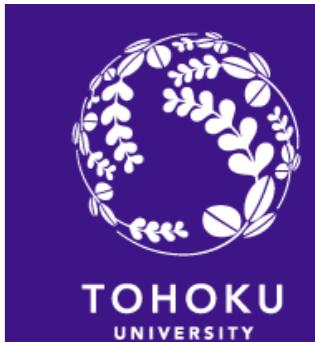


Lattice study of the trace anomaly contributions to the glueball masses

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Dept. of Phys., Tohoku Univ.



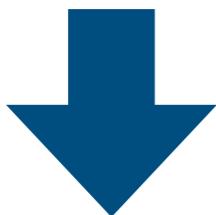
ConfXV., Aug. 1-6, 2022



Contents of this talk

What is the origin of the glueball masses?

in pure Yang-Mills theory,
breaking of scale invariance is induced by quantum effects



trace anomaly

classical

renormalization

$$T_{\mu\mu} = 0$$



quantized

$$T_{\mu\mu} \neq 0$$

purpose

to quantify the trace anomaly contribution
to the glueball masses by using lattice simulations

Trace anomaly

Decomposition
of EMT

$$T_{\mu\nu} = \bar{T}_{\mu\nu} + \hat{T}_{\mu\nu}$$

traceless trace $\neq 0$

X.-D. Ji, Phys. Rev. Lett. 74, 1071 (1995).

because $T_{00} = H$, $H = \bar{H} + \hat{H}$



Mass Decomposition

$$M = \bar{M} + \hat{M}$$

Trace anomaly is important to hadron mass generation

nucleon

$$M_N = \frac{M_q + M_g}{\text{kinetic energy}} + M_a + M_m$$

quark condensate

trace anomaly(gluon condensate)

Trace anomaly

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$$M_G = \cancel{M_q} + \boxed{M_g + M_a} + \cancel{M_m}$$

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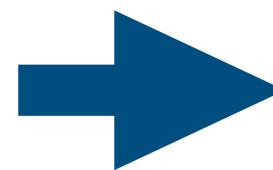
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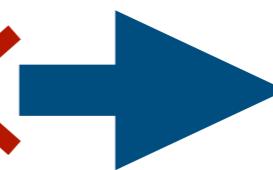
Renormalized energy-momentum tensor on the lattice
is necessary to measure each contribution

Gradient flow

Discretization
of space-time



Translational
invariance



It's difficult to
construct EMT

EMT can be constructed using the **gradient flow**

H. Suzuki, PTEP 2013, 083B03 (2013).



Diffusing the gauge fields as a function of $\tau (\geq 0)$

Flow equation

M. Lüscher, JHEP. 1008, 071 (2010).

$$\partial_\tau B_\mu(\tau, x) = D_\nu G_{\nu\mu}(\tau, x), \quad B_\mu(\tau = 0, x) = A_\mu(x)$$

$$G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu + [B_\mu, B_\nu], \quad D_\mu = \partial_\mu + [B_\mu, \cdot]$$

τ : diffusion time diffusion radius $R_d = \sqrt{8\tau}$

No UV divergence in operators
which are constructed using diffused gauge fields

Energy-momentum tensor (EMT)

by using flowed gauge fields at,

$G_{\mu\nu}^a$: field strength

$$U_{\mu\nu}(\tau, x) \equiv G_{\mu\rho}^a(\tau, x)G_{\nu\rho}^a(\tau, x) - \frac{1}{4}\delta_{\mu\nu}G_{\rho\sigma}^a(\tau, x)G_{\rho\sigma}^a(\tau, x)$$

$$E(\tau, x) \equiv \frac{1}{4}G_{\mu\nu}^a(\tau, x)G_{\mu\nu}^a(\tau, x) \quad \text{with flow time } \tau$$

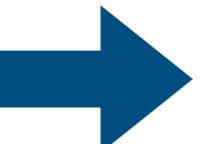
H. Suzuki, PTEP 2013, 083B03 (2013).

$$\left\{ T_{\mu\nu}(x) \right\}_R = \lim_{\tau \rightarrow 0} \left\{ \frac{1}{a_U(\tau)} U_{\mu\nu}(\tau, x) + \frac{1}{4a_E(\tau)} \delta_{\mu\nu} (E(\tau, x) - \langle E(\tau, x) \rangle_0) \right\}$$

traceless
trace anomaly

$\alpha_U(\tau), \alpha_E(\tau)$: perturbative coefficients

M. Asakawa, et al. Phys. Rev. D 90, 011501 (2014).

$\tau \rightarrow 0$ limit must be taken  on lattice, the minimal spatial distance is a

diffusion radius of gradient flow $R_d = \sqrt{8\tau}$

on lattice

R_d must be in the range of $a \ll R_d \ll (\text{box size})$

Mass and operator expectation values in lattice QCD

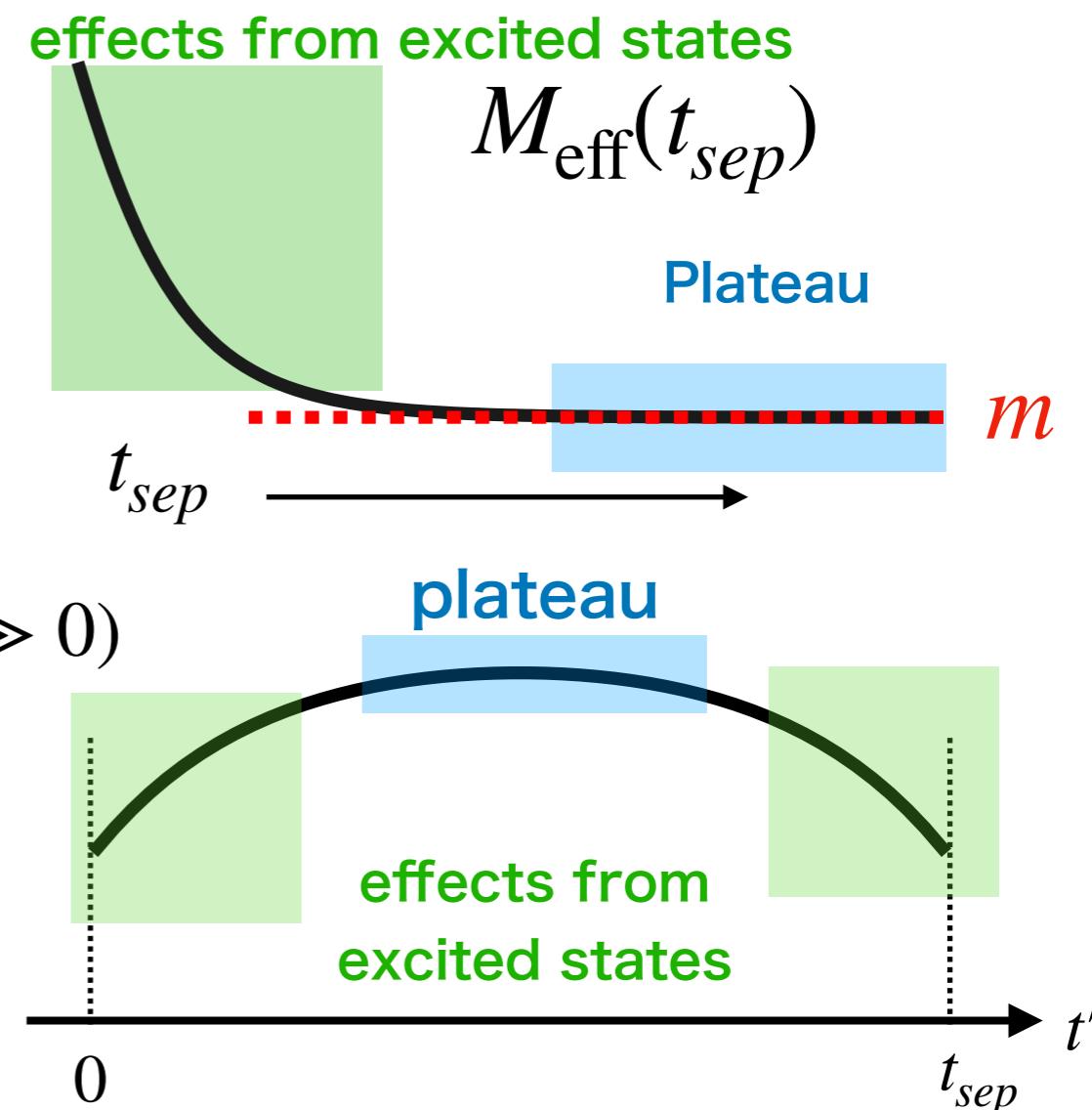
Glueball 2pt. func. $C(t_{sep})_{2pt} = \langle 0 | G(t_{sep})G^\dagger(0) | 0 \rangle$ G : Glueball operator

$$C(t_{sep})_{2pt} = \langle 0 | e^{Ht_{sep}} G(0) e^{-Ht_{sep}} G^\dagger(0) | 0 \rangle = \sum_i |\langle 0 | G | i \rangle|^2 e^{-E_i t_{sep}} \sim e^{-E_0 t_{sep}} \quad (t_{sep} \gg 0)$$

The ground state

∴ By defining effective mass as $M_{\text{eff}}(t_{sep}) = -\log \frac{C(t_{sep} + 1)}{C(t_{sep})}$,

$$M_{\text{eff}}(t_{sep}) \rightarrow E_0 = m, \quad (t_{sep} \gg 1)$$



expectation value of $T_{\mu\nu}$

$$\frac{\langle 0 | G(t_{sep})T_{\mu\nu}(t')G(0) | 0 \rangle}{C(t_{sep})_{2pt}} \rightarrow \langle T_{\mu\nu} \rangle, \quad (t_{sep} \gg t' \gg 0)$$

$$\propto M_{EMT}$$

Glueball two-point function and effective mass plot

Expectation value of operator $T_{\mu\nu} \rightarrow \langle T_{\mu\nu} \rangle \simeq \frac{\langle G(t_{sep})T_{\mu\nu}(t')G(0) \rangle}{\langle G(t_{sep})G(0) \rangle}, (t_{sep} \gg t' \gg 0)$

