Composite dynamics in $Sp(2N)$ gauge theories

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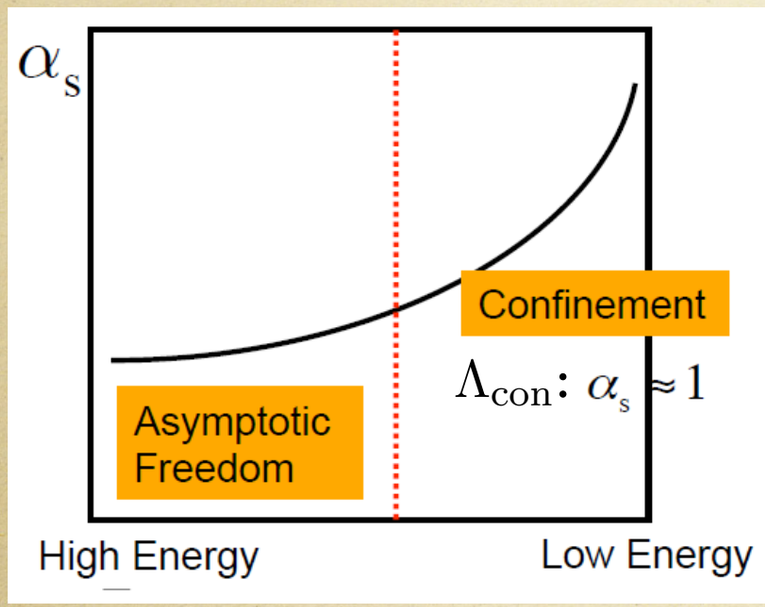
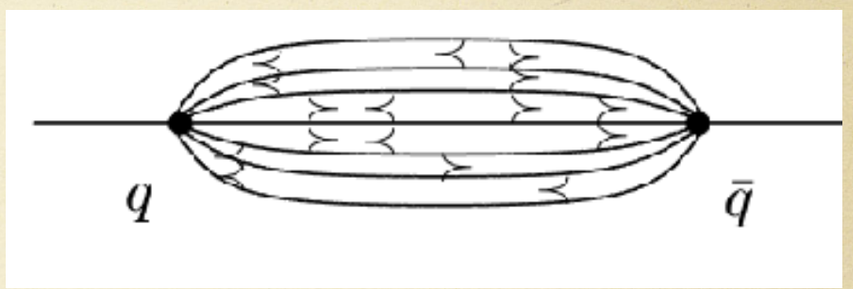
In collaboration with E. Bennett, D. K. Hong, H. Hsiao, C.-J. D. Lin, B. Lucini, M. Piai and D. Vadacchino

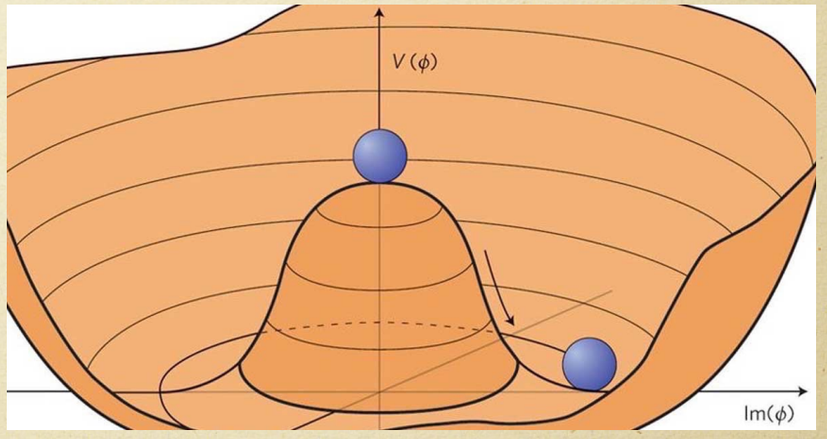
Confinement XV @ Stavanger, Norway
August 4, 2022

What's interesting about Sp(2N) gauge theory?

- Like *SU(N$_c$)* gauge theories (e.g. QCD), *Sp(2N)* gauge theories have interesting (non)perturbative features, such as *asymptotic freedom, confinement* and *chiral symmetry breaking*.







- Supplement *SU(N$_c$)* gauge theories by providing another sequence of N_c and support the large N_c equivalence in Yang-Mills theories.

- *Sp(2N)* gauge group is pseudoreal.
  - No sign problem in lattice simulations with finite chemical potential - provide new insights of QCD phase diagram
  - Global (flavor) symmetry is enhanced and its breaking pattern is different to *SU(N$_c$)* gauge theories.

if $\mathcal{G} \equiv SU(N_c), N_c > 2$

$$SU(N_f) \times SU(N_f) \to SU(N_f)$$

if $\mathcal{G} \equiv Sp(2N)$

$$SU(2N_f) \to Sp(2N_f)$$

- Finite temperature phase transition is first order for *Sp(2N)* with *N>1*.

Holland, Pepe & Wiese (2003)

Phenomenological applications for physics beyond SM based on novel strong dynamics: composite Higgs, partial top compositeness, composite dark matter (e.g. SIMP), gravitational waves, ...

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Phenomenological applications for physics beyond SM based on novel strong dynamics: composite Higgs, top partial compositeness, composite dark matter (e.g. SIMP), gravitational waves, ...

Composite Higgs

G. Cacciapaglia, plenary talk (Mon)

- **Composite Higgs:** PNGBs in the new strongly coupled gauge theories are identified by the Higgs doublets in SM.

  Kaplan & Georgi (1984)

  Key idea: EW symmetry is not broken by new strong interaction, but by vacuum misalignment

Composite Higgs + top-quark partial compositeness

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  Kaplan & Georgi (1984)

  Key idea: EW symmetry is not broken by new strong interaction, but by vacuum misalignment

- **Partial compositeness:** mixing between SM quarks and composite operators, formed by fermions in two different representations (chimera baryons), can explain quark mass hierarchy.

  Kaplan (1991)

  Key idea: large anomalous dim. of the chimera baryon, e.g. top-partner

- **Modern composite Higgs models**

  Contino, Nomura & Pomarol (2003); Agashe, Contino & Pomarol (2004)

SM matter and gauge fields

$$\mathcal{L}_{mix} = \lambda_{L,R}\, \bar{q}_{L,R}\, \mathcal{O}_q^{L,R} + g_V A^\mu J_\mu$$

strongly-coupled Higgs sector

- 4D UV models based on *Sp(2N)* gauge theories

  G. Ferretti & T. Karataev (2013), arXiv:1312:5330; J. Bernard, T. Gherghetta & T. S. Ray (2013), arXiv: 1311.6562

4D UV models for comp. Higgs + partial compositeness

Coset	HC	ψ	χ	$-q_\chi/q_\psi$	Baryon	Name
$\frac{SU(5)}{SO(5)} \times \frac{SU(6)}{SO(6)}$	SO(7)	$5 \times \mathbf{F}$	$6 \times \mathbf{Sp}$	5/6	$\psi\chi\chi$	M1
	SO(9)			5/12		M2
	SO(7)	$5 \times \mathbf{Sp}$	$6 \times \mathrm{F}$	5/6	$\psi\psi\chi$	M3
	SO(9)			5/3		M4
$\frac{SU(5)}{SO(5)} \times \frac{SU(6)}{Sp(6)}$	Sp(4)	$5 \times \mathbf{A}_2$	$6 \times \mathbf{F}$	5/3	$\psi\chi\chi$	M5
$\frac{SU(5)}{SO(5)} \times \frac{SU(3)^2}{SU(3)}$	SU(4)	$5 \times \mathbf{A}_2$	$3 \times (\mathbf{F}, \overline{\mathbf{F}})$	5/3	$\psi\chi\chi$	M6
	SO(10)	$5 \times \mathbf{F}$	$3 \times (\mathbf{Sp}, \overline{\mathbf{Sp}})$	5/12		M7
$\frac{SU(4)}{Sp(4)} \times \frac{SU(6)}{SO(6)}$	Sp(4)	$4 \times \mathbf{F}$	$6 \times \mathbf{A}_2$	1/3	$\psi\psi\chi$	M8
	SO(11)	$4 \times \mathbf{Sp}$	$6 \times \mathbf{F}$	8/3		M9
$\frac{SU(4)^2}{SU(4)} \times \frac{SU(6)}{SO(6)}$	SO(10)	$4 \times (\mathbf{Sp}, \overline{\mathbf{Sp}})$	$6 \times \mathbf{F}$	8/3	$\psi\psi\chi$	M10
	SU(4)	$4 \times (\mathbf{F}, \overline{\mathbf{F}})$	$6 \times \mathbf{A}_2$	2/3		M11
$\frac{SU(4)^2}{SU(4)} \times \frac{SU(3)^2}{SU(3)}$	SU(5)	$4 \times (\mathbf{F}, \overline{\mathbf{F}})$	$3 \times (\mathbf{A}_2, \overline{\mathbf{A}_2})$	4/9	$\psi\psi\chi$	M12

- M8: most lattice-friendly, e.g. exact # of matter content

$SU(4)/Sp(4)$

# of pNGBs = 5

4 of 5 PNGBs: Higgs doublets

$SU(2)_L \times U(1)_Y \subset Sp(4)$

SM EW

$SU(3)_c \times U(1)_Y \subset SO(6)$

SM Strong

Chimera baryon (top partner)

$$\hat{\Psi}^{a\alpha b} \equiv \left(\psi^a \chi^\alpha \psi^b\right)$$

carry color charge

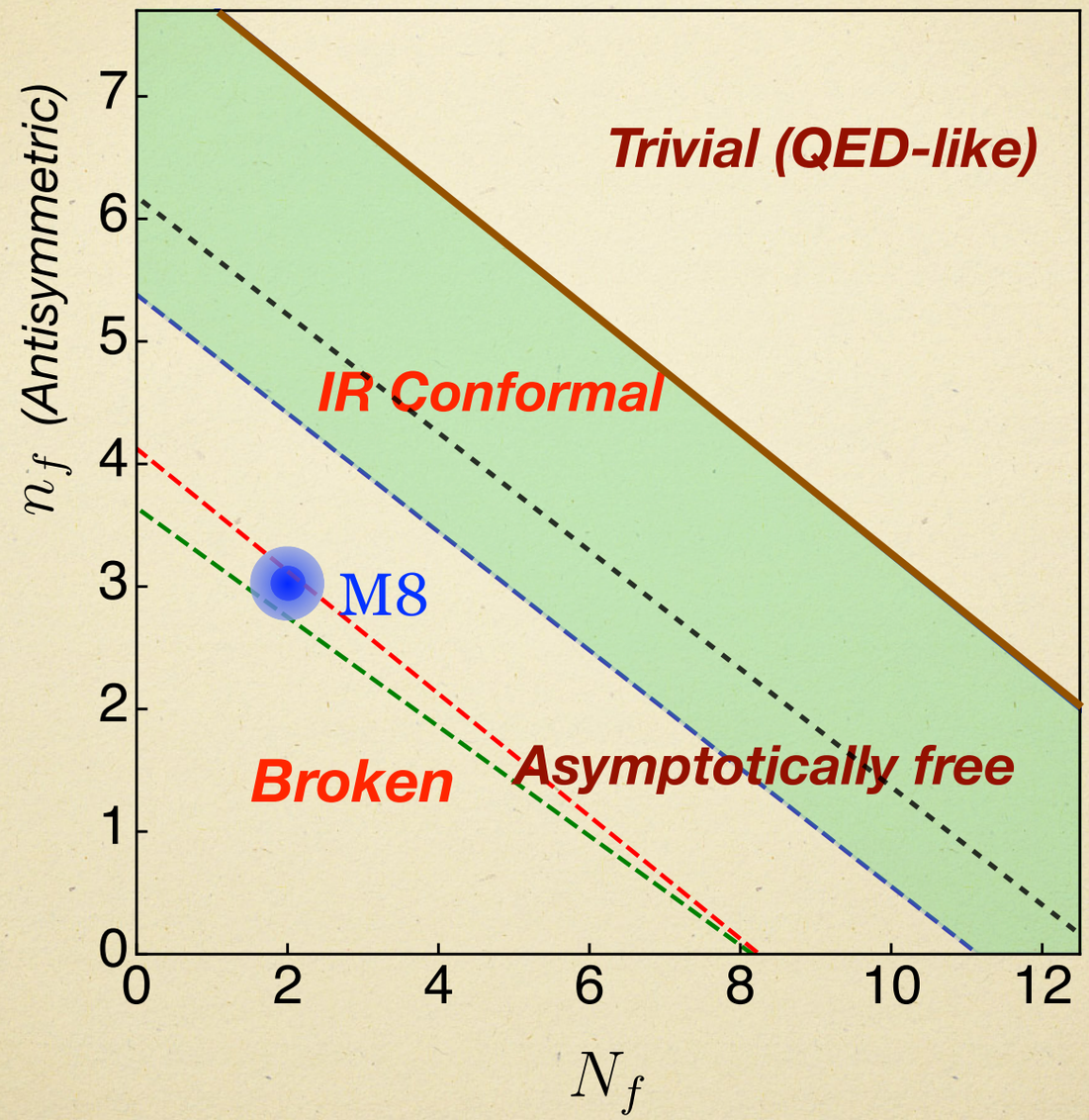
G. Cacciapaglia, G. Ferretti, T. Flacke & H. Serodio (2019), arXiv:1902.06890

Theory space of *Sp(4)* gauge group + fermion matter

B. Kim, D. Hong & JWL (2020), arXiv:2001.02690

Dashed lines: analytical estimates of the lower end of conformal window

Black: Schwinger-Dyson
Red: All-order beta func.
Green: 2-loop beta func.
Blue: BZ conformal exp.



