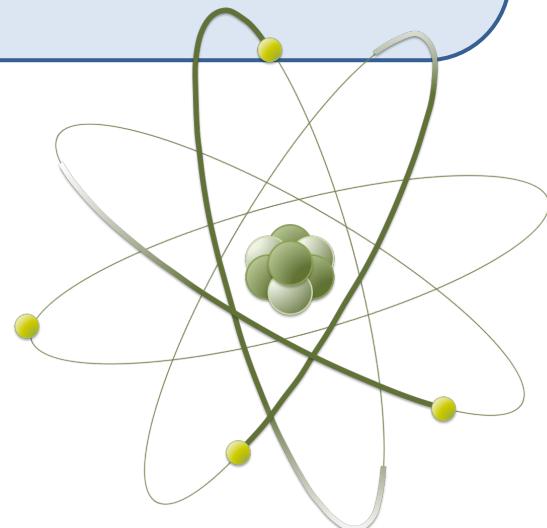


Phase structure of pure Yang-Mills theory in an anisotropic system: A new extreme condition of QCD

Daiki Suenaga (RIKEN in Japan)

Masakiyo Kitazawa (Osaka U. in Japan)



1. Introduction

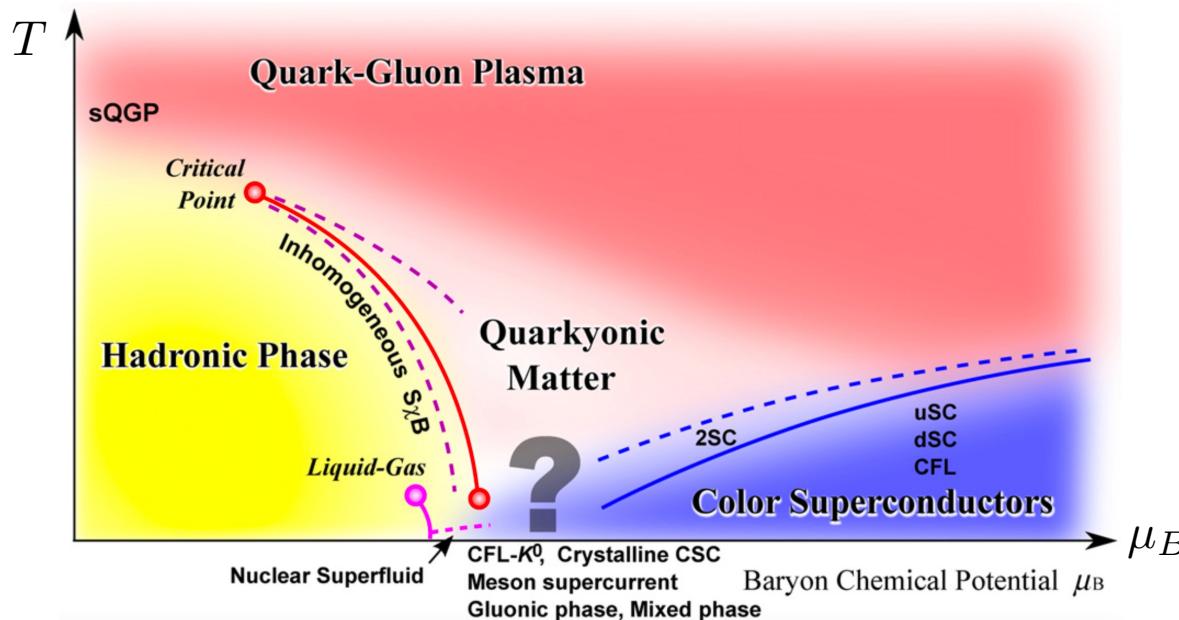
- QCD at extreme conditions

- Extreme conditions such as temperature and density are useful toward better understandings of QCD



- For instance, rich phase diagrams can be drawn

[eg Fukushima-Hatsuda (2017)]



- Quark deconfinement, quark-gluon plasma, inhomogeneous chiral broken phase, chiral restoration, color superconductivity, etc.

1. Introduction

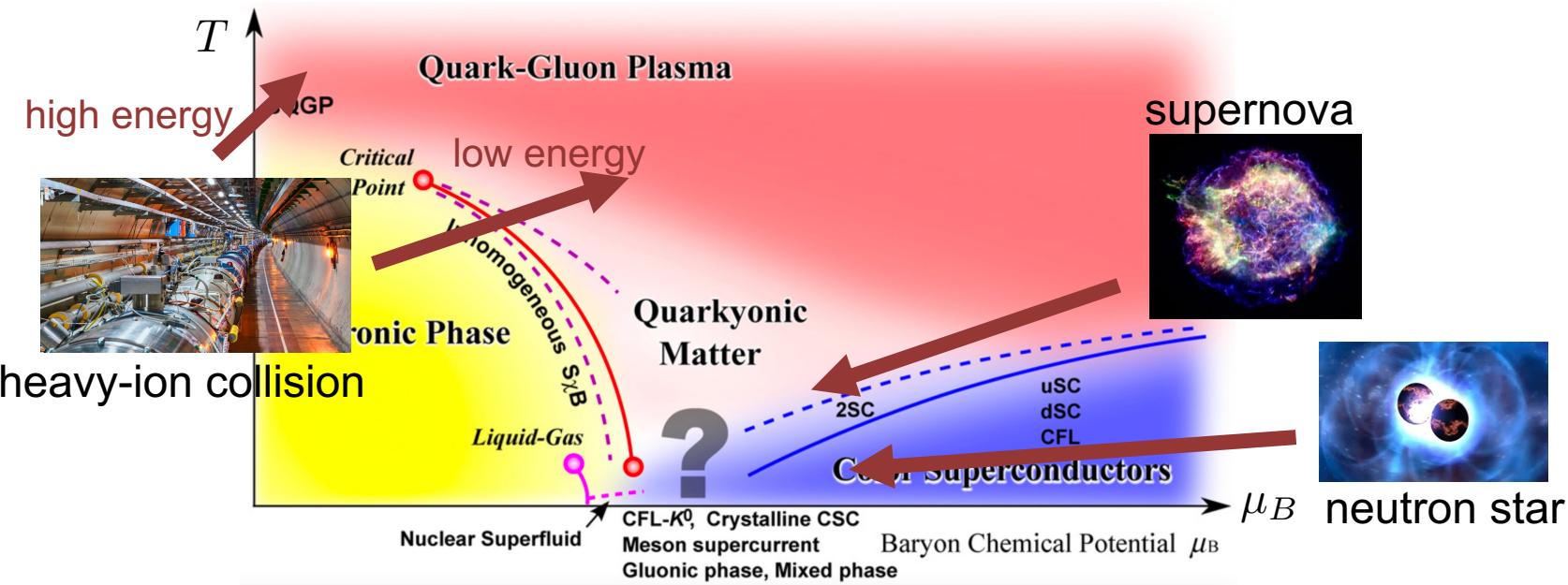
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- Heavy-ion collision experiments and neutron star/supernova observations can explore the phase structures