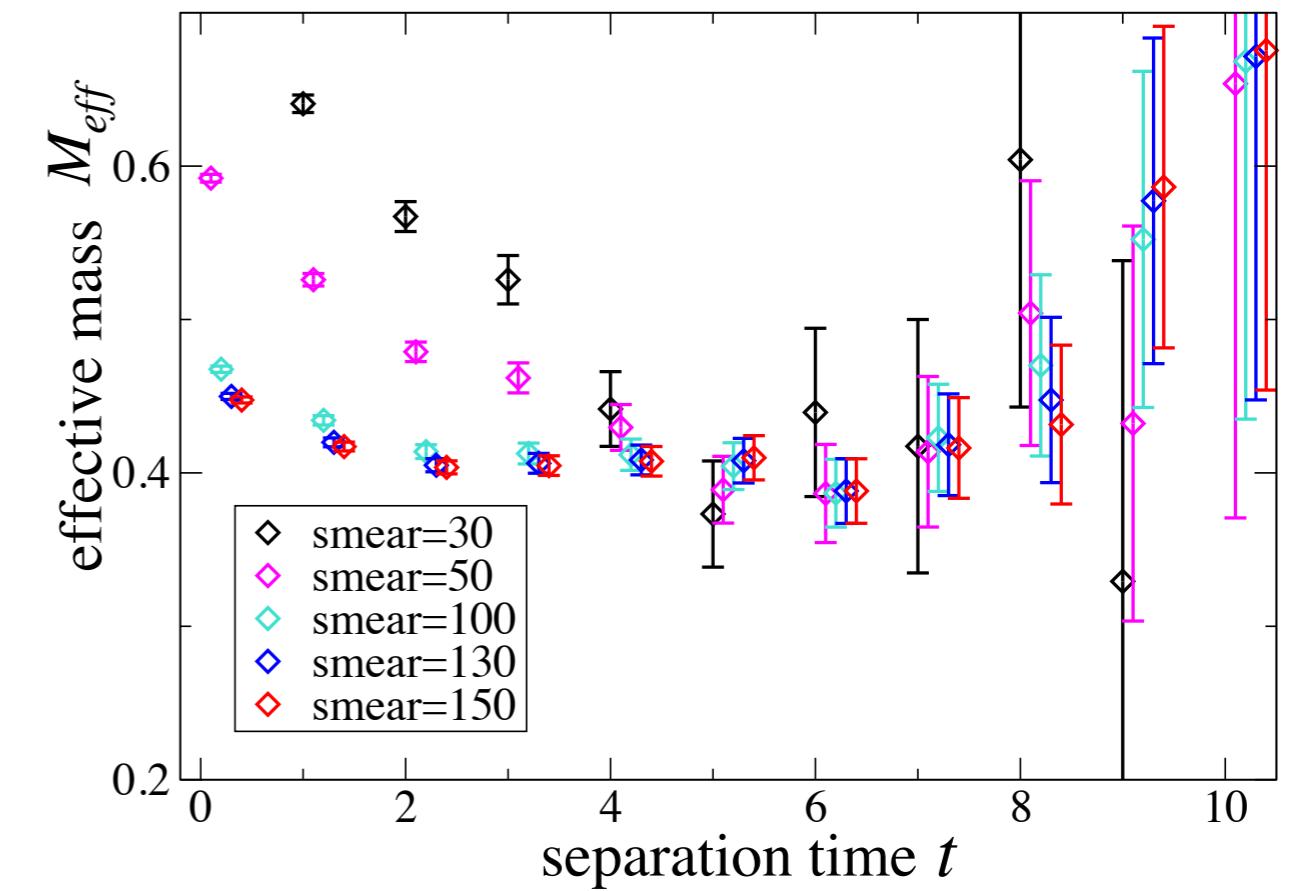
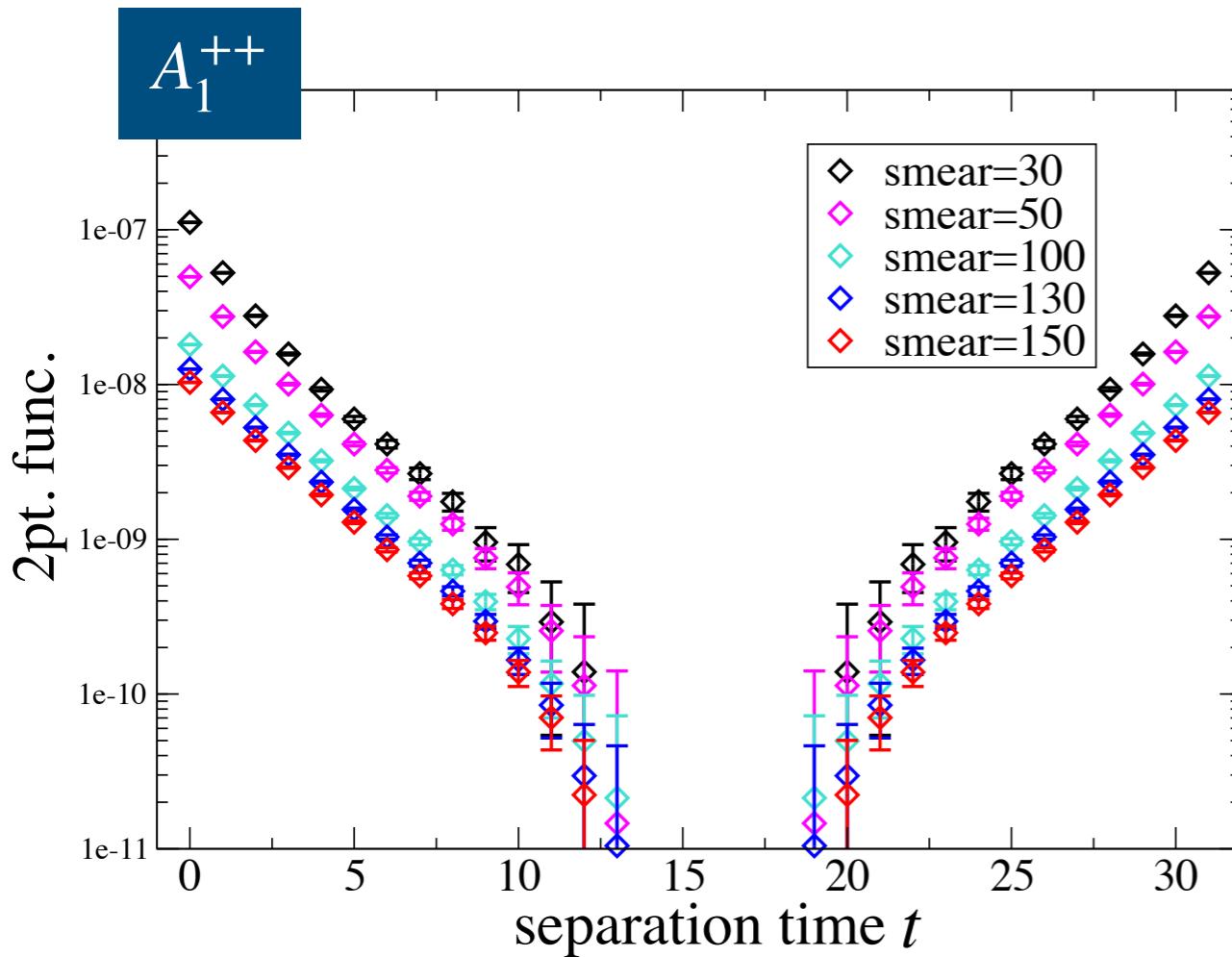
Precisions of 2pt. func. $\langle G(t_{sep})G(0) \rangle$ are very important

Stout smearing[1] is used to construct extended glueball operators[2]

[1] C. Morningstar and M. Peardon, Phys. Rev. D 69, 054501 (2004)

[2] K. S and S. Sasaki, PoS(LATTICE2021)333.

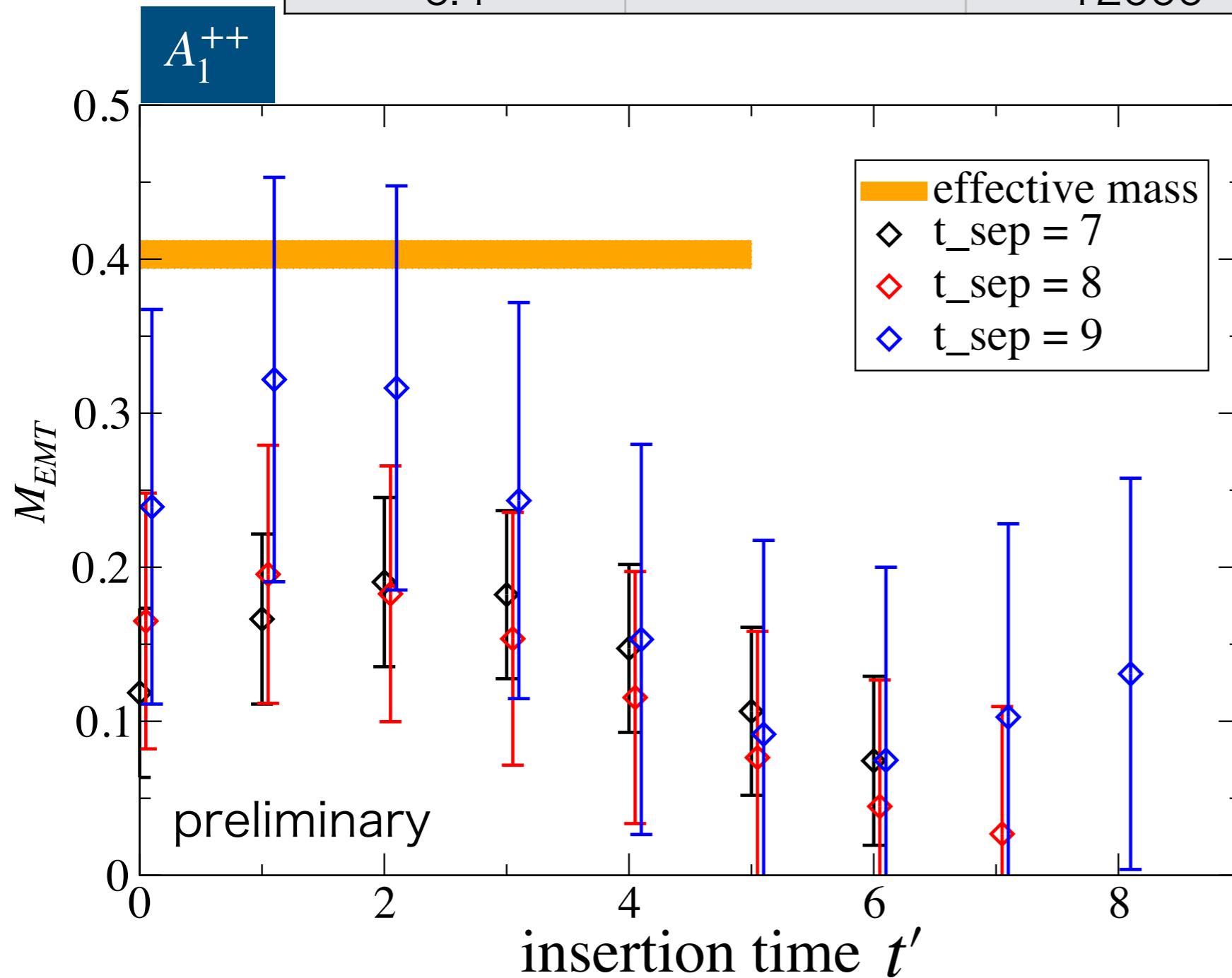
$\beta (= 6/g^2)$	$L^3 \times T$	Meas.	$1/a[\text{GeV}]$
6.4	$32^3 \times 32$	12000	3.84



