_{\mu\nu}p.k - p_\nu k_\mu)(p_\rho k.r - k_\rho p.r)$$
$$+ H(p^2,k^2,r^2)\left[-\delta_{\mu\nu}(p_\rho k.r - k_\rho p.r) + \frac{1}{3}(p_\rho k_\mu r_\nu - p_\nu k_\rho r_\mu)\right]$$
$$+ \text{cyclic permutations}$$

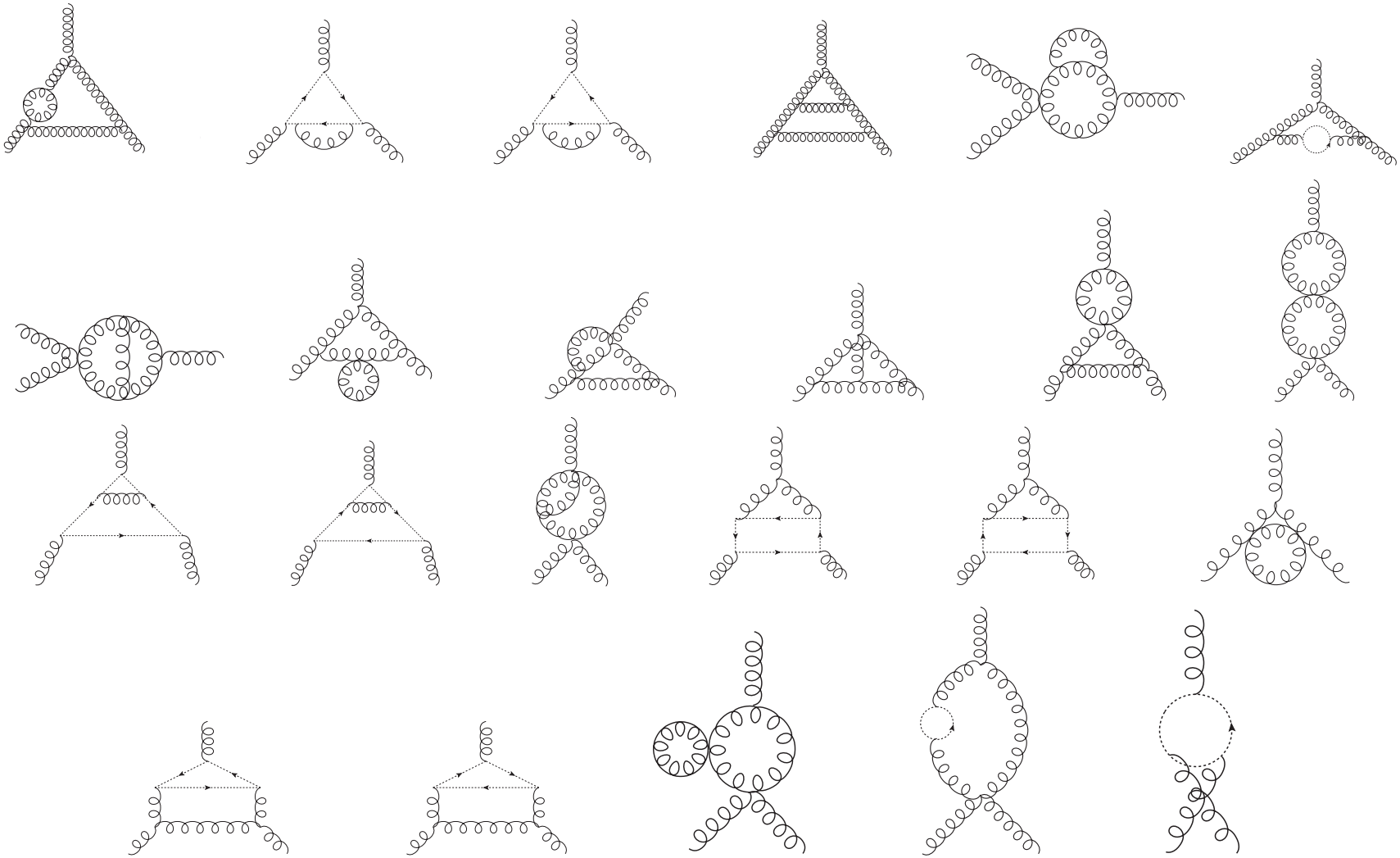






[M. Peláez, Tissier, Wschebor Phys.Rev.D 88 (2013) 125003]

