Non-invertible self-duality defects of Cardy-Rabinovici model and mixed gravitational anomaly

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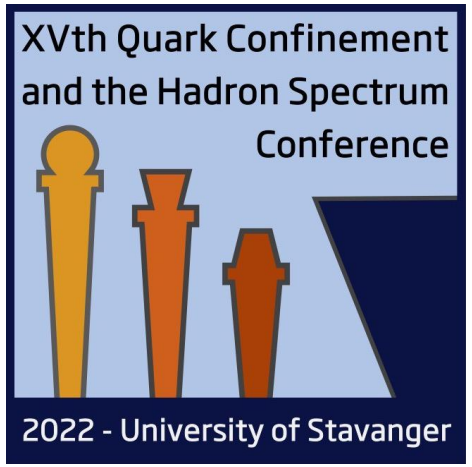

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Overview

- Model: Cardy-Rabinovici model (a toy model for YM with θ angle)
- Method: non-invertible symmetry from "duality" & its anomaly
- Results:
    1. $SL(2,\mathbb{Z})$ transformations of the CR model can be understood as "**dualities**" between the CR model and its (appropriately) $\mathbb{Z}_N^{[1]}$-gauged model.
    2. From these "dualities," at self-dual parameters, we construct **non-invertible symmetries** and determine their fusion rules.
    3. We find **a mixed gravitational anomaly** of this symmetry for some cases, which rules out the trivially-gapped vacuum.
    4. We also show that the conjectured phase diagram can match this anomaly nontrivially.

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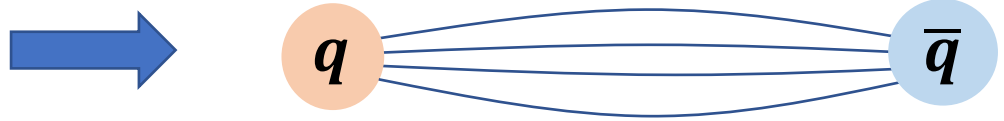
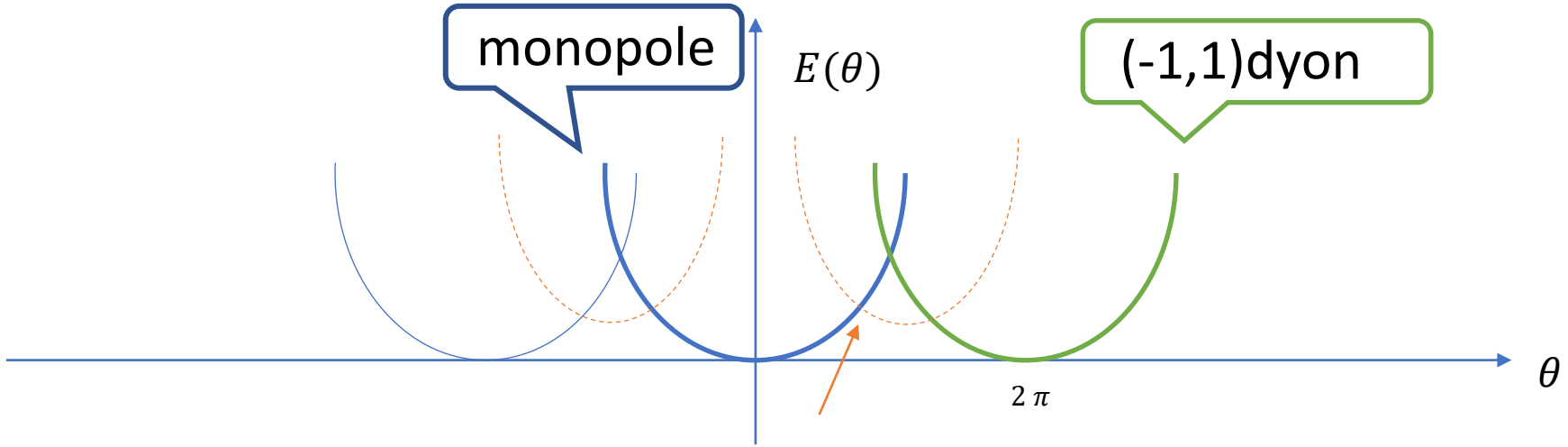
Motivation: confinement & θ angle

A popular understanding of quark confinement: dual superconductor picture

(monopole condensation)



