Energy-momentum tensor around a soliton in 1+1 dimensional ϕ^4 model and its regularization

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Energy momentum tensor (EMT)

- $T_{\mu\nu}$: Noether current with translational symmetry

Energy density

Momentum density

$$T_{\mu\nu} = \begin{pmatrix} T_{00} & T_{01} & T_{02} & T_{03} \\ T_{01} & T_{11} & T_{12} & T_{13} \\ T_{02} & T_{21} & T_{22} & T_{23} \\ T_{03} & T_{23} & T_{32} & T_{33} \end{pmatrix}$$

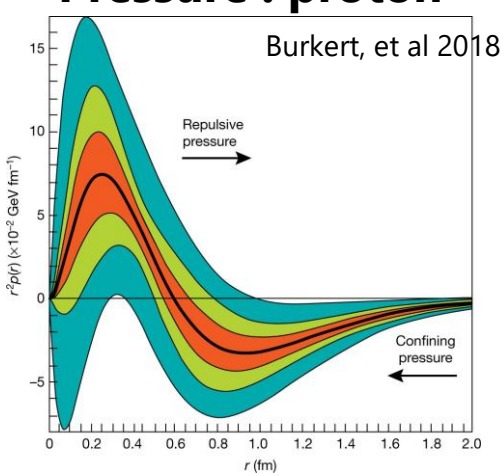
Stress tensor

Advantages to use EMT

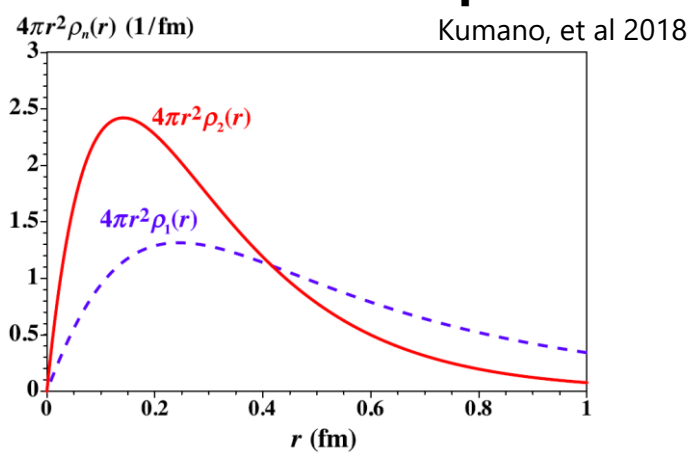
- ✓ Gauge invariance
- ✓ Conserved quantity
- ✓ Observable
- ✓ Local interaction

- EMT distribution inside hadrons

Pressure : proton



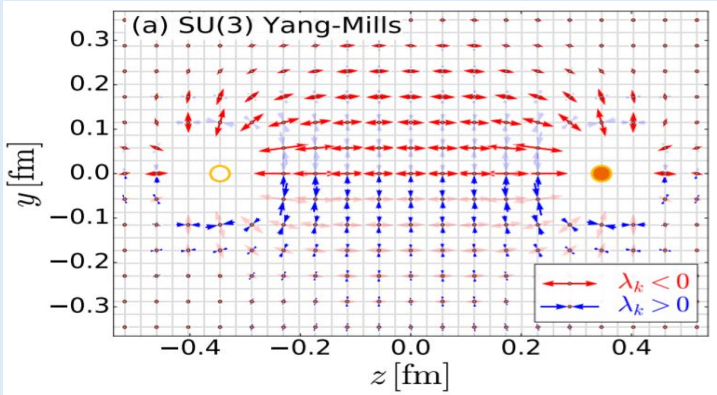
EMT distribution : pion



Quantum fluctuation of EMT

- **Distribution of EMT** in $Q\bar{Q}$

Stress distribution around $Q\bar{Q}$ on Lattice QCD



Yanagihara, et al. 2019

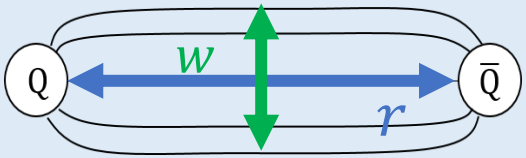
Pulling force (parallel to field)

