# *Crossover toward Quark Matter in Neutron Stars*

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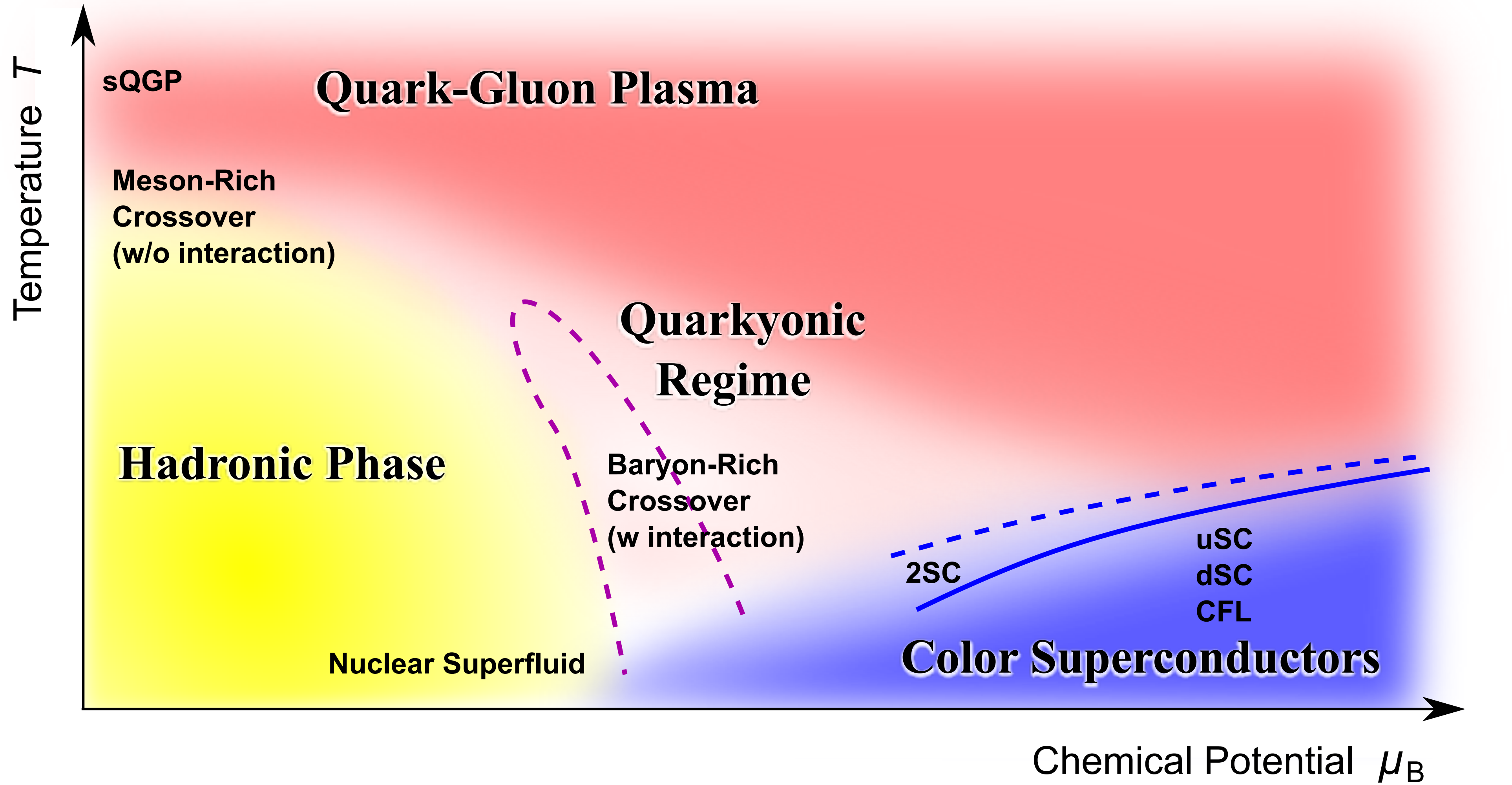
The University of Tokyo

— XVth Quark Confinement and the Hadron Spectrum —

# *Phase Diagram*




**Isospin rich matter may not have a 1st-order transition at all.**

# *Phase Diagram*

## Meson-Rich Crossover (at low density)

The picture of ideal hadron resonance gas works.
Thermodynamics dominated by non-interacting mesons.
Deconfinement = Hagedorn Transition
**Near $T_c$ conformality is the most broken.**

## Baryon-Rich Crossover (at high density)

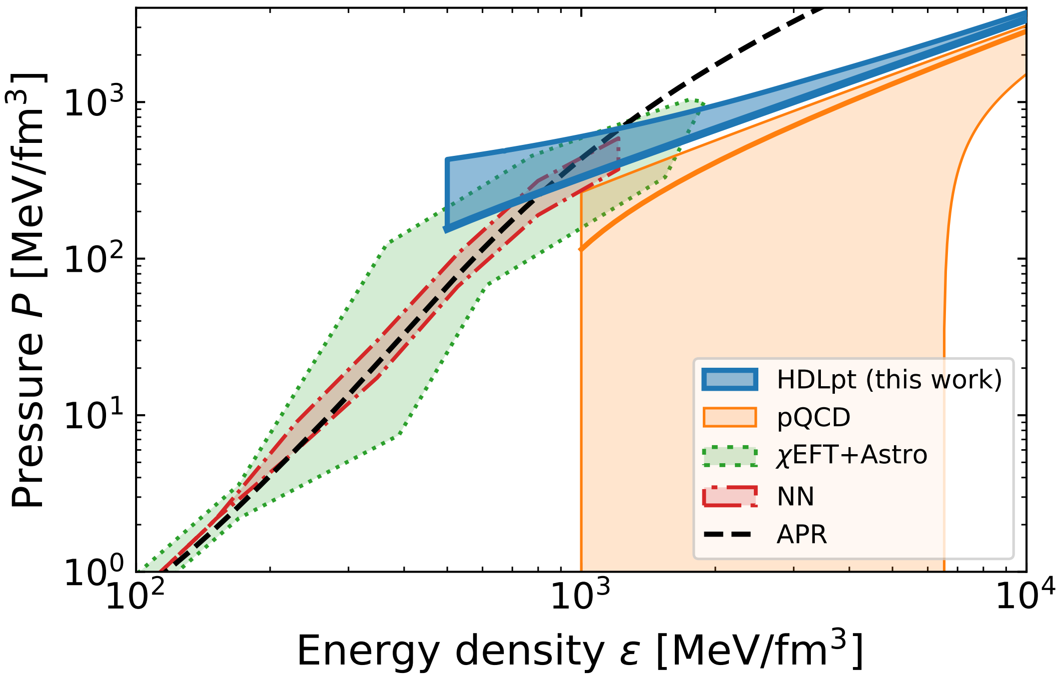
Baryons interact very strongly (in the large $N_c$ limit).
Quark degrees of freedom encoded in pion exchanges.
**Conformality restored very early via interactions.**

**Fujimoto-Fukushima-McLerran-Praszalowicz (2022)**
**→ Talk by Yuki Fujimoto on August 4th.**

# *Equation of State for Neutron Stars*

**Fujimoto-Fukushima (2020)**

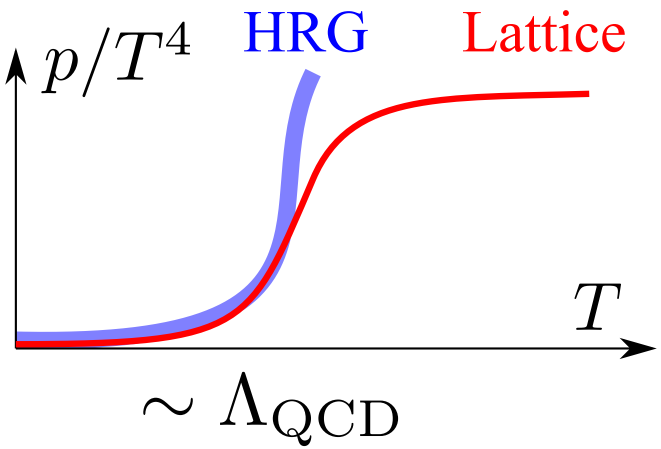
**Data-driven EOS agrees with an empirical EOS (APR) and seems to continue to the pQCD EOS with crossover.**

