Path integral study of the Casimir effect in a chiral medium

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Motivation

- Growing evidence of interplay between Casimir effect, confinement, and other nonperturbative effets from lattice[1]
- Introducing chiral media is known to result in potential repulsive Casimir effect[2]
- Tunable, repulsive Casimir force can be useful in micro(electro)mechanical systems
- Revisit parallel plate setup in gauge invariant path integral formalism

[1] Chernodub, Goy, Molochkov, and Nguyen 2018; Chernodub, Goy, and Molochkov 2019.
[2] Fukushima, Imaki, and Qiu 2019; Wilson, Allocca, and Galitski 2015.

Starting point

Euclidean QED with θ-term to model chiral media

$$S = \int \mathrm{d}^4x \left[\frac{1}{4} F_{\mu\nu} F_{\mu\nu} + i \frac{\theta(x)}{4} F_{\mu\nu} \widetilde{F}_{\mu\nu} \right]$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and $\widetilde{F}_{\mu\nu} = \frac{1}{2} \varepsilon_{\mu\nu\rho\sigma} F_{\rho\sigma}$.

In Feynman gauge this can be written as

$$S = -\frac{1}{2} \int \mathrm{d}^4x \, A_\mu K_{\mu\nu} A_\nu$$

with

$$K_{\mu\nu} = \delta_{\mu\nu} \partial^2 + i \varepsilon_{\mu\nu\rho\sigma} \partial_\sigma \theta(x) \partial_\rho$$

Boundary conditions on plates

Apply perfect electromagnetic conductor (PEMC)[3] boundary conditions to plates:

$$n_\mu F_{\mu\nu} + i\theta_\pm n_\mu \widetilde{F}_{\mu\nu} \Big|_{z=\pm\frac{L}{2}} = 0$$

where $n_\mu = (0, 0, 0, 1)$ is the normal vector of the plates.

Implement boundary conditions in the action using auxiliary fields[4]

$$S_{\text{bc}} = \int_{z=-\frac{L}{2}} \mathrm{d}^3\mathbf{x}\, \bar{b}_i \Big(F_{3i} + i\theta_- \widetilde{F}_{3i} \Big) + \int_{z=\frac{L}{2}} \mathrm{d}^3\mathbf{x}\, b_i \Big(F_{3i} + i\theta_+ \widetilde{F}_{3i} \Big)$$

[3] Lindell and Sihvola 2005.

[4] Bordag, Robaschik, and Wieczorek 1985.

Cases

EOM only depends on $\partial_\mu \theta(x)$ + choose $\theta(x)$ to only depend on z: $\beta(z) = \partial_z \theta(z)$

Linear $\theta(z)$ (constant $\beta(z)$) is known to be effective action of Weyl Semimetal (WSM)[5]

We consider the following cases:

1. QED: $\beta(z) = 0$
2. Chiral QED/WSM: $\beta(z) = \beta$ constant
3. Chiral QED between, QED outside: $\beta(z) = \begin{cases} \beta & \text{if} \quad z \in \left[-\frac{L}{2}, \frac{L}{2}\right] \\ 0 & \text{otherwise} \end{cases}$

[5] Goswami and Tewari 2013.

Schematic overview

$$Z = \int \mathcal{D}A\,\mathcal{D}b\, e^{-S-S_{\text{bc}}}$$

S_{bc} is linear in the fields $\rightarrow A_\mu$ can be integrated out

$$Z = \frac{1}{\sqrt{\det(K)}} \int \mathcal{D}b\, e^{-\frac{1}{2}\int \mathrm{d}^3\mathbf{x} b\mathbb{K}b} = \frac{1}{\sqrt{\det(K)\det(\mathbb{K})}}$$

where $\mathbb{K}$ follows from the bcs and the Green's function K^{-1}
$\rightarrow$ effects of restricted A_μ field modelled as (nonlocal) 3D effective action
The energy follows as

$$\begin{aligned}\mathcal{E} &= \mathcal{E}_A + \mathcal{E}_b \\ &= \frac{1}{2V}\log\det(K) + \frac{1}{2V}\log\det(\mathbb{K})\end{aligned}$$

Calculating the Green's function

Fourier transform $t, x, y \to \mathbf{k}$

$$\left[\delta_{\mu\rho}\left(\partial_z^2 - \mathbf{k}^2\right) - g\beta(z)\epsilon_{\mu\rho j3}k_j\right] K_{\rho\nu}^{-1}(\mathbf{k}, z, z') = \delta_{\mu\nu}\delta(z - z')$$