Pushing force (perpendicular to field)

Describing stress

Viewing fluxtube

- **Quantum effect** in $Q\bar{Q}$



Lüscher, Münster, and Weisz 1981

Theoretical model

Fattening due to string vibration

$$w^2 \sim \log r$$

Motivate theoretical analysis of quantum correction to EMT distribution in $Q\bar{Q}$ system

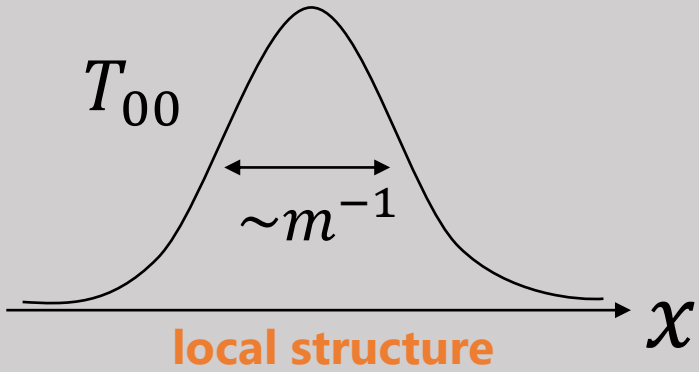
Quantum fluctuation of a soliton

Purpose: **Analysis of quantum correction to EMT distribution around a soliton in 1+1d real scalar ϕ^4 model**

classical soliton in ϕ^4 model

$$S = \int dx^2 \left\{ \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - \frac{\lambda}{4}\left(\phi^2 - \frac{m^2}{\lambda}\right)^2 \right\}$$

kink $\phi_{cl} = +\frac{m}{\sqrt{\lambda}} \tanh\left(\frac{m(x-X)}{\sqrt{2}}\right)$

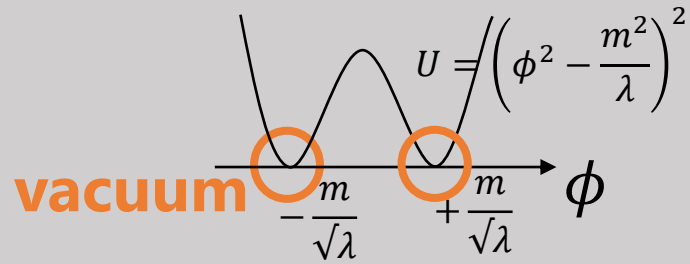


Classical EMT: Trivial

Quantum EMT: Nontrivial, **First trial**

ϕ^4 Model in 1+1 d

$$S = \int dx^2 \left\{ \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{\lambda}{4}\left(\phi^2 - \frac{m^2}{\lambda}\right)^2 \right\}$$

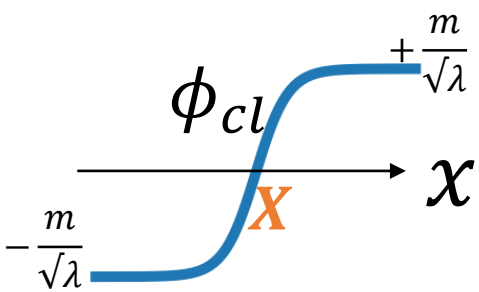


Classical

- Kink solution of EOM

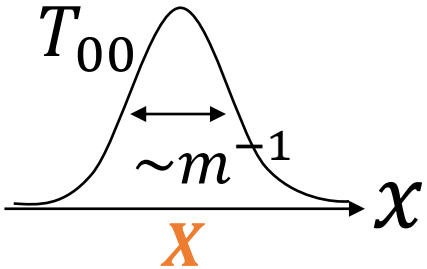
$$\phi_{cl} = +\frac{m}{\sqrt{\lambda}}\tanh\left(\frac{m(x-X)}{\sqrt{2}}\right)$$

X: location of kink



- T_{00} under kink solution

$$T_{00} = \frac{m^4}{\lambda}\cosh^{-4}\left(\frac{m(x-X)}{\sqrt{2}}\right)$$



QM

$E_{LO} \sim O(\lambda^0)$ Dashen, et al 1974

$T_{00}^{LO} \sim O(\lambda^0)$ Goldhaber, et al. 2003 ?

