London model of dual color superconductor
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My understanding: The experimental fact of permanent confinement of the colored quarks and the colored gluons in colorless hadronic jails is supported by the numerical lattice computations. It is attributed to peculiar properties of the nonperturbative, strong-coupling vacuum medium of QCD : it does not allow for penetration of the chromo-electric field $\vec{E}_a$ of which quarks and gluons are the sources: The chromo-dielectric function of the QCD vacuum is $\epsilon=0$.
Natural idea: The Meissner effect for $\vec{E}_a$ is dual to the Meissner effect observed for $\vec{B}$ in ordinary superconductors ($\mu=0$). As $\epsilon.\mu = 1$ in Lorentz-invariant medium the QCD vacuum behaves simultaneously as a perfect color paramagnet.

My attitude: Follow humbly the big masters (and quote from them).

London replaced in Maxwell eqs the Ohm current by super-current $\vec{J}$

- rot $\vec{J}$ describes the Meissner effect for the magnetic field $\vec{B}$ in superconductivity
- $\frac{\partial}{\partial t}\vec{J}$ describes superconductivity

$$\boxed{\vec{J} = -\kappa \vec{A}}$$

- Later: London theory derived from GL, GL derived from BCS

Our program: Find the form of the current $\vec{J}_a$ within the effective strong-coupling QCD Maxwell equations (without quarks) such that:

(1) $\frac{\partial}{\partial t}\vec{J}_a$ describes the Meissner effect for the chromo-electric field $\vec{E}_a$ in the QCD vacuum medium

(2) Such an approach raises the question : Does $\vec{J}_a$ describe some chromo-magnetic superfluidity associated with the confining medium ?

(3) Try to find the generic form of the GL-like strong-coupling effective QCD which gives rise to the London QCD current $\vec{J}_a$ upon appropriate chromo-magnetic color gluon condensate.

1. London QCD current for the dual Meissner effect

Definition of the fields in the canonical $A_a^0 = 0$ gauge kept also in strong-coupling regime

$$\vec{E}_a = -\partial \vec{A}_a/\partial t$$

$$\vec{B}_a = \text{rot}\,\vec{A}_a - \frac{1}{2} g f_{abc}\, \vec{A}_b \times \vec{A}_c$$

QCD Maxwell equations:

Gauss' law (constraint) $\quad \text{div}\,\vec{E}_a = -g f_{abc}\, \vec{A}_b . \vec{E}_c = j^0{}_a$

Ampere's law (eq. of motion) $\quad \text{rot}\,\vec{B}_a - \partial \vec{E}_a/\partial t = -g f_{abc}\, \vec{A}_b \times \vec{B}_c = \vec{j}_a$

Bianchi identities (non-trivial in QCD):

$$\text{div}\,\vec{B}_a \equiv g f_{abc}\, \vec{A}_b . \text{rot}\,\vec{A}_c = k^0{}_a$$

$$(*) \qquad \text{rot}\,\vec{E}_a + \partial \vec{B}_a/\partial t \equiv g f_{abc}\, \vec{E}_b \times \vec{A}_c = \vec{k}_a$$

What F. London did in SC (on blackboard)

Replace at strong coupling $\boxed{j^0{}_a \to J_a^0 \qquad \vec{j}_a \to \vec{J}_a}$ and apply rot to (*)

- $\vec{\nabla} J^0{}_a + (\partial^2/\partial t^2 - \nabla^2)\vec{E}_a + \frac{\partial}{\partial t}\vec{J}_a = g f_{abc}\,\mathrm{rot}\,(\vec{E}_b \times \vec{A}_c) = \mathrm{rot}\,\vec{k}_a$

Impose the dual Meissner effect in the form $(\partial^2/\partial t^2 - \nabla^2)\vec{E}_a = \mu^2\,\vec{E}_a$

$$\boxed{\frac{\partial}{\partial t}\vec{J}_a + \vec{\nabla} J^0{}_a = -\,\mu^2\,\vec{E}_a + g f_{abc}\,\mathrm{rot}\,(\vec{E}_b \times \vec{A}_c)} \qquad (1)$$

Perform rot (1) and take off the time derivative:

Responsible for dual color superconductivity???

$$\boxed{\mathrm{rot}\,\vec{J}_a = +\,\mu^2\,\vec{B}_a - \frac{1}{2} g f_{abc}\,[\,\mu^2 + \mathrm{rot\,rot}\,](\vec{A}_b \times \vec{A}_c)} \qquad (2)$$

Take off rot:

$$\boxed{\vec{J}_a = +\,\mu^2\,\vec{A}_a - \frac{1}{2} g f_{abc}\,\mathrm{rot}\,(\vec{A}_b \times \vec{A}_c)}$$

- If the analogy does not falter this current should follow from a manifestly gauge-invariant GL-like effective strong-coupling QCD upon peculiar gauge-dependent mean field approximation
- No trace of the chromo-magnetic charge density $k^0{}_a$.

2. The London QCD current for the chromo-magnetic superfluid ???

- Ordinary London macroscopic superconductivity:
  - response to $\vec{B}$ – Meissner effect
  - response to $\vec{E}$ – superconductivity
- QCD vacuum medium (analogies may falter!):
  - response to $\vec{E}_a$ – dual Meissner effect – done !!!
  - response to $\vec{B}_a$ – chromo-magnetic superfluidity ???
- In QCD the strong coupling = large distances: The macroscopic quantum long-range order of chromo-magnetic (super)fluidity is conceivable. $\vec{B}_a$ is not confined
- Can we associate the flow $\vec{J}_a$ with the observed almost perfect fluidity in droplets of the strongly interacting quark- gluon plasma created in heavy-ion collisions? To be analyzed. E. Shuryak, V. Zakharov, …

3/1 The GL-like effective QCD at strong coupling
[Work in progress (A. Smetana, P. Benes, J.H.)]

