Study of hadron masses with Faddeev-Popov eigenmode projection in the Coulomb gauge

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Outline

- Motivation
  - How is quark model related to QCD?
  - Quark model from Coulomb gauge QCD?

- Projections in the Coulomb gauge and Results
  - $\vec{A}$ independence on the static $q\bar{q}$ potential
  - $\vec{A} = 0$ projection
  - Light hadron masses under the $\vec{A} = 0$ projection
  - Generalization of the $\vec{A} = 0$ projection
  - Faddeev-Popov (FP) eigenmode projection
  - Light hadron and glueball masses under the FP eigenmode projection

- Summary and Conclusion
Motivation
How is quark model related to QCD?

The quark (potential) model

$$V_{\text{inter quark}} = V_{\text{static q\bar{q}}} + V_{\text{color-magnetic}}, \quad m_Q \approx 0.3 \text{ GeV}$$

is successful in phenomenological description of hadrons. But, why?

Big differences between quark model and QCD

- absence of dynamical gluons
- global color SU(3) symmetry, not local SU(3) symmetry
- spatial-rotation symmetry, not full Lorentz symmetry

The large constituent quark mass $m_Q \approx 0.3 \text{ GeV}$ seems to be a gauge-dependent quantity.

cf. studies on quark propagator in the Landau gauge

[Skullerud, Williams, Leinweber, Bowman, Heller, Bonnet, Zhang, Parappilly, ...]
Quark model from Coulomb gauge QCD?

Quark model may be an effective theory of QCD in the Coulomb gauge.

Global definition of the Coulomb gauge

Minimization of

\[ R_C[A] = \sum_{i=x,y,z} \sum_{a=1}^{8} \int dt d^3s \left( A_{s,i}^a(t) \right)^2 \]

under the local color SU(3) gauge transformation

- minimize spatial gauge-field fluctuations
  \leftrightarrow \text{absence of dynamical gluons?}
- leave global color SU(3) symmetry
- leave spatial-rotation symmetry

We seek the relation between quark model and Coulomb gauge QCD by new-type projections of gauge configurations.

cf. studies on QCD vacuum by Abelian or center projections
Projections in the Coulomb gauge and Results
In quark model, static $q\bar{q}$ potential $V_{\text{static } q\bar{q}}$ is a basic constituent.

In QCD, $V_{\text{static } q\bar{q}}$ can be calculated from Polyakov loops.

$\vec{A}$ independence on $V_{\text{static } q\bar{q}}$ at the gauge configuration generated in lattice QCD in the Coulomb gauge.

For an explicit check, we here define $\vec{A} = 0$ projection

$$\left\{A^{a,\text{projected}}_{s,\mu} := \left(A^{a}_{s,0}, 0\right)\right\}$$

and apply to the static $q\bar{q}$ potential.

- calculated from Wilson loop
- very good agreement
Our strategy

In lattice QCD simulations in the Coulomb gauge, we can keep the original $V_{\text{static } q\bar{q}}$ by just leaving the temporal gauge fields $A_0$ unchanged.

We utilize the $\vec{A}$ independence on $V_{\text{static } q\bar{q}}$ to investigate how other basic constituents in the quark model are encoded in QCD.

e.g. color-magnetic interactions, $m_Q \simeq 0.3 \, \text{GeV}$

What we do

We perform projections of spatial gauge fields in gauge configurations, generated in quenched lattice QCD simulations in the Coulomb gauge on a $16^3 \times 32, \beta = 6.0$ lattice.
Light hadron masses under the \( \vec{A} = 0 \) projection:

\[ m_\rho = 0.409(17) \text{ GeV}, \quad m_N = 0.607(44) \text{ GeV}, \quad m_\Delta = 0.658(49) \text{ GeV} \]

Suggestions from the results:

\[ m_N \approx m_\Delta \implies \text{Vanishing of the color-magnetic interactions} \]

Overall decrease of hadron masses

\( \implies \text{constituent quark with smaller mass of } \approx 0.2 \text{ GeV} \)
Generalization of $\vec{A} = 0$ projection

Actually, it is natural to consider that color-mag. int. vanishes under the $\vec{A} = 0$ projection because

$$\vec{A} = 0 \quad \Rightarrow \quad B_i^a = \epsilon_{ijk} \left( \partial_j A_k^a - \partial_k A_j^a - g f^{abc} A_j^b A_k^c \right) = 0.$$ 

In transformation from $\vec{A} = 0$ configuration to the original one,

- Emergence of color-magnetic interactions
- Generation of constituent quark mass: $m_Q \approx 0.2 \text{ GeV} \rightarrow 0.3 \text{ GeV}$ should happen.

Eigenmode projection: a generalization of $\vec{A} = 0$ projection

We develop a generalization of the $\vec{A} = 0$ projection, which smoothly connects these configurations, utilizing the Faddeev-Popov (FP) operator.

cf. FP operator is studied in the context of Gribov horizon, instantaneous color-Coulomb potential, and gluon-chain picture.