We calculate $T_{00}^{LO}(x)$ and $T_{11}^{LO}(x)$ in $O(\lambda^0)$!
(Comparison with previous studies ➡ last slide)

Eigenmode of quantum fluctuation

Dashen, Hasslacher and Neveu 1974

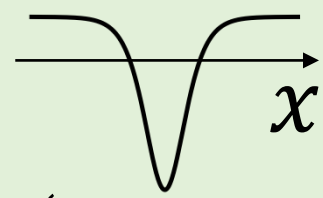
Expanding ϕ^4 action $S[\phi]$ around a kink

Substituting $\phi(x) = \phi_{cl} + \eta(x)$

η:quantum fluctuation

$$S[\eta] = S_{cl} + \int dx^2 [\frac{1}{2}(\partial_0 \eta)^2 - \frac{1}{2}\eta \left(-\frac{\partial^2}{\partial x^2} - m^2 + \frac{3\phi_{cl}^2}{\lambda}\right)\eta - \lambda\phi_{cl}\eta^3 - \frac{\lambda}{4}\eta^4]$$

$$\left(-\frac{\partial^2}{\partial x^2} - m^2 + \frac{3\phi_{cl}^2}{\lambda}\right)\eta_n = \omega_n^2 \eta_n$$



Eigenvalues

$$\omega_q^2 = q^2 + 2m^2$$

$$\omega_1^2 = \frac{3}{2}m^2$$

$$\boldsymbol{\omega_0^2 = 0}$$

Eigenfunctions

$$\eta_q(x) \xrightarrow{x\to\pm\infty} \exp[i\left(qx \pm \frac{1}{2}\delta(q)\right)]$$

Phase shift

$$\eta_1(x)$$

$$\boldsymbol{\eta_0(x) = \partial_x \phi_{cl}}$$

Translational mode➡IR divergence

Collective coordinate method

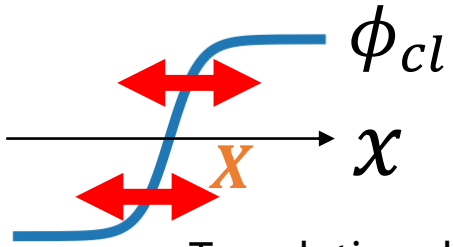
Gervais, Jevicki, Sakita 1975
Tomboulis 1975

Rewriting Lagrangian

Remove translational mode($\eta_0(x)$) and IR divergence

$$\phi(x,t) = \phi_{cl}(x - X) + \eta(x - X)$$

We regard X as a dynamical variable.



Translational mode

$X \to X(t)$ **Dynamical variable**

$P \leftrightarrow X$ **Canonical conjugate momentum**

$$L[\pi, \phi] \to L[\tilde{\pi}, \tilde{\eta}, X, P]$$

$\tilde{\pi}, \tilde{\eta}$:without translational mode

Convert

Degree of freedom of $\eta_0(x)$ ➡ **Degree of freedom of Soliton center of motion**

New Lagrangian

$$L[\tilde{\pi},\tilde{\eta},X,P] = P\dot{X} - M_0 - \frac{\left(P+\int dx\,\pi\eta'\right)^2}{2M_0\left[1+\left(\frac{1}{M_0}\right)\xi\right]^2} + \boldsymbol{L}[\tilde{\pi},\tilde{\eta}]$$

Center of motion $O(\lambda^1)$ **Negligible**

We consider this part

- Translational invariance
- Lorentz symmetry

Gervais, Jevicki, Sakita 1975
Goldstone and Jackiw 1975
Tomboulis 1975
Christ and Lee 1975