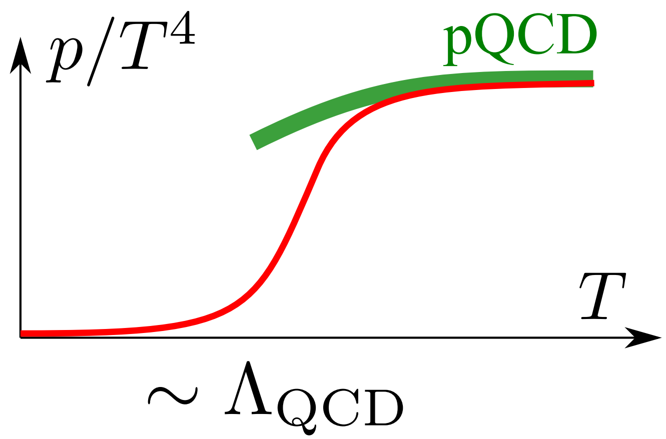



**See: Fujimoto-Fukushima-Murase (2018-2021) for ML (NN) inference.**

# *Underlying Physics for Crossover*

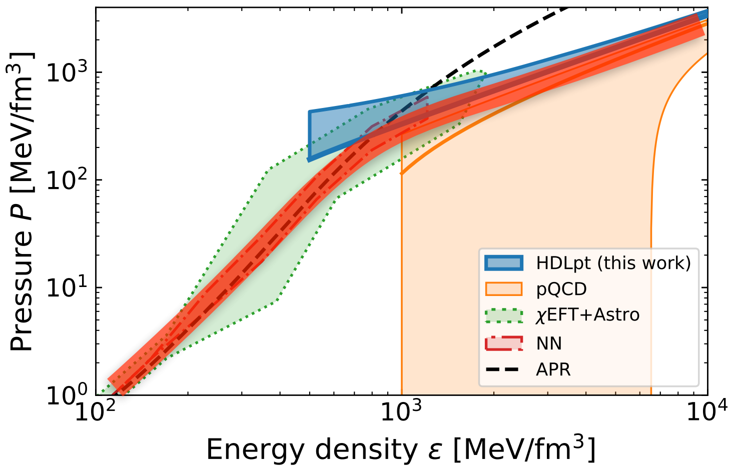
**High-*T* has been understood by HRG + pQCD**




**High-Density**

**A duality region where the hadrons and quarks may coexist (quarkyonic).**

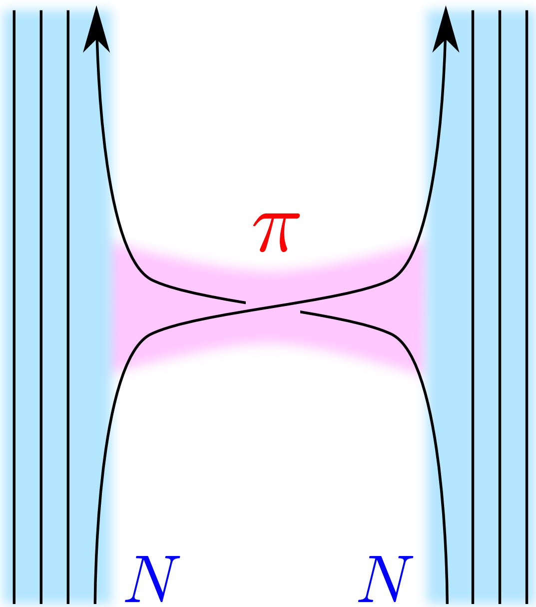



**pQCD**

# *Underlying Physics for Crossover*

**Baryonic (Nuclear) Matter and Quark Matter are not distinguishable = Quarkyonic**

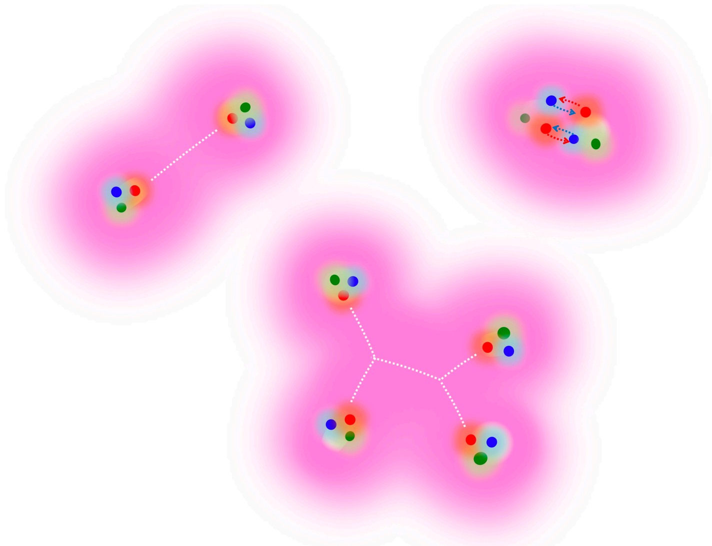
**McLerran-Pisarski (2008)**




**Pressure of Quark Matter**
**Kinetic Energy ~ $O(N_c)$**

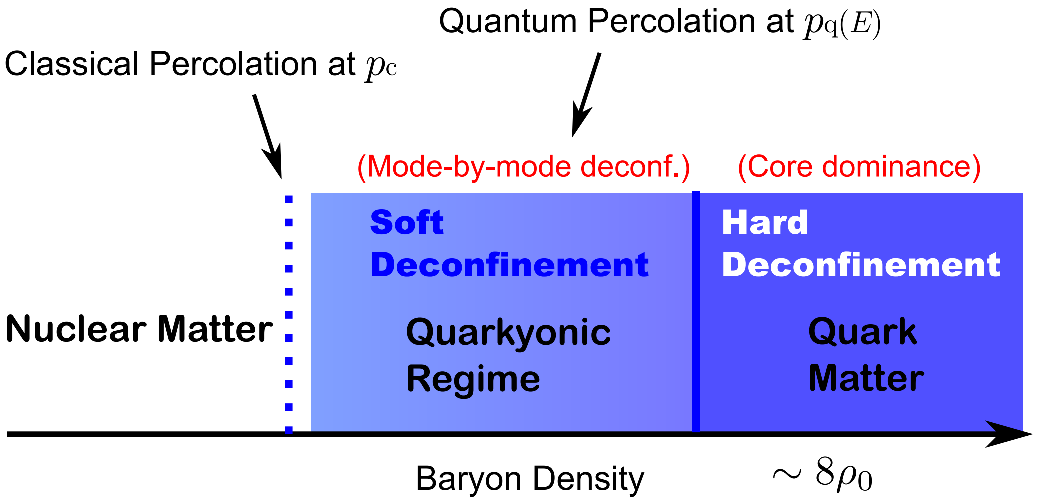
**Pressure of Baryonic Matter**
**Interaction Energy ~ $O(N_c)$**

# *Underlying Physics for Crossover*



**Fukushima-Kojo-Weise (2020)**

***NN*, *NNN*, *NNNN*, ….**
**all many-body interactions become the same order of O($N_c$) around ~ 2$n_0$**




**Quantum percolation has a critical density as a function of energy.**

**Partial deconfinement!**

# *Underlying Physics for Crossover*

## Matching of Global Symmetry Breaking Patterns

**Baryons: 8+1 (low-lying)**

**Quarks: 3color × 3flavor = 9**

**Flavor Triplet**


**Condensate**


**Excitation**

$\langle ud \rangle \ \langle ds \rangle \ \langle su \rangle$ **Diquark condensates break chiral symmetry in the same way as the hadronic phase.**

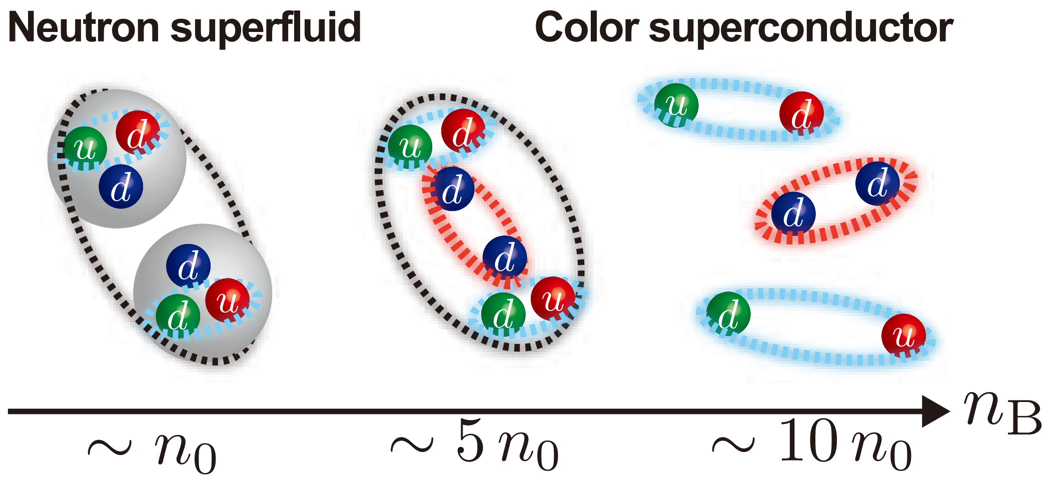
**Diquarks realize duality between baryons and quarks!**

**Physical degrees of freedom have 1-1 correspondence.**
**(Quarks = Baryons / Gluons = Vector Mesons / etc…)**

# *Underlying Physics for Crossover*

**Fujimoto-Fukushima-Weise (2020)**

In the neutron star nuclear matter is neutron rich and the neutron pairing is formed.