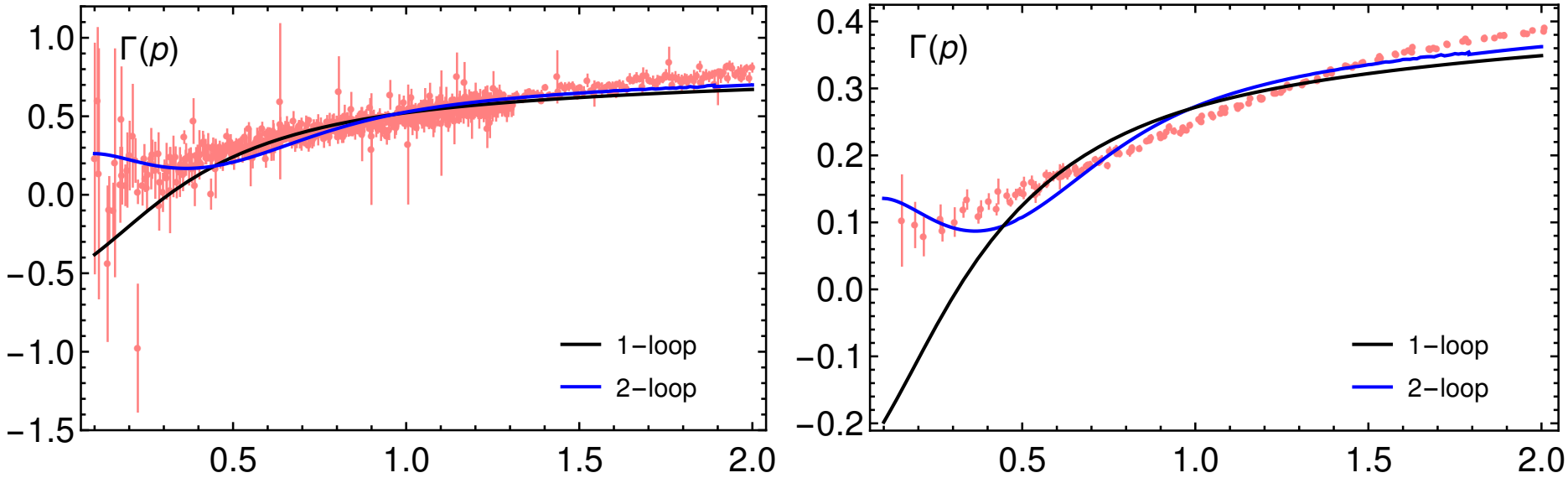
Three-gluon vertex: Two-loops



Three-gluon vertex

$$\Gamma_{\mu\nu\rho}(p,\mu) = 2\Gamma_a(p^2,\mu)\delta_{\mu\nu}p_\rho + \Gamma_b(p^2,\mu)(\delta_{\mu\rho}p_\nu + \delta_{\nu\rho}p_\mu)\,.$$

$$\Gamma(p^2,\mu) = \frac{\Gamma^{\text{tree}}_{\mu'\nu'\rho}(p)P^{\perp}_{\mu'\mu}(p)P^{\perp}_{\nu'\nu}(p)\Gamma_{\mu\nu\rho}(p,\mu)}{\Gamma^{\text{tree}}_{\mu'\nu'\rho}(p)P^{\perp}_{\mu'\mu}(p)P^{\perp}_{\nu'\nu}(p)\Gamma^{\text{tree}}_{\mu\nu\rho}(p)}\,,$$



kinematics: One Gluon vanishing momentum
data from: [A.C. Aguilar et al. Phys.Lett.B 818 (2021) 136352] data from: [G. Catumba et al,EPJ Conf. 258,02008(2022)]
figure from [Barrios, Peláez, Reinosa 2207.10704 [hep-ph]]

- The three-gluon vertex diverges as $\log p$ in the very deep infrared. This $\log p$ comes from one-loop corrections.

$$\Gamma(p^2,\mu) = 1 + \frac{\lambda(\mu)}{24}\left[\ln\left(\frac{p^2}{\mu^2}\right) + C_1(m^2/\mu^2)\right] + \frac{\lambda^2(\mu)}{24}\left[-3\tau_1 \ln\left(\frac{p^2}{\mu^2}\right) + C_2(m^2/\mu^2)\right] + \mathcal{O}\left(\frac{p^2}{m^2}\right),$$

with

$$\tau_1 = \frac{1}{4}\frac{\mu^2}{m^2}\left(\frac{m^4}{\mu^4} + \frac{5}{2}\frac{m^2}{\mu^2} + \ln\frac{\mu^2}{m^2} - \left(1+\frac{m^2}{\mu^2}\right)^3 \ln\left[1+\frac{\mu^2}{m^2}\right]\right)$$