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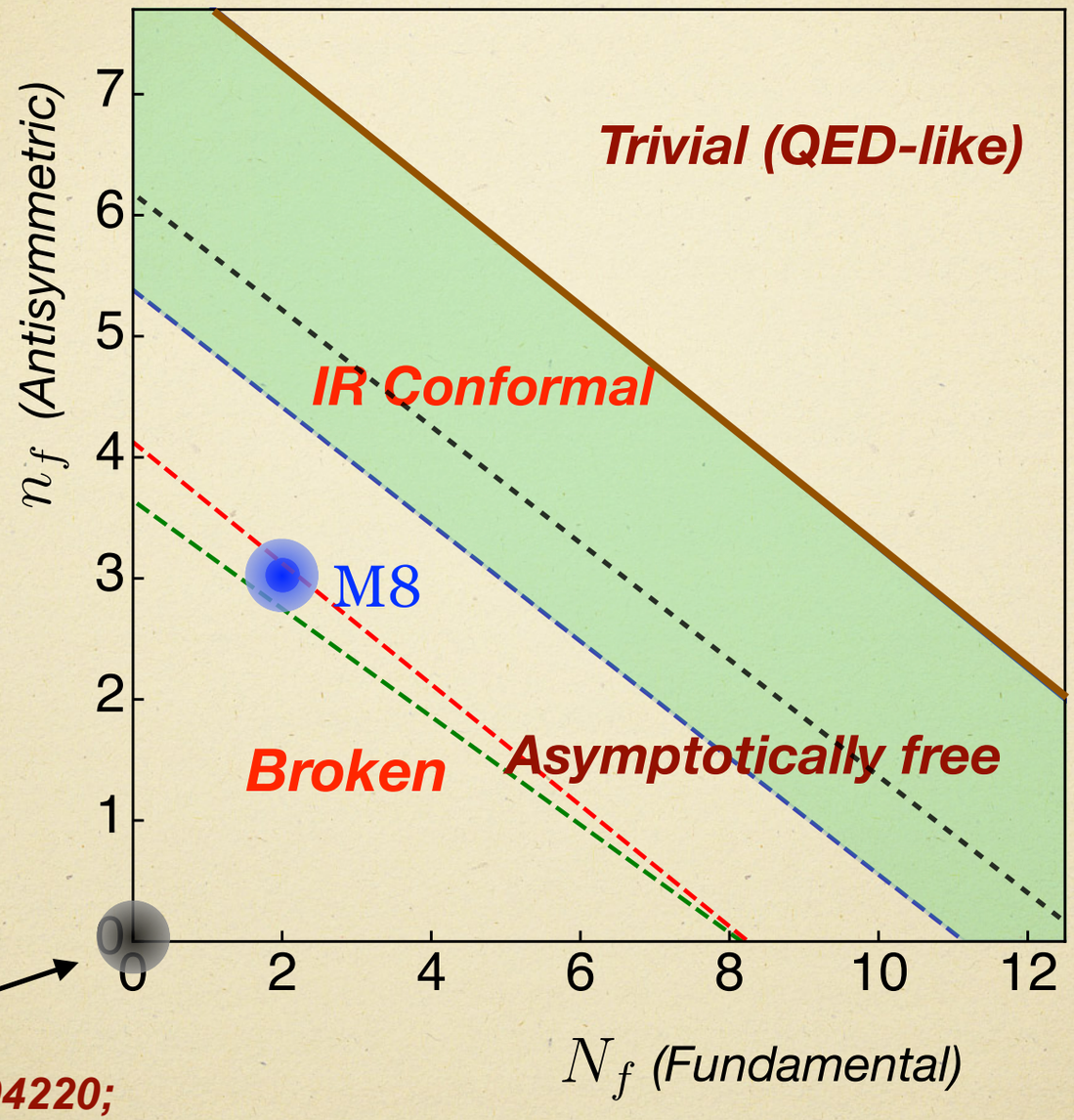

E. Bennett et al, arXiv:1712.04220; (2017, 2019) arXiv:1912.06505.

- ☑ Glueballs & quenched meson spectrum

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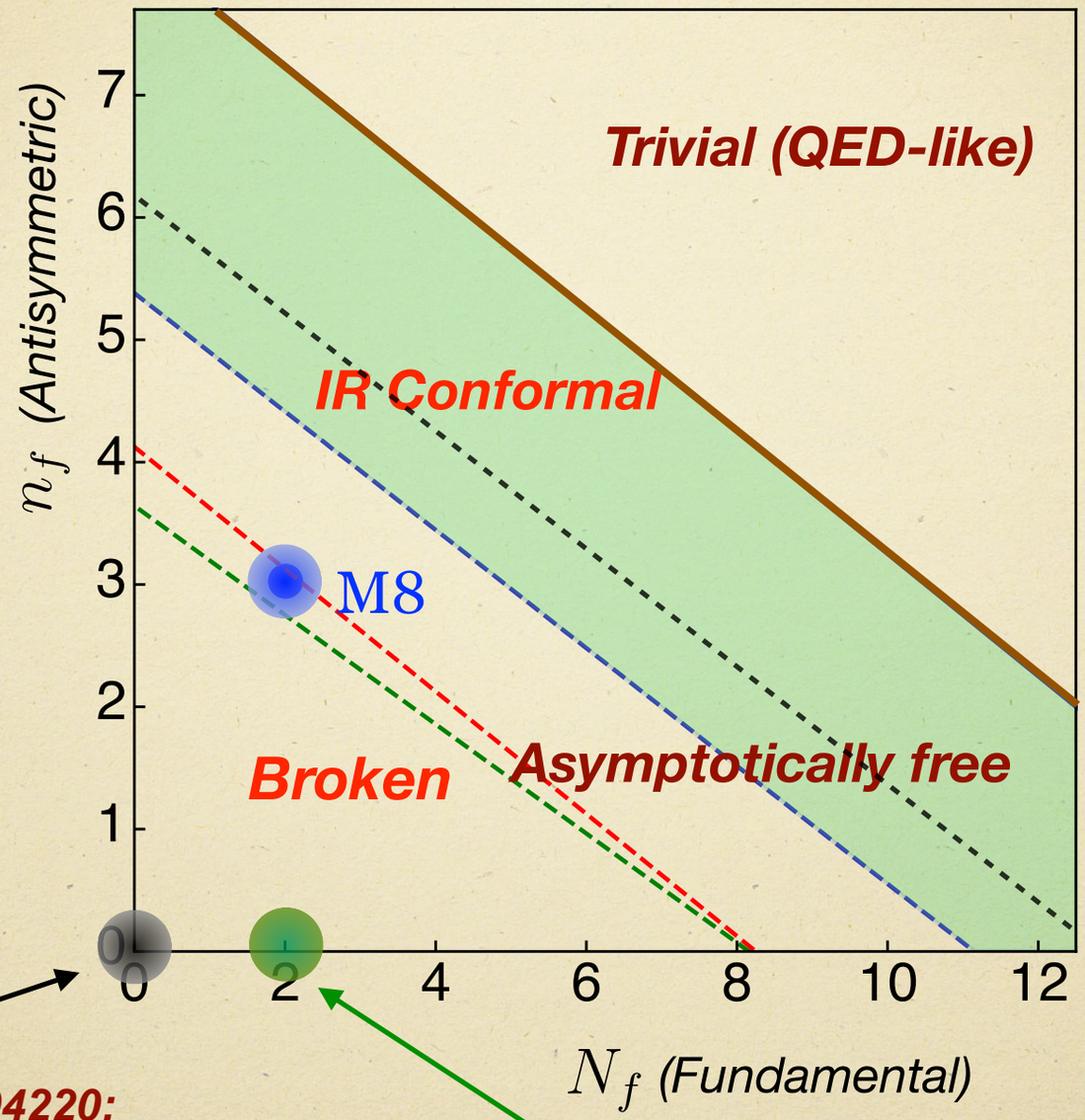

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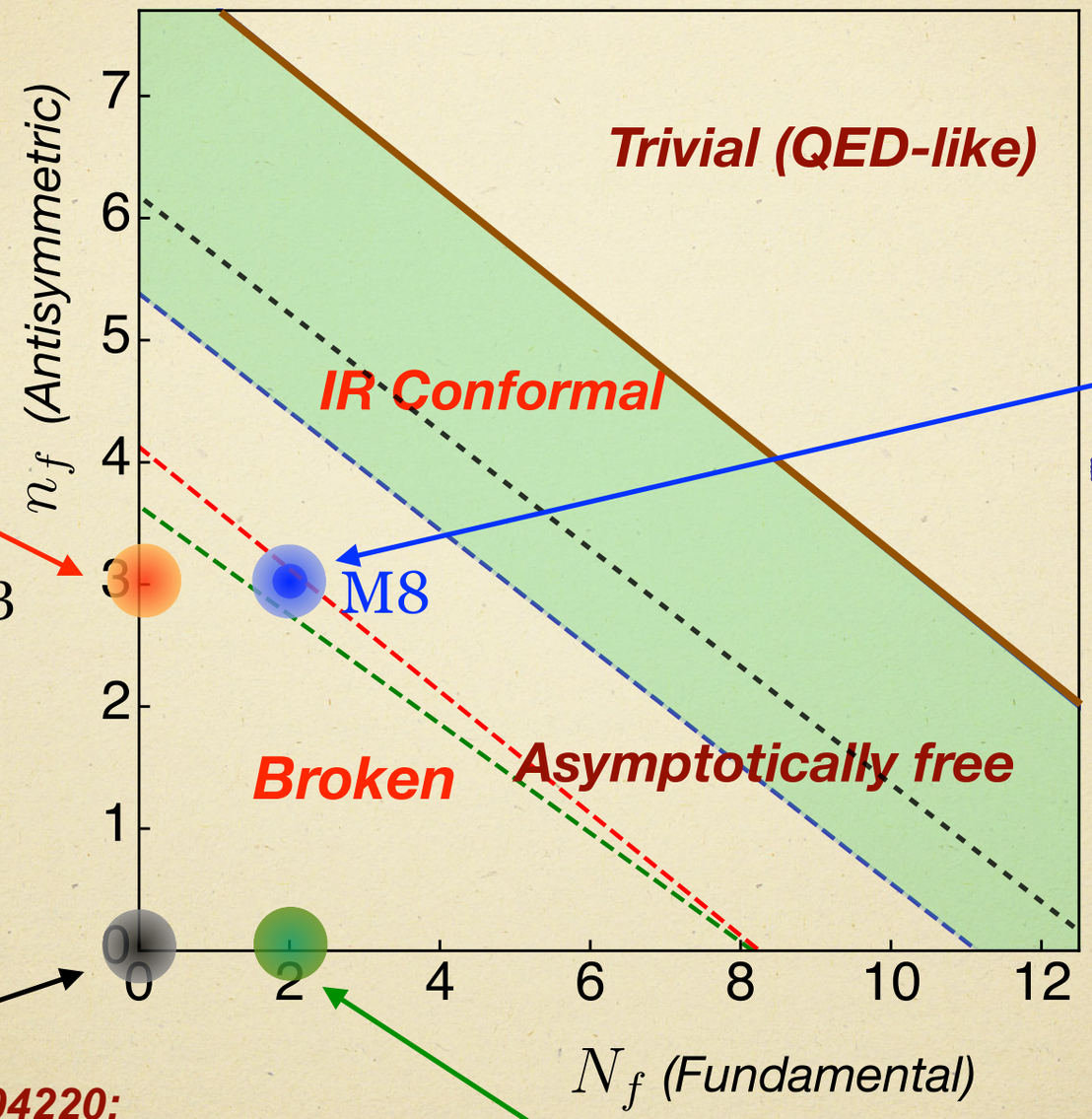
E. Bennett et al (2019), arXiv:1909.12662

- ☑ Meson spectrum in N_f=2 dynamical simulations

Theory space of *Sp(4)* gauge group + fermion matter



E. Bennett et al (2022), work in progress

- Meson spectrum in n_f=3 dynamical simulations

E. Bennett et al (2022), arXiv:2202.05516

- Exploratory studies of model M8: mesons & chimera baryons

E. Bennett et al, arXiv:1712.04220; (2017, 2019) arXiv:1912.06505.

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E. Bennett et al (2019), arXiv:1909.12662

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Lattice setup

- Lattice formulation with the standard Wilson gauge & fermion actions

$\mathcal{G} \equiv Sp(2N)$ — pure gauge theories

$\mathcal{G} \equiv Sp(4),\ N_f = 2\,(\mathrm{F}),\ n_f = 3\,(\mathrm{AS})$ — theories with dynamical fermions

- (bare) lattice parameters: $\beta = 4N/g^2$, m_0^f & m_0^{as}

- Simulations by employing Heat-bath algorithm for pure gauge theories and the (rational) hybrid Monte Carlo ((R)HMC) for the theories with dynamical fermions.