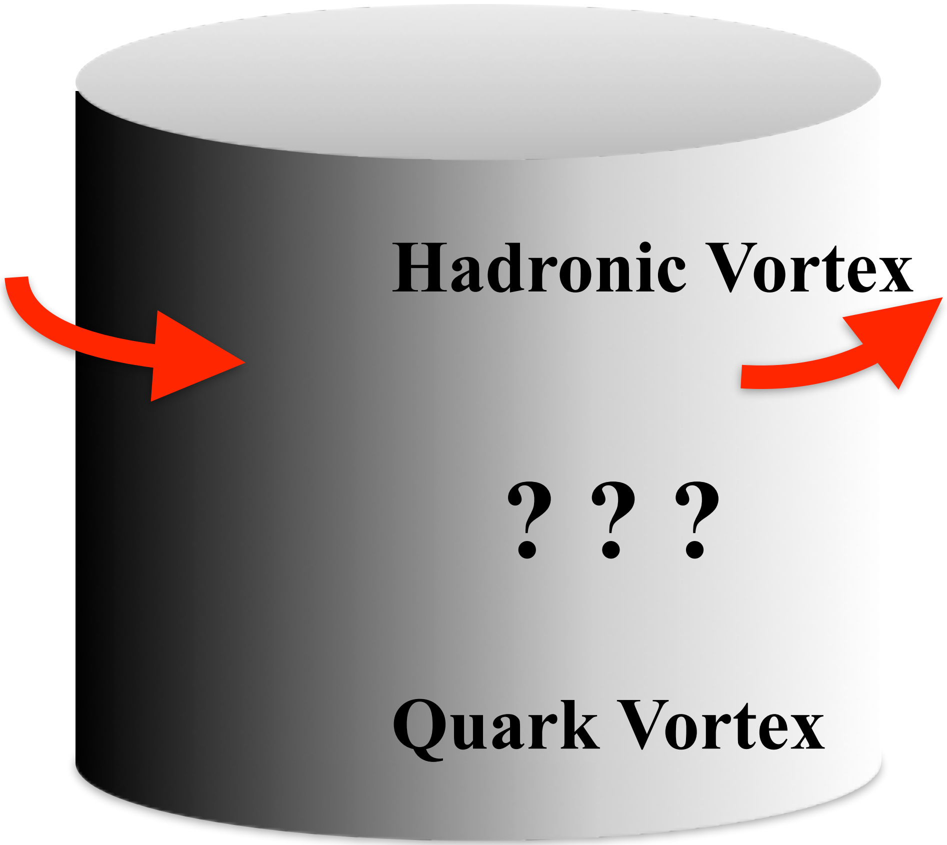

**Chiral symmetry can be broken by this component.**

**Neutron pairing:** $\langle nn \rangle \sim \langle udd\, udd \rangle$

**Corresponding diquark pairing:** $\langle ud \rangle \langle ud \rangle \langle dd \rangle$

# *Underlying Physics for Crossover*

## Controversy




**Rotate the bucket filled with quarks**

Upper part : Hadronic Vortex

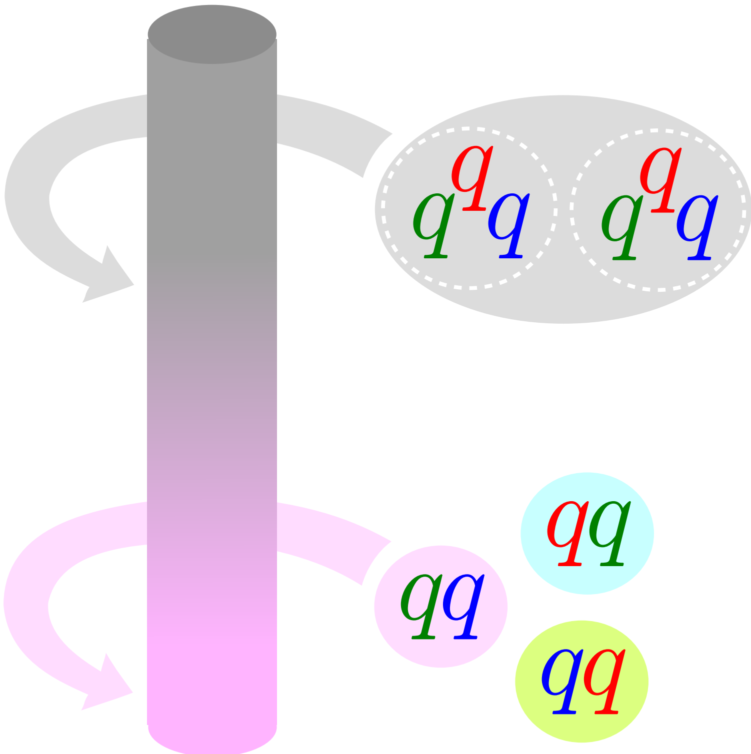
Lower part : Quark Vortex

**How can they be connected?**

# *Underlying Physics for Crossover*

## Controversy

**Alford-Baym-Fukushima-Hatsuda-Tachibana (2018)**



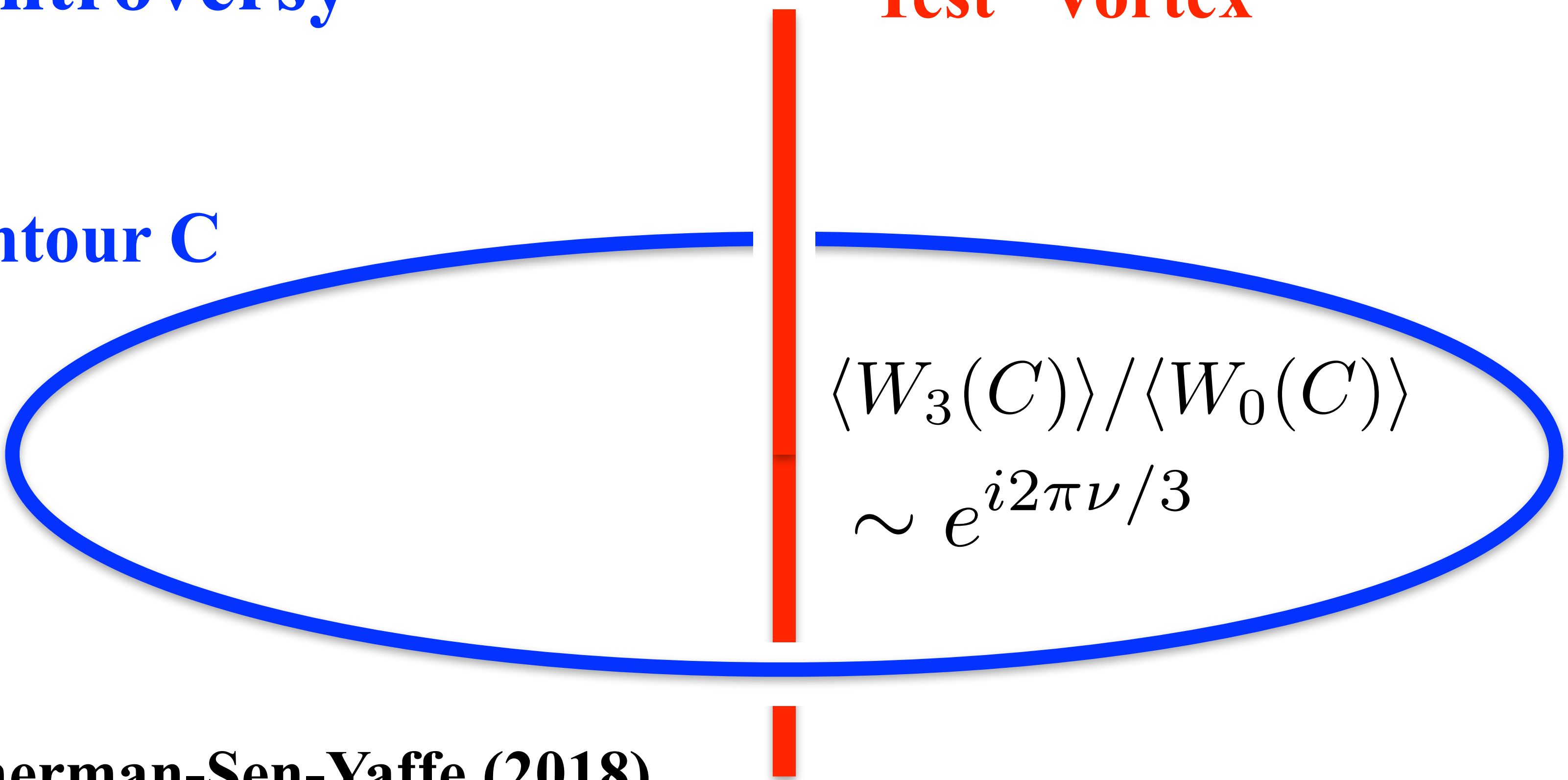

**We proposed a scenario of the vortex continuity, but…**

# *Underlying Physics for Crossover*

## Controversy

**"Test" Vortex**

**Contour C**



$$\langle W_3(C)\rangle / \langle W_0(C)\rangle \sim e^{i2\pi\nu/3}$$

**Cherman-Sen-Yaffe (2018)**

**Hadronic phase has no color flux and no phase…**
**Distinguishable?**

# *Underlying Physics for Crossover*

**Gap matrix in color-flavor space**

$$\Phi = \begin{pmatrix} \Phi^{\bar{r}\bar{u}} & 0 & 0 \\ 0 & \Phi^{\bar{g}\bar{d}} & 0 \\ 0 & 0 & \Phi^{\bar{b}\bar{s}} \end{pmatrix}$$