Witten effect: monopole acquires electric charge $\theta/2\pi$ by increasing θ



cf.) Large-N → multi-branch quadratic function structure [Witten '80]

(-1,2) dyon? Oblique conf. ['t Hooft '81]

Cardy-Rabinovici model

A toy model mimicking such structure:
Cardy-Rabinovici model [Cardy and Rabinovici '82, Cardy '82]

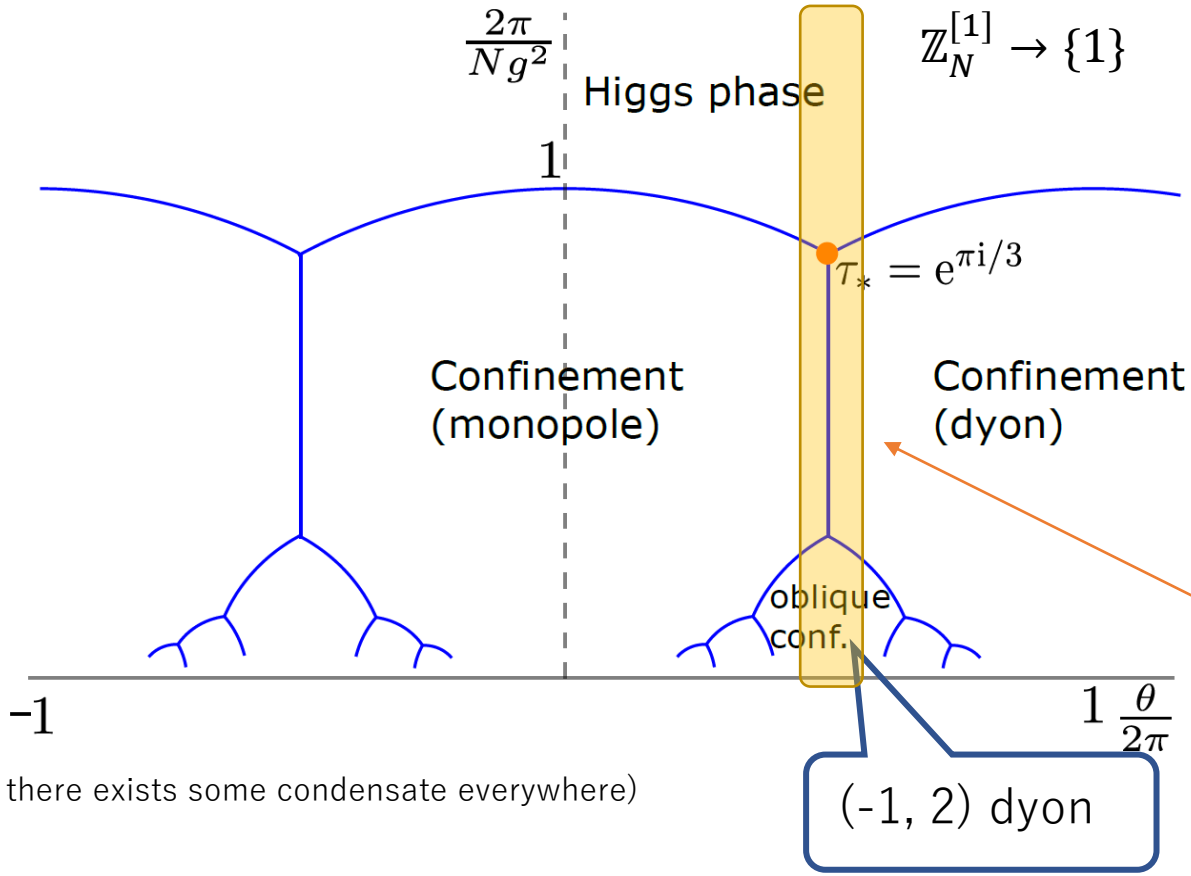
- **U(1) gauge** + **charge-N Higgs** + **monopole**
- $\mathbb{Z}_N^{[1]}$ symmetry ($\sim \mathbb{Z}_N^{[1]}$ center symmetry in $SU(N)$ YM)
- Formulated as a Villain-type lattice gauge theory. Symbolically,

$$Z_{CR} = \int \mathcal{D}a \, e^{-S_{U(1)}[da]} \sum_{C,C':\text{loops}} W^N(C)\, H(C')$$

where $S_{U(1)}[da] = \frac{1}{2\,g^2}\int da \wedge * da + \frac{iN\theta}{8\,\pi^2}\int da \wedge da$,
$W(C)$: Wilson loop, $H(C)$: 't Hooft loop