Glueball mass calculated from EMT operator

$$\langle T_{00} \rangle \simeq \frac{\langle G(t_{sep}) T_{00}(t') G(0) \rangle}{\langle G(t_{sep}) G(0) \rangle}, \quad (t_{sep} \gg t' \gg 0) \quad T_{00}(t') \text{ given at } \tau = 2.5$$

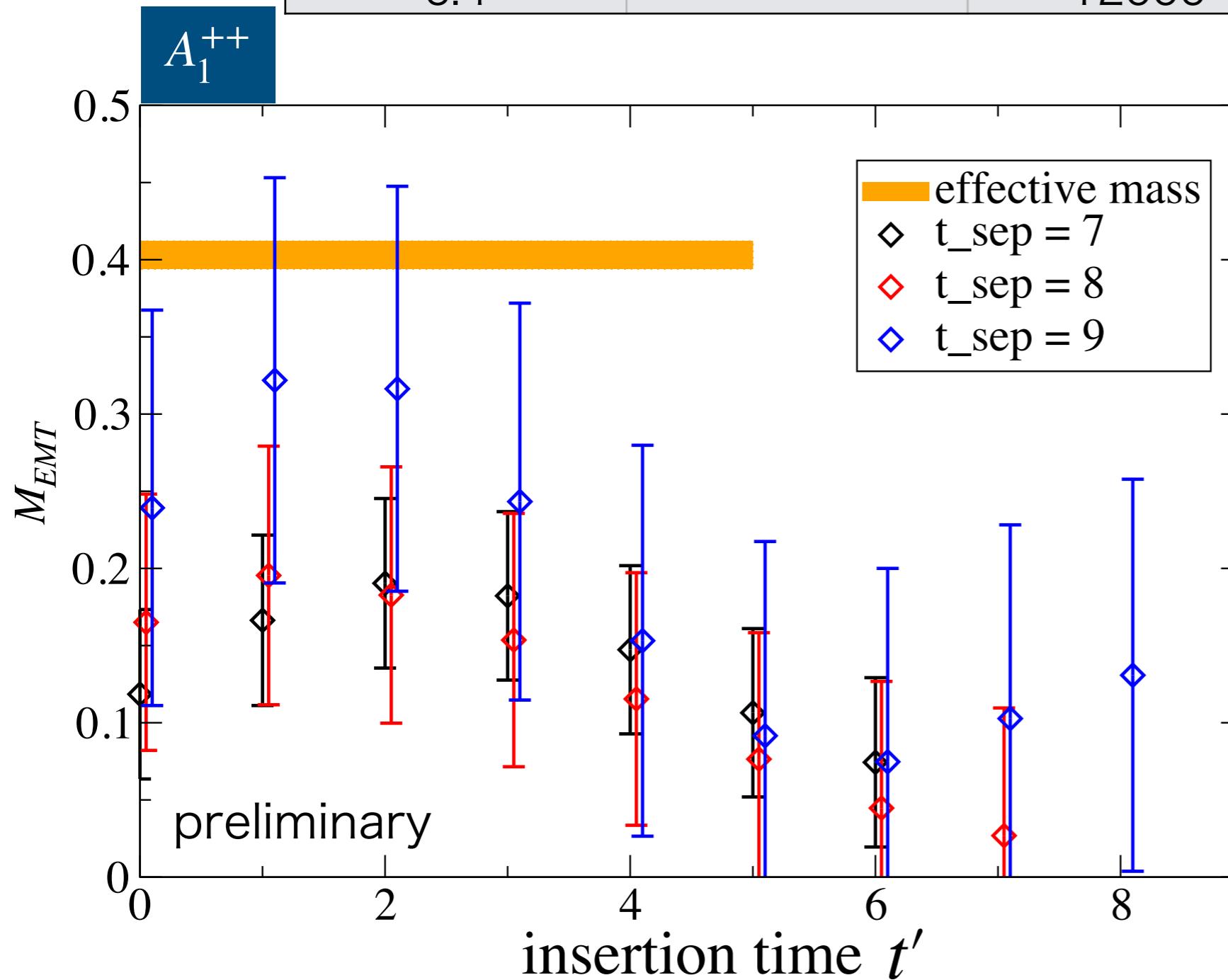
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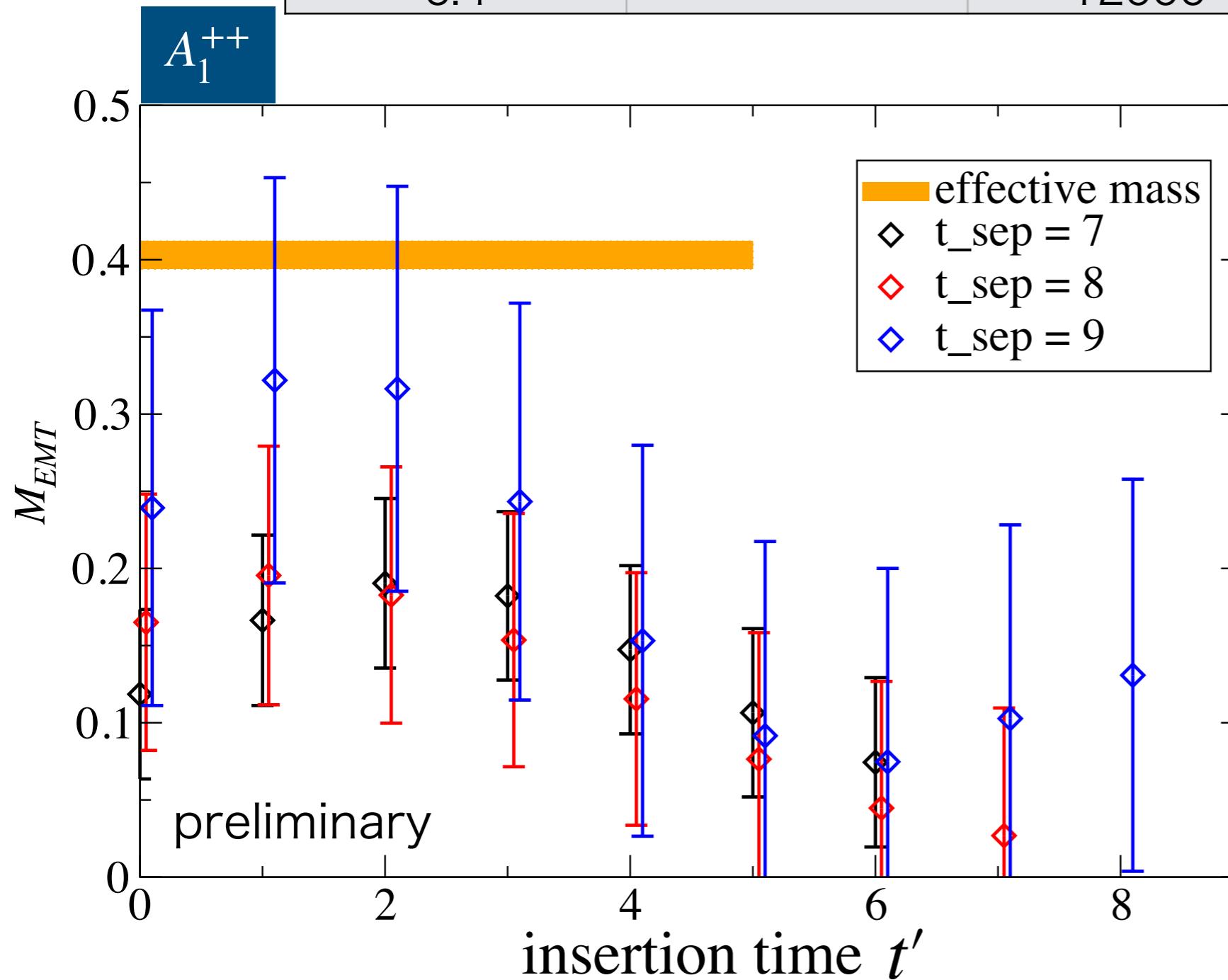