Using a suitable polarization basis this reduces to a scalar equation

$$\left(\partial_z^2 - k_c^2(z)\right) D(z, z') = \delta(z - z')$$

with

$$k_c^2(z) = |\mathbf{k}|^2 + i\beta(z)|\mathbf{k}|$$

Calculating the Green's function

Case 1: $k_c(z) = |\mathbf{k}|$ and case 2: $k_c(z) = k_c = \sqrt{|\mathbf{k}|^2 + i\beta|\mathbf{k}|}$ have constant $k_c(z)$

$$\begin{matrix} \left(\partial_z^2 - |\mathbf{k}|^2\right) f(z-z') = \delta(z-z') \\ \left(\partial_z^2 - k_c^2\right)\varphi(z-z') = \delta(z-z') \end{matrix}, \quad \begin{matrix} f(z-z') = -\dfrac{1}{2|\mathbf{k}|} e^{-|z-z'||\mathbf{k}|} \\ \varphi(z-z') = -\dfrac{1}{2k_c} e^{-|z-z'|k_c} \end{matrix}$$

Case 3: $k_c(z)$ is piecewise constant, glue solutions of cases 1 and 2 together by requiring continuity and smoothness.

$$\begin{aligned} \overline{D}(z,z') &= \varphi(z-z') + C_1(z')\sinh(zk_c) + C_2(z')\cosh(zk_c) \\ D^{\pm}(z,z') &= f(z-z') + C^{\pm}(z')e^{-|z||\mathbf{k}|} \end{aligned}$$

Casimir force

The Casimir force follows as

$$F = F_A + F_b = -\frac{\mathrm{d}\mathcal{E}_A}{\mathrm{d}L} - \frac{\mathrm{d}\mathcal{E}_b}{\mathrm{d}L}$$

The force component from bcs F_b is finite
while F_A requires regularization.

Look at dimensionless Casimir force relative to the QED Casimir force

$$\widetilde{F} = \frac{F}{F_{\mathrm{qed}}}, \quad F_{\mathrm{qed}} = -\frac{\pi^2}{240L^4}$$

Case 1: QED with PEMC plates

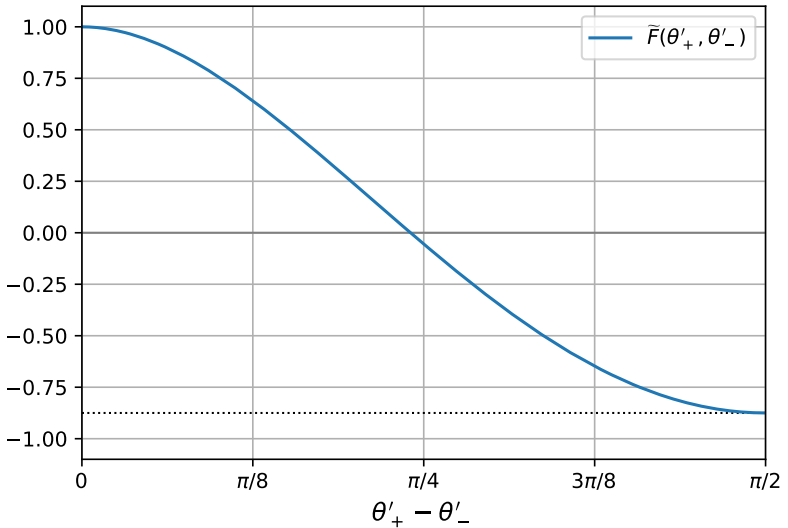
In terms of "duality angles" $\theta'_{\pm} = \arctan(\theta_{\pm})$

Casimir force only depends on the difference $\theta'_{+} - \theta'_{-}$

The dotted line is $-\frac{7}{8}$

Agrees with[a]

[a] Rode, Bennett, and Buhmann 2018.

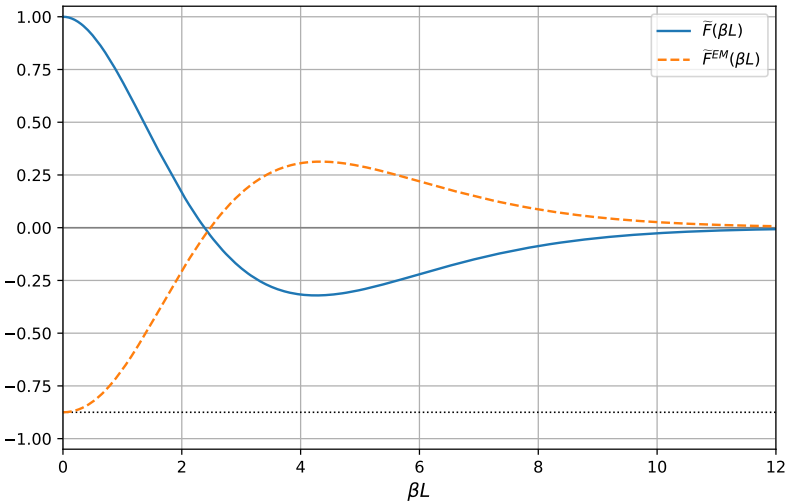


Case 2: Chiral QED with PEMC plates

$\widetilde{F}(\beta L)$: $\theta_+ = \theta_-$
$\widetilde{F}^{\mathrm{EM}}(\beta L)$: $\theta_\pm = \infty, \theta_\mp = 0$