Conjectured phase diagram

An energy vs. entropy argument for $W^{nN}(C)\,H^{m}(C)$ [Cardy and Rabinovici '82, Cardy '82]
→



(assuming that there exists some condensate everywhere)

Complex coupling

$$\tau := \frac{\theta}{2\pi} + i\,\frac{2\pi}{Ng^2}$$

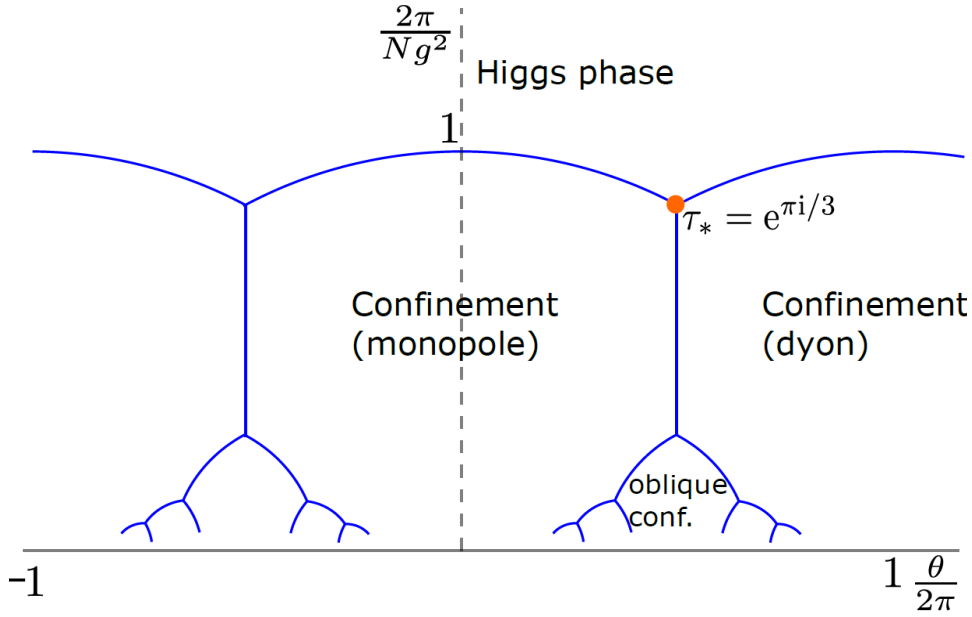
The same CP& $\mathbb{Z}_N^{[1]}$ mixed anomaly as $SU(N)$ YM [Honda and Tanizaki '19]

Conjectured phase diagram

Complex coupling

$$\tau := \frac{\theta}{2\pi} + i\frac{2\pi}{Ng^2}$$

- **This phase diagram has $SL(2,\mathbb{Z})$ invariance: S (electromagnetic duality) and T $(\theta \to \theta + 2\pi)$ transformations.**



$$\begin{cases} S: \tau \mapsto -\frac{1}{\tau}, & \begin{pmatrix} e \\ m \end{pmatrix} \mapsto \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} e \\ m \end{pmatrix} = \begin{pmatrix} -m \\ e \end{pmatrix} \\ T: \tau \mapsto \tau + 1, & \begin{pmatrix} e \\ m \end{pmatrix} \mapsto \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} e \\ m \end{pmatrix} = \begin{pmatrix} e-m \\ m \end{pmatrix} \end{cases}$$

- However, the S transformation is not the duality of the CR model itself, because S-transformed model has electric charge-1 & magnetic charge-N matters.→ **duality between the CR model and its $\mathbb{Z}_N^{[1]}$ gauged model**

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Generalized Symmetry

Important
~conservation law.

(ordinary symmetry) = (co-dim. 1 topological defect with group-like structure)

Higher-form symmetry

describing symmetry acting on higher-dimensional objects (e.g., Wilson loop)
Many applications to gauge theories