- Scale setting: Gradient flow method *Luscher (2010)* *Luscher & Wiese (2011)*

$$\hat{a} \equiv a/w_0 \qquad \hat{m} \equiv m^{\mathrm{lat}} w_0^{\mathrm{lat}} = m w_0$$

Borsarnyi et al (2012), arXiv: 1203.4469

What did we learn about Sp(2N) gauge theory so far?

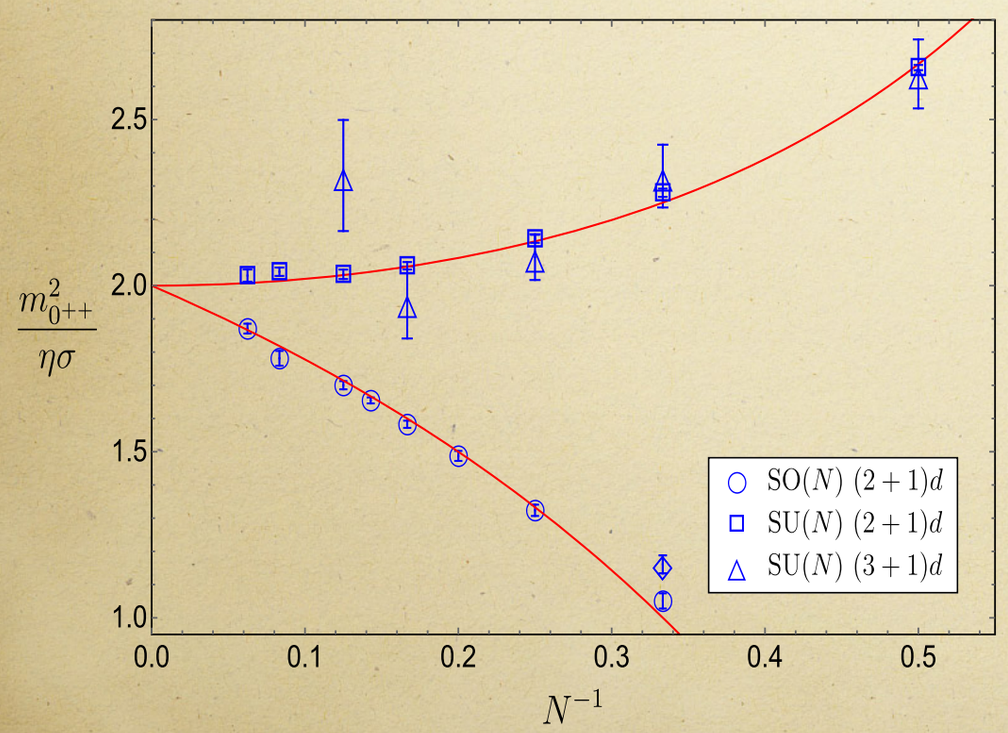
Universalities in Yang-Mills: *SU(N$_c$), SO(N$_c$)* & *Sp(2N)*

☑ Casimir scaling $\dfrac{m_{0^{++}}^2}{\sigma} = \eta \dfrac{C_2(A)}{C_2(F)}$

$$\frac{C_2(A)}{C_2(F)} = \begin{cases} \frac{2N^2}{N^2-1} & \text{for SU}(N) \\ \frac{2(N-2)}{N-1} & \text{for SO}(N) \\ \frac{4(N+1)}{2N+1} & \text{for Sp}(2N)\,, \end{cases}$$

$$\eta(0^{++}) \equiv \frac{m_{0^{++}}^2}{\sigma} \cdot \frac{C_2(F)}{C_2(A)} = \begin{cases} 5.41(12), & (d=3+1)\,, \\ 8.440(14)(76), & (d=2+1)\,. \end{cases}$$

D. Hong, JWL, B. Lucini, M. Piai & D. Vadacchino (2017)



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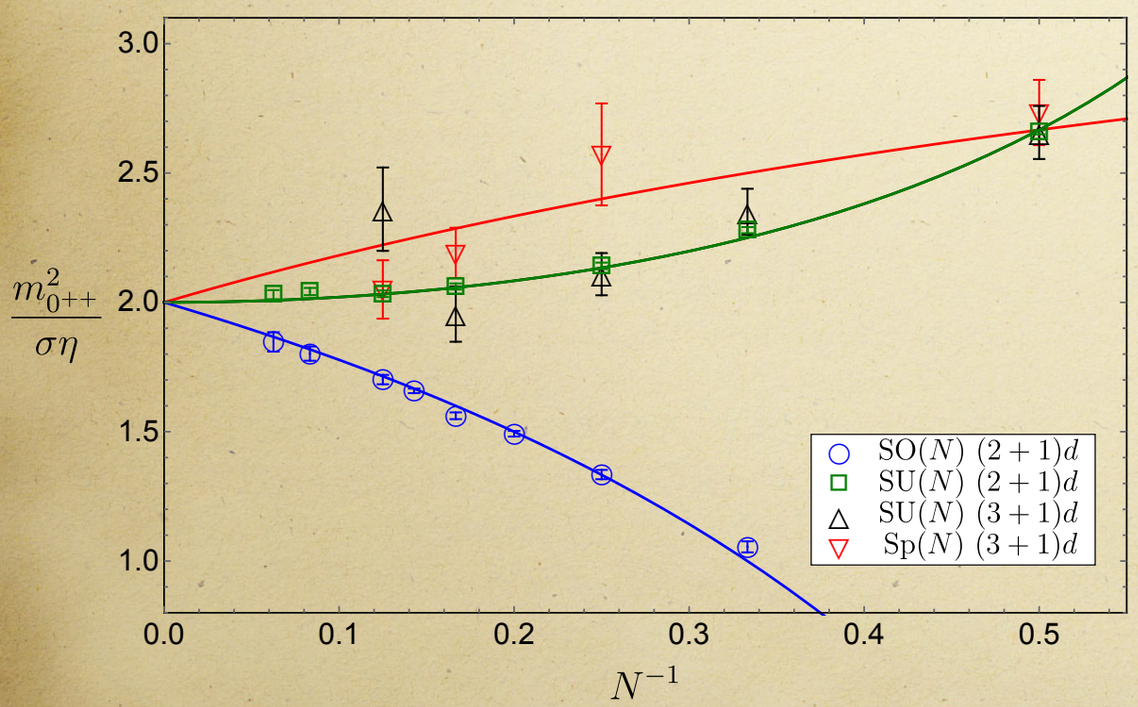
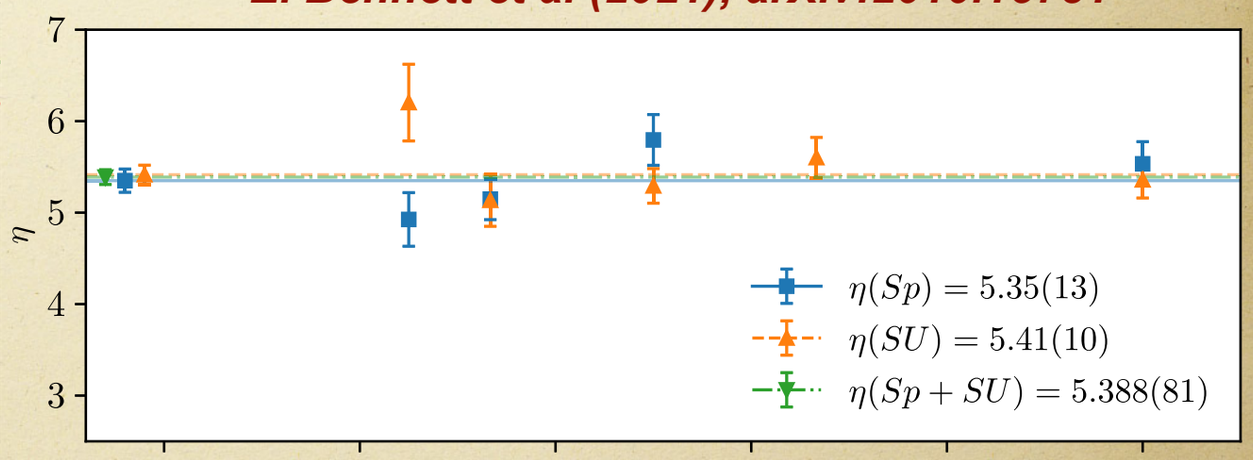


E. Bennett et al (2021), arXiv:2010.15781

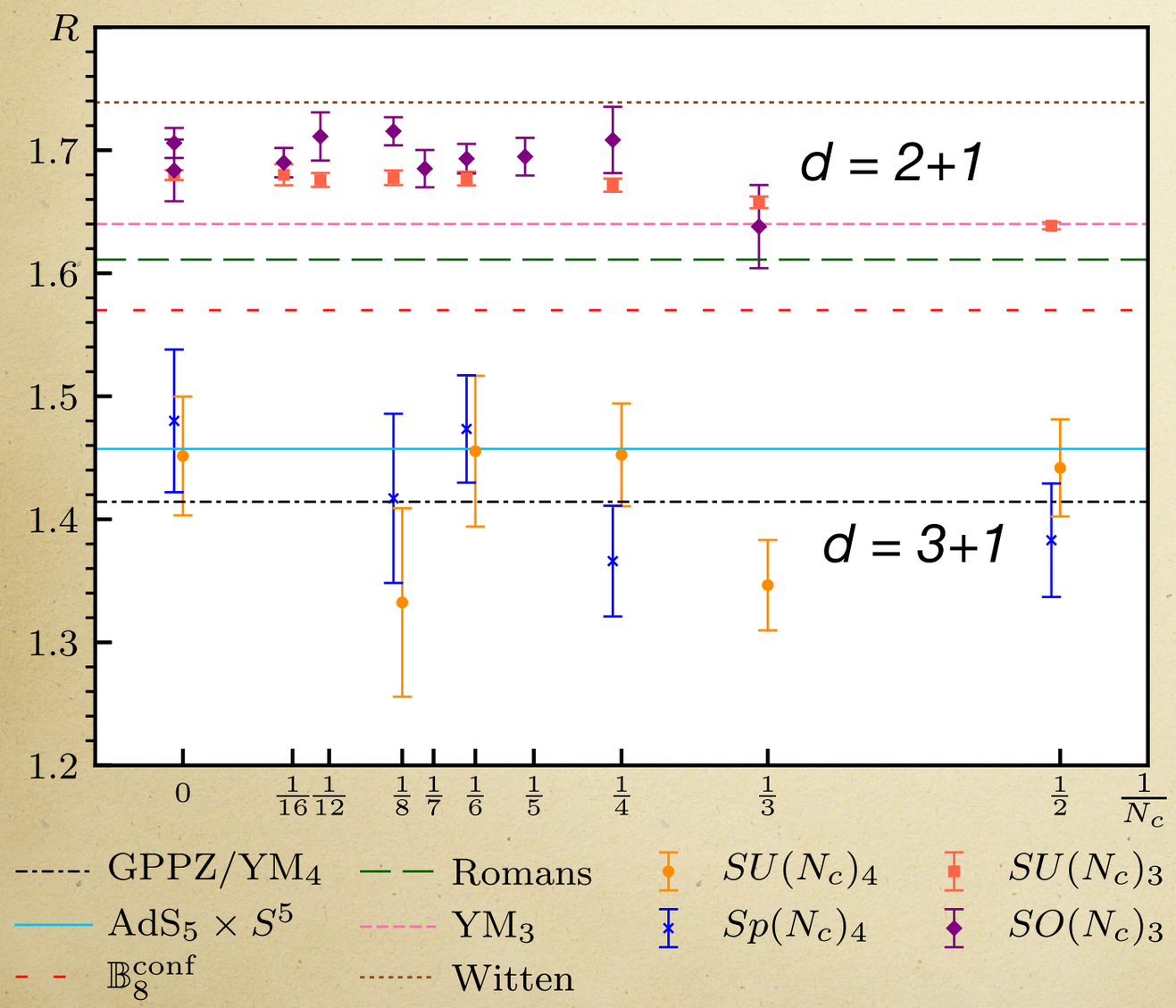


The value of the universal constant updated by combining the results from lattice calculations of *Sp(2), Sp(4), Sp(6)* and *Sp(8)* YMs.