# *Underlying Physics for Crossover*

**Abelian CFL vortex**

$$\Phi^{\mathrm{A}} = \Delta_{\mathrm{CFL}}\, e^{i\nu_{\mathrm{A}}\varphi} \begin{pmatrix} f(r) & 0 & 0 \\ 0 & f(r) & 0 \\ 0 & 0 & f(r) \end{pmatrix}$$

**Non-Abelian CFL vortex**

$$\Phi^{(1)} = \Delta_{\mathrm{CFL}} \begin{pmatrix} e^{i\nu_1\varphi} f(r) & 0 & 0 \\ 0 & g(r) & 0 \\ 0 & 0 & g(r) \end{pmatrix}$$

# *Underlying Physics for Crossover*

## Why Non-Abelian ?

$$\Phi^{(1)} = \Delta_{\mathrm{CFL}} e^{\frac{i}{3}\nu_1\varphi} \begin{pmatrix} e^{\frac{2i}{3}\nu_1\varphi} f(r) & 0 & 0 \\ 0 & e^{-\frac{i}{3}\nu_1\varphi} g(r) & 0 \\ 0 & 0 & e^{-\frac{i}{3}\nu_1\varphi} g(r) \end{pmatrix}$$

**Abelian Phase (Superfluid Vortex)**

**Non-Abelian Phase ($T_3$ and $T_8$) (Gauged Vortex)**

$$A_\varphi^{(1)} = -\frac{\nu_1}{g_{\mathrm{c}} r} [1 - h(r)] \begin{pmatrix} -\frac{2}{3} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} \end{pmatrix}$$

**Non-Abelian vortex carries Non-Abelian Magnetic Flux**

# *Underlying Physics for Crossover*

If $C$ is large enough, $f \to 1$, $g \to 1$, $h \to 0$

$$\langle W_3(C) \rangle \;\sim\; \mathrm{tr} \begin{pmatrix} e^{-\frac{4\pi i}{3}\nu_1} & 0 & 0 \\ 0 & e^{\frac{2\pi i}{3}\nu_1} & 0 \\ 0 & 0 & e^{\frac{2\pi i}{3}\nu_1} \end{pmatrix}$$

$$\sim\; e^{\frac{2\pi i}{3}\nu_1}$$

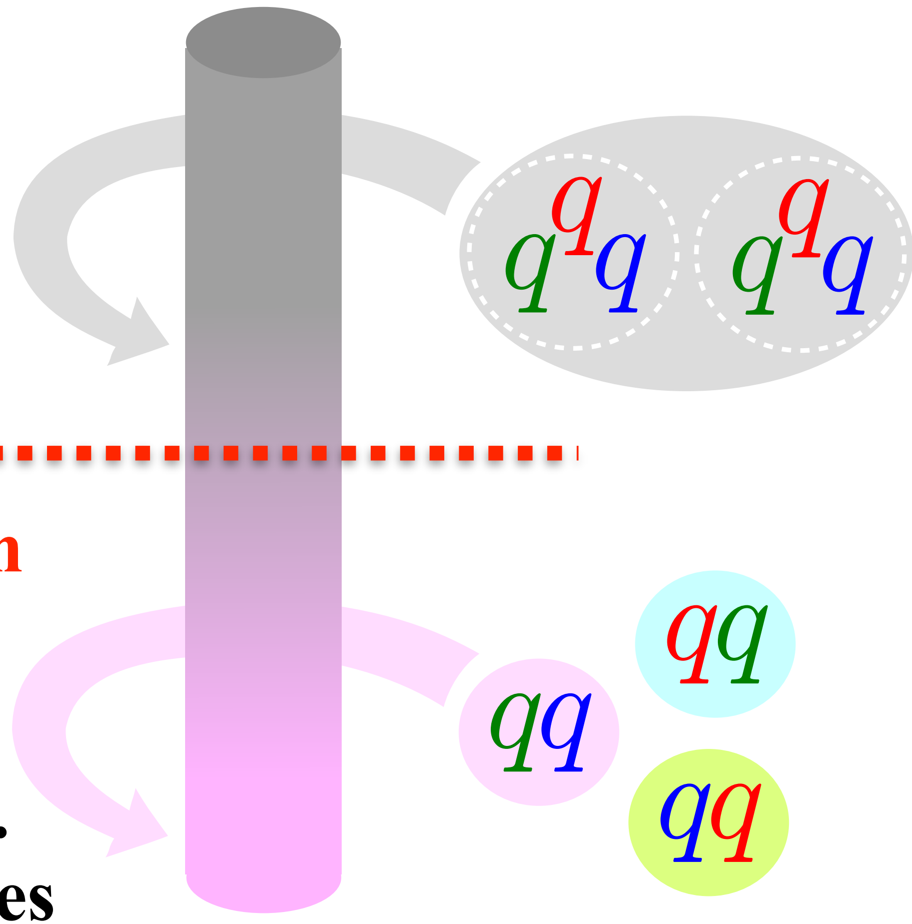
**Center element of the non-Abelian magnetic flux appears (making the vortices "anyons")**

**Cherman-Sen-Yaffe (2018)**

**See also Hirono-Tanizaki (2019)**

# *Underlying Physics for Crossover*

## Recent controversy



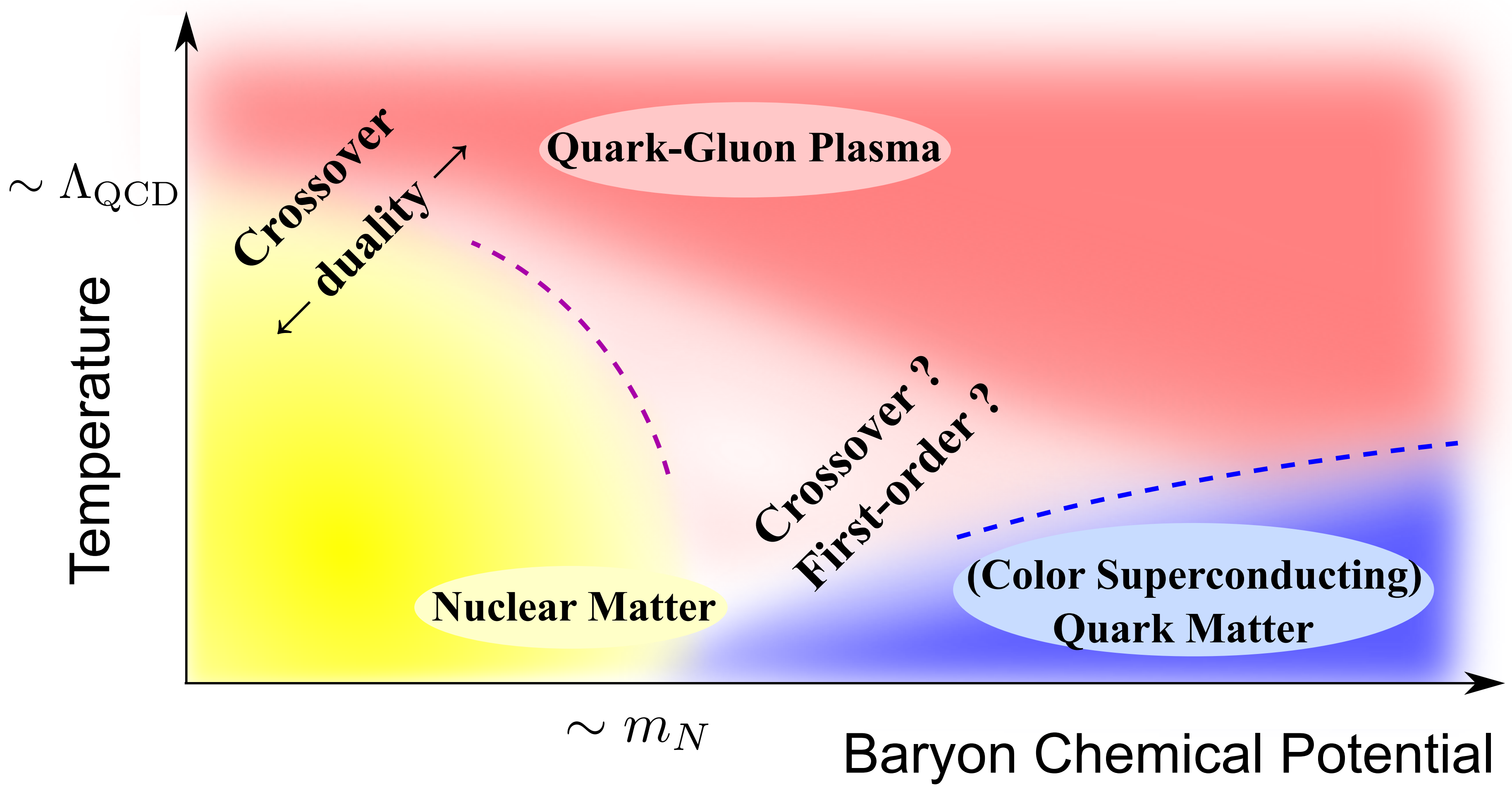

**There must be a discontinuous interface!?**

**→ 1st-order phase transition**

**We can cook up hadronic vortices to realize continuity.
Implying that more structures of hadronic vortices should be considered.**