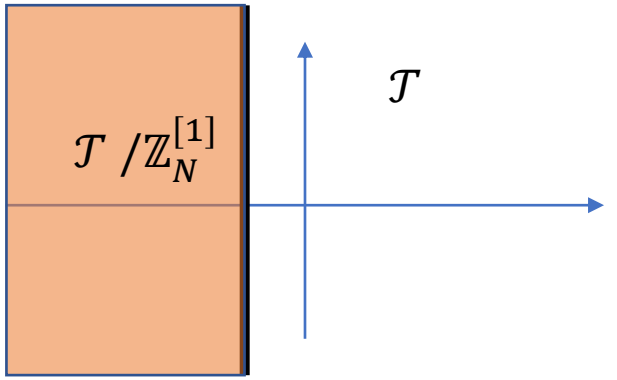
e.g.) $SU(N)$ YM theory has $\mathbb{Z}_N^{[1]}$ symmetry
→ CP & $\mathbb{Z}_N^{[1]}$ mixed "anomaly" @$\theta = \pi$
constrains the phase diagram [Gaiotto, Kapustin, Komargodski, Seiberg '17]

"noninvertible" symmetry

Well-studied in 2d QFTs
less known in higher-dimensions

Non-invertible duality defect

Recently, construction of duality defects in 4d has been developed.
[Koide, Nagoya, Yamaguchi '21; Choi et. al. '21; Kaidi, Ohmori, Zheng '21]



Rough idea:

Famous 2d example: Kramers-Wannier duality in Ising model.

$$\mathcal{T}\ /\ \mathbb{Z}_2 \simeq \mathcal{T} \quad \Longrightarrow \quad \textbf{KW duality defect line} = \textbf{"half-space gauging"}$$

Generalization to 4d: **self-duality by 1-form symmetry $\mathbb{Z}_N^{[1]}$ gauging leads to a similar defect**

$$\mathcal{T}\ /\mathbb{Z}_N^{[1]} \simeq \mathcal{T} \quad \Longrightarrow \quad \text{"half-space } \mathbb{Z}_N^{[1]} \text{ gauging"}$$
: 3-dim topological defect

Note.) Gauging a p-form discrete symmetry causes a dual (d-p-2)-form symmetry
→ When d=4, only 1-form symmetry gauging can be self-dual.

Results

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Notations

- (The spacetime manifold is spin and torsion-free).
- The partition function with $\mathbb{Z}_N^{[1]}$ background B:

$$Z_{CR}^{\tau}[B] \coloneqq \int \mathcal{D}a\, e^{-S_{U(1)}[da+B]} \sum_{C,C':\text{loops}} W_{da+B}^{N}(C)\, H_{da+B}(C')$$

- The partition function of level-p $\mathbb{Z}_N^{[1]}$-gauged CR model with (dual) $\mathbb{Z}_N^{[1]}$ background B:

$$Z_{CR\,/\left(\mathbb{Z}_N^{[1]}\right)_p}^{\tau}[B] \coloneqq \int \mathcal{D}b\, Z_{CR}^{\tau_*}[b]\, e^{\frac{iNp}{4\pi}\int b\wedge b}\, e^{\frac{iN}{2\pi}\int b\wedge B}$$