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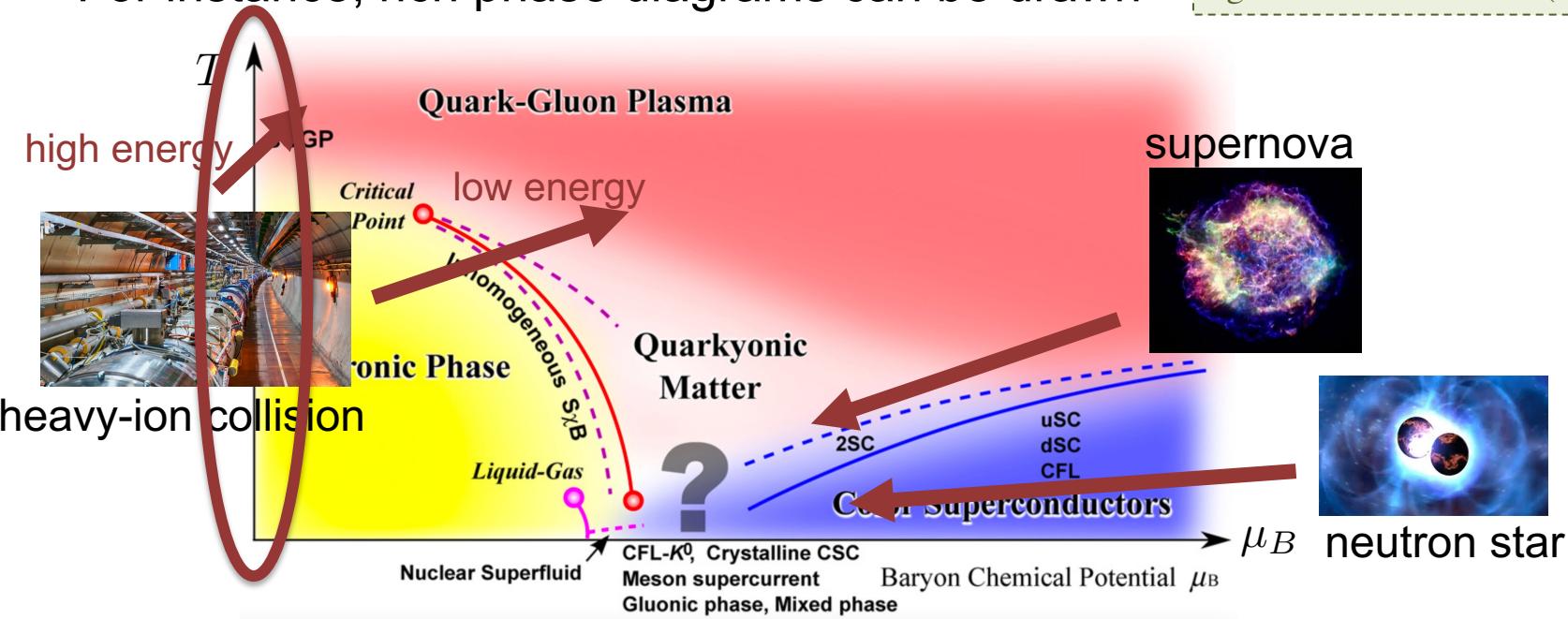
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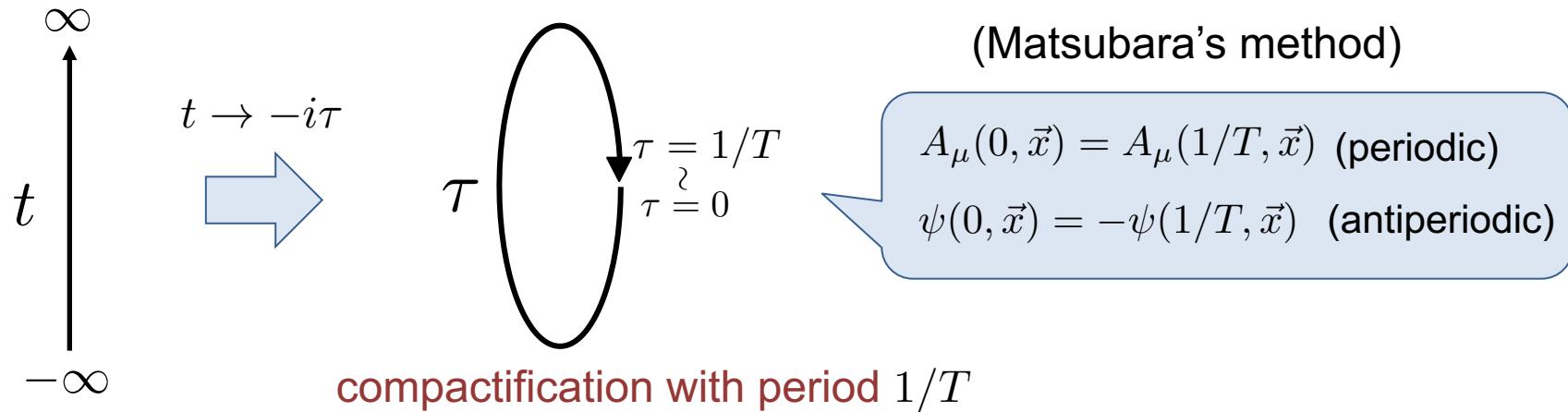


- Heavy-ion collision experiments and neutron star/supernova observations can explore the phase structures