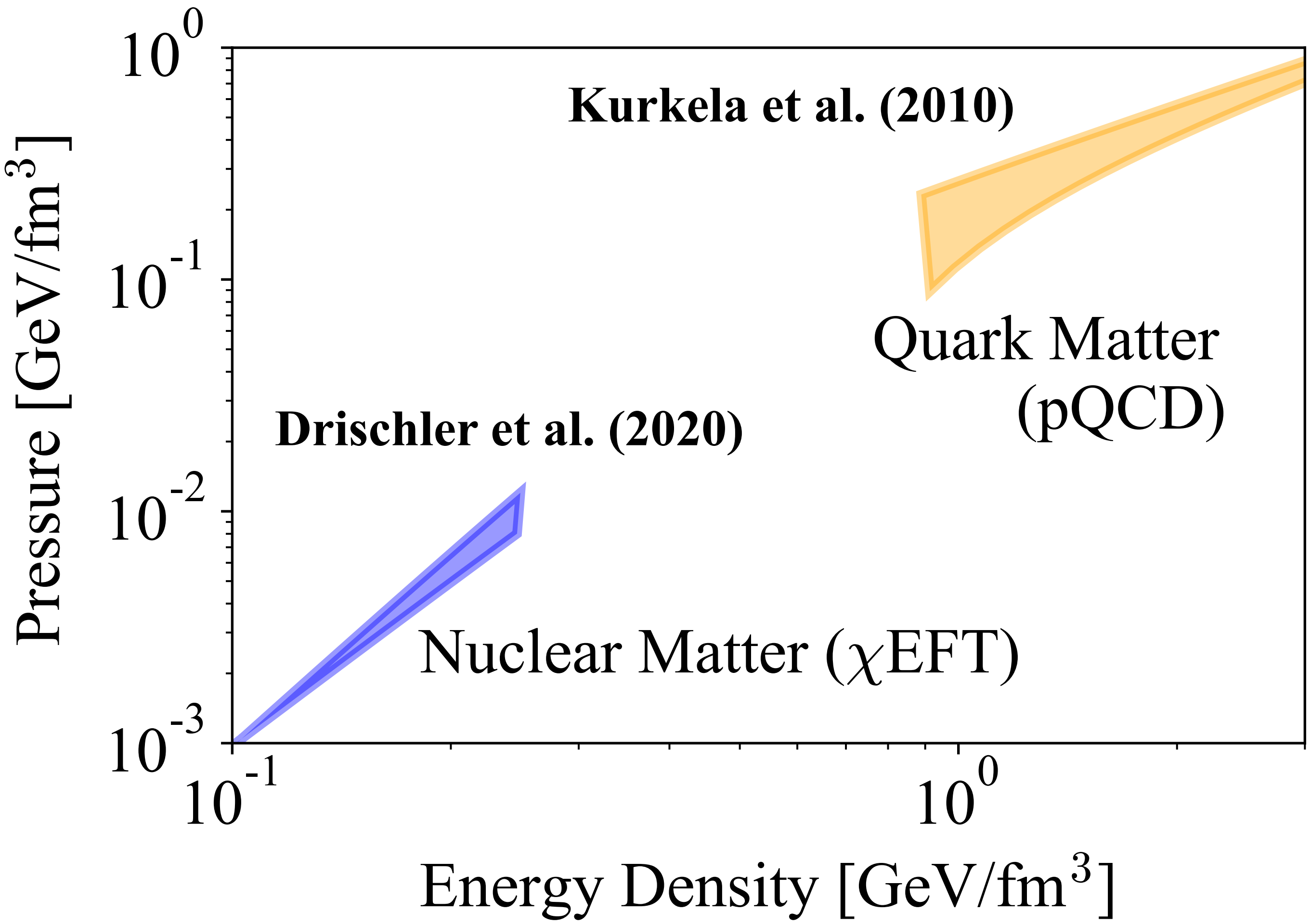
# *Constraining EOS*

## Crossover or not?



# *Constraining EOS*

## Low and High Density Regions

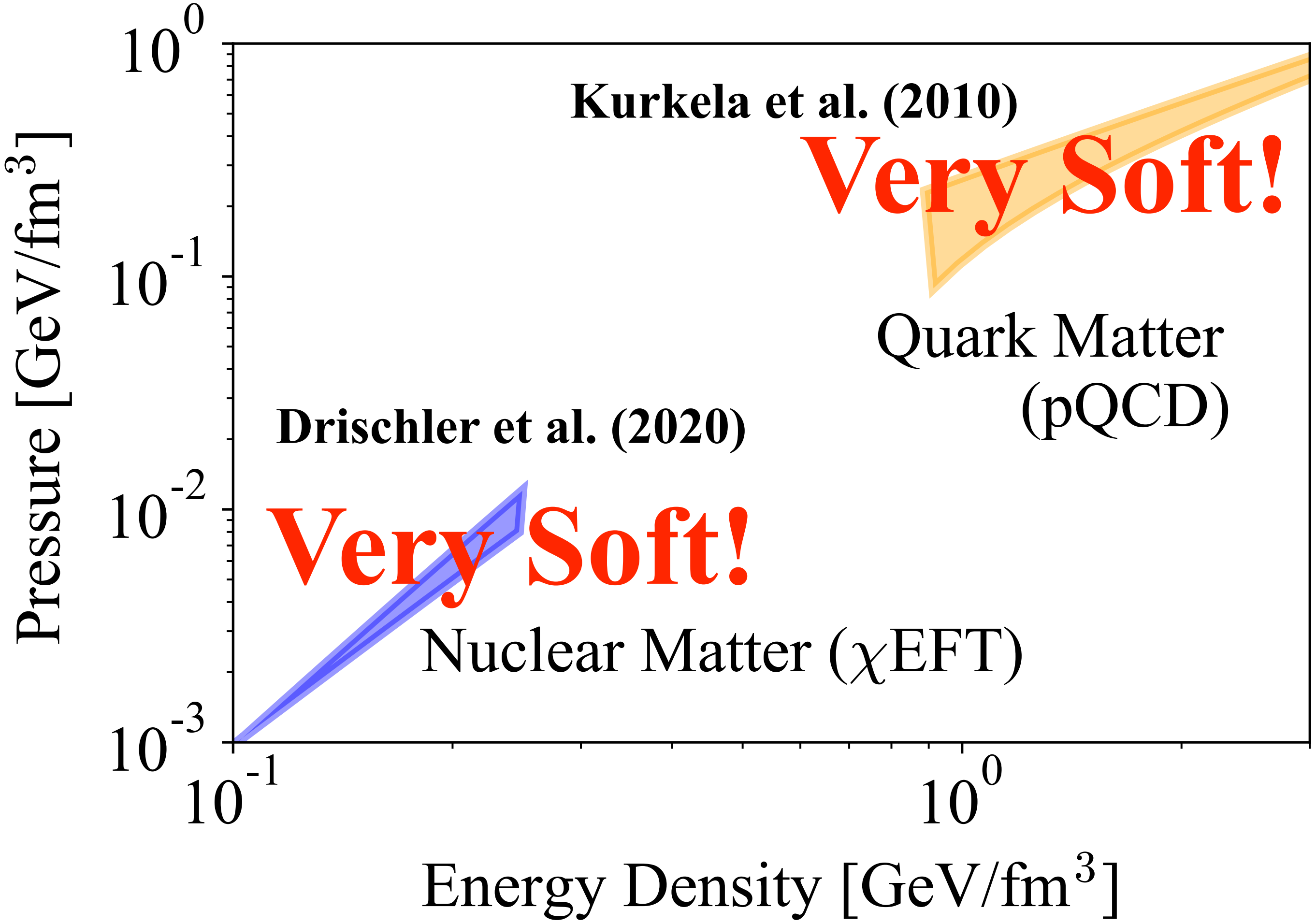



$\bar{\Lambda}/\mu_q = 2 \sim 4$

**Upper edge is favored by resummed pQCD; see, Fujimoto-Fukushima (2020)**

# *Constraining EOS*

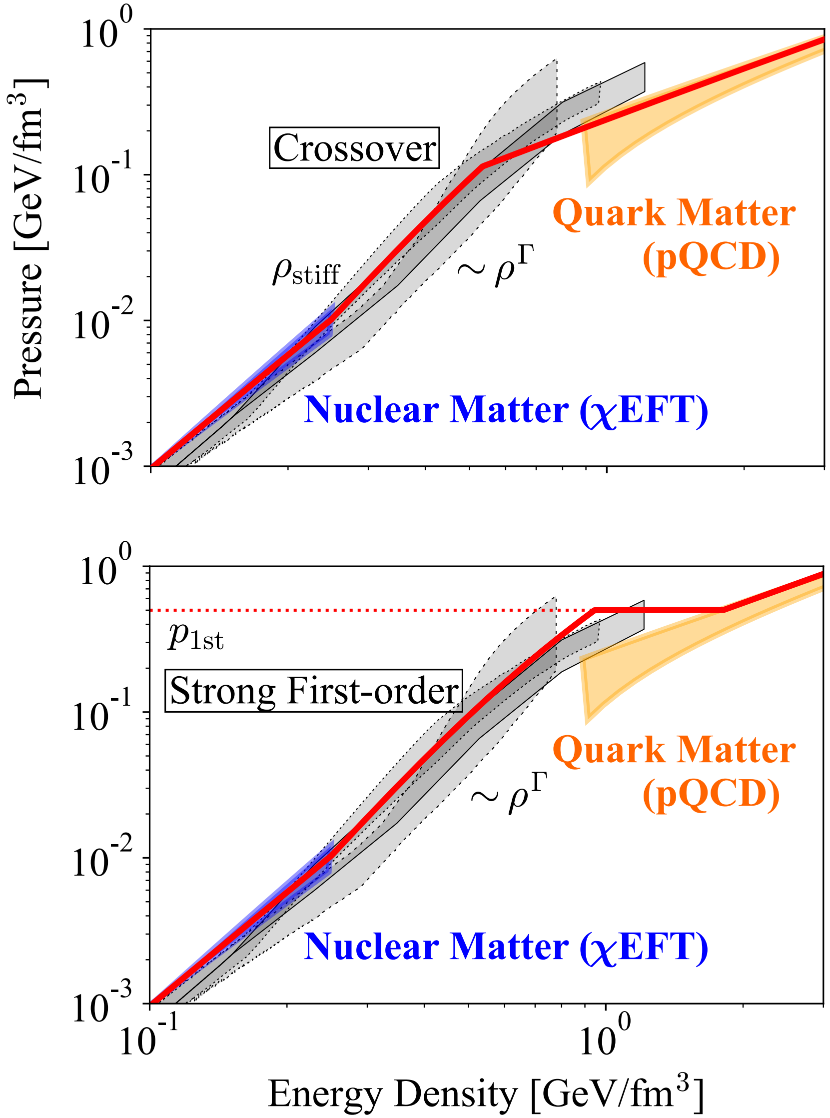
## Low and High Density Regions




$\bar{\Lambda}/\mu_q = 2 \sim 4$