It shows a consistency
with effective mass

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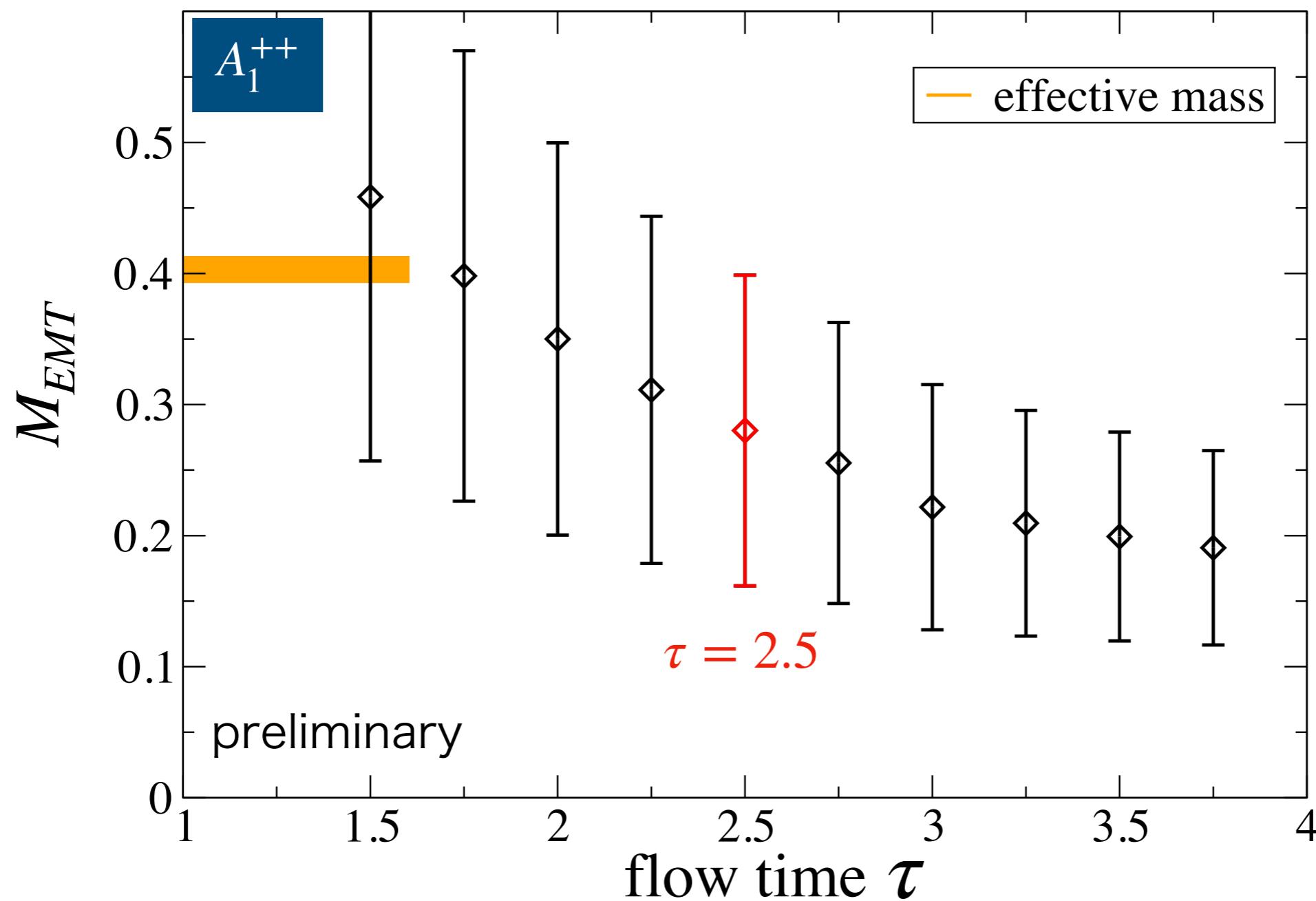
$t_{sep} = 9$ possibly isn't
long enough to reach
saturation, though
 $t_{sep} = 10$ doesn't have
enough precision

Flow time dependence

To get correct EMT, flow time $\tau \rightarrow 0$ limit has to be taken eventually
→ It needs to be done after the continuum limit

H. Suzuki, PTEP 2013, 083B03 (2013)

Flow time dependence becomes mild as $\tau \rightarrow$ large

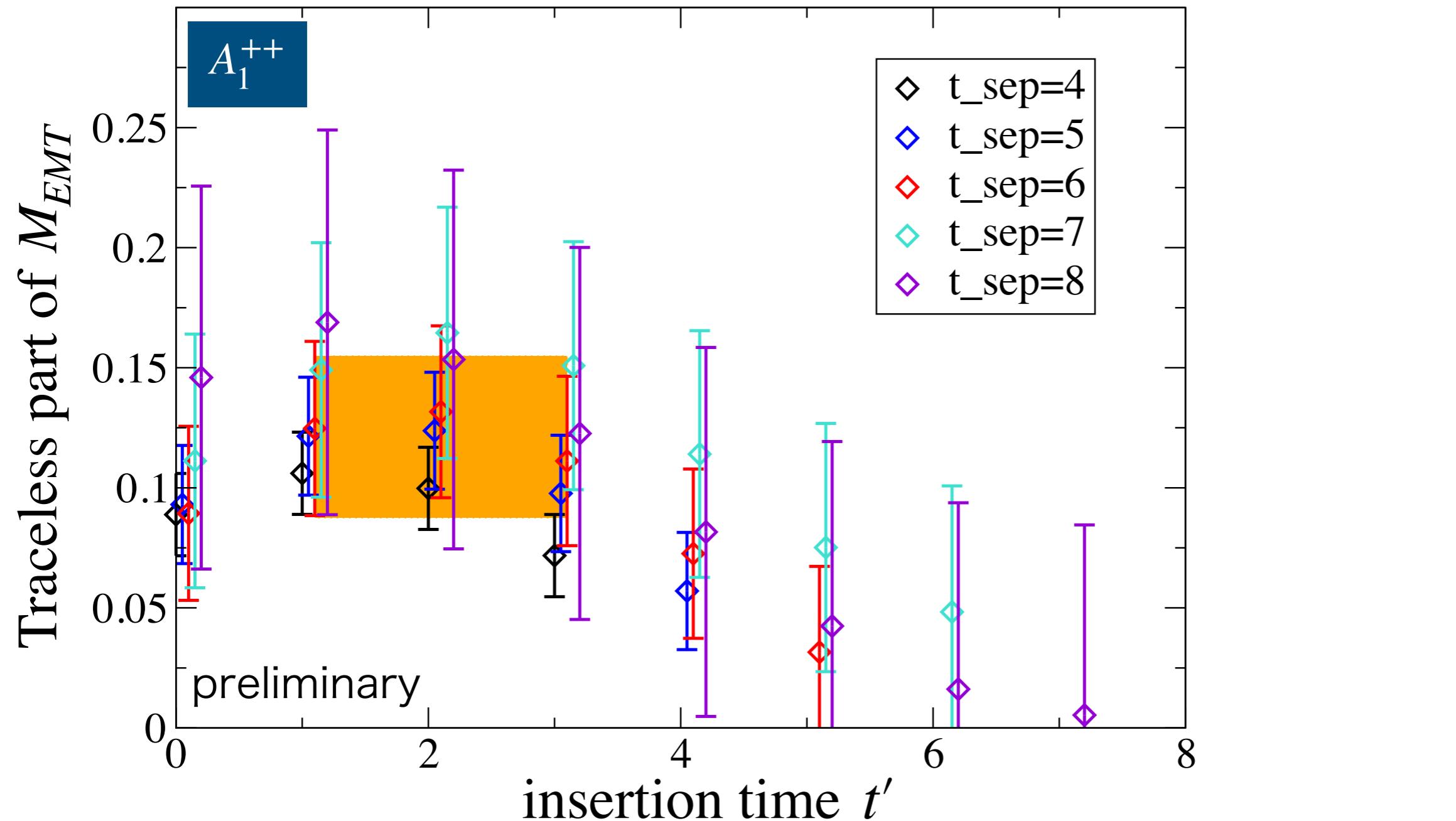


Traceless part

$\beta (= 6/g^2)$	$L^3 \times T$	Meas.	$1/a[\text{GeV}]$
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Behavior of traceless part

