Asymptotic freedom using a gluon mass as regulator

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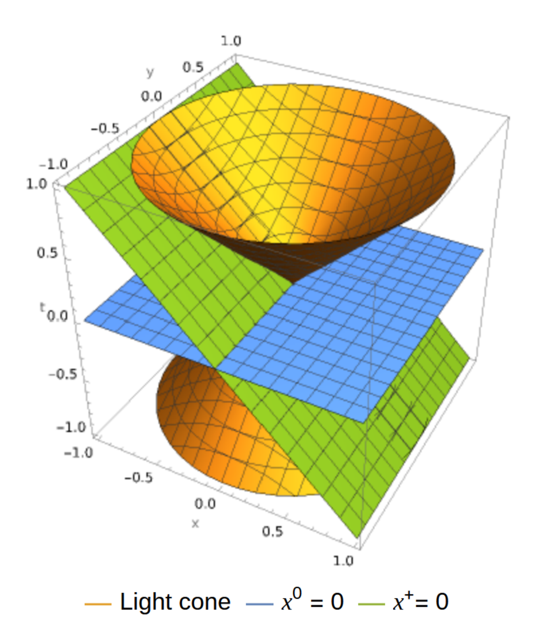
Introduction

Asymptotic freedom and the non-trivial structure of the QCD vacuum prevents a perturbative characterization of strongly-bounded systems based on equal-time commutation relations. The Front Form of Hamiltonian dynamics is boost invariant, has a simpler vacuum and presents a more adequate formalism to this end, but suffers from severe divergences. In this context Stanisław D. Głazek and Kenneth G. Wilson developed the Similarity Renormalization group (SRG) and Renormalization Group Procedure for Effective Particles (RGPEP) to obtain non-divergent results by means of effective Hamiltonians with counterterms. RGPEP has passed the test of producing asymptotic freedom, though with some undesired dependence on regularization at 3rd order in perturbation theory, so new options need to be considered. Here we test if a canonical infinitesimal mass for gluons can be used as regulator.

1. Front Form of dynamics



Light-front coordinates

$$x^{\pm} = x^0 \pm x^3 \quad x^{\perp} = \left(x^1, x^2\right)$$

Light-front momentum

$$p^{\mu} = \left(p^+, p^-, p^{\perp}\right)$$

Norm and energy

$$p^2 = p^+p^- - p^{\perp 2}$$

$$p^- = \frac{p^{\perp 2} + m^2}{p^+}$$

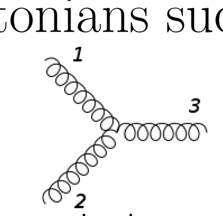
Pure gluonic QCD

With (anti)commutation relations in the front $x^+ = 0$:

$$\left[a_1, a_{1'}^{\dagger}\right] = 16\pi^3 p^+ \delta\left(p_1 - p_{1'}\right) \delta_{\sigma_{1'}}^{\sigma_1} \delta_{r_{1'}}^{r_1} = \left\{b_1, b_{1'}^{\dagger}\right\}$$

and gauge $A^+ = 0$, we obtain Hamiltonians such as:

$$H_0 = \sum_1 \int [1] \frac{\left(p^{\perp}\right)^2 + \xi^2}{p^+} a_1^{\dagger} a_1$$



$$H_1 = g\left(\sum \int\right)_{123} f_{t_r} \tilde{\delta}\left(p^{\dagger} - p\right) Y_{123} a_1^{\dagger} a_2^{\dagger} a_3 + h.c.$$

where we used a cononical gluon mass ξ and a gaussian form factor $f_{t_r} = \exp\left[-t_r\left(\mathcal{M}_i^2 - \mathcal{M}_f^2\right)^2\right]$ as regulators.

2. Effective Hamiltonians from RGPEP

The RGPEP

- Relates dynamics at different scales through equation

$$H_0\left(a_0\right) = H_t\left(a_t\right),$$

- H_0 initial Hamiltonian with counterterms, H_t effective one at scale $t = s^4 = \frac{1}{\lambda^4}$ with $[s]$ = length and $[\lambda]$ = energy.
- a_t effective operators satisfying:

$$a_t = U_t a_0 U_t^{\dagger},$$

with U_t unitary.

- Evolution with t is

$$H_t\left(a_0\right)' = -\left[U_t^{\dagger} U_t', H_t\left(a_0\right)\right]$$

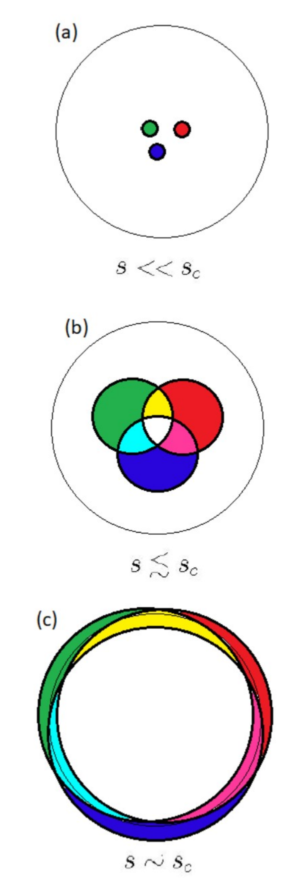


Figure 1: Figure adapted from Patryk Kubiczek

- The procedure provides ways of relaxing cutoffs by the introduction of *form factors*

$$f_t = \exp\left[-t\left(\mathcal{M}_i^2 - \mathcal{M}_f^2\right)^2\right]$$

with $\mathcal{M}$ invariant masses.