**Upper edge is favored by resummed pQCD; see, Fujimoto-Fukushima (2020)**
**Uncertainty extends to the softer direction, which is nearly excluded by massive neutron stars.**

# Constraining EOS




**Interpolated by a polytrope EOS**

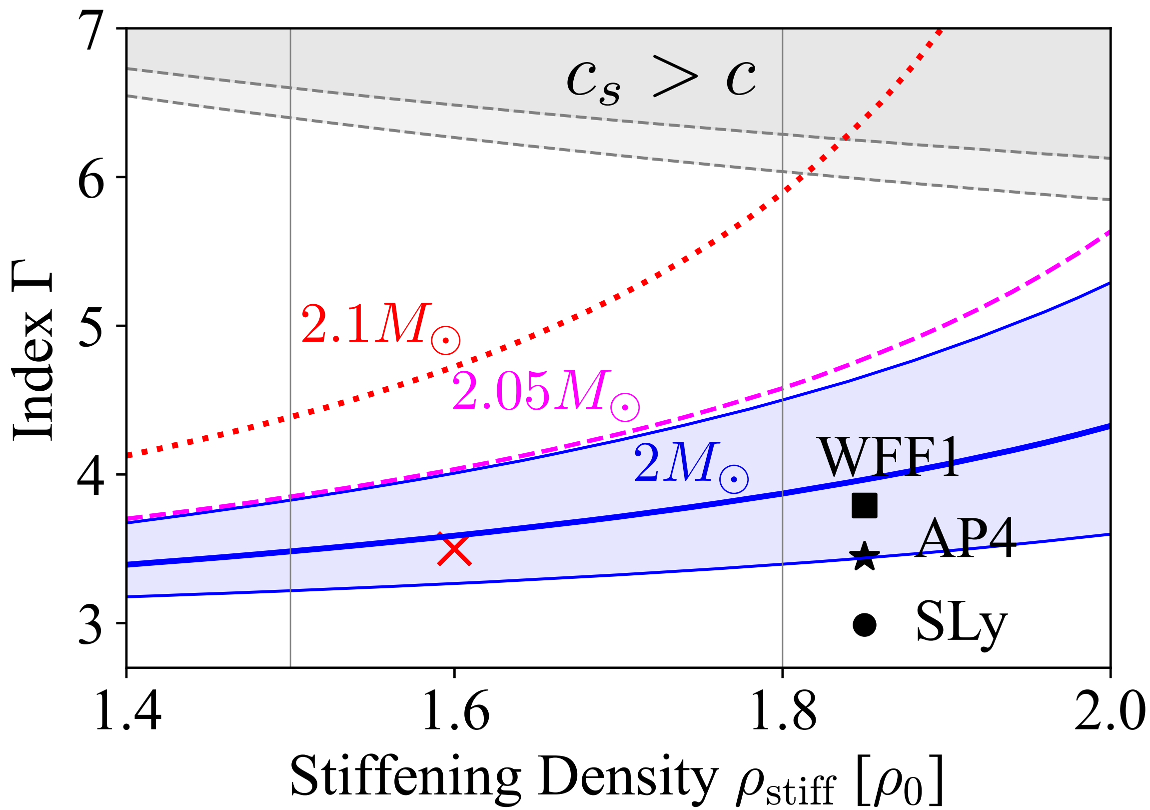
$$p(\rho) \sim K\,\rho^{\Gamma}$$

To support the massive neutron stars the intermediate EOS must be stiff, and where it becomes stiff is parametrized by $\rho_{\rm stiff}$.

For the first-order scenario one more parameter, $p_{\rm 1st}$, is necessary.