Universalities in Yang-Mills: $SU(N_c)$, $SO(N_c)$ & $Sp(2N)$

☑ Mass ratio of gluebells $R \equiv \frac{m_{2^{++}}}{m_{0^{++}}}$

A. Athenodorou et al (2016), arXiv:1605.04258

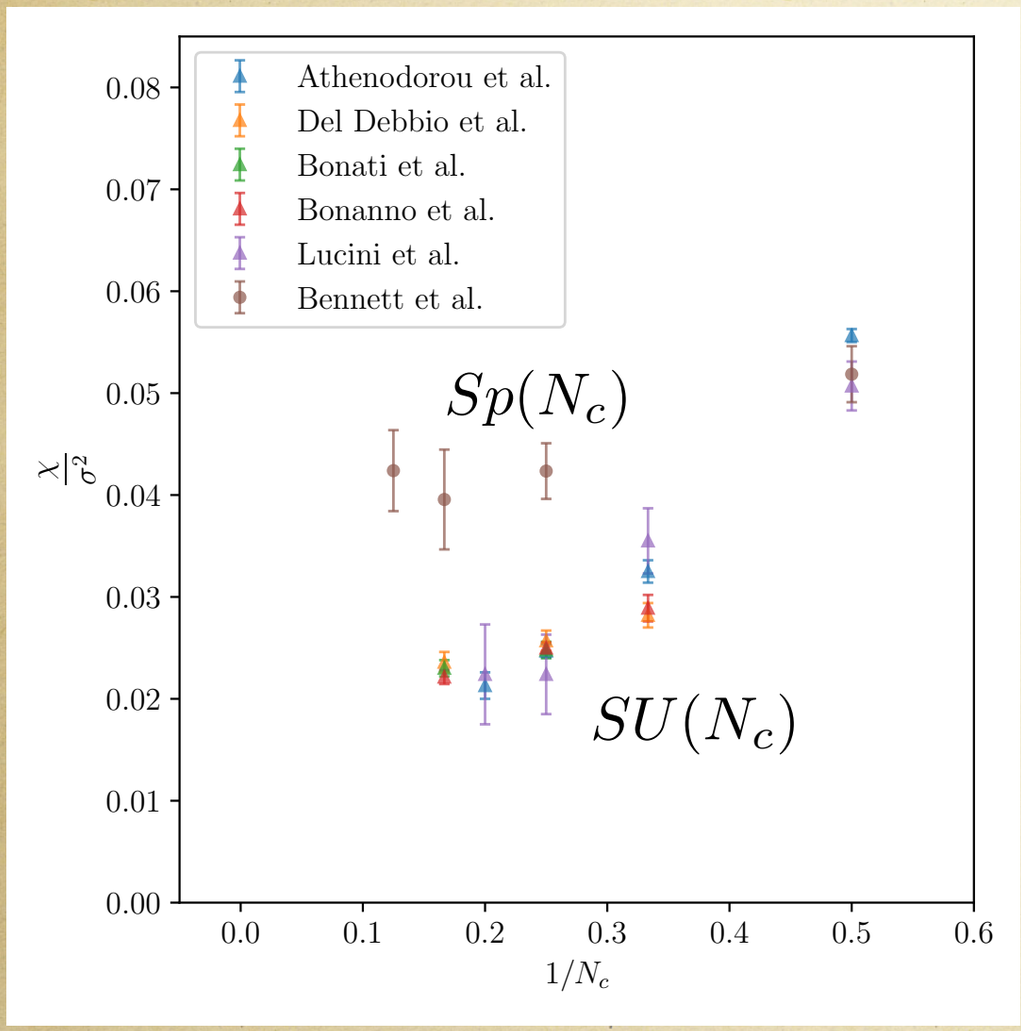


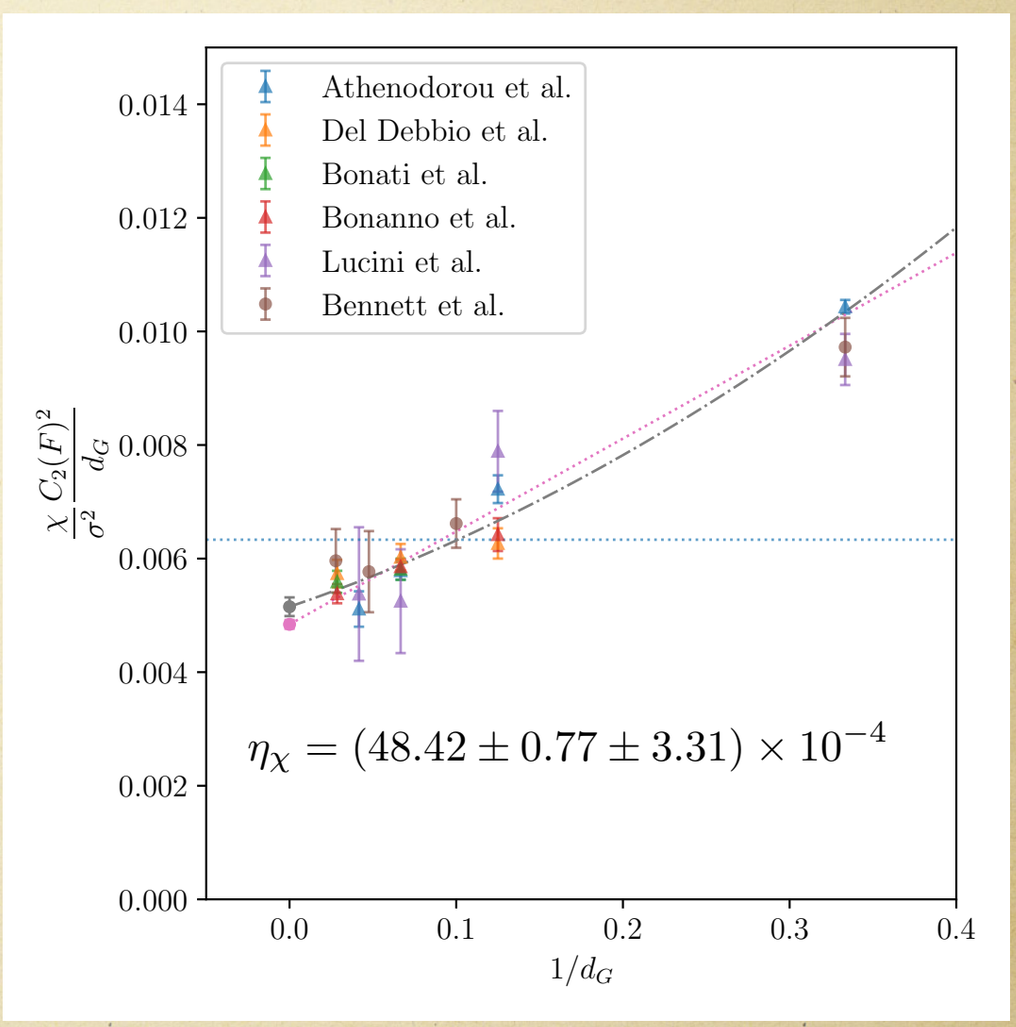
Lattice results of *SU(N)*, *SO(N)* & *Sp(2N)* YMs at finite *N* and analytical results from gauge-gravity dualities and alternative field methods support the conjecture of the universal mass ratio R.

E. Bennett et al (2020), arXiv:2004.11063

Topological susceptibility in Yang-Mills: *SU(N$_c$)* & *Sp(2N)*

$$\chi \equiv \int \mathrm{d}^4x \, \langle q(x) q(0) \rangle \qquad q(x) \equiv \frac{1}{32\pi^2} \varepsilon^{\mu\nu\rho\sigma} \, \mathrm{Tr} \, F_{\mu\nu}(x) F_{\rho\sigma}(x)$$



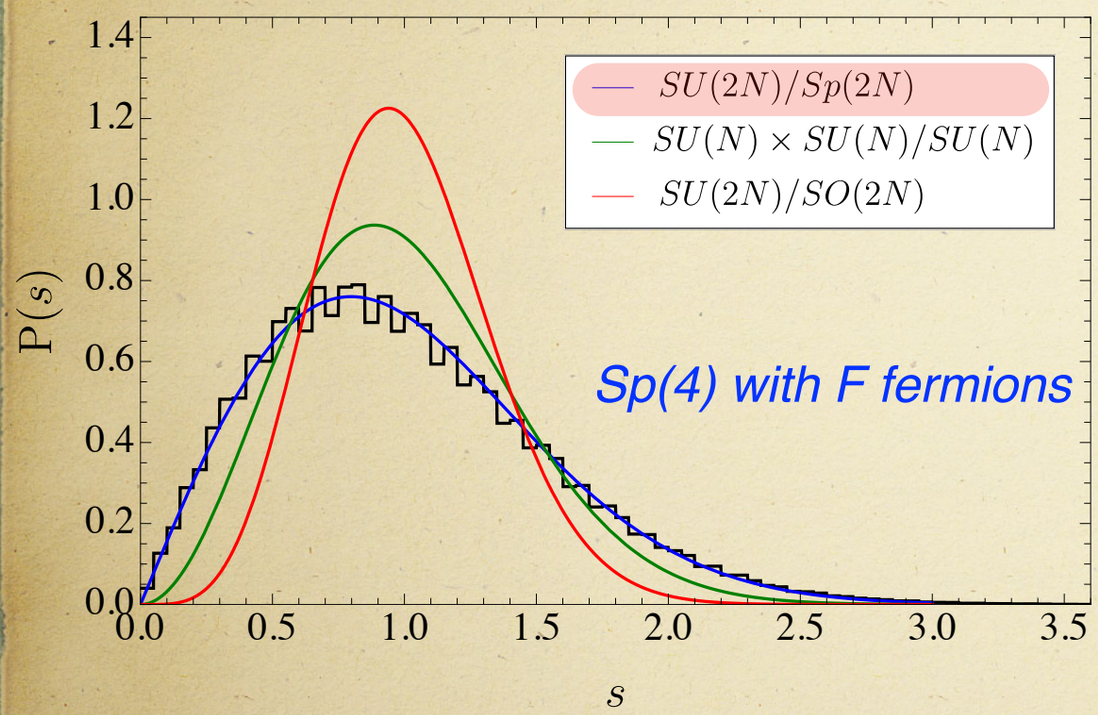


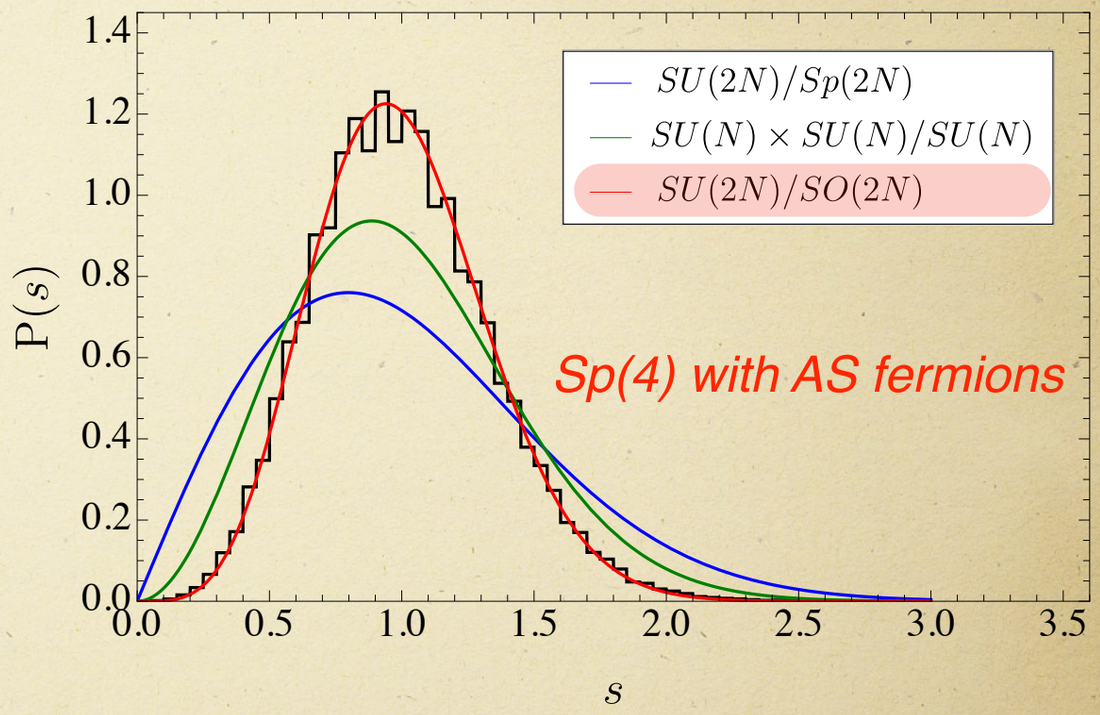
$$\eta_\chi \equiv \frac{\chi C_2(F)^2}{\sigma^2 d_G} = \frac{\chi}{\sigma^2} \cdot \begin{cases} \frac{N_c^2-1}{4N_c^2} & \text{for } SU(N_c) \\ \frac{N_c+1}{8N_c} & \text{for } Sp(N_c) \end{cases}$$

E. Bennett et al (2022), arXiv:2205.09254

Numerical results: Global symmetry breaking

- Symmetry breaking patterns are encoded in the Dirac spectrum





chRMT prediction: $P(s) = N_{\bar\beta} s^{\bar\beta} e^{-c_{\bar\beta} s^2}$, with $N_{\bar\beta} = 2\dfrac{\Gamma^{\bar\beta+1}\left(\frac{\bar\beta}{2}+1\right)}{\Gamma^{\bar\beta+2}\left(\frac{\bar\beta+1}{2}\right)}$, $c_{\bar\beta} = \dfrac{\Gamma^2\left(\frac{\bar\beta}{2}+1\right)}{\Gamma^2\left(\frac{\bar\beta+1}{2}\right)}$

- Fermions in the F and AS representations are correctly implemented.