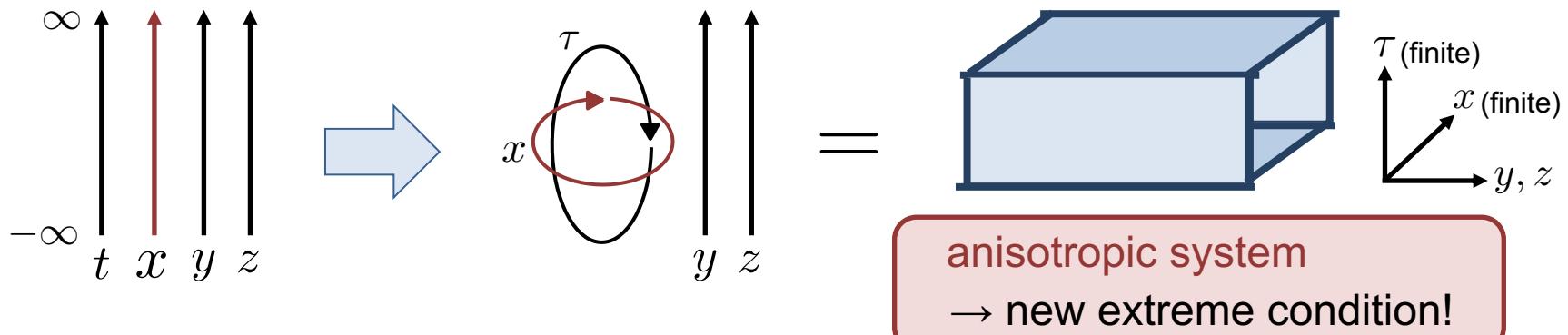
1. Introduction

• Compactification

- Finite temperature system is realized by compactifying imaginary time



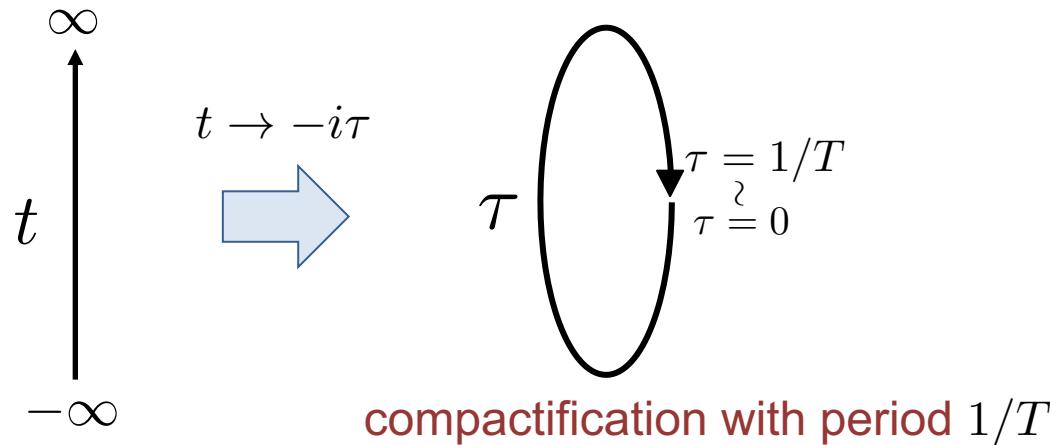
- What happened when another spatial axis is also compactified?



1. Introduction

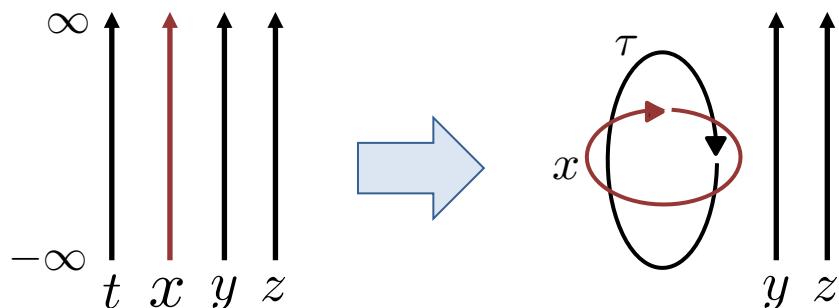
• Compactification

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$$\mathbb{S}^1 \times \mathbb{R}^3$$

- What happened when another spatial axis is also compactified?

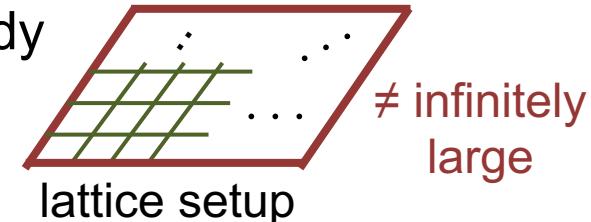


$$\mathbb{T}^2 \times \mathbb{R}^2$$

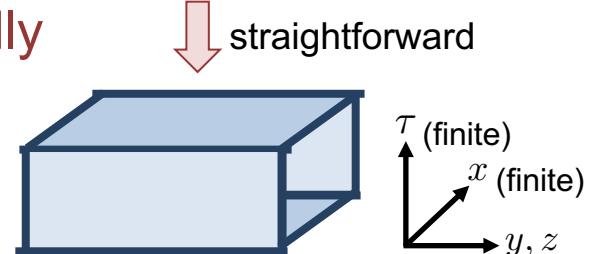
1. Introduction

• Anisotropic system as a new QCD environment

- Lattice simulation is nothing but a finite-volume study



- Simulation in anisotropic system is straightforwardly done by imposing certain boundary conditions



For example

- Polyakov loops in pure YM theory in anisotropic system were simulated

Chernodub-Goy-Molochkov, PRD (2019)

- Pressure/energy in pure YM theory in anisotropic system were simulated

Kitazawa-Mogliacci-Kolbe-Horowitz, PRD (2019)

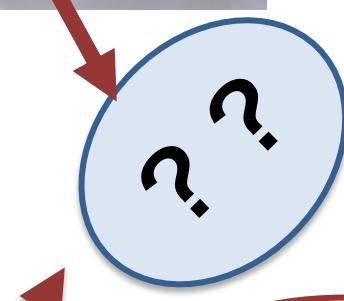
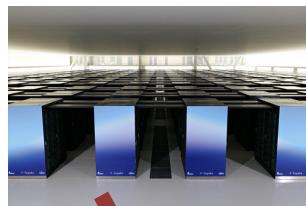


- We explored phase structures of pure YM theory in anisotropic system with an effective theory