$$M_{EMT} = M_{\text{traceless}} + M_{\text{anomaly}}$$

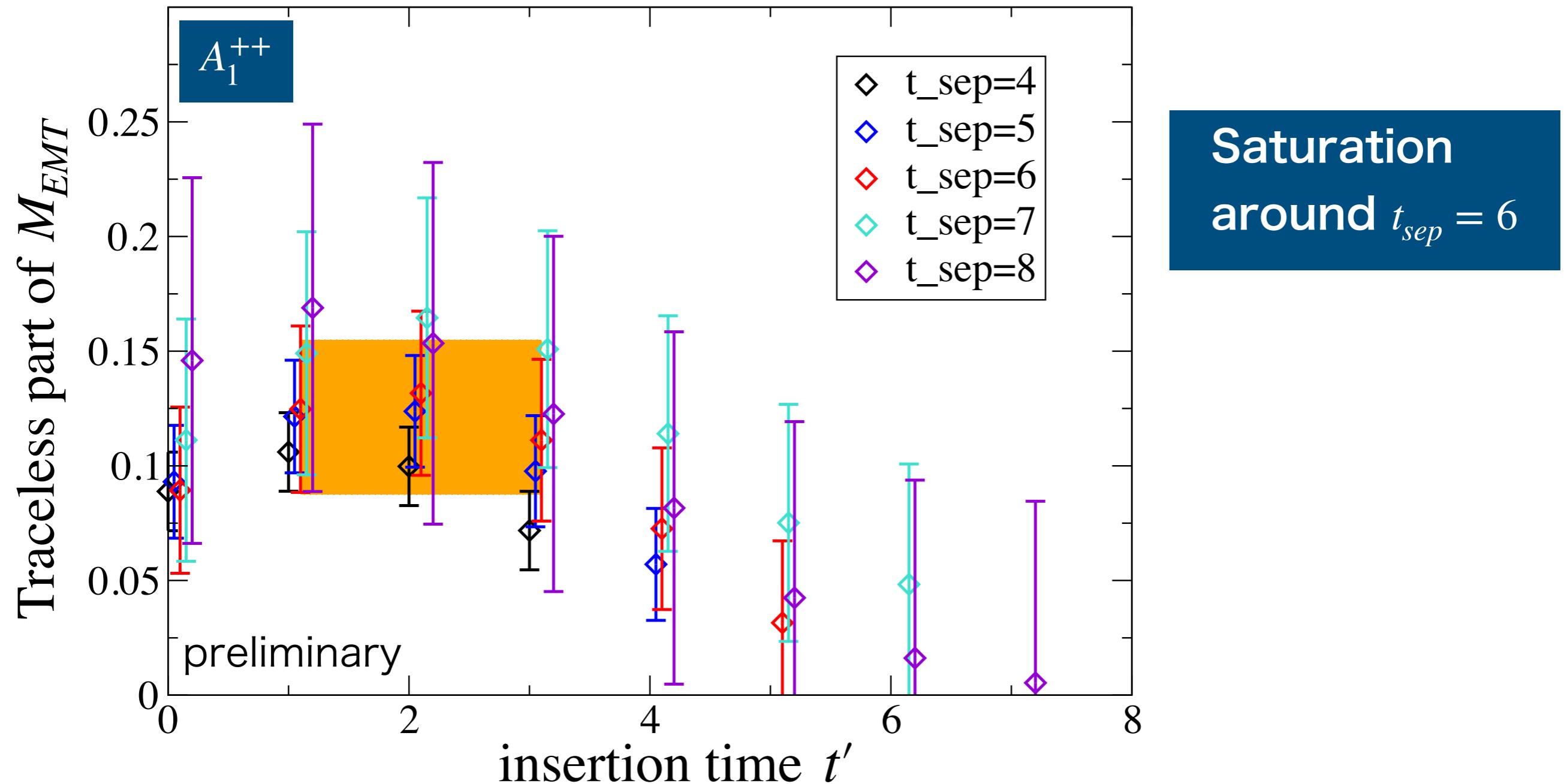


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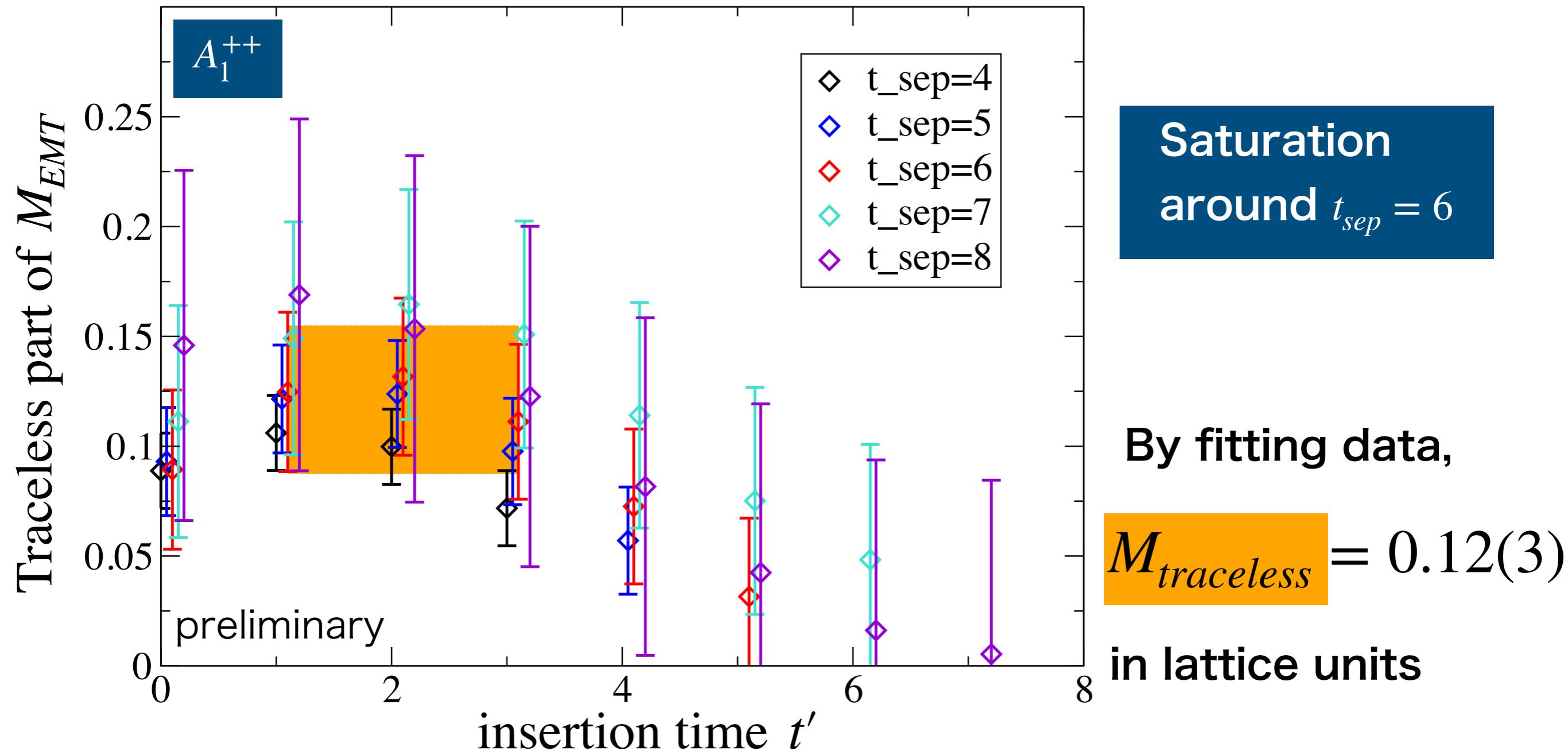


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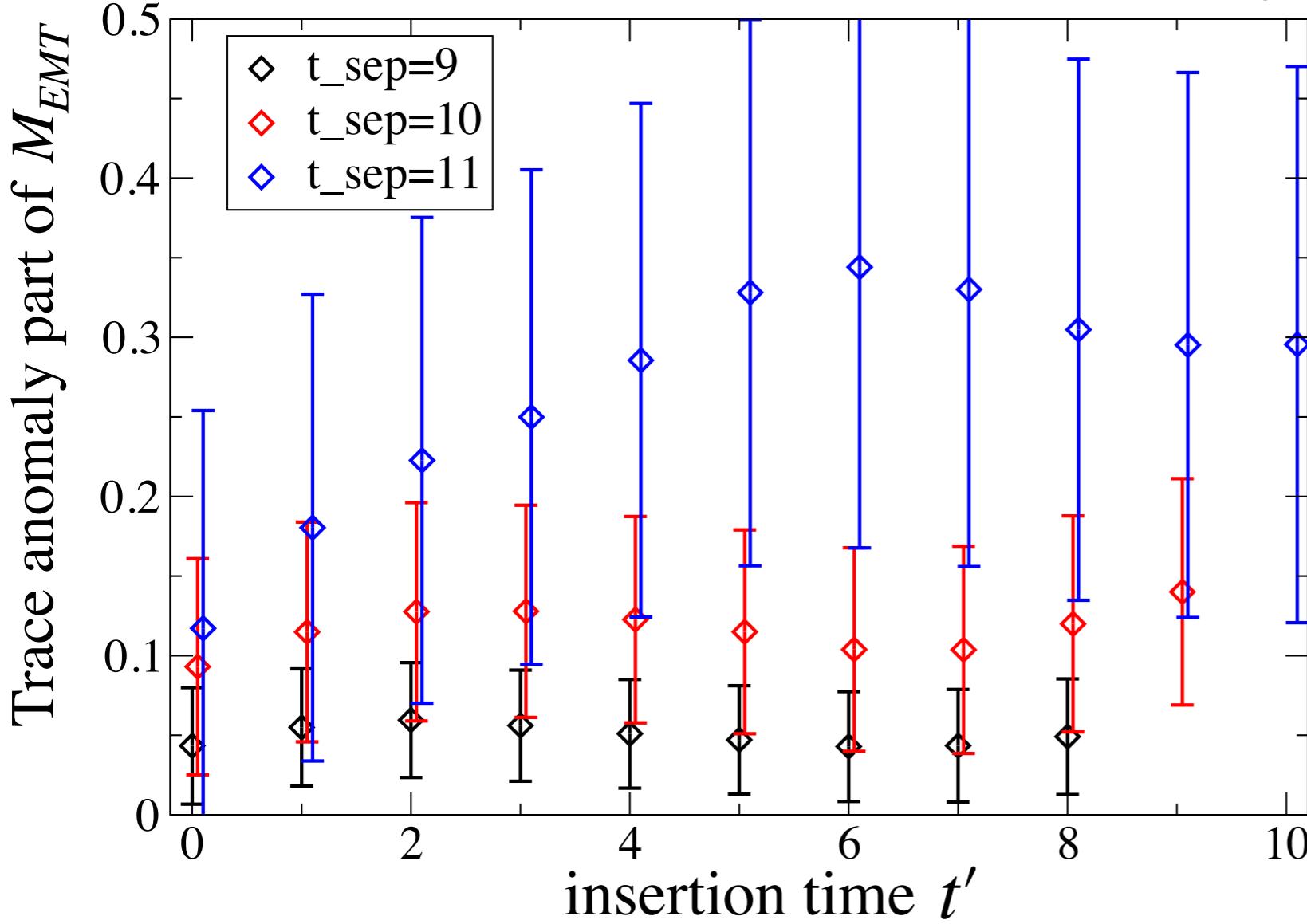
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A_1^{++}