E. Bennett el al (2022), arXiv: 2202.05516

Numerical results: Global symmetry breaking

- Key requirement for composite Higgs: spontaneous symmetry breaking

Pseudo Nambu Goldstone Bosons (pNGBs)

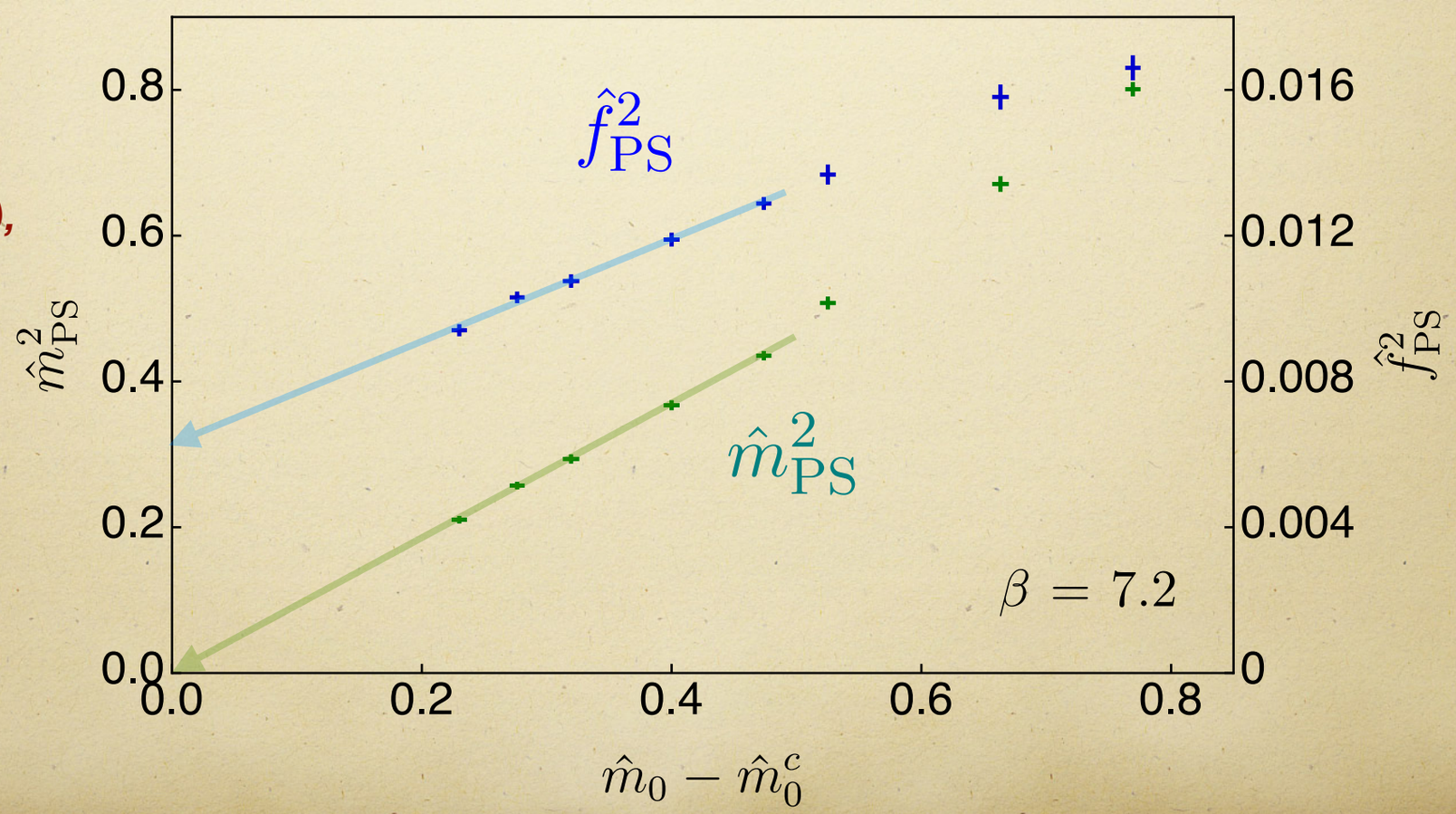
$SU(4)/Sp(4)$

# of pNGBs = 5

GMOR relation: $m_{\rm PS}^2 f_{\rm PS}^2 = m_f \langle \bar{\psi}\psi \rangle$ *Gell-Mann, Oakes, Renner (1968)*

N_f=2 (F) Sp(4)

E. Bennett el al (2019), arXiv:1909.12662

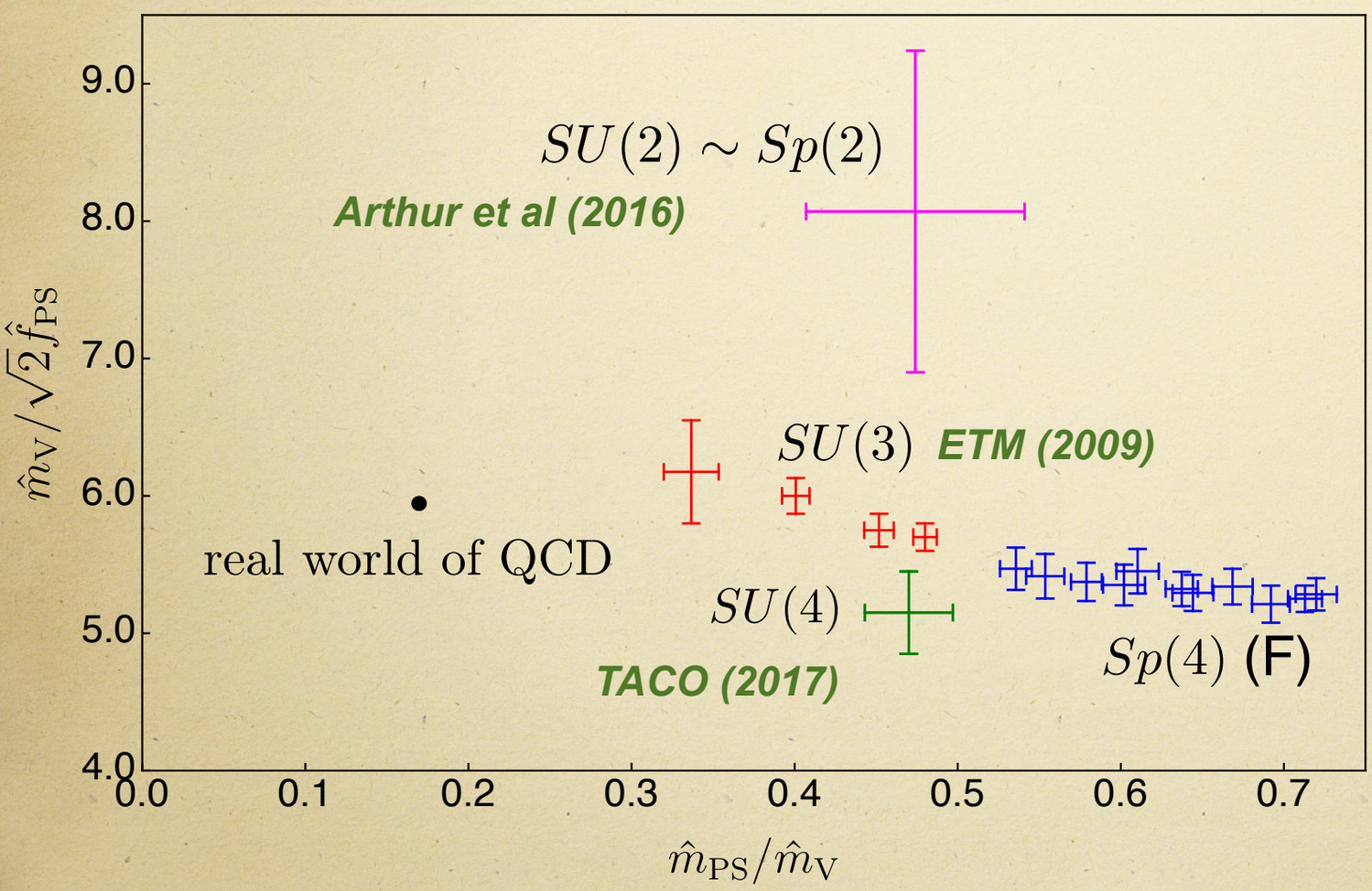


Numerical results: Vector meson

- Lattice results of various gauge theories coupled to N_f=2 fund. Dirac flavors
- The hypothesis vector meson dominance leads to the KSRF relation

Kowarabayashi & Suzuki (1966)
Riazuddin & Fayyazuddin (1966)

$$g_{VPP} = \frac{m_{\rm V}}{\sqrt{2} f_{\rm PS}}$$



E. Bennett el al (2019), arXiv:1909.12662

HLS EFT fit results:
in the massless limit

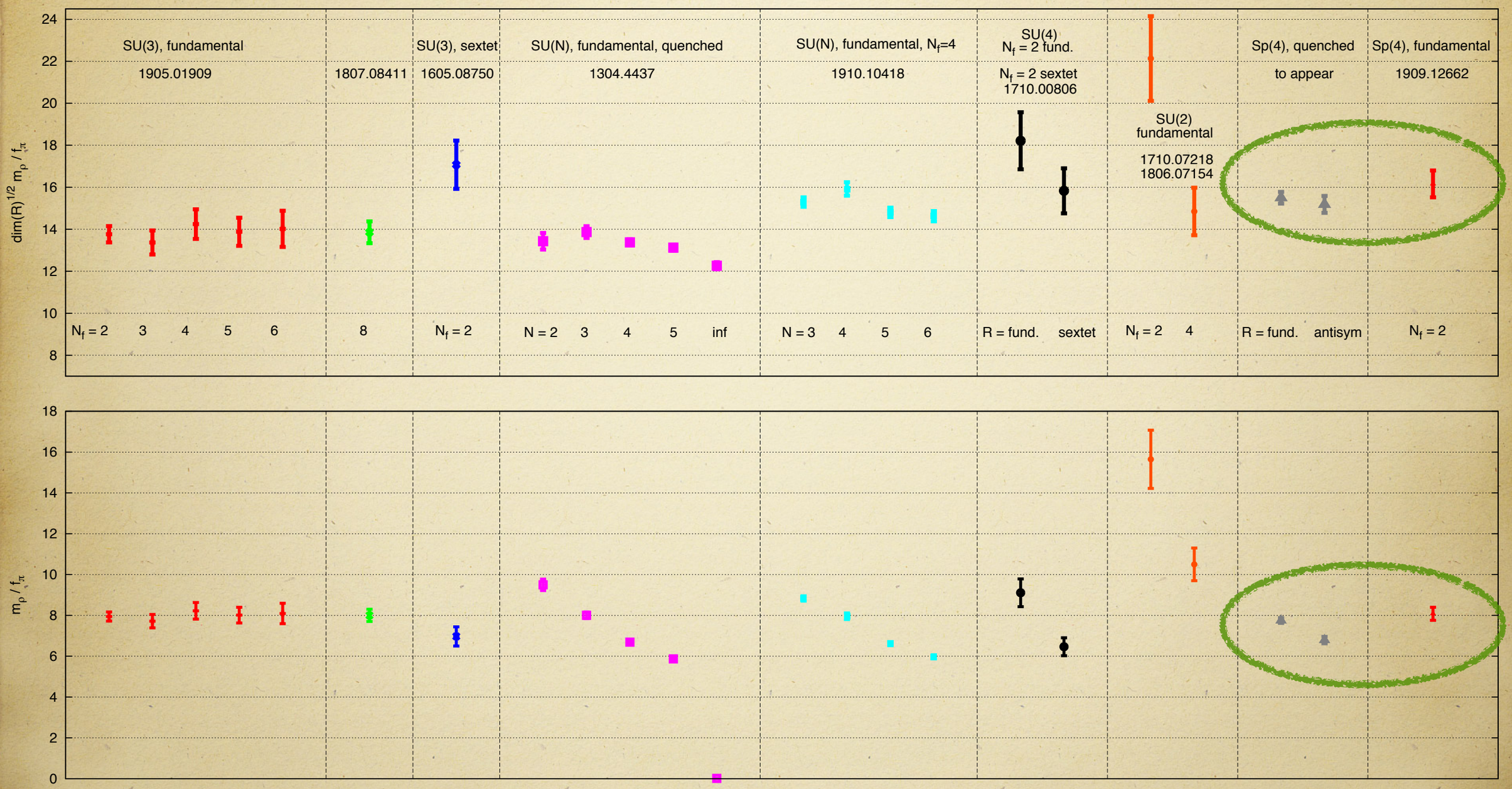
$$g^{\chi}_{\rm VPP} = 6.0(4)(2)$$

Large N argument:

$$f_{\rm PS} \sim \sqrt{N_c}$$

$$m_{\rm V}/\sqrt{2} f_{\rm PS} \times \sqrt{N_c/3} \sim 6$$

Vector mesons in strongly coupled gauge theories

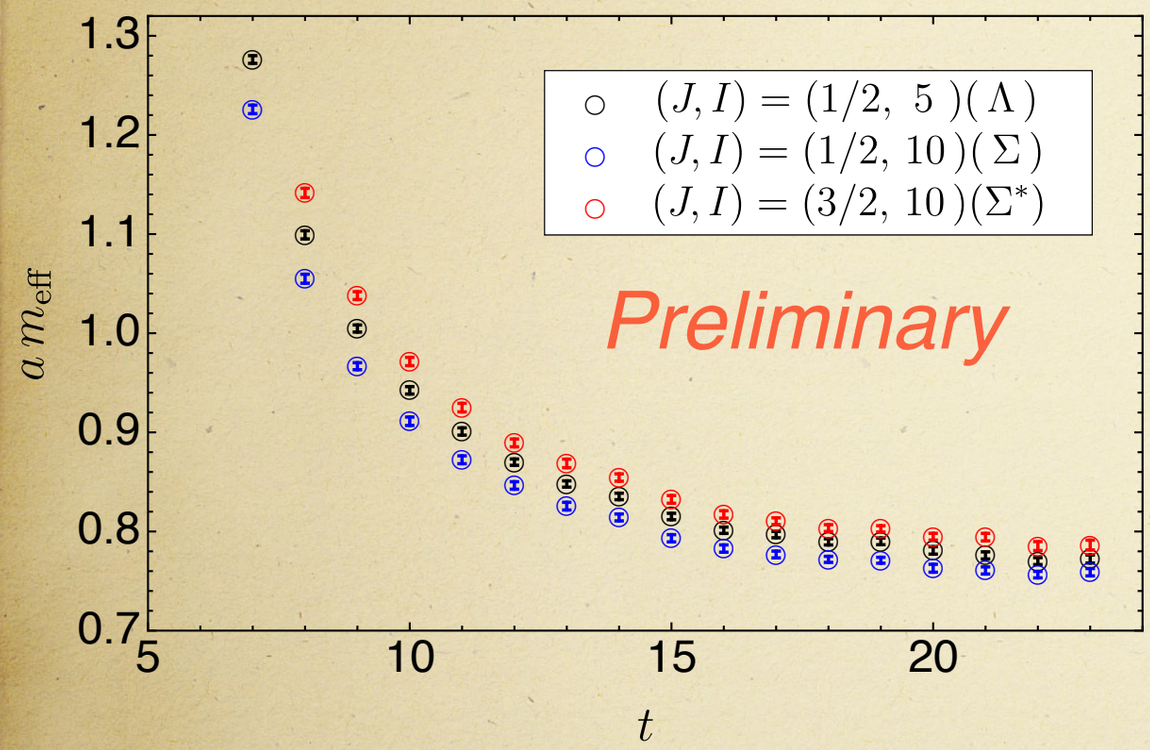


Nogradi & Szikszai, arXiv:1912.04114

- $m_{\rm V}/f_{\rm PS}$ depends on N and the fermion representation, but not the # of flavors

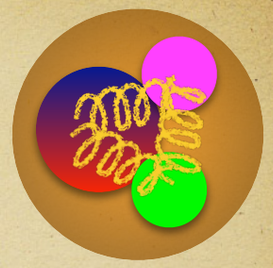
Numerical result: Chimera baryon

$$m_{\rm PS}^{(as)}/m_{\rm PS}^{(f)} \sim 1.67$$



- (J, I) = (1/2, 5) chimera baryon (top partner) is not the lightest state, but still stable since $m_\Lambda - m_\Sigma \ll m_{\rm PS}^{(f)}$.

Numerical result: Chimera baryon



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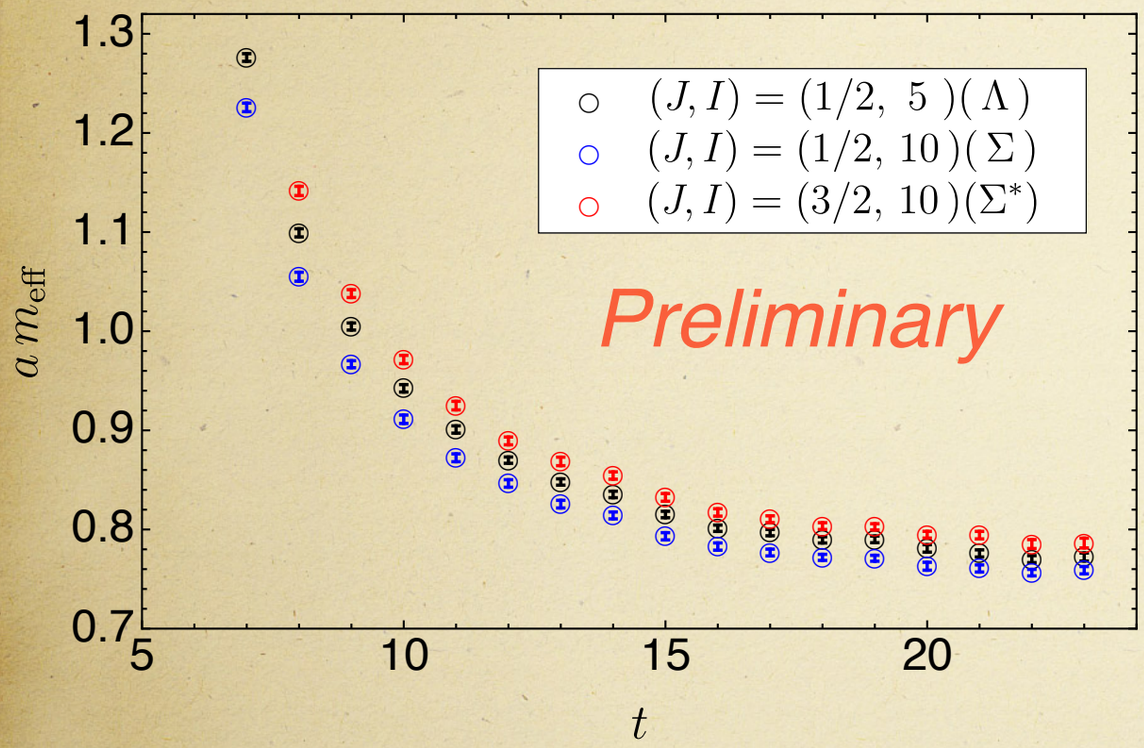
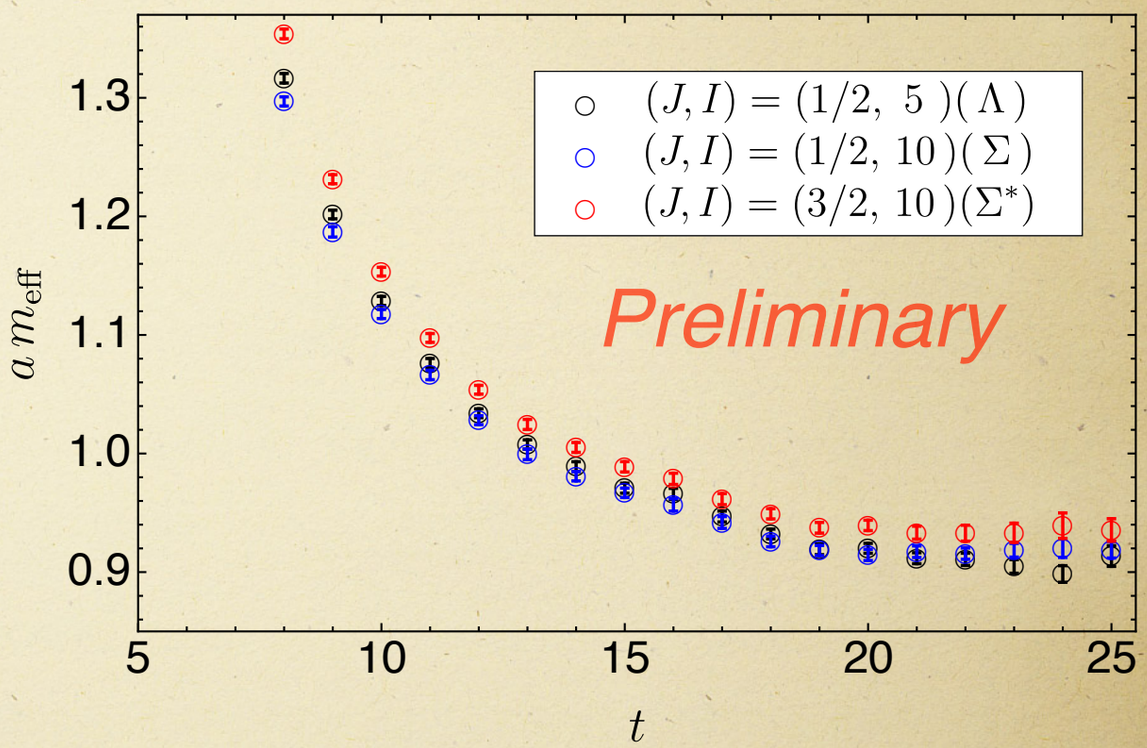