1. Introduction

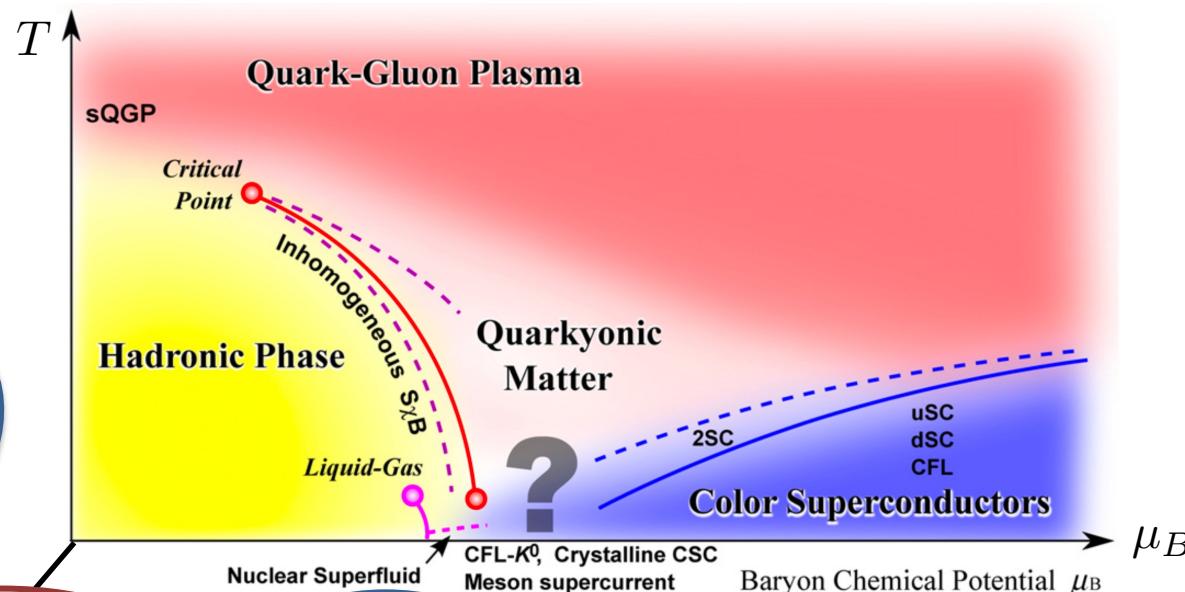
• Anisotropic system as a new QCD environment

lattice simulations

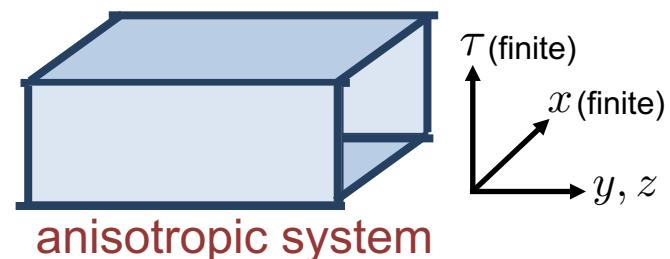


new axis!

$1/L_x$
(L_x : extent of x axis)

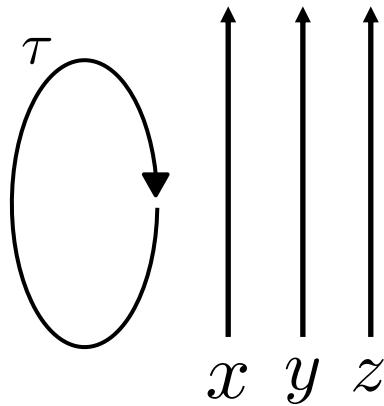


As the first step toward completing the diagram,
we explored here for pure YM theory



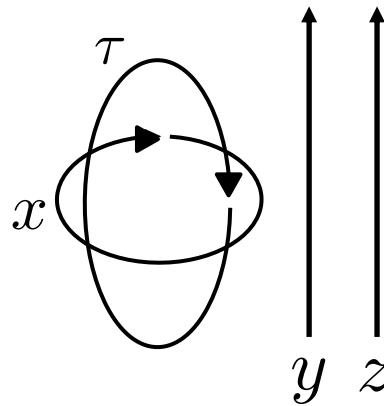
2. Model

- Strategy



Finite temperature

$$\mathbb{S}^1 \times \mathbb{R}^3$$



Anisotropic system

$$\mathbb{T}^2 \times \mathbb{R}^2 \cong \mathbb{S}^1 \times \mathbb{S}^1 \times \mathbb{R}^2$$



- It is useful to extend a model for pure YM in finite temperature toward study in anisotropic system

2. Model

• Model at finite temperature

- Approaches for pure YM theory at finite temperature $\mathbb{S}^1 \times \mathbb{R}^3$

- Massive quasiparticle models

eg, Gorenstein-Yang (1995), Peshier-Kampfer-Pavlenko-Soff (1996)

$$p(T) = \frac{d}{6\pi^2} \int_0^\infty dk f(k) \frac{k^4}{\sqrt{k^2 + m^2(T)}} - B(T)$$

vacuum pressure
 T dep. gluon mass

- Polyakov loop models

review: Fukushima-Skokov (2017)

$$A_\tau = T \operatorname{diag}(\theta_1, \dots, \theta_{N_c}) \quad \text{with} \quad \sum_{i=1}^{N_c} \theta_i = 0$$

- Glueball as a dilaton

eg, Carter-Scavenius-Mishustin-Ellis (2000)

$$\mathcal{L} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - U(\phi) - \frac{1}{4} \mathbf{A}_{\mu\nu} \cdot \mathbf{A}^{\mu\nu} + \frac{1}{2} G^2 \phi^2 \mathbf{A}_\mu \cdot \mathbf{A}^\mu$$

dilaton potential

⋮

- In this talk, we employ a Polyakov loop model of Ref.[1] which is simple and intuitively understandable for study on $\mathbb{T}^2 \times \mathbb{R}^2$

[1] Meisinger-Miller-Ogilvie, PRD65, 034009 (2002)

2. Model

• Model at finite temperature

- The model for pure YM of Ref.[1] is defined by

$$F = F_{\text{perp}} + F_{\text{pot.}} \quad (\text{free energy density})$$

$$\left[\begin{array}{l} \text{I)} \quad F_{\text{perp}} = \frac{2}{L_\tau} \sum_{j,k=1}^{N_c} \left(1 - \frac{\delta_{jk}}{N_c}\right) \sum_{l_\tau} \int \frac{d^3 p}{(2\pi)^3} \ln \left[\left(\omega_\tau - \frac{(\Delta\theta_\tau)_{jk}}{L_\tau} \right)^2 + \mathbf{p}^2 \right] \\ \qquad \qquad \qquad \boxed{L_\tau = 1/T, \omega_\tau = (2\pi l_\tau)/L_\tau} \\ \qquad \qquad \qquad \boxed{(\Delta\theta_\tau)_{jk} = (\theta_\tau)_j - (\theta_\tau)_k} \\ \qquad \qquad \qquad \text{- from perturbative calculation with } A_\tau = T \text{ diag}((\theta_\tau)_1, \dots, (\theta_\tau)_{N_c}) \text{ and } \sum_{i=1}^{N_c} (\theta_\tau)_i = 0 \\ \\ \text{II)} \quad F_{\text{pot.}} = -\frac{1}{L_\tau R^3} \ln \left[\prod_{j < k} \sin^2 \left(\frac{(\Delta\theta_\tau)_{jk}}{2} \right) \right] \\ \qquad \qquad \qquad \leftarrow \text{Haar measure potential by strong-coupling expansion} \\ \qquad \qquad \qquad \boxed{\text{Polonyi-Szlachanyi (1982)}} \\ \qquad \qquad \qquad \text{- } R \text{ is size of colorful domain} \end{array} \right]$$