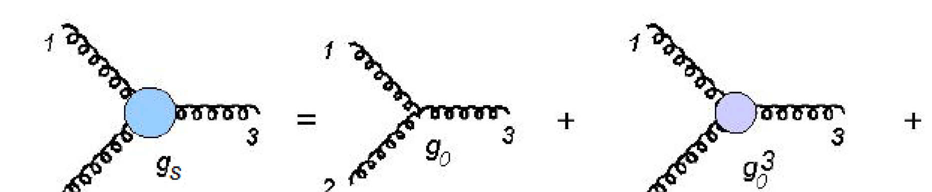
3. The 3-gluon vertex and the running coupling

- The running coupling is obtained through the 3-gluon vertex function $\mathcal{V}_{21,t} = gH_{21,g,t} + g^3 H_{21,t}$
- $\mathcal{V}_{21,t}$ is obtained solving the RGPEP equation by means of a parturbative expansion of H_t

$$H_t = H_{11,0,t} + (H_{21,g,t} + h.c) + H_{11,g^2,t} + H_{22,g^2,t} + (H_{31,g^2,t} + h.c.) + \ldots$$

where H_t with $t = 0$ is the regularized Front-Form Hamiltonian with counterterms.

- We need to consider at least first and third order terms,



where the third order diagrams are

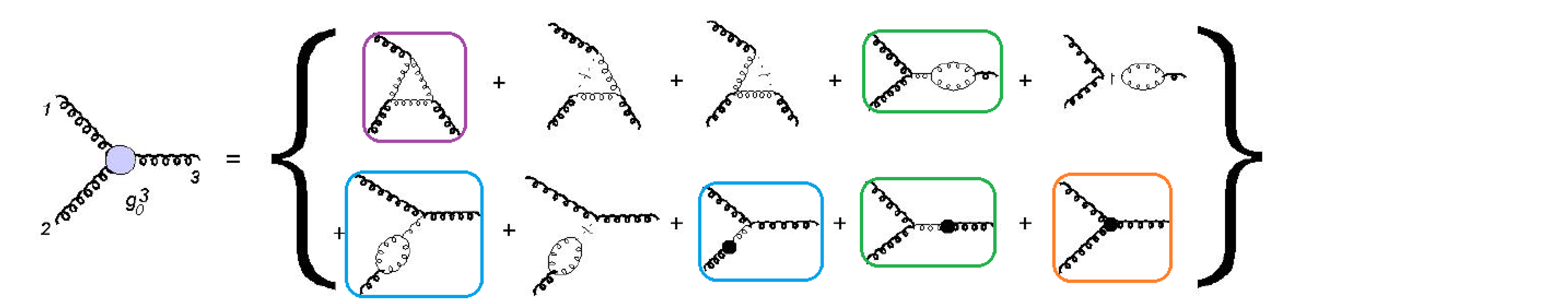


Figure 3: Third-order terms of the 3-gluon vertex, black dots denote counterterms. Green and blue enclosed counterterms cancel the corresponding 2nd-order divergences, the 3rd-order counterterm is enclosed with a orange rectangle.

- Dynamics are studied in terms of
  Longitudinal momentum fractions: $x_{1/3} \equiv x_1 = p_1^+/p_3^+$
  Relative transverse momenta: $\kappa_{12}^{\perp} \equiv \kappa_{1/3}^{\perp} = p_1^{\perp} + x_{1/3} p_3^{\perp}$
- Ultraviolet divergences are cancelled by counterterms and a reference scale t_0 is introduced.
- The coupling constant is the function in front of Y_{123} for an arbitrary value of x_1 and $\kappa_{12} \to 0$:

$$\mathcal{V}_{21,t}\left(x_{1/3}, \kappa_{12}^{\perp}, \epsilon\right) = gY_{123} + g^3 Y_{123}\left[c_t\left(x_{1/3}, \kappa_{12}^{\perp}\right) - c_{t_0}\left(x_{1/3}, \kappa_{12}^{\perp}\right) - c_0\left(x_{1/3}, \kappa_{12}^{\perp}, \epsilon\right)\right].$$

- Setting the value of g_t to be g_0 at scale t_0

$$g_t = g_0 + g_0^3 \lim_{\kappa_{12} \to 0}\left[c_t\left(x_1, \kappa_{12}^{\perp}\right) - c_{t_0}\left(x_1, \kappa_{12}^{\perp}\right)\right].$$

- The function c_t receives contributions from enclosed diagrams below. Each one is divergent when $\xi \to 0$, but the sum is finite.