- Quote the prescient Fritz London:

$$\vec{J}_{GL} = i\,\frac{e}{m}(\Phi^* \,\vec{\nabla}\Phi - \Phi\, \vec{\nabla}\Phi^*) - \frac{4e^2}{m^2}\,\Phi^*\Phi\, \vec{A} \quad \text{->} \quad -\kappa\, \vec{A}$$

In QCD the gauge fields can be large at large distances, tending to condense. Hence, higher order polynomials in $F_{a\mu\nu}$ equally important as the second order in L_{QCD}

- L_{eff} ; Effective field theory appropriate: S. Weinberg, R. Friedberg, T. D. Lee (quote), H. Pagels and E. Tomboulis, …
- Dimension 4 is unique
- $K = \frac{1}{2}\, F_{a\mu\nu}\, F_a{}^{\nu\mu} = \vec{E}_a{}^2 - \vec{B}_a{}^2$
- Dimension 6 term is unique
- $t = \frac{1}{6} f_{abc}\, F_{a\mu\nu}\, F_b{}^{\nu\lambda}\, F_{c\lambda}{}^{\mu} = \frac{1}{2} f_{abc}\, \vec{B}_a\,(\frac{1}{3}\vec{B}_b \times \vec{B}_c - \vec{E}_b \times \vec{E}_c\,)$

3/2 The GL-like effective QCD at strong coupling

- In general $$H_{eff} = \vec{\nabla}(A_a^0 . \vec{\varepsilon_a}) + \vec{E}_a \, \partial L_{eff} / \partial \vec{E}_a - L_{eff}$$
- Basic ingredient: constant gauge field described by constant non-commuting gauge potentials (Coleman; Brown&Weissberger)
- In the canonical $A_a^0 = 0$ gauge
- $\vec{E}_a = - \partial \vec{A}_a / \partial t$ is a "small" field
- $\vec{B}_a = \text{rot}\, \vec{A}_a - \frac{1}{2} g f_{abc} \vec{A}_b \times \vec{A}_c$ is a 'large" field

$$< H_{eff} >_{vac} = -< L_{eff} >_{vac} = < \frac{1}{2} \vec{B}_a^2 + c \frac{1}{M^2} f_{abc} \vec{B}_a \overrightarrow{(B_b \times \vec{B}_c)} + \ldots >_{vac}$$

- which invariants are non-zero when $\vec{E}_a = 0$???
- vacuum is Lorentz-invariant ($\epsilon.\mu = 1$ is valid)
- Hence vacuum is a perfect color paramagnet characterized by colorless spinless condensate $f_{abc} \vec{B}_a \overrightarrow{(B_b \times \vec{B}_c)}$ with fixed $\vec{B}_a$

3/3 The GL-like effective QCD at strong coupling

- Which gauge-dependent mean field*, if any, applied to L_{eff} yields the London QCD current
- $J^i_a = + \mu^2 A^i_a - \frac{1}{2} g f_{abc} (\mathrm{rot}\,(\vec{A}_b \times \vec{A}_c))^i =$
- $= \mu^2 A^i_a - g f_{abc} [(\nabla^j A^i_b) A^j_c - (\nabla^j A^j_b) A^i_c]$

- We suggest $\langle A^i_a A^j_b \rangle = \Delta^2 \delta^{ij} \delta_{ab}$ (Gubarev, Zakharov; Lee).
- Keep only A^i_a and its space derivatives in the first power.

$$L_{GL} = A\, F_{a\mu\nu} F_a^{\nu\mu} + (B/M^2)\, f_{abc} F_{a\mu\nu} F_b^{\nu\lambda} F_{c\lambda}^{\mu} + \ldots$$

$$J^0_a = -4Ag\, f_{abc} A^i_b E^i_c - \frac{6B}{M^2} g \varepsilon^{ijk} B^i_a A^j_c E^k_c + \ldots$$

$$J^i_a = -\, 4Ag f_{abc} (\vec{A}_b \times \vec{B}_c)^i - \frac{6B}{M^2} g f_{ebc} f_{eag} [B^i_b (\vec{B}_c \cdot \vec{A}_g) - \vec{E}^i_b (\vec{E}_c \cdot \vec{A}_g)] + \ldots$$

candidate for the GL-like current of a dual color superfluid

partial success (div $\vec{A}_b$ not reproduced) – work in progress

Conclusion and outlook

- Done: The Meissner effect for $\vec{E}_a$ by the phenomenological strong-coupling QCD gluonic current $\vec{J}_a$
- The path to the gauge-invariant GL-like effective QCD at strong coupling outlined – apparently promising.
- Can the medium above the confining QCD vacuum manifest itself experimentally also by some sort of superfluidity as the ordinary superconductor? In any case the gauge-invariant L_{GL} contains the candidate current.
- Outlook: It is a long way to understanding confinement if the main analogy does not falter: Quote T.D. Lee (1978): "… to derive quark confinement from QCD directly…"

Thanks for your attention