preliminary



Trace anomaly part

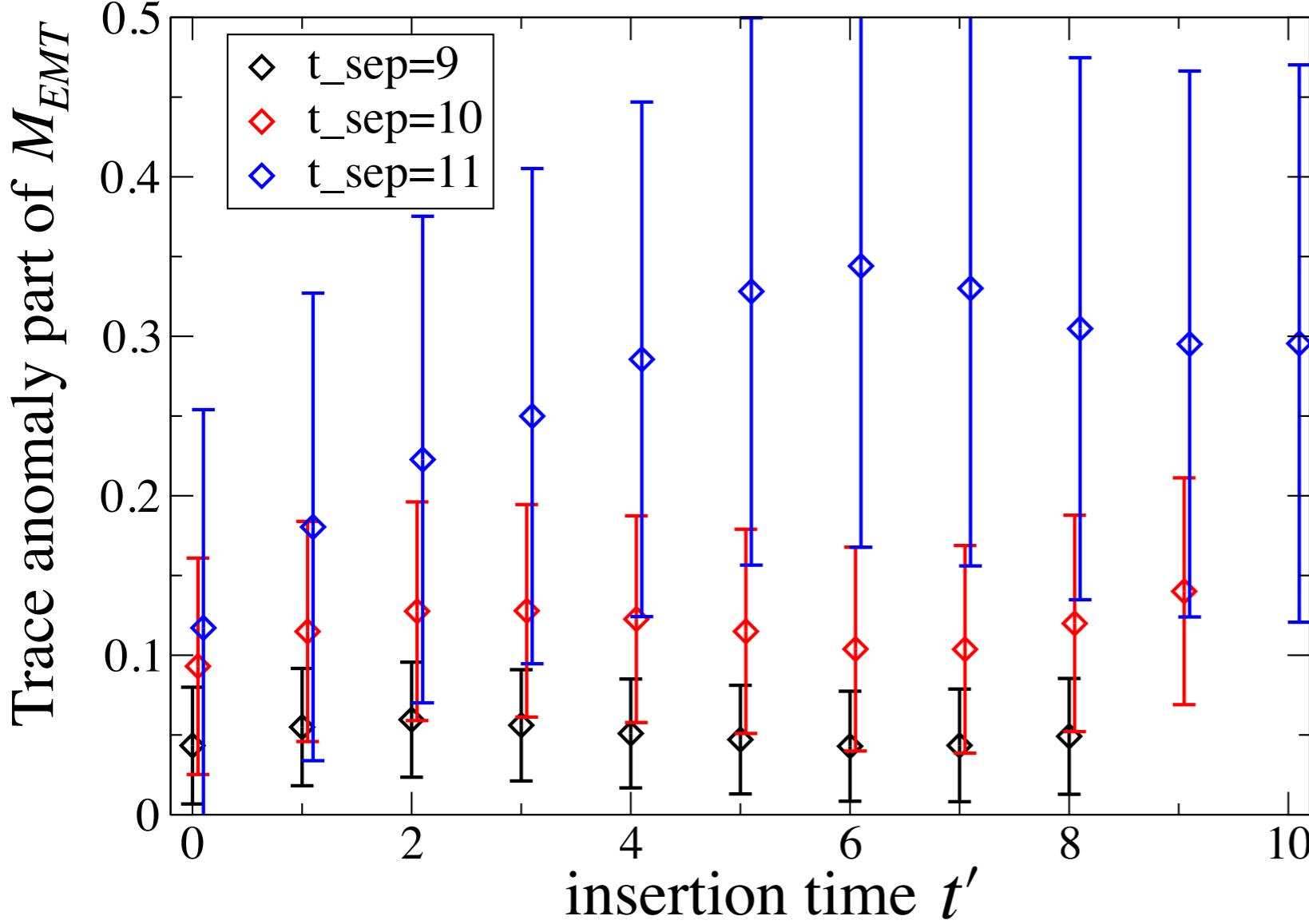
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$$\langle T_{00}^{\text{anomaly}} \rangle = \frac{\langle G(t_{\text{sep}}) T_{00}^{\text{anomaly}}(t') G(0) \rangle}{\langle G(t_{\text{sep}}) G(0) \rangle}, (t_{\text{sep}} - t' \rightarrow \infty, t' \rightarrow \infty)$$

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$t_{\text{sep}} = 11$ isn't
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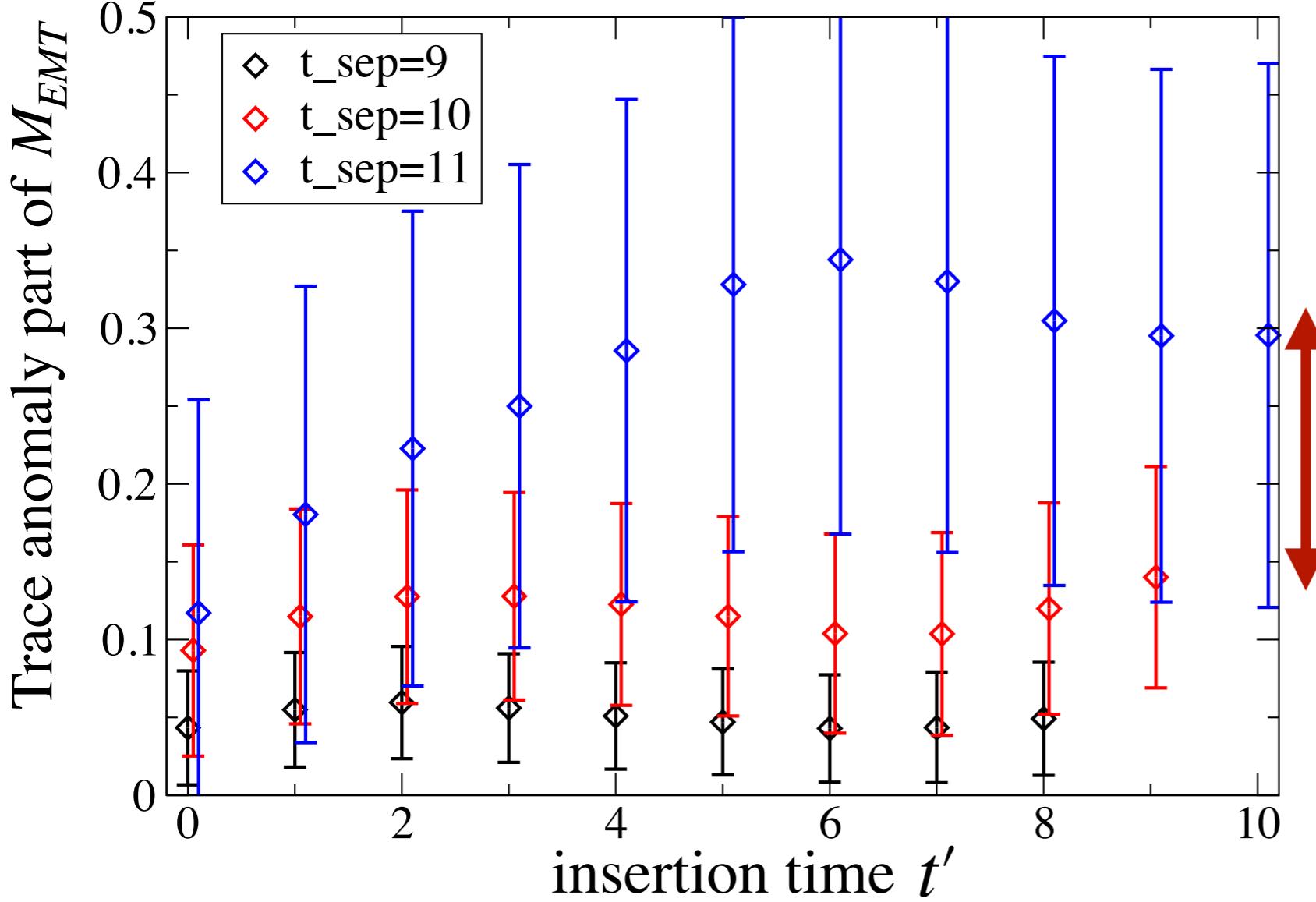
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Big change between
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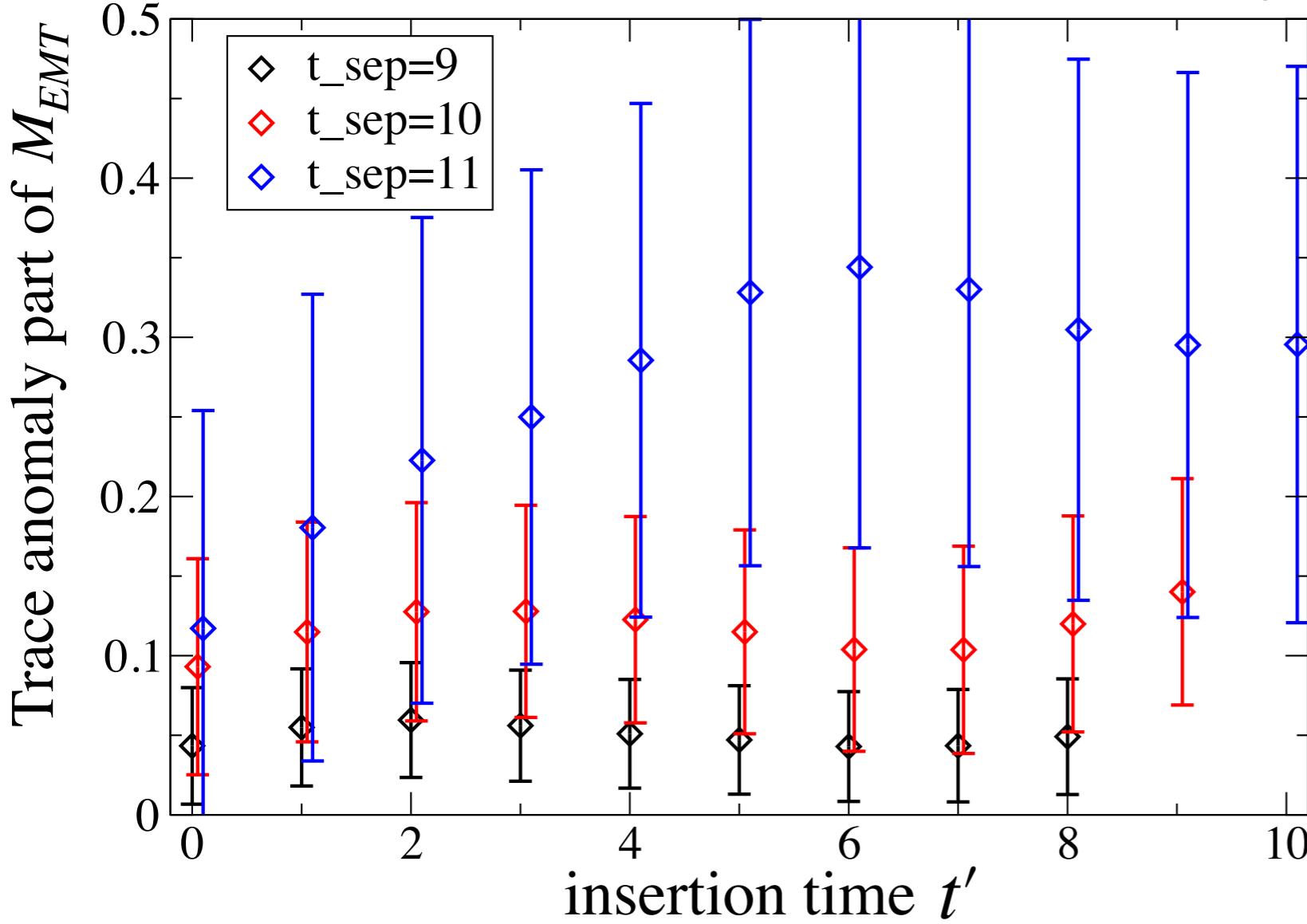
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$t_{\text{sep}} = 11$ isn't
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Big change between
 $t_{\text{sep}} = 10$ and 11

2pt. func. doesn't
have enough precision?