Agrees with[a]

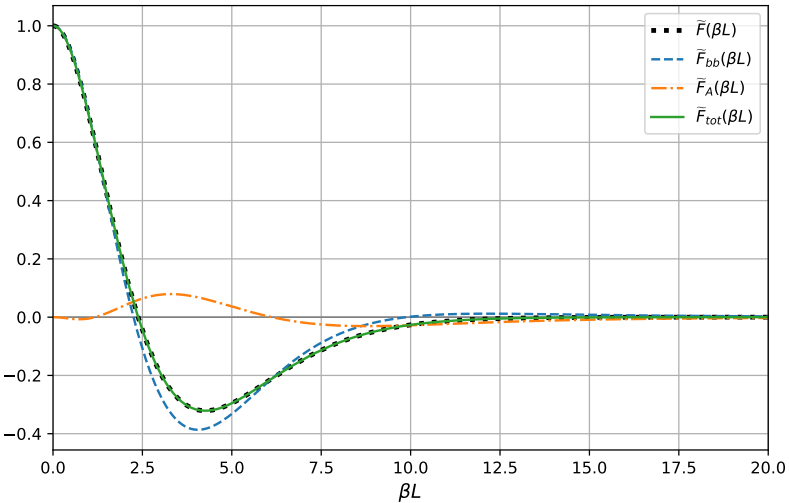
[a] Fukushima, Imaki, and Qiu 2019.



Case 3: Chiral between, QED outside

$\widetilde{F}(\beta L)$: Case 2 for $\theta_+ = \theta_-$
$\widetilde{F}_{bb}(\beta L)$: Force component from bcs
$\widetilde{F}_A(\beta L)$: Force component from WSM

Case 2 and 3 are identical for all $\theta_\pm$



Outlook

- Construct different, experimentally viable, setups
- Consider finite temperatures
- Moving plates
- Dynamic axion field
- Move towards models with interesting nonperturbative behaviour such as (1+1)d $\mathbb{C}P^{N-1}$ and (2+1)d Yang Mills

Polarization basis

Longitudinal and polarization in z direction

$$E^0_\mu = \frac{1}{|\mathbf{k}|}\begin{pmatrix}\mathbf{k}\\0\end{pmatrix}, \quad E^3_\mu = \begin{pmatrix}\mathbf{0}\\1\end{pmatrix}$$

Transversal polarizations

$$E^1_i = \frac{1}{\sqrt{2}}(e^1_i + ie^2_i), \quad E^2_i = \frac{1}{\sqrt{2}}(e^1_i - ie^2_i)$$

with $e^{1,2}_i$ real and obeying

$$e^2_i = \varepsilon_{ijk}\frac{k_k}{|\mathbf{k}|}e^1_j$$

Polarization basis

K is diagonal in polarization basis

$$K_{rs} = \begin{pmatrix} \partial_z^2 - |\mathbf{k}|^2 & & & \\ & \partial_z^2 - (k_c^\star)^2(z) & & \\ & & \partial_z^2 - k_c^2(z) & \\ & & & \partial_z^2 - |\mathbf{k}|^2 \end{pmatrix}$$

with

$$k_c^2(z) = |\mathbf{k}|^2 + i\beta|\mathbf{k}|$$

Nonlocal effective action

The nonlocal effective theory is given by the action

$$\frac{1}{2}\int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3} b_i^{\dagger a}\mathbb{K}_{ij}^{ab} b_j^b$$

with

$$\mathbb{K}_{ij}^{ab} = \overline{V}_{i\mu}^a(\partial_z)V_{\nu j}^b(\partial_{z'})K_{\mu\nu}^{-1}(z,z')\Big|_{z=z_a,z'=z_b}$$

and

$$V_{rs}^a(\partial_z) = \begin{pmatrix} \partial_z & 0 & 0 & i|\mathbf{k}| \\ 0 & \partial_z + ig\theta_a|\mathbf{k}| & 0 & 0 \\ 0 & 0 & \partial_z - ig\theta_a|\mathbf{k}| & 0 \end{pmatrix}$$

$$\overline{V}_{rs}^a(\partial_{z'}) = \begin{pmatrix} \partial_{z'} & 0 & 0 \\ 0 & \partial_{z'} + ig\theta_a|\mathbf{k}| & 0 \\ 0 & 0 & \partial_{z'} - ig\theta_a|\mathbf{k}| \\ -i|\mathbf{k}| & 0 & 0 \end{pmatrix}$$