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$$\begin{aligned} L_\tau &= 1/T, \quad \omega_\tau = (2\pi l_\tau)/L_\tau \\ (\Delta\theta_\tau)_{jk} &= (\theta_\tau)_j - (\theta_\tau)_k \end{aligned}$$

- from perturbative calculation with $A_\tau = T \text{ diag}((\theta_\tau)_1, \dots, (\theta_\tau)_{N_c})$ and $\sum_{i=1}^{N_c} (\theta_\tau)_i = 0$

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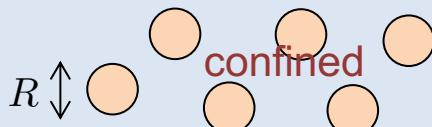
← Haar measure potential by strong-coupling expansion

Polonyi-Szlachanyi (1982)

- R is size of colorful domain

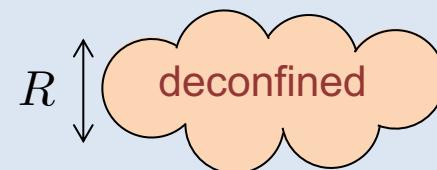
$$R \ll L_\tau$$

$F_{\text{pot.}}$ dominates (\rightarrow confined)



$$R \gg L_\tau$$

$F_{\text{pert.}}$ dominates (\rightarrow deconfined)



2. Model

- **Model in anisotropic system**

- Our model in $\mathbb{T}^2 \times \mathbb{R}^2$ is defined by

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- The simplest extension of “Meisinger-Miller-Ogilvie, PRD65, 034009 (2002)”

2. Model

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- The simplest extension of “Meisinger-Miller-Ogilvie, PRD65, 034009 (2002)”
- Two types of “Polyakov loops” are defined (order parameter of two Z_{N_c} symm.)

$$P_\tau(x, \mathbf{r}_L) = \frac{1}{N_c} \text{tr} \left[\mathcal{P}_\tau \exp \left(i \int_0^{L_\tau} A_\tau(\tau, x, \mathbf{r}_L) d\tau \right) \right] \quad P_x(\tau, \mathbf{r}_L) = \frac{1}{N_c} \text{tr} \left[\mathcal{P}_x \exp \left(i \int_0^{L_x} A_x(\tau, x, \mathbf{r}_L) dx \right) \right]$$

2. Model

• Techniques

- F_{pert} includes complicated “Matsubara summations \sum_{l_τ, l_x} ”

$$F_{\text{pert}} = \frac{2}{L_\tau L_x} \sum_{j,k=1}^{N_c} \left(1 - \frac{\delta_{jk}}{N_c}\right) \sum_{l_\tau, l_x} \int \frac{d^2 p_L}{(2\pi)^2} \ln \left[\left(\omega_\tau - \frac{(\Delta\theta_\tau)_{jk}}{L_\tau}\right)^2 + \left(\omega_x + \frac{(\Delta\theta_x)_{jk}}{L_x}\right)^2 + \mathbf{p}_L^2 \right]$$

with $\omega_\tau = (2\pi l_\tau)/L_\tau$, $\omega_x = (2\pi l_x)/L_x$ ($l_\tau, l_x \in \mathbb{Z}$)

- Handled by Epstein-Hurwitz zeta function

textbook: Elizalde, “Ten physical applications of spectral zeta functions” (LNPMGR, volume 35)

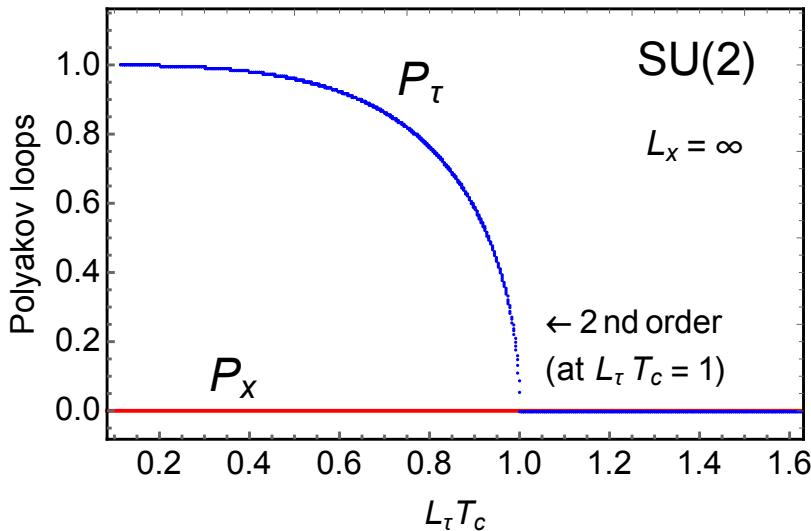
- 
- Vacuum contributions including UV divergences are easily subtracted
 - The very nonlinear sums are translated into sums with modified Bessel functions $K_\nu(x)$
- easy to be evaluated!

3. Results

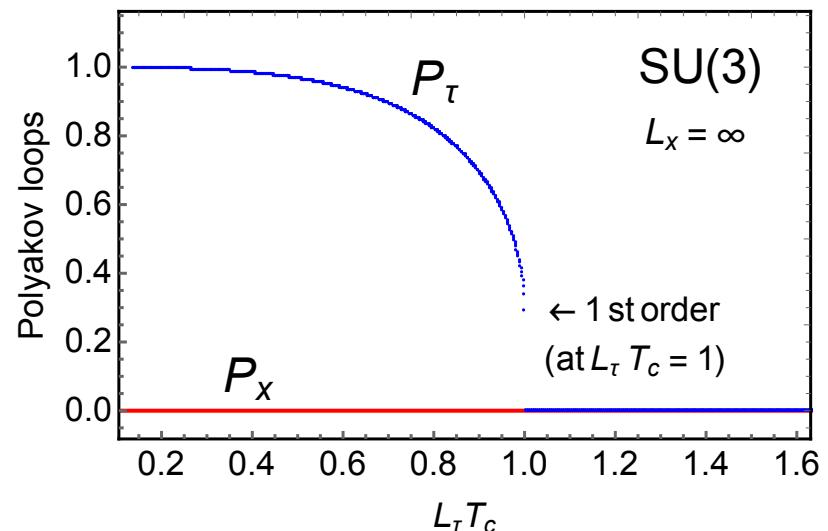
- **Results for $L_x = \infty$**

- When $L_x = \infty$, the model reduced to that in finite temperature $\mathbb{S}^1 \times \mathbb{R}^3$
- Find the stationary points of the free energy