Separating center of motion part from other part

$$L[\tilde{\pi},\tilde{\eta}] = \frac{1}{2}\tilde{\pi}^2 - \frac{1}{2}(\partial_x\tilde{\eta})^2 + \frac{1}{2}\left(-m^2 + \frac{3\phi_{cl}^2}{\lambda}\right)\tilde{\eta}^2 - \lambda\phi_{cl}\tilde{\eta}^3 + O(\lambda^1)$$

New EMT

ϕ^4 Model in 1+1 d

$$T^{\mu}{}_{\nu}[\phi] = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \partial_\nu \phi - \mathcal{L}\delta^{\mu}{}_{\nu}$$

Collective coordinate method →

$$T^{\mu}{}_{\nu}[\tilde{\pi}, \tilde{\eta}, X, P] = \boldsymbol{T^{\mu}{}_{\nu}[\tilde{\pi}, \tilde{\eta}] + O(\lambda^1)}$$

We calculate $\langle T^{\mu}{}_{\nu} \rangle$ in $O(\lambda^0)$

$$\boxed{\langle T^{\mu}{}_{\nu} \rangle \equiv \frac{1}{Z} \int \mathcal{D}\tilde{\pi}\mathcal{D}\tilde{\eta}\, T^{\mu}{}_{\nu}[\tilde{\pi}, \tilde{\eta}]\, e^{-i\int dt\, L[\tilde{\pi}, \tilde{\eta}]}}$$

EMT form factor

$$\langle P' | T^{\mu}{}_{\nu}(x, 0) | P \rangle = \int dX\, e^{-i(P-P')X} \boldsymbol{T^{\mu}{}_{\nu}(x - X)} + O(\lambda^{\frac{1}{2}})$$

$|P\rangle$:soliton 1 particle state

Expectation value of EMT distribution

Diagram

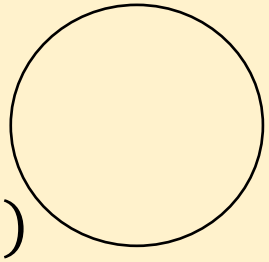
$$\langle T_{00}\rangle = T_{00}(\phi_{cl}) + \frac{1}{2}\langle(\partial_0\eta)^2\rangle + \frac{1}{2}\langle(\partial_1\eta)^2\rangle + \underbrace{(\partial_1\phi_{cl})}_{O(\lambda^{-\frac{1}{2}})}\langle\partial_1\eta\rangle + \underbrace{\lambda\phi_{cl}\left(\phi_{cl}^2 - \frac{m^2}{\lambda}\right)}_{O(\lambda^{-\frac{1}{2}})}\langle\eta\rangle$$

$$+ \underbrace{\frac{\lambda}{2}\left(3\phi_{cl}^2 - \frac{m^2}{\lambda}\right)}_{O(\lambda^0)}\langle\eta^2\rangle + O(\lambda^1)$$

$$\langle\eta^2\rangle = \langle\eta(x)\eta(x)\rangle$$

$$= G(x,x)$$

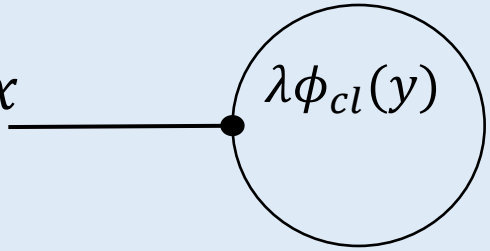
$=$ x



$\sim O(\lambda^0)$

UV divergent!

$\langle\eta\rangle =$ x



$$= \int dy\, \lambda\phi_{cl}(y)G(x,y)G(y,y)$$

$\sim O(\lambda^{\frac{1}{2}})$

UV divergent!