with the following normalization,

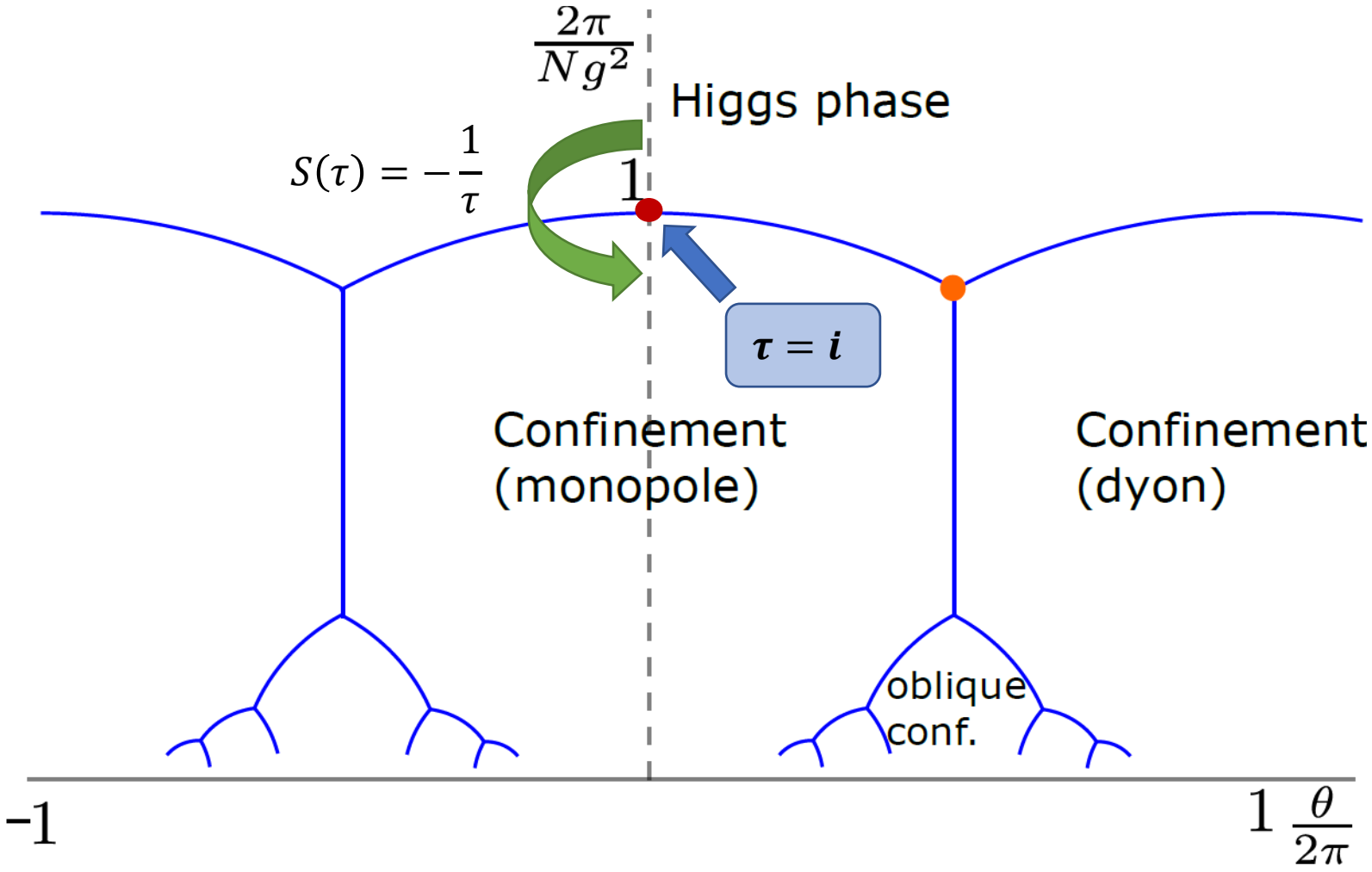
$$\int \mathcal{D}b \ldots \coloneqq \frac{|H^0(X;\mathbb{Z}_N)|}{|H^1(X;\mathbb{Z}_N)|} \sum_{b\in H^2(X;\mathbb{Z}_N)} \ldots$$

Warm-up: S-defect

For Maxwell theory, constructed in [Choi et. al. 2021]

Complex coupling

$$\tau := \frac{\theta}{2\pi} + i\,\frac{2\pi}{Ng^2}$$



The S "self-duality" at $\tau = i$ can be realized as

$$\boldsymbol{CR} \,/\, \left(\mathbb{Z}_{\boldsymbol{N}}^{[\boldsymbol{1}]}\right)_{\boldsymbol{0}} \simeq \boldsymbol{CR}$$

[U(1) gauge + (1,0) matter + (0,N) matter] system with coupling $\tau = i/N^2$

[U(1) gauge +(N,0) matter + (0,1) matter] system with coupling $\tau = i$

electromagnetic S transform

CP symmetry

We can construct non-invertible defects by half-space gauging

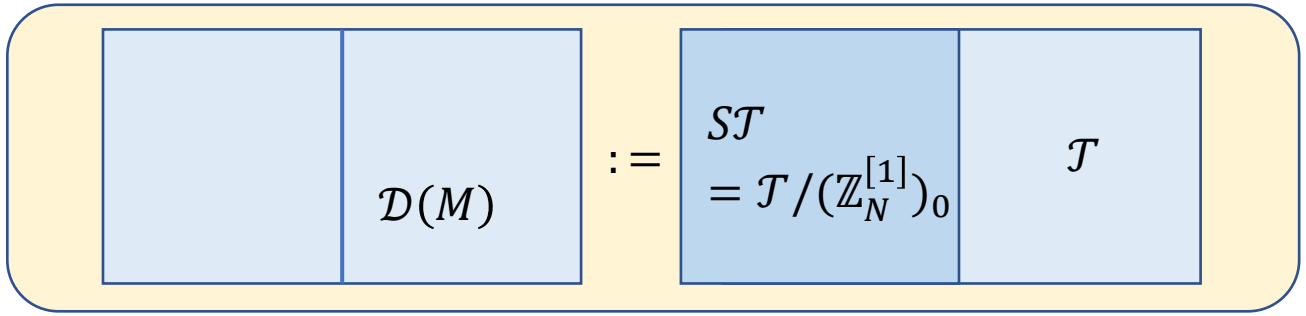
$$\mathcal{D}(M) \times \mathcal{D}(M) = C(M)\,\frac{1}{N} \sum_{\Sigma \in H_2(M,\mathbb{Z}_N)} \eta\,(\Sigma)$$