$$SU(2) : T_c = 1/(0.873R)$$



$$SU(3) : T_c = 1/(0.733R)$$

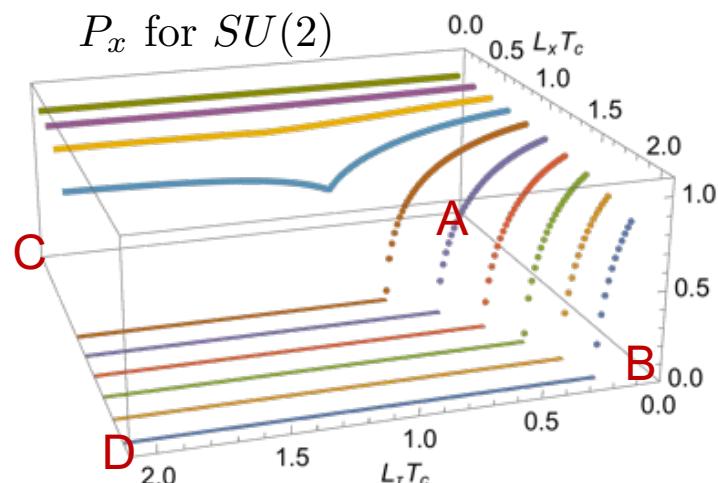
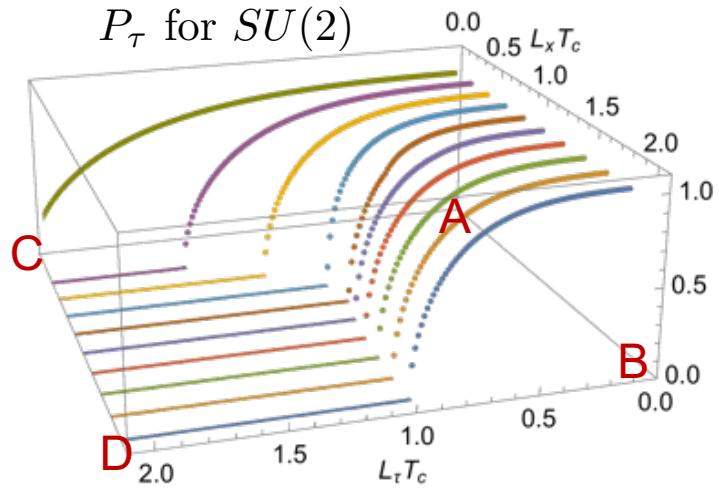


- These results are consistent with analysis in Ref.[1]

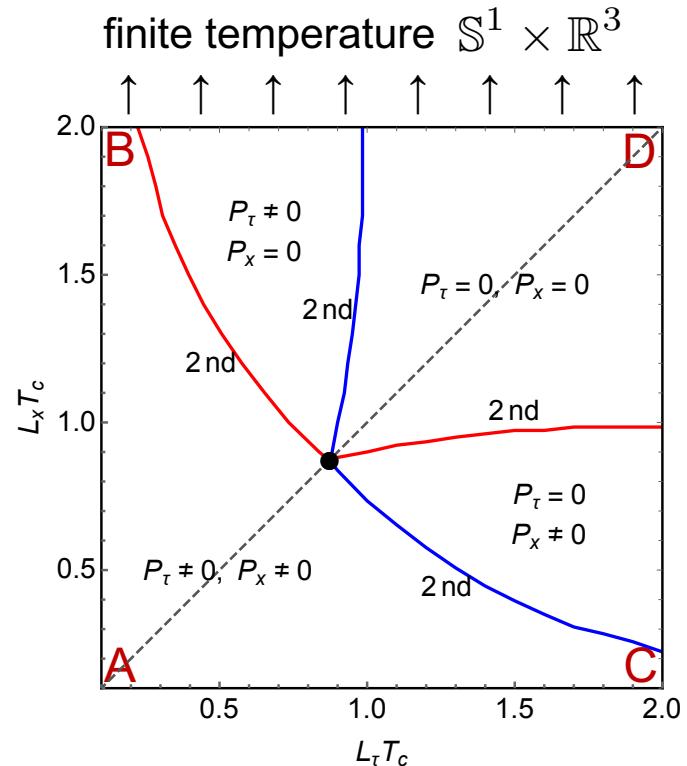
[1] Meisinger-Miller-Ogilvie, PRD65, 034009 (2002)

3. Results

- Results for $SU(2)$ in $\mathbb{T}^2 \times \mathbb{R}^2$



$$T_c = 1/(0.873R)$$

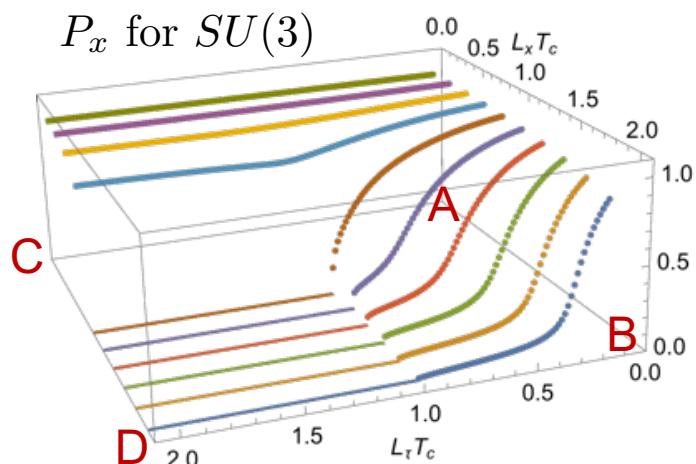
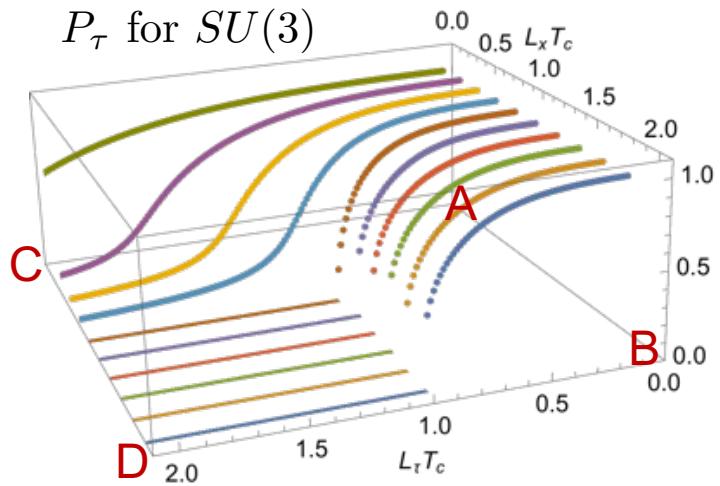