Conclusions

To sumarize:

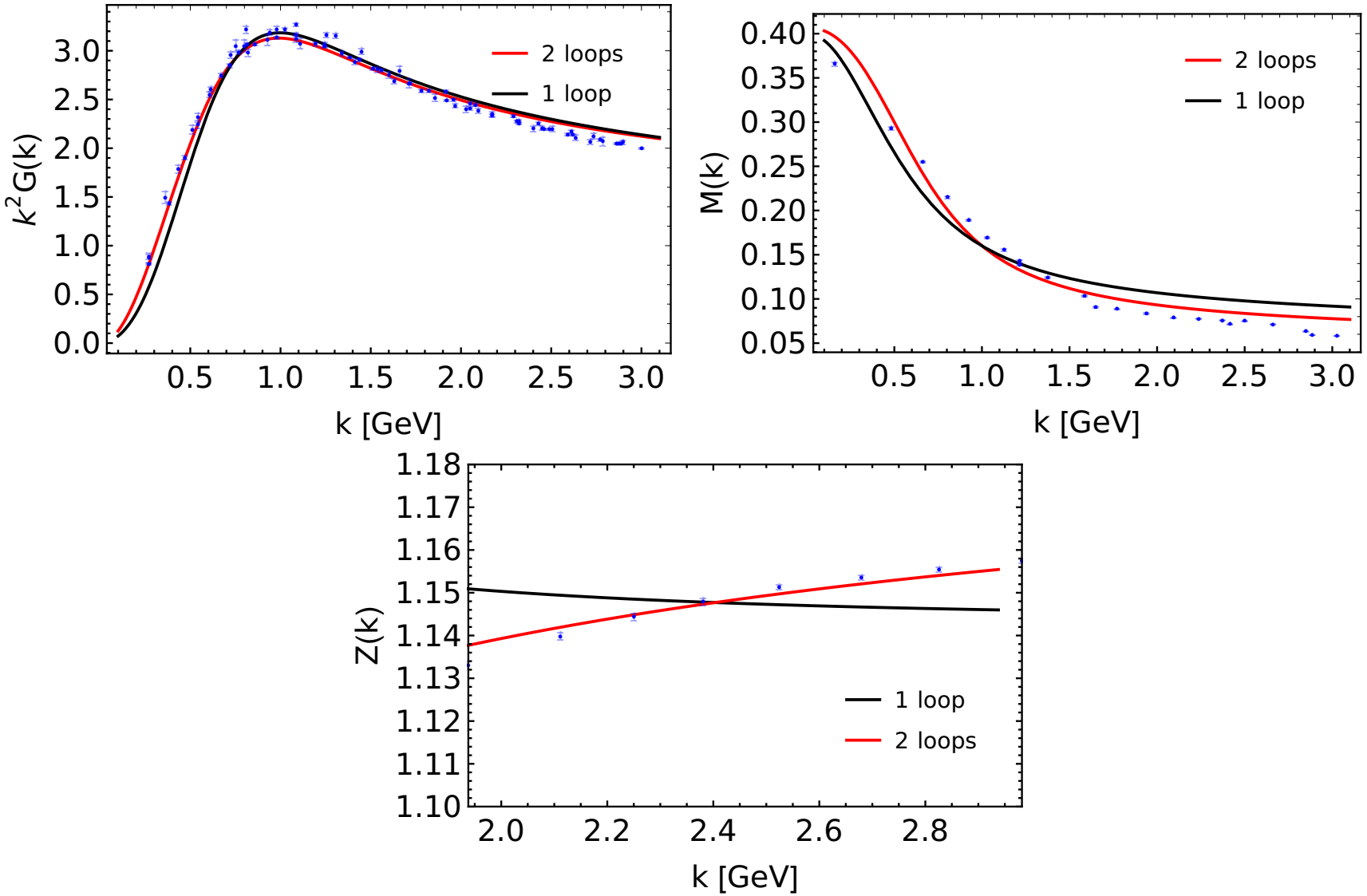
- We work in Curci-Ferrari model in Landau gauge, that includes a gluon mass term. This gluon mass could be thought as the result of a gauge fixing procedure taking into account the Gribov copies problem.
- This mass term regularizes the infrared so the renormalization group flow does not present a Landau pole.

Conclusions

- **pCF** gives very accurate results for two and three-point correlation function in **Yang-Mills sectors**.
- Within this model the Yang-Mills sector can be treated perturbately.
- We would like to include the observation that Yang-Mills sector can be studied perturbately in the computation of observables.

Thanks!

Unquenched results



Gluon dressing function, quark mass functions and scalar quark function