$$\eta\,(\Sigma) \times \mathcal{D}(M) = \mathcal{D}(M) \times \eta\,(\Sigma) = \mathcal{D}(M)$$

The trivially-gapped phase is ruled out for $N > 2$.

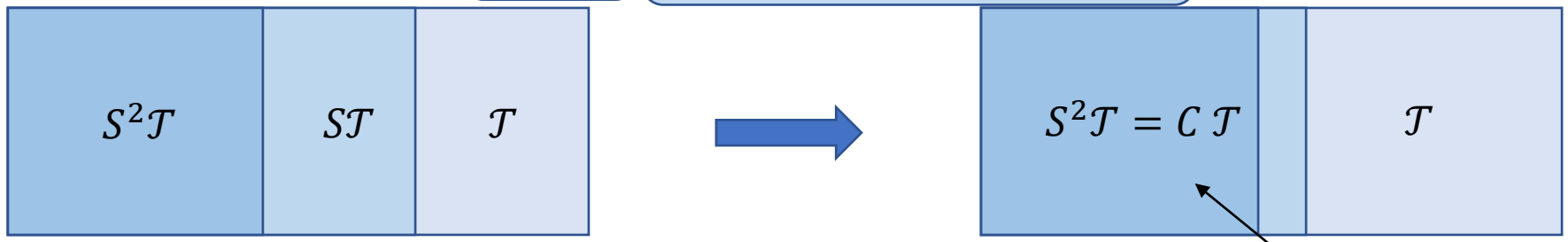
A simple guess $Z_{mono}[B] + Z_{Higgs}[B]$ is consistent with these constraints: consistent with the conjectured phase diagram!

Fusion rules

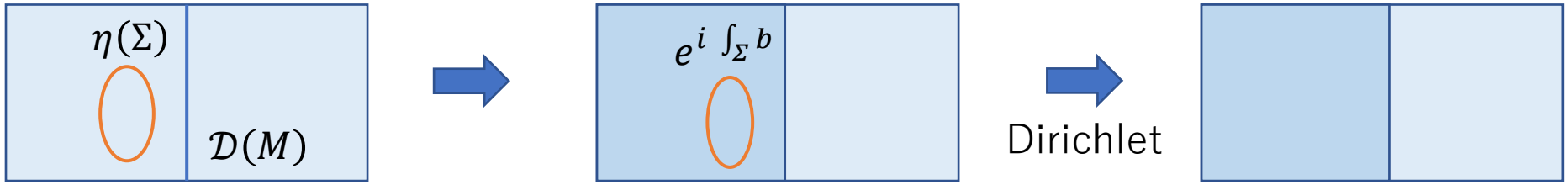


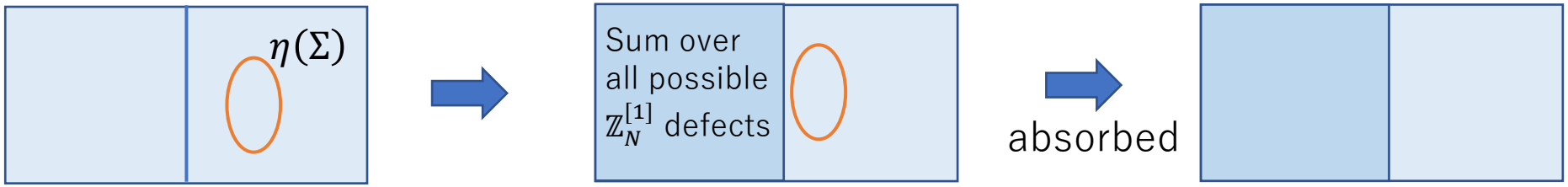
1. $\mathcal{D}(M) \times \mathcal{D}(M) = C(M) \frac{1}{N} \sum_{\Sigma \in H_2(M,\mathbb{Z}_N)} \eta(\Sigma)$