Vacuum subtraction

EMT regularization

Rebhan and Nieuwenhuizen 1997
Dashen, et al 1974

Counter terms

$$\langle T_{\mu\nu}\rangle_{soliton} - \langle T_{\mu\nu}\rangle_{vac} + (\langle \delta T_{\mu\nu}\rangle_{soliton} - \langle \delta T_{\mu\nu}\rangle_{vac}) = \text{finite}$$

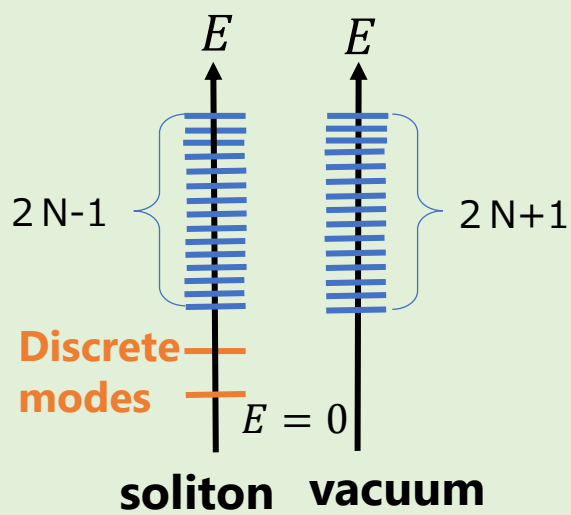
Mode number cutoff (MNC)

- ✓ Finite system whose length is R
- ✓ Subtract "vacuum" from "soliton"
  - -Cutoff number is determined following that "Mode number" is the same for soliton and vacuum

Soliton continuous mode vacuum

$$\sum_{n=-(N-1)}^{N-1} \sqrt{q_n^2 + 2m^2} - \sum_{n=-N}^{N} \sqrt{k_n^2 + 2m^2}$$

- ✓ After vacuum subtraction, $R \to \infty$



Mass renormalization

Dashen, Hasslacher and Neveu 1974

EMT regularization

Counter terms

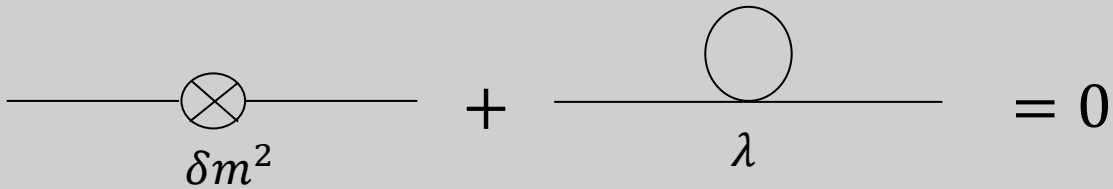
$$\langle T_{\mu\nu}\rangle_{soliton} - \langle T_{\mu\nu}\rangle_{vac} + (\langle \delta T_{\mu\nu}\rangle_{soliton} - \langle \delta T_{\mu\nu}\rangle_{vac}) = \text{finite}$$

Mass renormalization

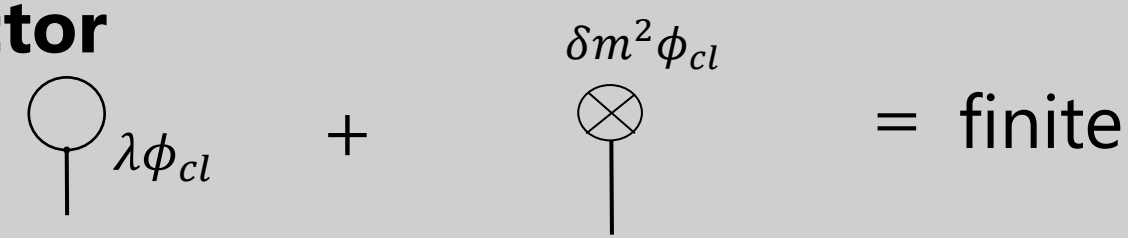
- ✓ Only 1-loop mass renormalization
- ✓ Mass renormalization in **vacuum** sector