# *Constraining EOS*

**Parameter regions excluded by physical conditions**




**Bounded by causality.**

**Stiff**

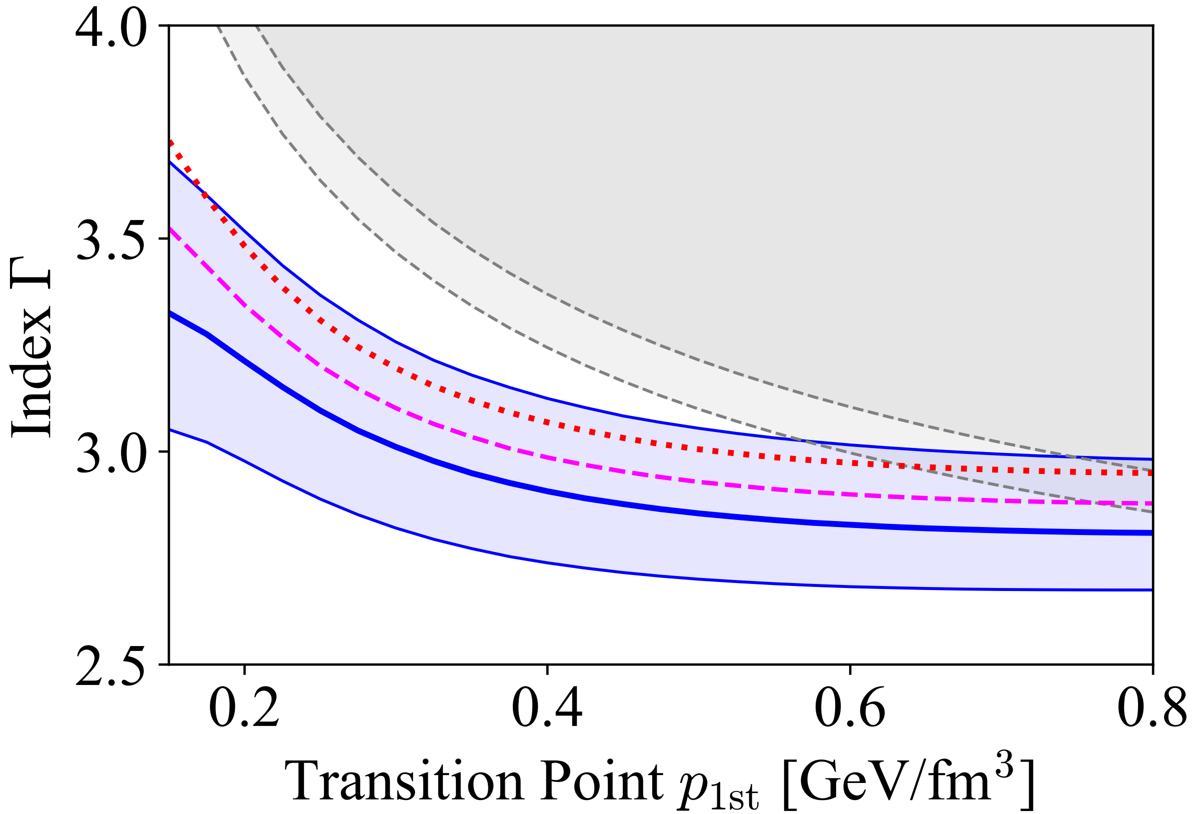
**Soft**

**Bounded by massive neutron stars.**

**For a conservative choice we use $\Gamma = 3.5$ in our simulation.**

# *Constraining EOS*

**Parameter regions excluded by physical conditions**




**Bounded by causality.**

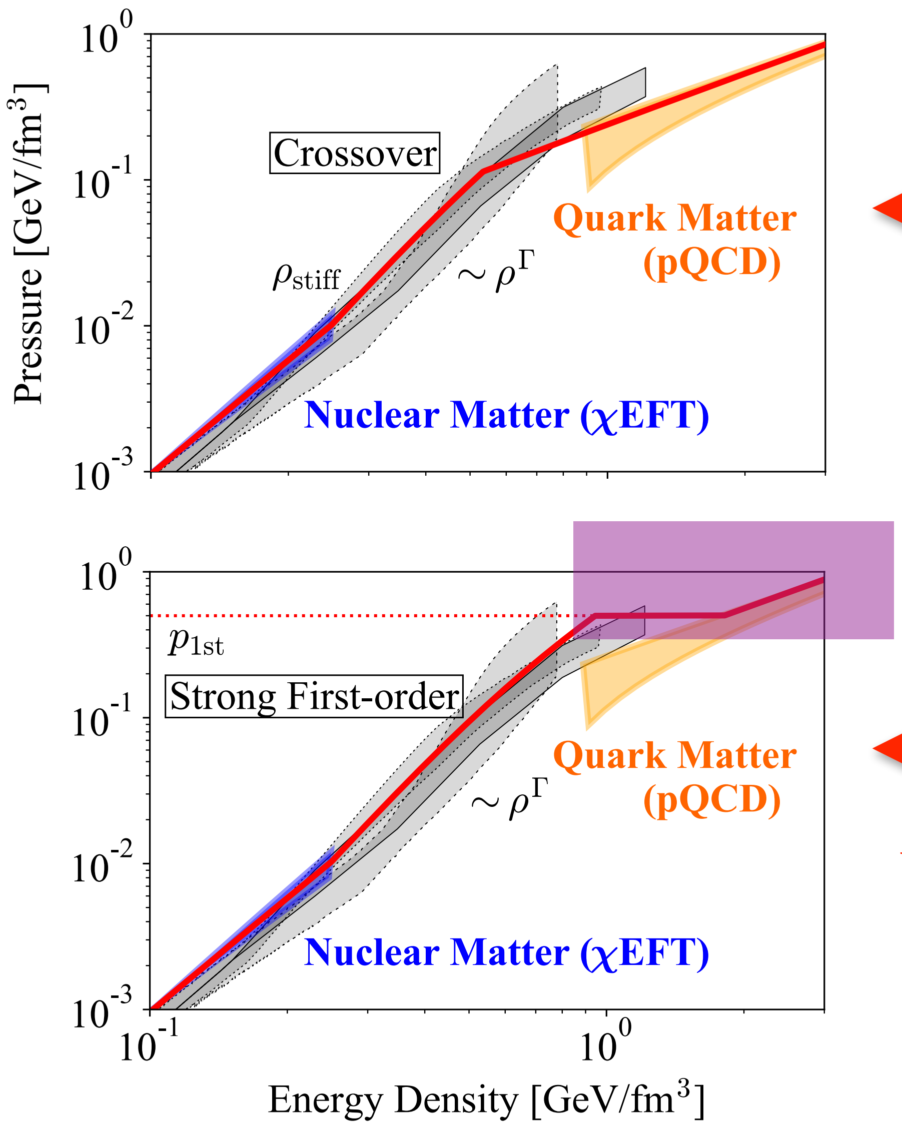
**Stiff**

**Soft**

**Bounded by massive neutron stars.**

**Assuming a transition at far high density, $p_{1st}$ is irrelevant.**

# *Constraining EOS*

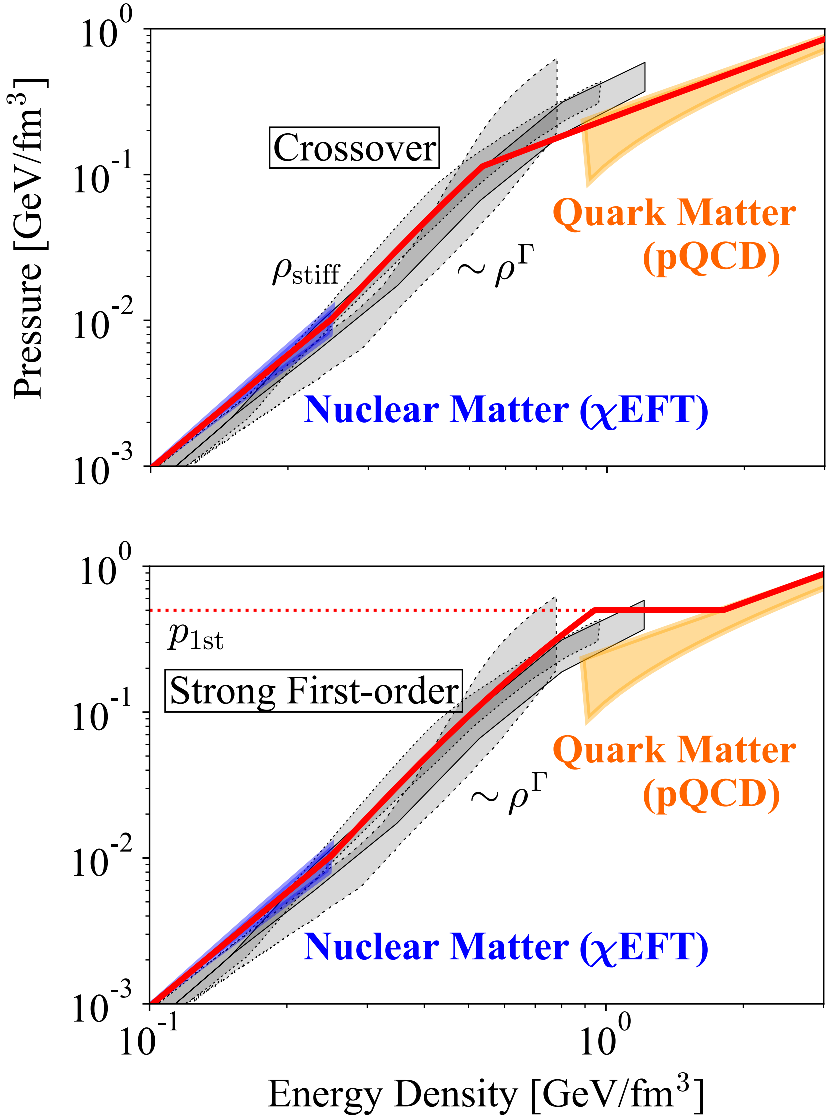



**We call this "With Crossover".**

**Discard the phase transition (too high density to probe)**

**We call this "Without Crossover".**

# *Constraining EOS*



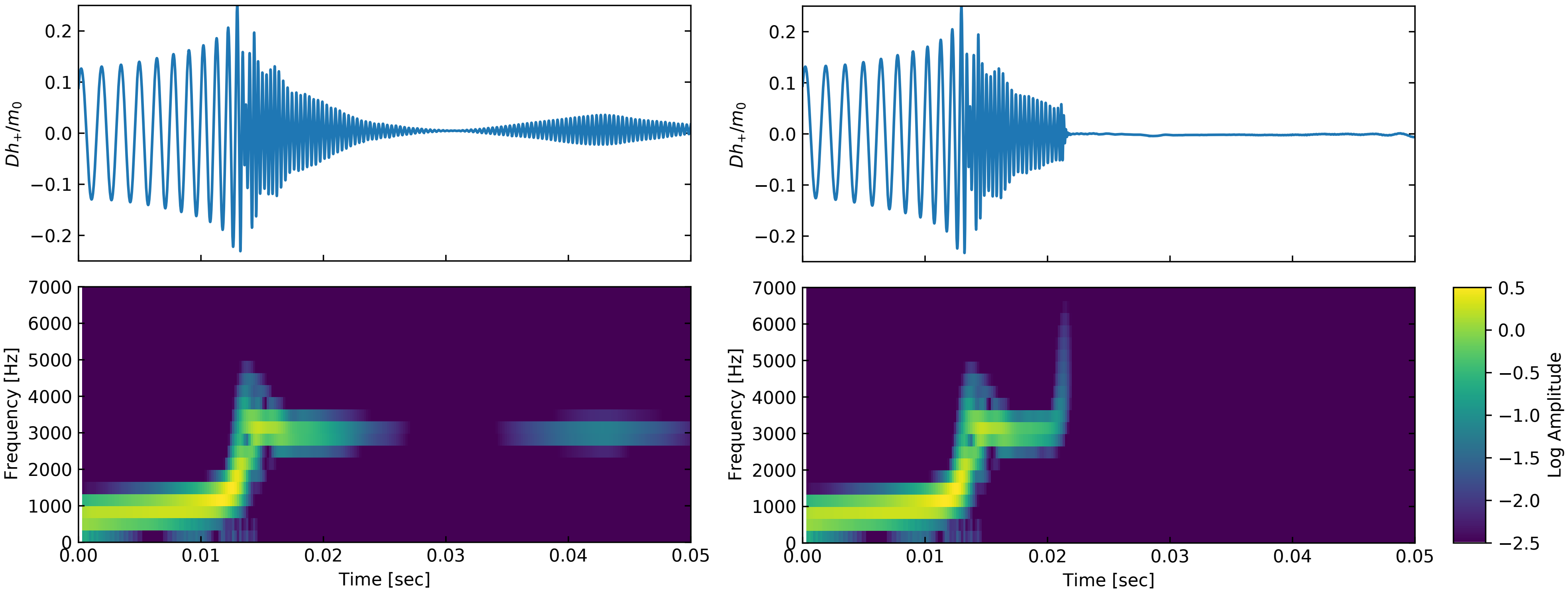

(Remark)

Weak 1st-order at intermediate density is still possible, but in this construction to match pQCD, it is inevitably very weak 1st, and there is no qualitative difference from the crossover.

# *Gravitational Waves with Crossover*

## Fujimoto-Fukushima-Hotokezaka-Kyutoku (2022)

**Without Crossover** **With Crossover**

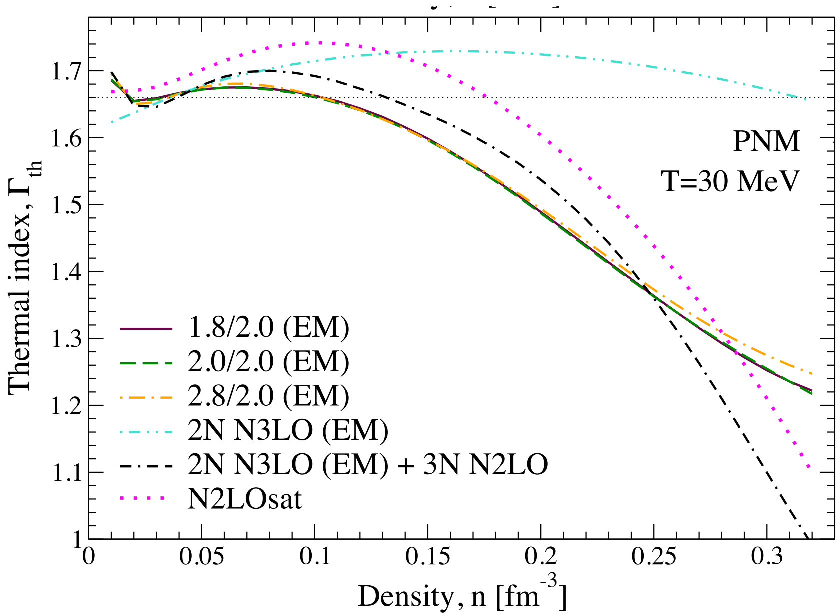



**Fujimoto-Fukushima-Hotokezaka-Kyutoku (appearing)**

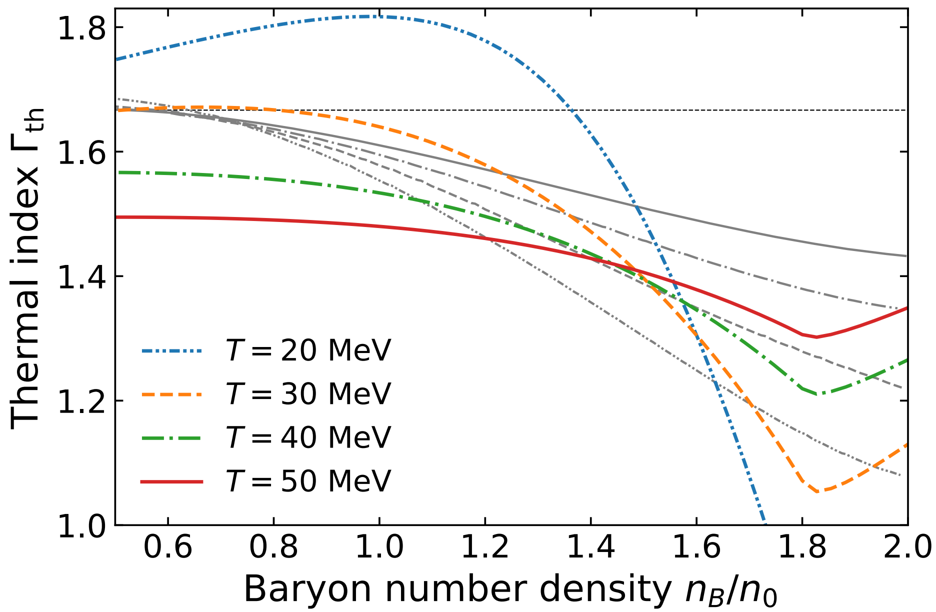