$\mathbb{Z}_N^{[1]}$ gauging on M



2. $\eta(\Sigma) \times \mathcal{D}(M) = \mathcal{D}(M) \times \eta(\Sigma) = \mathcal{D}(M)$

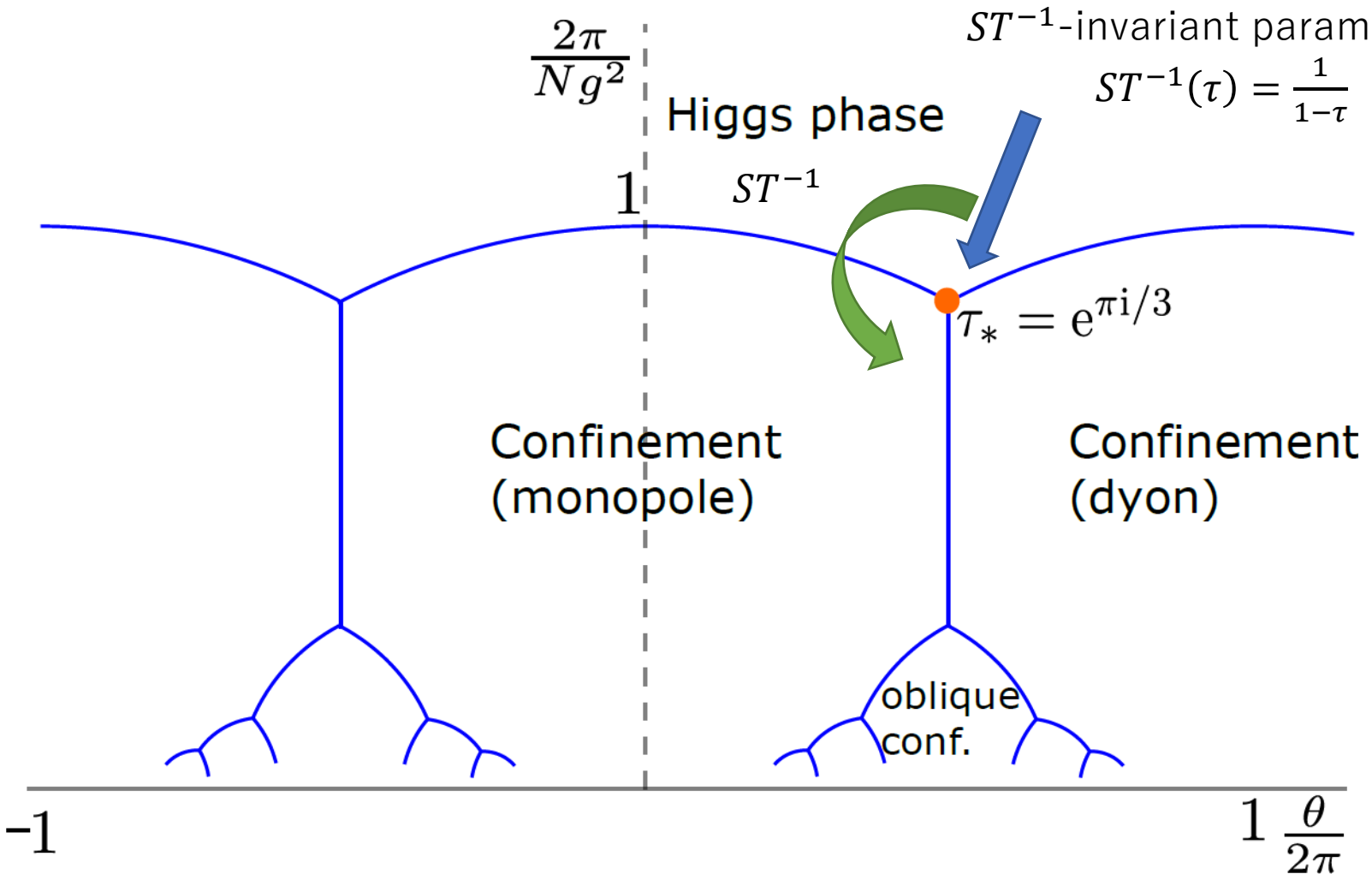




Nontrivial example: ST^{-1} defect

Complex coupling

$$\tau := \frac{\theta}{2\pi} + i\,\frac{2\pi}{Ng^2}$$



The ST^{-1} "self-duality" at $\tau = \tau_* = e^{i\pi/3}$ can be realized as

[U(1) gauge + (1,0) matter + (0,N) matter] system with coupling $\frac{\tau_*-1}{N^2}$