phase diagram for $SU(2)$

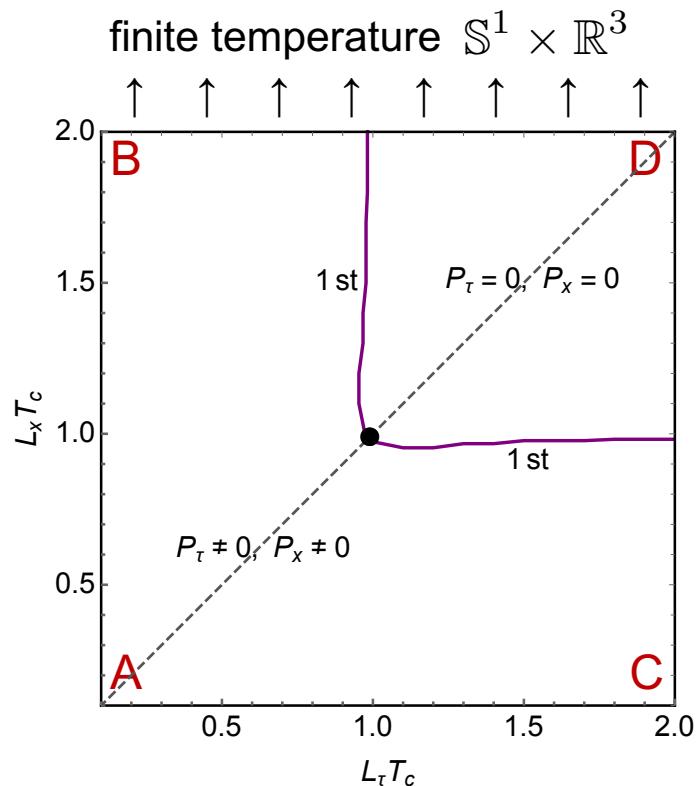
- 2nd order for all transitions

3. Results

- Results for $SU(3)$ in $\mathbb{T}^2 \times \mathbb{R}^2$



$$T_c = 1/(0.733R)$$

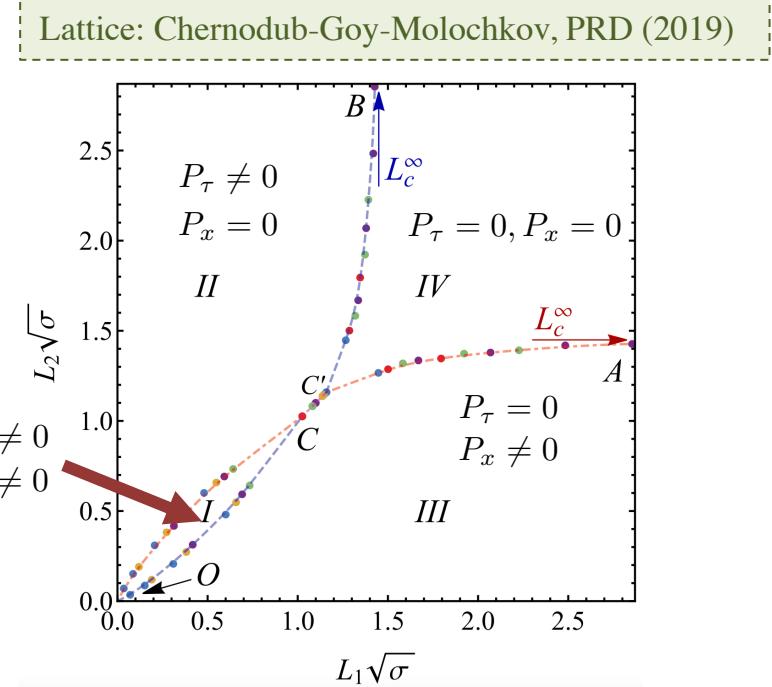
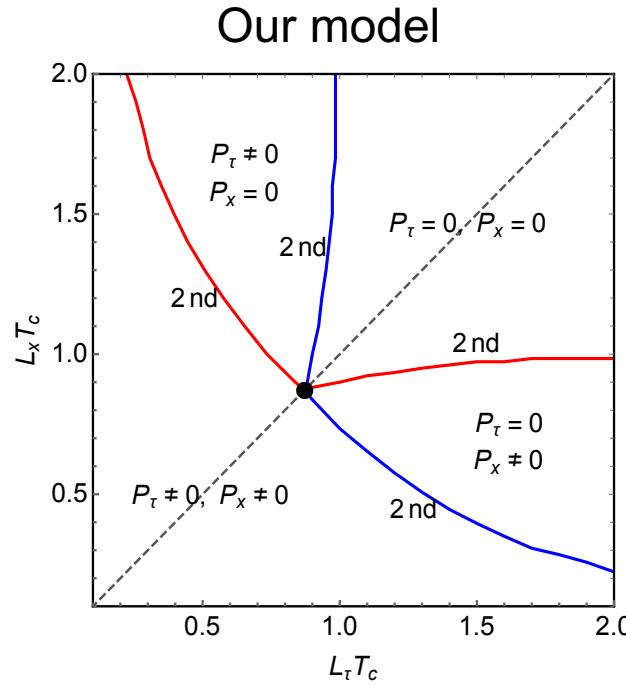


phase diagram for $SU(3)$

- 1nd order for all transitions

3. Results

- Comparison with lattice results for $SU(2)$



- The shape of region for $P_\tau \neq 0, P_x \neq 0$ is very different → need improvement

→ Possibility of finding new ingredients that cannot be seen
in ordinary QCD phase diagram

- The usefulness of focusing on the novel extreme condition: $\mathbb{T}^2 \times \mathbb{R}^2$

4. Conclusions

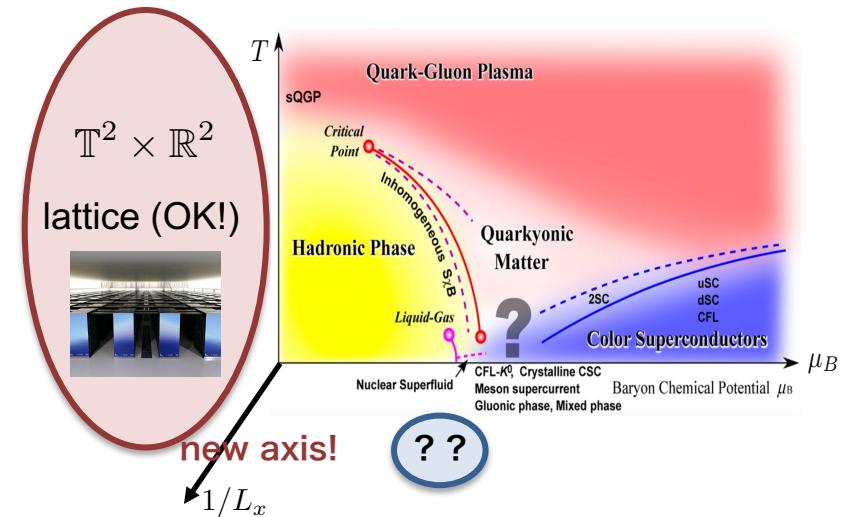
Suenaga-Kitazawa, in preparation

20/20

• Conclusions

- I proposed novel QCD phase diagram focused on anisotropic system: $\mathbb{T}^2 \times \mathbb{R}^2$ lattice (OK!)
- Lattice simulations for pure Yang-Mills theory is being done recently

Chernodub-Goy-Molochkov, PRD (2019)
Kitazawa-Mogliacci-Kolbe-Horowitz, PRD (2019) etc.