  ➡Counter terms appear in **soliton** sector

Vacuum sector $m^2 \to m^2 + \delta m^2$



Soliton sector

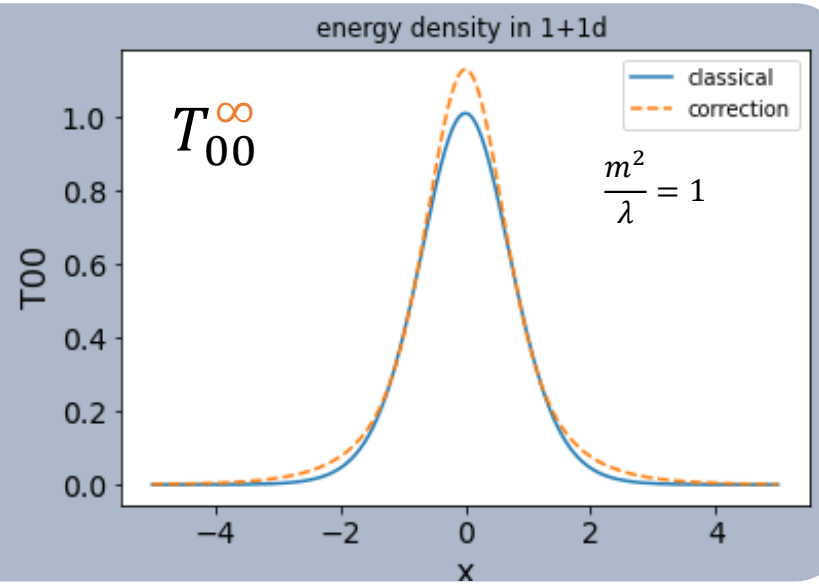


Result

In finite system whose length is R

$$T_{00}^{R}(x) = -\frac{3\sqrt{2m}}{2\pi}\frac{1}{R} + T_{00}^{\infty}(x)$$

$$T_{11}^{R} = -\frac{3\sqrt{2m}}{2\pi}\frac{1}{R}$$



- T_{00} distribution

$$\lim_{R\to\infty}\int_{-R/2}^{R/2} dx\, T_{00}^{R} = E \qquad \times \int_{-\infty}^{\infty} dx \lim_{R\to\infty} T_{00}^{R}$$

reproduce the E_{MNC} calculated by Dashen et al.

- T_{11} distribution

T_{11}^{R} is constant ➡ $\partial_0 T^{01} + \partial_1 T^{11} = 0$

EMT conservation law is satisfied

Comparison with Goldhaber et al.

Goldhaber, et al. 2003

Different regularization

Our study: **finite system**

$$T_{00}^{R}(x) = -\frac{3\sqrt{2}m}{2\pi}\frac{1}{R} + T_{00}^{\infty}(x)$$

$$T_{11}^{R} = -\frac{3\sqrt{2}m}{2\pi}\frac{1}{R}$$

Previous study: **infinite system**

$$T_{00}^{Prev}(x) = -\frac{3m^2}{4\pi}\frac{1}{\cosh^2(mx/\sqrt{2})} + T_{00}^{\infty}(x)$$

$$T_{11}^{Prev}(x) = -\frac{3m^2}{4\pi}\frac{1}{\cosh^2(mx/\sqrt{2})}$$

- Different result for **"finite"** and **"infinite"** regularizations
- $\int_{-\infty}^{\infty} dx\, T_{00}^{R} = \int_{-\infty}^{\infty} dx\, T_{00}^{Prev} = E_{\text{Dashen+}}$
- $\partial_x T_{11}^{Prev} \neq 0$

  T_{11}^{Prev} **is inconsistent with EMT conservation law.**

Our result is more reliable.

Summary

1-loop calculation of EMT distribution around a soliton in 1+1d real scalar ϕ^4 model

- Removing IR divergences by collective coordinate method
- Removing UV divergences by
  vacuum subtraction(Mode number cutoff)
  and mass renormalization
- The spatial integrals of obtained T_{00} reproduce the known total energies.
  T_{11} are consistent with the EMT conservation laws

Future work:

2+1d ϕ^4model · finite temperature · sine-Gordon model