$m_{\rm PS}^{(as)}/m_{\rm PS}^{(f)} \sim 4.30$



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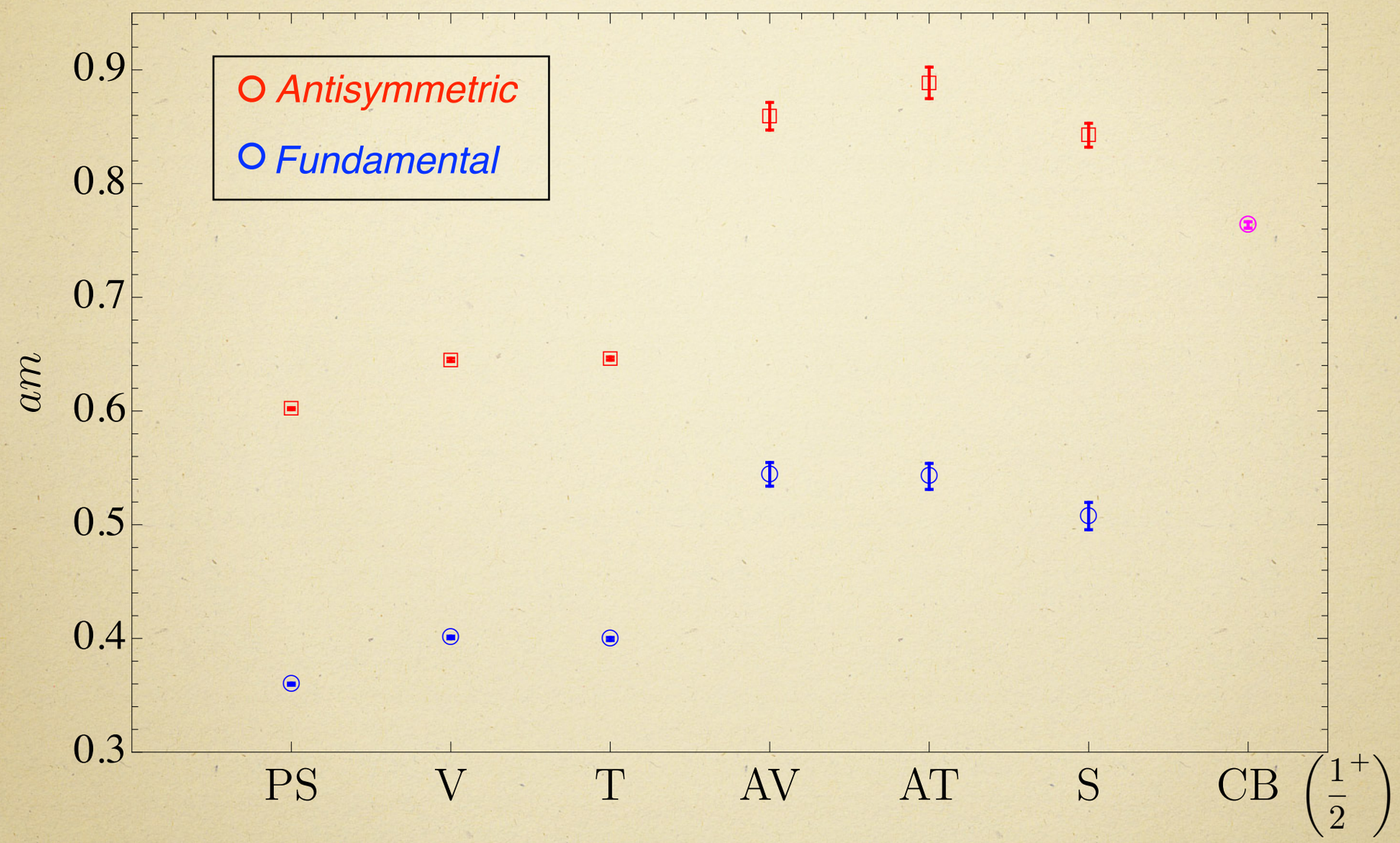
- (J, I) = (1/2, 5) chimera baryon (top partner) becomes the lightest state when $m_{\rm PS}^{(as)}$ is about 4 times larger than $m_{\rm PS}^{(f)}$.

Numerical result: mass spectrum of the model M8

$$\beta = 6.5,\ a\,m_0^{as} = -1.01,\ a\,m_0^{f} = -0.71,\ \ N_t \times N_s^3 = 54 \times 28^3$$

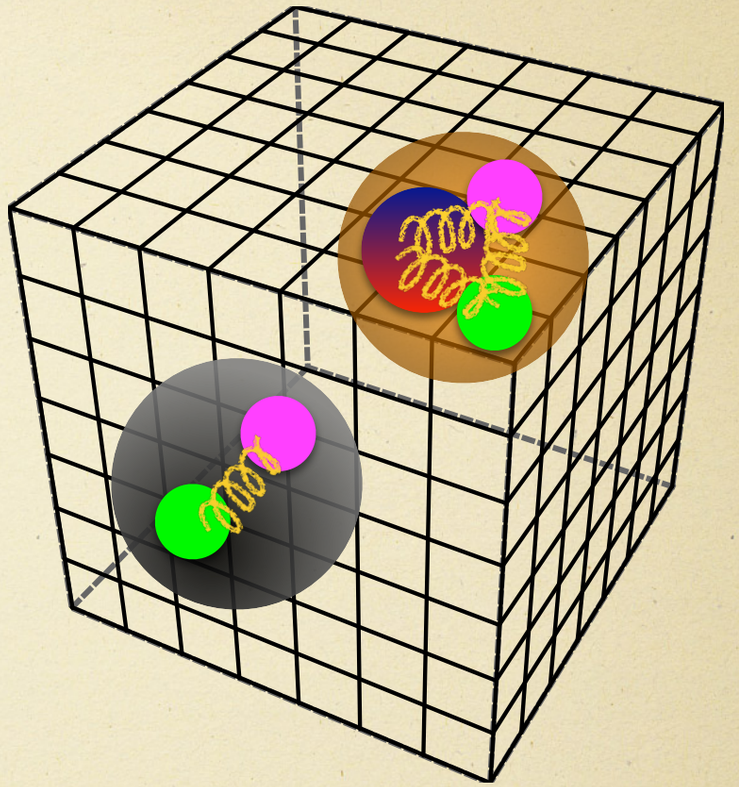


E. Bennett el al (2022), arXiv: 2202.05516

- Premature to discuss any physics, yet : single lattice, large mass, ...

Concluding remarks

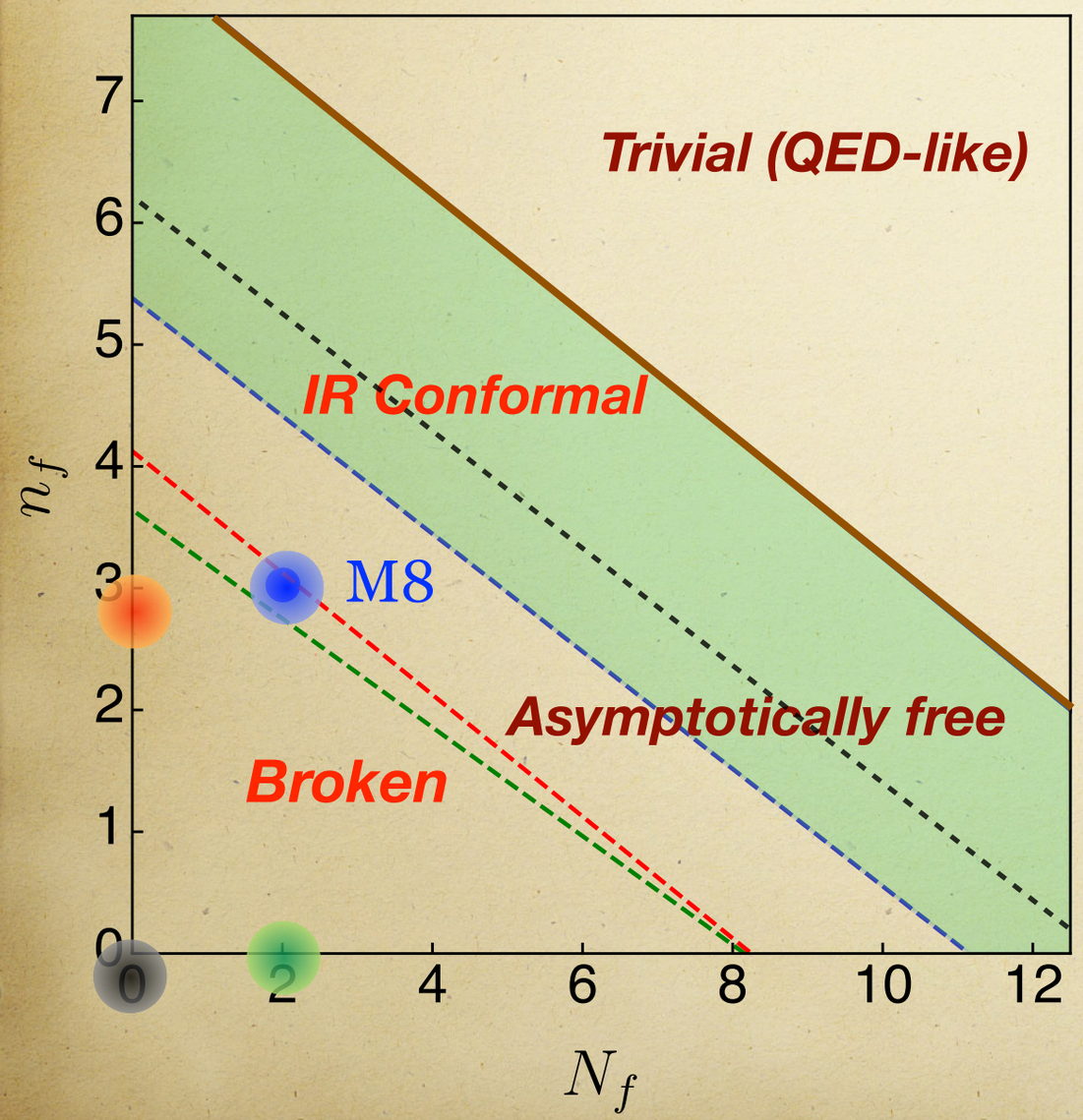
Sp(2N) gauge theories with/without fermions on the lattice



- **Theoretical point of view:** new insights of composite dynamics, such as large N universalities in Yang-Mills and (near-)conformal phase in theories with large numbers of fermion.
- **Phenomenological point of view:** Due to a nonperturbative nature of novel strong dynamics in search for physics beyond SM, numerical lattice calculations can play a crucial role by providing various phenomenological inputs, such as mass, form factor, low-energy constants, etc.

On going work

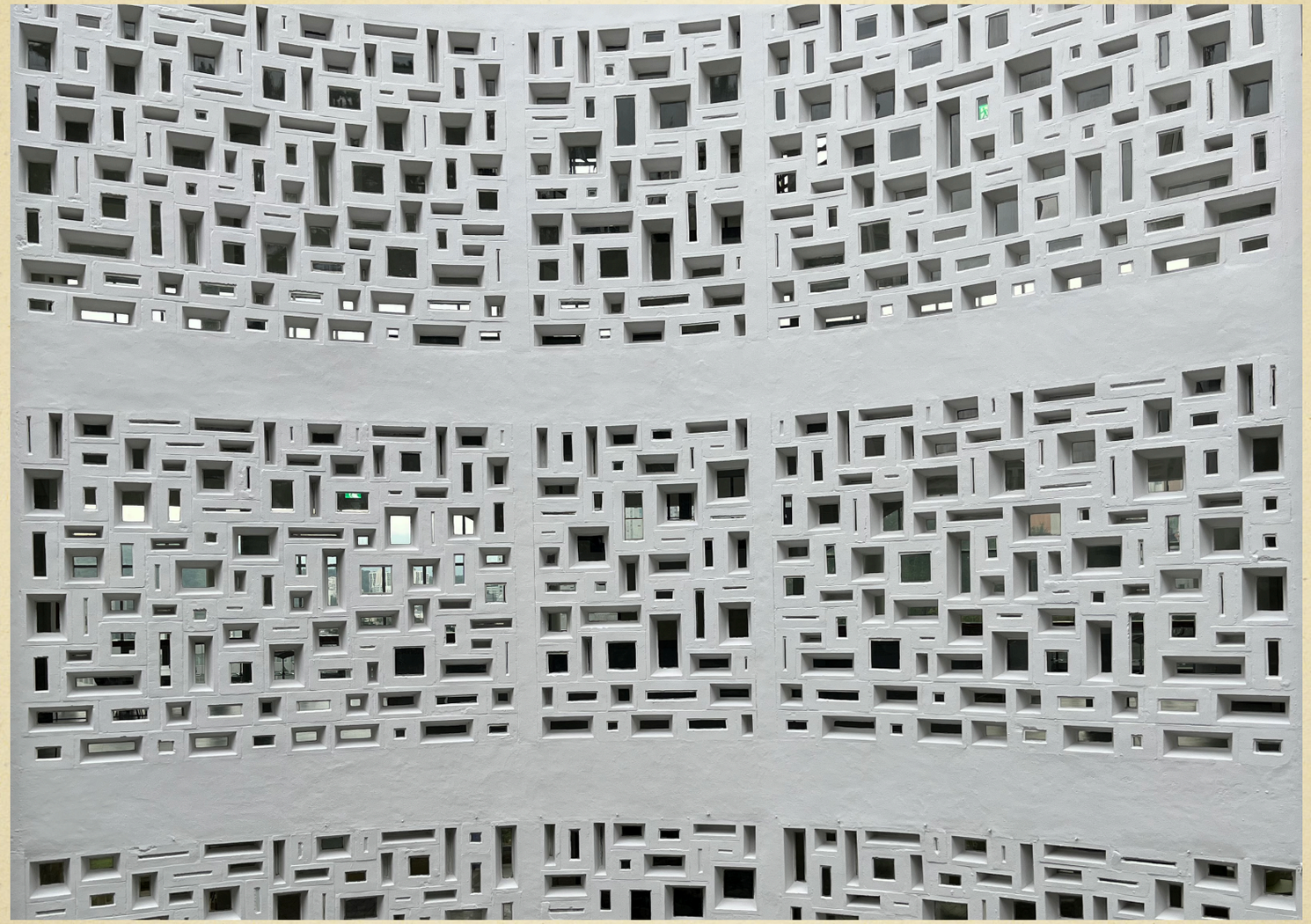
- $Sp(4)$ gauge theory with $n_f=3$ (dynamical) anti-sym. Dirac fermions
- $Sp(4)$, $Sp(6)$, $Sp(8)$ gauge theory with quenched fund., anti-sym. & sym. Dirac fermions



- Meson spectra of $Sp(4)$ with $N_f=2$ fund. & $n_f=3$ anti-sym. dynamical Dirac fermions
- Spectrum of chimera baryon (top partner)

Future work (long term)

- Model M8 is broken or (nearly) conformal?
- If it is (nearly) conformal, how large is the anomalous dimension?
- Phase structure of *Sp(2N)* gauge theories at finite temperature and/or density



Thank you for your attention!