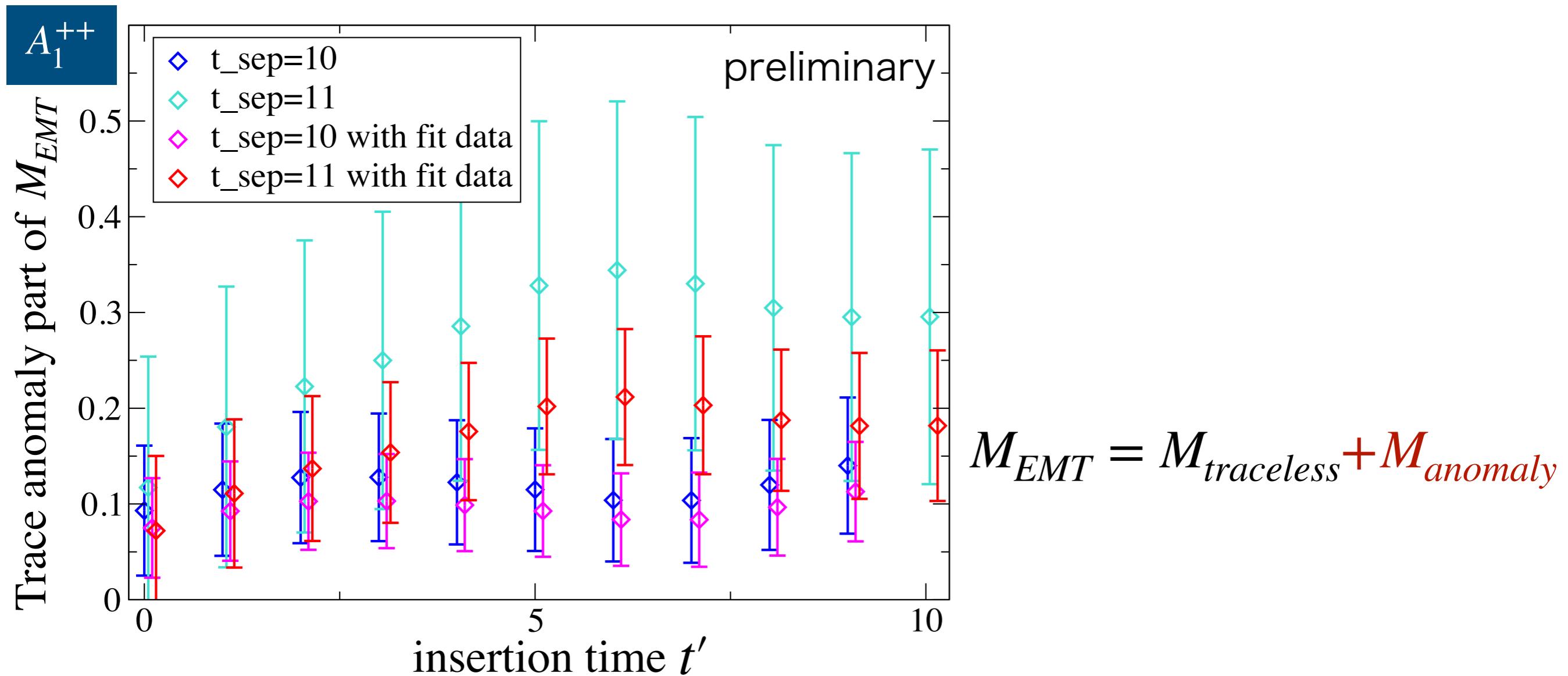
Improvement of long separation time behavior

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1. Getting A and M from fitting of 2pt. func. data

2. Using $A_{fit} e^{-M_{fit} t}$ func as 2pt. func. in calculations of $\langle T_{00} \rangle$



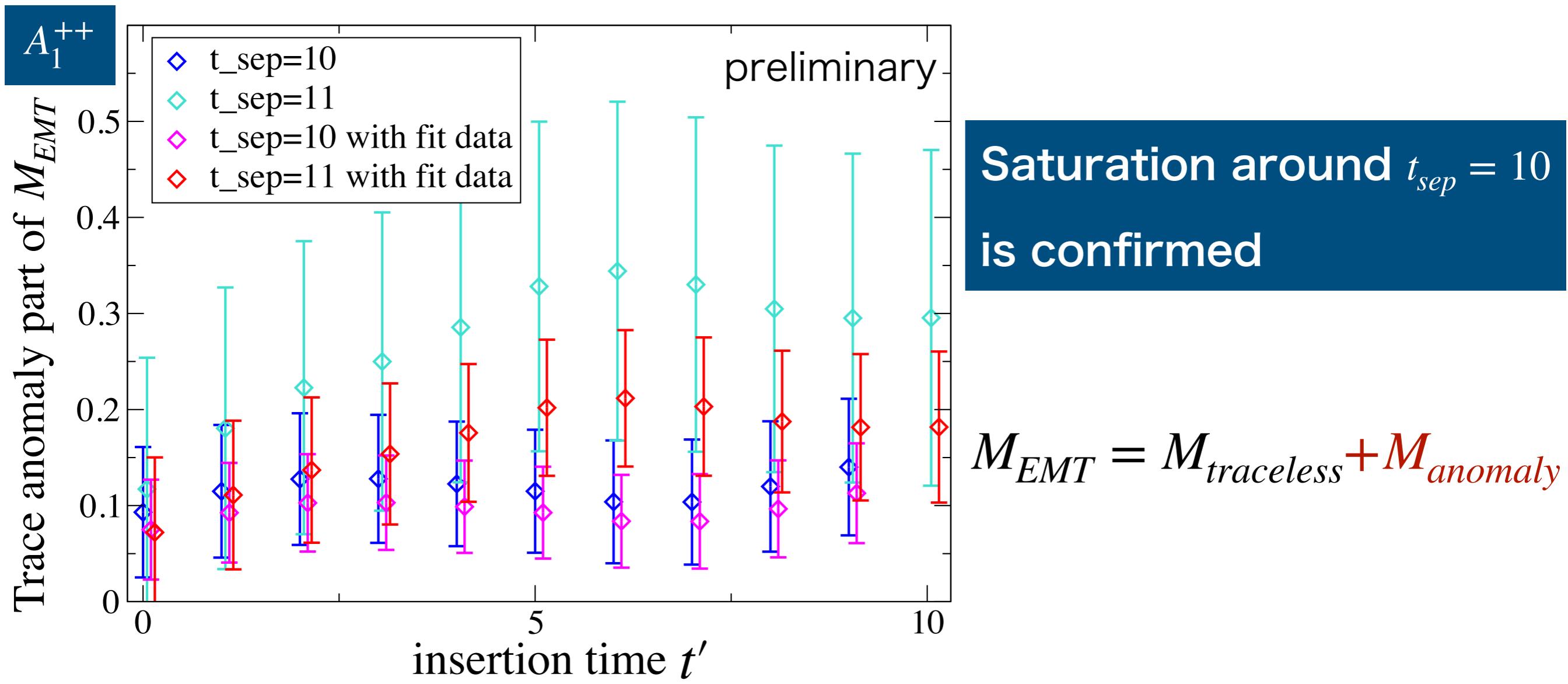
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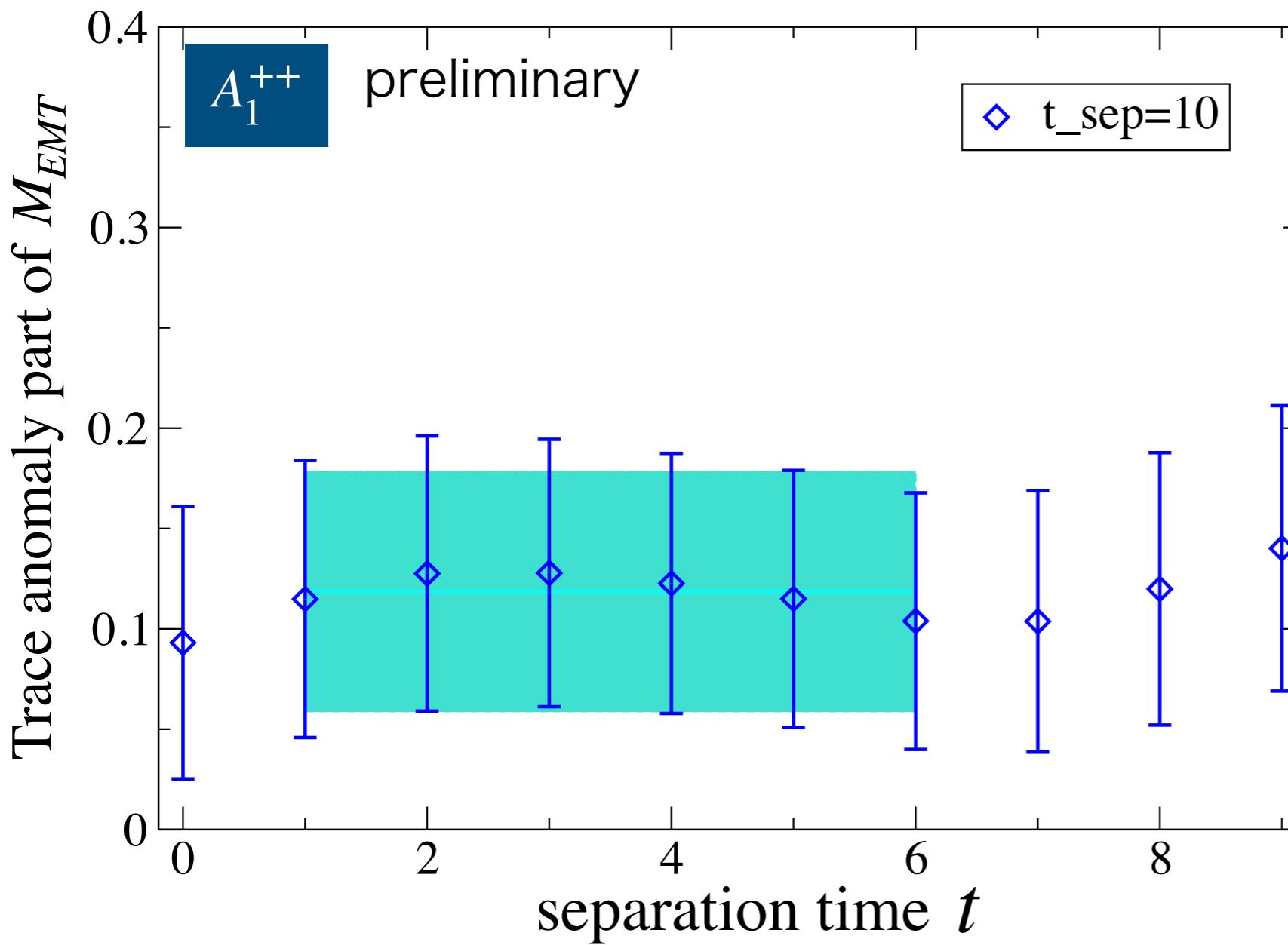
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Trace anomaly part