4. Regularizations and results

The initial Hamiltonian has UV ($\kappa \to \infty$) and small-x ($x \to 0$) divergences. The counterterms cancel UV divergences and different options are considered for small-x:

- **Old reg.**: Several regulating functions of the small-x divergences are studied in [1]. The coupling constant is independent of their cutoffs, but a finite dependence on x_1 remains in most cases (see figure 1) and asymptotic freedom is not always observed at this order.

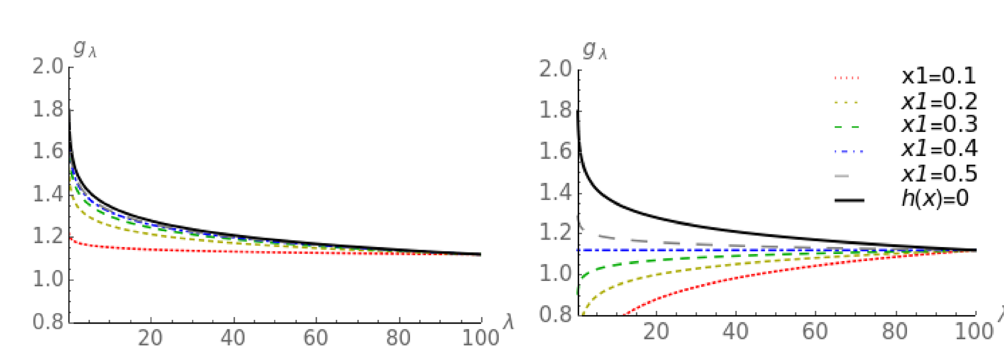


Figure 2: g_λ versus scale λ for different small-x regularizations, g_λ is set to 1.1 at $\lambda = 100$ GeV. The black line is the result for Green's functions analysis, also obtained in [1] when $h(x) = 0$.

- **New reg.**: A gluon mass ξ also serve as a small-x regulator. Form factors f_t, which appear once the RGPEP equations are solved, dump divergent terms once they have non-zero gluon masses, rendering the integral finite without the introduction of extra functions besides f_{t_r}, neccesary to regularize the bare Hamiltonian. Asymptotic freedom is produced down to $x_1 \approx 0.13$ with a finite dependence on this variable:

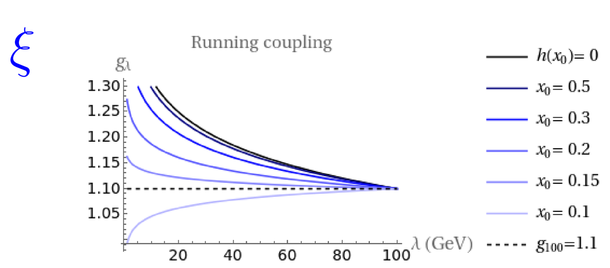


Figure 4: g_λ versus scale λ using a gluon mass as a small-x regulator, g_λ is set to 1.1 at $\lambda = 100$ GeV. The black line ($h(x) = 0$) is the result from renormalization group equations.

$$g_t = g_0 - N_c \frac{g_0^3}{48\pi^2} \log\left(\frac{\lambda}{\lambda_0}\right)\left[11 + h\left(x_1\right)\right]$$

where

$$h\left(x_1\right) = -6\left\{\frac{1 + x_2^2}{1 - x_2^2} \log\left(\frac{\left(1 + x_2^2\right)^2}{x_2^2}\right) + \frac{1 + x_1^2}{1 - x_1^2} \log\left(\frac{\left(1 + x_1^2\right)^2}{x_1^2}\right) - \frac{1 - x_1^2 x_2^2}{\left(1 + x_1^2\right)\left(1 + x_2^2\right)} + \left(1 - \frac{1}{1 - x_1^2} - \frac{1}{1 - x_2^2}\right) \log\left(\frac{8\left(1 + x_2^2\right)\left(1 + x_1^2\right)}{\left(x_1^2 + x_2^2\right)}\right)\right\}.$$

References

[1] María Gómez-Rocha and Stanisław D. Głazek. Asymptotic freedom in the front-form hamiltonian for quantum chromodynamics of gluons. *Physical Review D*, 92, 2015.

[2] Stanisław D. Głazek. Dynamics of effective gluons. *Physical Review D*, 63, 2001.

Acknowledgements

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Remarks

- Ad 1. σ_i denote spin and r_i color. Integer subscript in operators and sum simbols stand for quantum numbers: $a_1 = a_{p_1,\sigma_1,r_1}$. $\int[1] = \int \frac{\theta\left(p_1^+\right) dp_1^+ d^2 p_1^{\perp}}{16\pi^3 p_1^+}$ and $\left(\sum \int\right)_{123}$ denote sum and integration over quantum numbers denoted by 1, 2 and 3.
- Ad 2. Invariant masses are $\mathcal{M}_i = \left(\sum_{p \in i} p\right)^2$.
- Ad 3. Integer subscrips ij in Hamiltonians denote i-creation and j-annihilation operators. ϵ denotes polarization dependence and c_0 is the finite part of the 3rd-order counterterm.
- Ad 4. Function $h(x_1)$ depends only on x_1 since $x_2 = 1 - x_1$; $x_1 \in (0, 1)$.
- Ad 5. The funtion h_{x_1} presented at the conference is corrected in this version.