[Gribov, Zwanziger, Szczepaniak, Swanson, Greensite, Thorn, ’t Hooft,...]
Faddeev-Popov (FP) eigenmode projection

Since FP operator $M(t) = \partial \cdot D$ is real symmetric, its eigenmodes

\[ M(t)\psi_n(t) = \lambda_n(t)\psi_n(t), \quad \lambda_n, \psi_n(t) \in \mathbb{R} \]

form a complete set

\[ \sum_{n=1}^{8L_s^3} \psi_{n,s}^a(t)\psi_{n,s'}^a(t) = \delta_{s,s'}\delta^{a,a'}. \]

Spatial gauge fields can be expanded as

\[ A_{s,i}^a(t) = \sum_{n=1}^{8L_s^3} c_{n,i}(t)\psi_{n,s}^a(t), \quad c_{n,i}(t) := \sum_{s,a} A_{s,i}^a(t)\psi_{n,s}^a(t). \]

We here define eigenmode projection

\[ A_{s,\mu}^{a,\text{projected}} := \left( A_{s,0}^a, \sum_{n \leq N_{\text{cut}}} c_{n,i}\psi_{n,s}^a \right), \quad \lambda_1 \leq \cdots \leq \lambda_{8L_s^3} \]

with a cut-off number $N_{\text{cut}}$. 
Meaning of the eigenmode projection

Eigenmode projection

\[
\begin{align*}
\{ A_{s,\mu}^{a,\text{projected} } \} := & \left( A_{s,0}^{a}, \sum_{n \leq N_{\text{cut}}} c_{n,i} \psi_{n,s}^{a} \right), \quad \lambda_1 \leq \cdots \leq \lambda_{8L_{s}^{3}}
\end{align*}
\]

The eigenmode projection smoothly connects the $\vec{A} = 0$ projected gauge configuration with the original one:

\[
N_{\text{cut}}/8L_{s}^{3} = 0 \% \implies \vec{A} = 0 \text{ projected conf.}
\]

\[
\vdots
\]

\[
N_{\text{cut}}/8L_{s}^{3} = 100 \% \implies \text{the original conf.}
\]

The $n$-th smallest eigenvalue $\sqrt{\lambda_n}$ plotted against $n/8L_{s}^{3}$

Eigenmode projection introduces energy scales in the analysis.
Under the $\vec A = 0$ projection, N – $\Delta$ mass splitting approximately vanishes.

With $N_{\text{cut}}/8L_s^3 = 0.1\%$ low-lying eigenmodes, N – $\Delta$ mass splitting starts to emerge.

With only 1% low-lying eigenmodes, light hadron masses are reproduced.

An important role of low-lying eigenmodes on hadron masses.
Glueball mass under the eigenmode projection

Near the $\vec{A}=0$ projection, $0^{++} - 2^{++}$ glueball mass splitting seems to vanish.

With $N_{\text{cut}}/8L_s^3 = 0.1\%$ low-lying eigenmodes, $0^{++} - 2^{++}$ glueball mass splitting starts to emerge.

With only 1% low-lying eigenmodes, glueball masses are approximately reproduced.

Similar results also in the case of glueball masses.
Summary and Conclusion
Summary and Conclusion

We have performed the projections of spatial gauge fields with low-lying Faddeev-Popov eigemodes.

What we have found

- Near the \( \vec{A} = 0 \) projection, mass splitting between different spin states approximately vanishes.
- With \( 0.1\% (N_{\text{cut}} = 33) \) low-lying eigenmodes, mass splitting starts to emerge.
- With \( 1\% (N_{\text{cut}} = 328) \) low-lying eigenmodes, hadron masses are approximately reproduced.

The energy scales of emergence of color-magnetic interactions and generation of constituent quark mass are estimated to be 0.5 GeV and 1.3 GeV, respectively.

Quark model might be derived from Coulomb gauge QCD by a reduction of high-lying spatial gluons above 1.3 GeV.
Backup
Future studies

- Check the effects of dynamical quarks using full lattice QCD simulations

- Check the effects of Gribov copies by comparing the results obtained from gauge configurations with different $R_C[A]$.

- Perform the eigenmode projections with different operators, such as the Laplacian, to check the FP operator is special or not in the current approach.

- Study the eigenmode projection in the Landau gauge, where one can investigate the mechanism of quark confinement in the Landau gauge.
Vanishing of $0^{++} - 2^{++}$ glueball mass splitting

Glueball mass measurement under the eigenmode projection with 33 (0.10\%) low-lying eigenmodes

Correlation functions are very alike near the $\vec{A} = 0$ projection.
Energies of low-lying eigenmodes

The \( n \)-th smallest eigenvalue \( \lambda_n \) as a function of \( n/8L_s^3 \)

- The 33-smallest eigenvalue corresponds to the low-lying eigenmode of about \((0.5 \text{ GeV})^2\).
- The 328-smallest eigenvalue corresponds to the low-lying eigenmode of about \((1.3 \text{ GeV})^2\).
Eigenmode projection with high-lying eigenmodes

We have also applied the eigenmode projection with high-lying eigenmodes on light hadron masses.

Hadron masses can be hardly reproduced with high-lying eigenmodes, unlike the case of low-lying eigenmodes.
To perform the eigenmode projection with $N_{\text{cut}}$ low-lying eigenmodes, one needs to obtain $N_{\text{cut}}$ low-lying eigenmodes or $8L_s^3 - N_{\text{cut}}$ high-lying eigenmodes.

In our study, we calculated 1638 low-lying and high-lying eigenmodes for each gauge configuration and time slice using the Lanczos algorithm. Actually, we solved the large-scale eigenvalue problems

$$500(N_{\text{conf}}) \times 32(L_t) \times 2(\text{smallest and largest}) \text{ times.}$$

Since $1638/8L_s^3 \approx 5\%$, we cover

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