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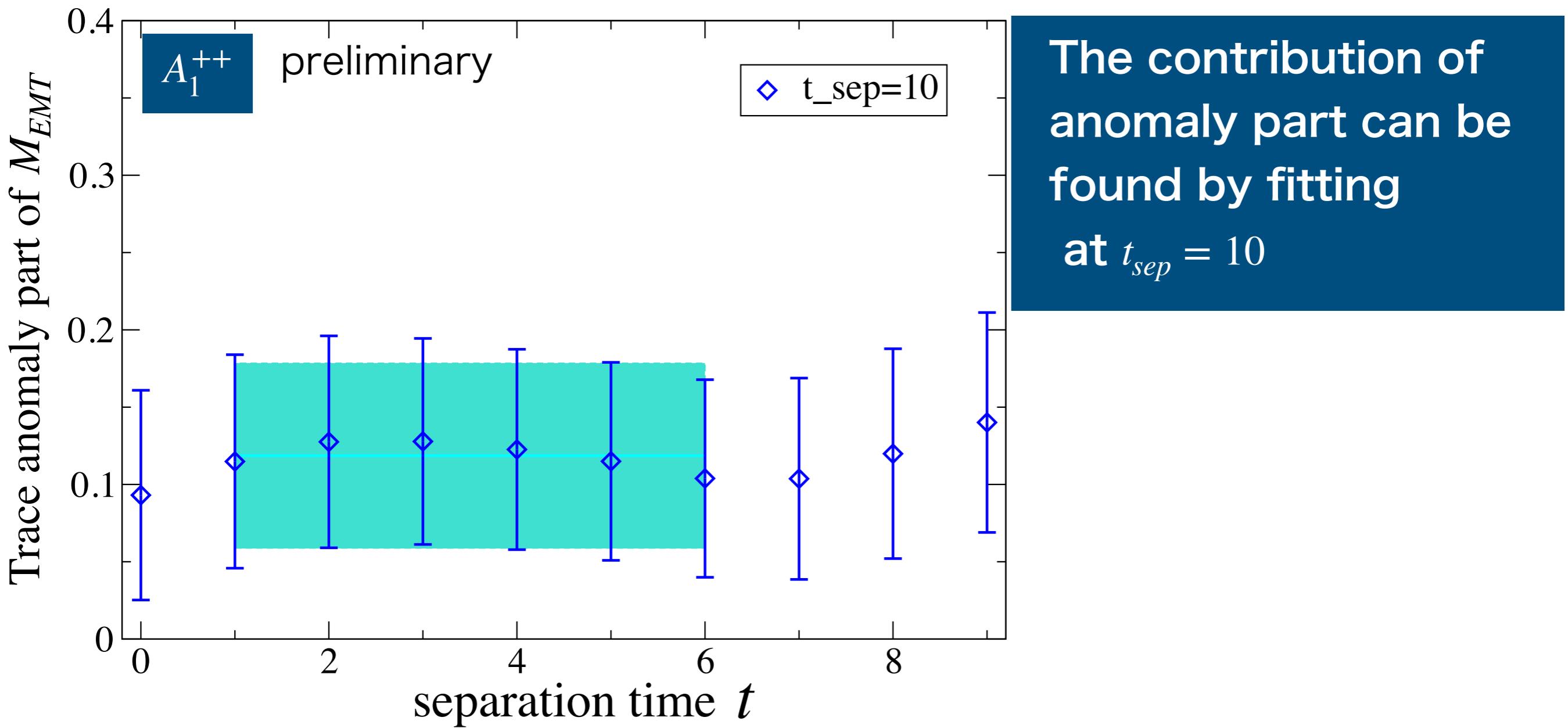
Behavior of trace anomaly part



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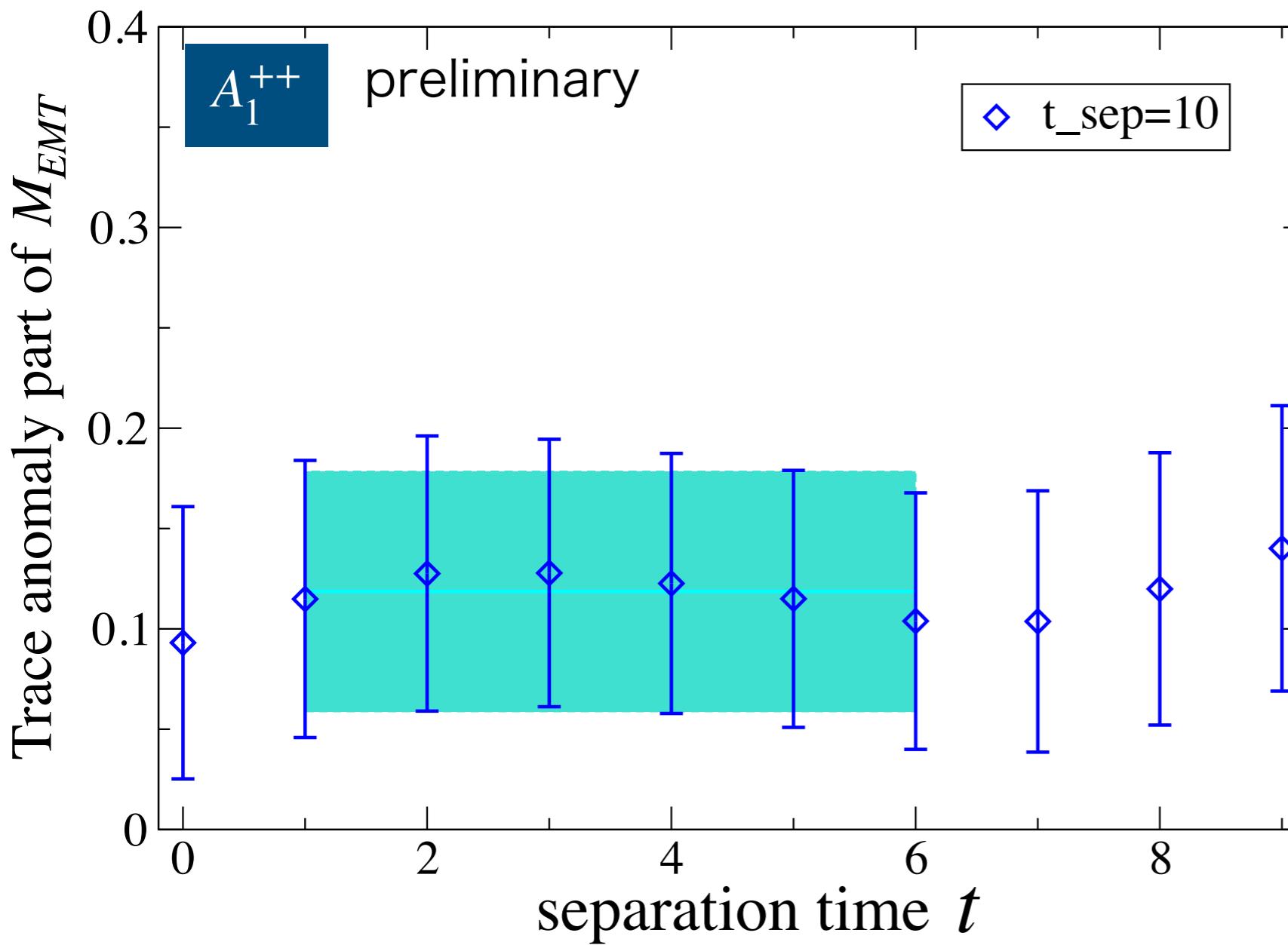
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Behavior of trace anomaly part



The contribution of anomaly part can be found by fitting
at $t_{sep} = 10$

By fitting data,
 $M_{anomaly} = 0.12(6)$
in lattice units

Mass decomposition of glueball

A sum of the traceless and trace anomaly part gives

$$M_{\text{traceless}} + M_{\text{anomaly}} = 0.24(7) \quad \text{at } \tau = 2.5$$

whose value agrees with the full EMT result $M_{EMT} = 0.28(12)$.

In comparison to the effective mass,

$$M_{EMT}/M_{\text{eff}} = 0.60(17)$$

This could approach to 1
with $\tau \rightarrow 0$ limit

By taking the ratio of $M_{\text{traceless}}/M_{\text{anomaly}} = 1.0(6)$

$$M_{EMT} = M_{\text{traceless}}(50 \pm 20\%) + M_{\text{anomaly}}(50 \pm 29\%) \text{ at } \tau = 2.5$$

We confirm that trace anomaly certainly plays an important role on glueball mass generation

Summary and future work

Summary

- We have studied trace anomaly contribution to glueball mass.
- The renormalization energy-momentum tensor (EMT) operator is constructed by using the gradient flow.
- Glueball mass M_{EMT} are calculated by using EMT operator through the ratio of 3pt. and 2pt. functions.
- We found that $M_{EMT}/M_{eff} = 0.60(17)$ at $\tau = 2.5$ of which value approaches unity toward $\tau \rightarrow 0$.
- The trace anomaly certainly contributes the mass of glueball as $M_{EMT} = M_{anomaly}(50 \pm 20\%) + M_{traceless}(50 \pm 29\%)$

Future work

- Taking continuum limit $a \rightarrow 0$ (in progress)
- Taking flow time $\tau \rightarrow 0$ (after continuum limit)
- Analysis of 2^{++} state glueball