# A side remark: Thermal Index

$$P_{\text{thermal}} \approx \rho\varepsilon_{\text{thermal}}(\Gamma_{\text{th}} - 1)$$

**Fujimoto-Fukushima-Hidaka-Hiraguchi-Iida (2021)**



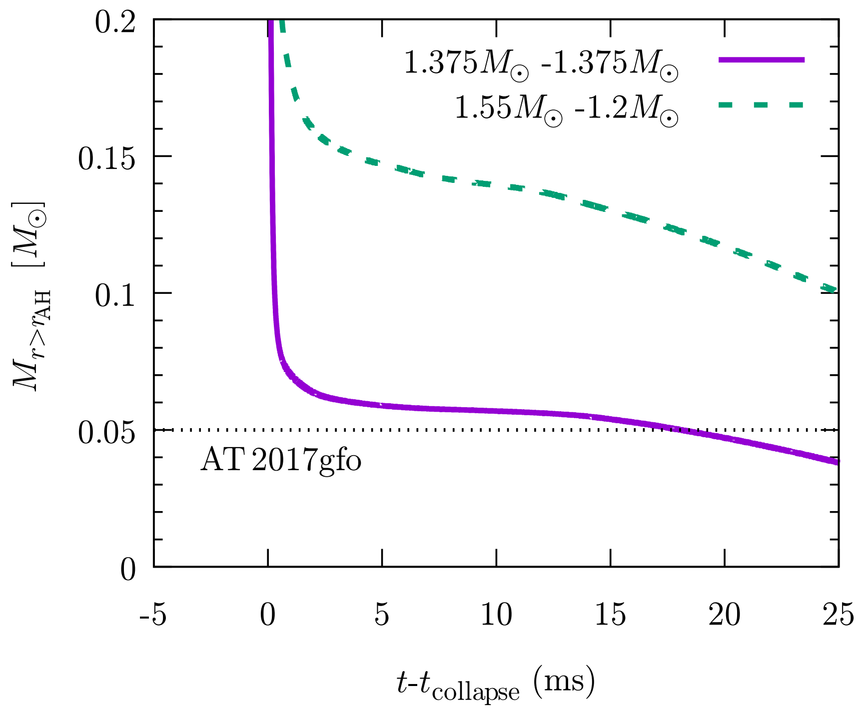

**Carbone-Schwenk (2019)**




**Thermal index is not larger than 1.8 and could be ~1.5 at high density.**

# *Important Consistency Check*

## Fujimoto-Fukushima-Hotokezaka-Kyutoku (2022)




From the kilonova data the ejecta mass is known to be $0.05M_\odot$

The remaining mass outside the apparent horizon after the BH formation should be larger than this mass.

**Equal mass case is disfavored, but unequal mass is possible. If the mass ratio is determined independently (in the inspiral stage), the ejecta mass imposes a useful constraint on EOS.**

# *Summary*

- **Gravitational waves from the post-merger stage should be sensitive to EOS at high density.**
  - Needs further upgrade but should be coming if the (reliable) theoretical prediction is made.
- **Crossover (or weak 1st-order PT at low density) vs. strong 1st-order PT at high density**
  - Life-time till the BH collapse signifies the sudden softening of EOS associated with quark matter.
- **Kilonova and ejecta mass give another constraint.**
  - Some discussions on the maximum NS mass could be changed with realistic EOS.