- Naïve extension of “Meisinger-Miller-Ogilvie, PRD65, 034009 (2002)” is not enough to reproduce the lattice result

{ e.g. treatment consistent with a certain gauge condition?
more sophisticated model? }

Ruggieri-Alba-Castorina-Plumari-Ratti-Greco (2012)

Polonyi-Szlachanyi (1982)

Study of QCD phase diagram in anisotropic system has just begun!

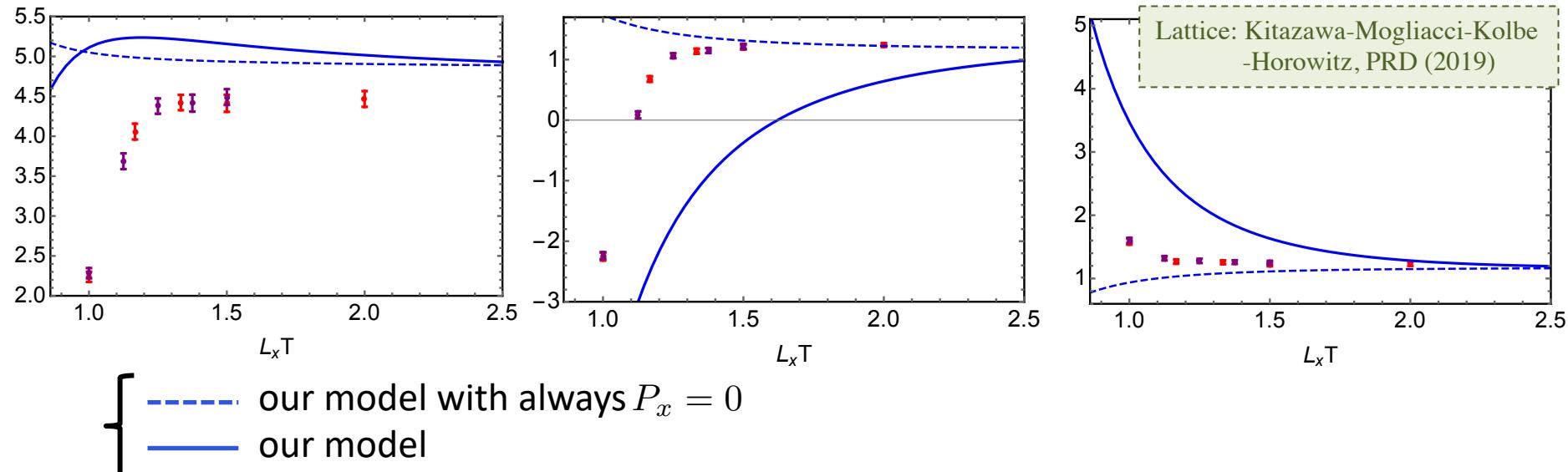
- Further investigation from both theoretical and lattice studies are needed

join us!

Backup

• Thermodynamic quantities

- L_x dependence of energy density ϵ/T^4 , and pressures p_x/T^4 and p_z/T^4

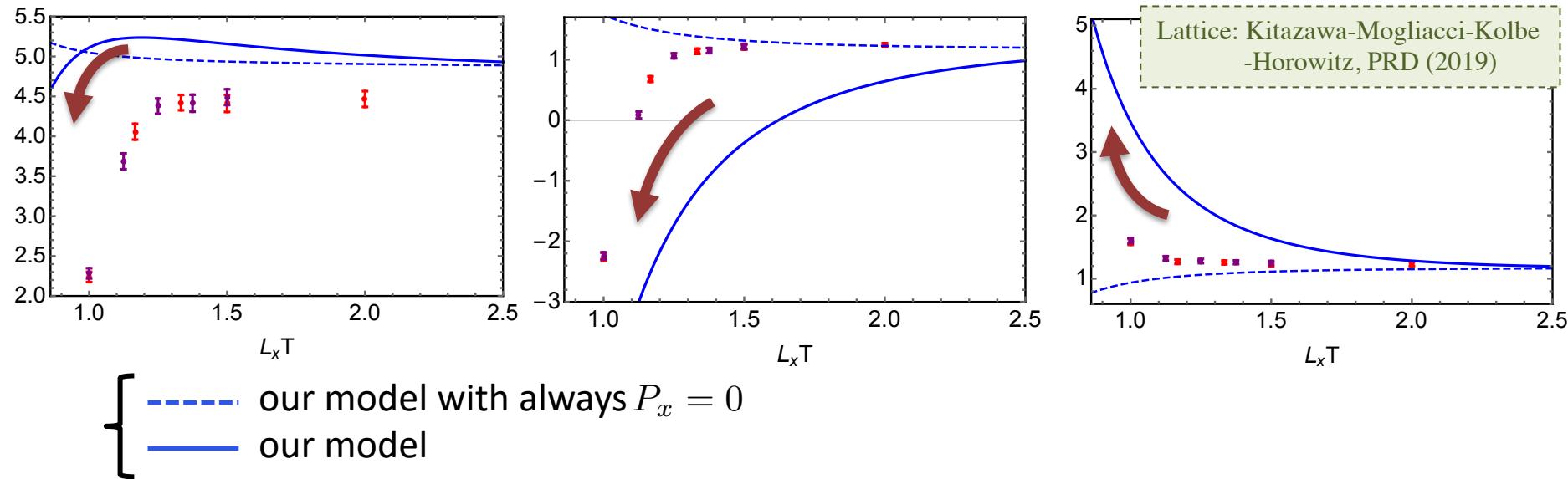


- At larger volume $L_x T \gtrsim 1.3$, effects from P_x may be small (----- is better)

Backup

• Thermodynamic quantities

- L_x dependence of energy density ϵ/T^4 , and pressures p_x/T^4 and p_z/T^4



- At larger volume $L_x T \gtrsim 1.3$, effects from P_x may be small (--- is better)
- For $L_x T \lesssim 1.3$, P_x plays an important role to get *qualitatively* correct behavior
- I showed that naïve extension of “Meisinger-Miller-Ogilvie (2002)” is not enough but a model having both the above properties should be the correct one!