$$\boldsymbol{CR \,/ \left(\mathbb{Z}_N^{[1]}\right)_{-1} \simeq CR}$$

[U(1) gauge + (N,0) matter + (0,1) matter] system with coupling $-\frac{1}{\tau_*-1} = \tau_*$

$$\left(Z^{\tau_*}_{CR\,/\left(\mathbb{Z}_N^{[1]}\right)_{-1}}[B] = N^{\frac{\chi}{2}} e^{-\frac{i\pi}{3}\sigma} Z^{\tau_*}_{CR}[B]\right)$$

cf.) [Witten '95] for Maxwell theory

We can construct non-invertible defects by half-space gauging

$$\mathcal{D}(M) \times \mathcal{D}(M) \times \mathcal{D}(M) \propto \mathrm{C}(M) \sum_{\Sigma \in H_2(M,\mathbb{Z}_N)} \eta\,(\Sigma)$$

$$\eta\,(\Sigma) \times \mathcal{D}(M) = \mathcal{D}(M) \times \eta\,(\Sigma) = \mathcal{D}(M)$$

The trivially-gapped phase is ruled out.

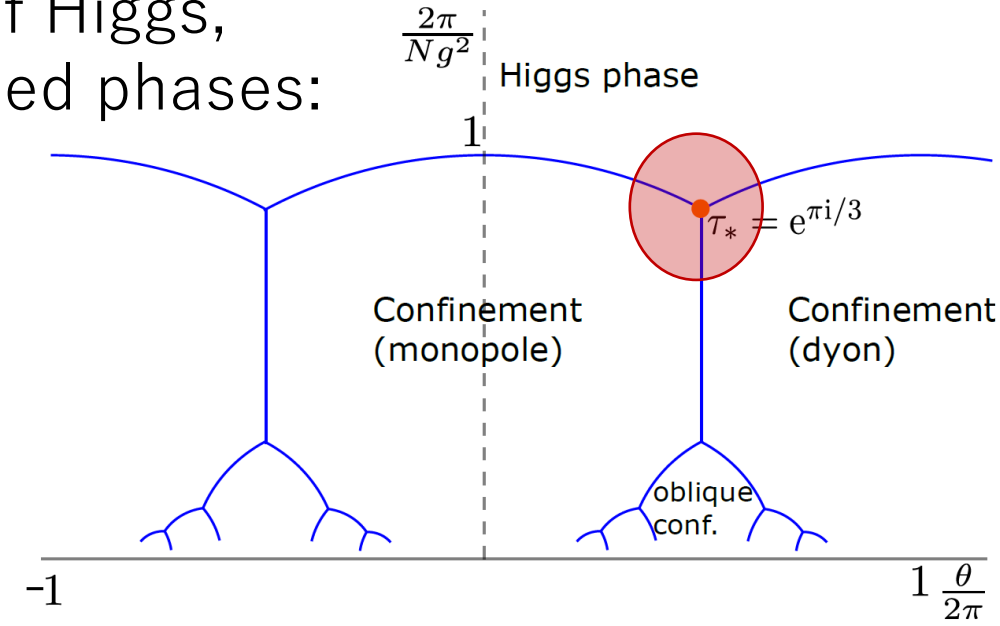
Anomaly and conjectured phase diagram

A natural guess for low-energy theories of Higgs, monopole-condensed, and dyon-condensed phases:

$$Z_{Higgs}[B] = \int \mathcal{D}a\mathcal{D}b \; e^{\frac{iN}{2\pi}\int b\wedge(da+B)}$$

$$Z_{mono}[B] = 1,$$

$$Z_{dyon}[B] = e^{\frac{iN}{4\pi}\int B\wedge B}$$



A linear combination of them:

$$Z[B] = Z_{mono}[B] + e^{\frac{\pi i}{3}\sigma} Z_{dyon}[B] + N^{-\frac{\chi}{2}} e^{\frac{2\pi i}{3}\sigma} Z_{Higgs}[B]$$

matches the anomaly: $Z^{\tau_*}_{CR/\left(\mathbb{Z}_N^{[1]}\right)_{-1}}[B] = N^{\frac{\chi}{2}} e^{-\frac{i\pi}{3}\sigma} Z^{\tau_*}_{CR}[B]$!

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