Backup slides

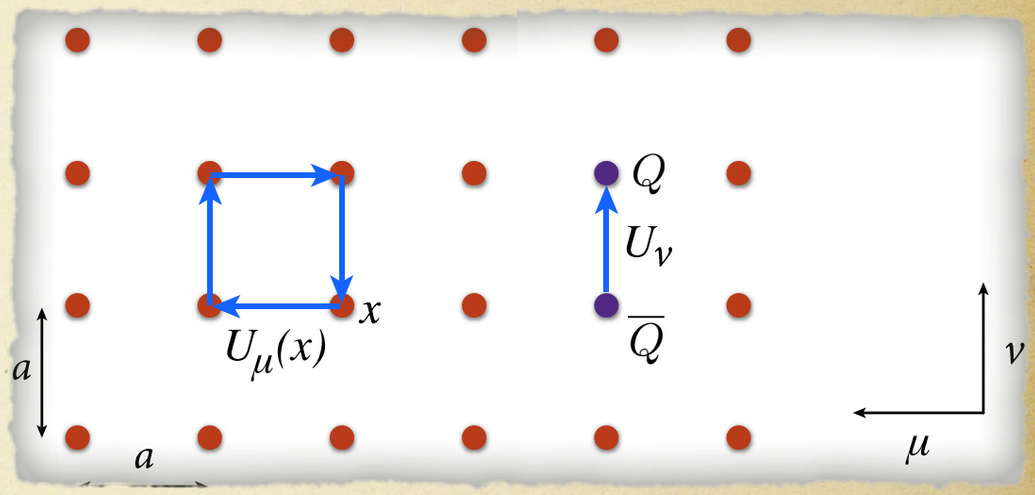
Lattice *Sp(2N)* gauge theories

- Lattice formulation with the standard Wilson gauge & fermion actions

$$S \equiv \beta \sum_x \sum_{\mu<\nu} \left(1 - \frac{1}{4}\mathrm{Re}\,\mathrm{Tr}\, U_\mu(x) U_\nu(x+\hat{\mu}) U_\mu^\dagger(x+\hat{\nu}) U_\nu^\dagger(x)\right)$$

$$+a^4 \sum_x \overline{Q}_j(x) D^F Q_j(x) + a^4 \sum_x \overline{\Psi}_k(x) D^{AS} \Psi_k(x)$$

fundamental (F) *antisymmetric (AS)*



with $\beta = 4N/g^2$

$$D^F Q_j(x) \equiv (4/a + m_0^f) Q_j(x) - \frac{1}{2a} \sum_\mu \left\{ (1-\gamma_\mu) U_\mu^F(x) Q_j(x+\hat{\mu}) + (1+\gamma_\mu) U_\mu^F(x-\hat{\mu}) Q_j(x-\hat{\mu}) \right\},$$

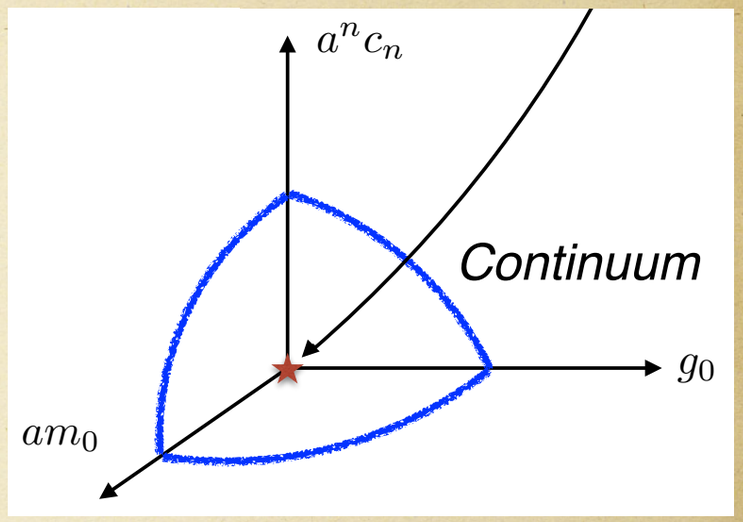
$$D^{AS} \Psi_k(x) \equiv (4/a + m_0^{as}) \Psi_k(x) - \frac{1}{2a} \sum_\mu \left\{ (1-\gamma_\mu) U_\mu^{AS}(x) \Psi_k(x+\hat{\mu}) + (1+\gamma_\mu) U_\mu^{AS}(x-\hat{\mu}) \Psi_k(x-\hat{\mu}) \right\}$$

where $U_\mu(x) = U_\mu^F(x) \in Sp(4)$ and

$$\left(U_\mu^{AS}\right)_{(ab)(cd)}(x) \equiv \mathrm{Tr}\left[(e_{AS}^{(ab)})^\dagger U_\mu(x) e_{AS}^{(cd)} U_\mu^{\mathrm{T}}(x) \right], \quad \text{with } a<b,\ c<d.$$

Determination of the lattice parameters

- Sp(4) theory with fermions: Weak and strong coupling regimes are separated by 1st order phase transition.

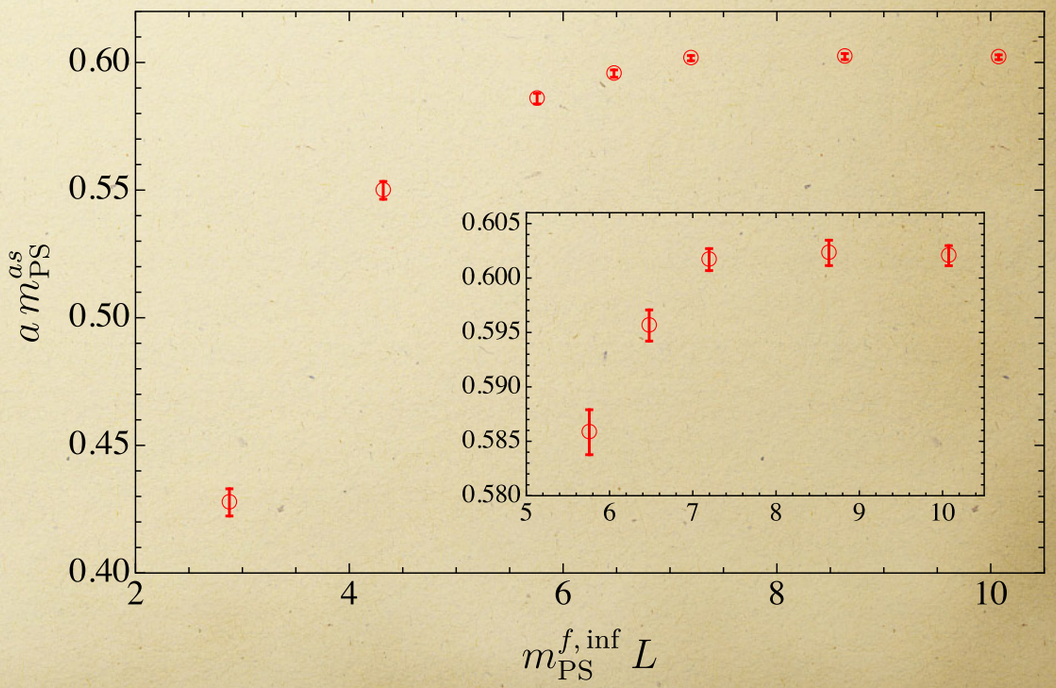


$$N_f{=}2\ F\ Sp(4): \quad \beta \gtrsim 6.8$$

$$n_f{=}3\ AS\ Sp(4): \quad \beta \gtrsim 6.5$$

$$N_f{=}2\ \&\ n_f{=}3\ AS\ Sp(4): \quad \beta \gtrsim 6.3$$

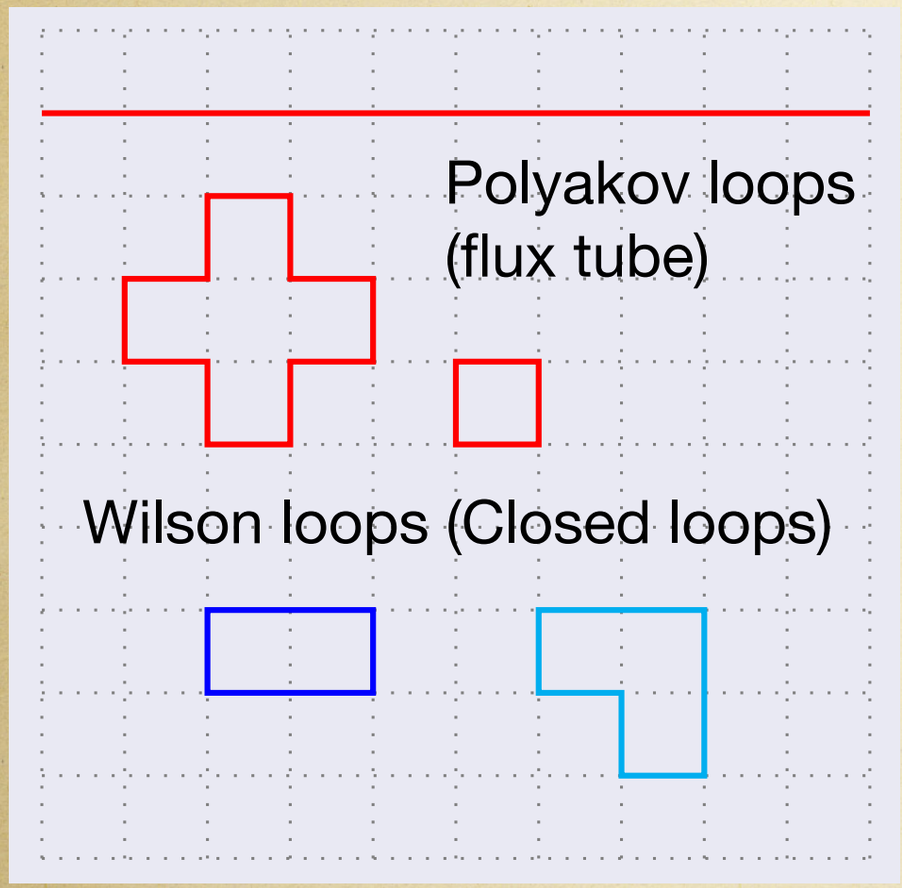
- Finite volume corrections are statistically negligible if $m_{\rm PS} L \gtrsim 7.5$.



E. Bennett et al (2022), arXiv:2202.05516

Observables I: Glueballs & string tension in pure *Sp(2N)*

- Gauge invariant gluonic operators



- String tension

$$\Phi(\vec{x}) \equiv \mathrm{Tr}\left(\mathrm{P}e^{ig\oint_{\mathcal{C}} A_0(t,\vec{x})\mathrm{d}t}\right)$$

$$\langle \Phi^\dagger(0)\Phi(z)\rangle = \sum_n c_n^l e^{-m_n^l z} \qquad \sigma = \lim_{\tau\to\infty}\frac{m_0^l}{\tau}$$

- Glueballs

$$\mathcal{O}_{\mathcal{C}}(t,\vec{x}) \equiv \mathrm{Tr}\left(\prod_{\mathcal{C}} \mathrm{U_I}\right)$$

$$\langle O^\dagger(0)O(t)\rangle = \sum_n |c_{J^P,n}|^2 e^{-m_{J^P,n}t}$$

Irreducible reps. of the octahedral group

$$R = A_1, A_2, E, T_1, T_2$$

Subdued reps. of continuum rotational group

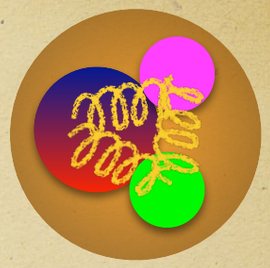
J	A_1	A_2	E	T_1	T_2
0	1	0	0	0	0
1	0	0	0	1	0
2	0	0	1	0	1
3	0	1	0	1	1
4	1	0	1	1	1

Observables II: spin-0 & 1 mesons in *Sp(4)*

- Global symmetry breaking: $SU(4)/Sp(4) \times SU(6)/SO(6)$
- Gauge invariant, flavor non-singlet, i.e. $i \neq j$ or $k \neq m$

Label M	Interpolating operator $\mathcal{O}_M$	Meson in QCD	J^P	$Sp(4)$	$SO(6)$
PS	$\overline{Q^i}\gamma_5 Q^j$	π	0^-	$5(+1)$	1
S	$\overline{Q^i} Q^j$	a_0	0^+	$5(+1)$	1
V	$\overline{Q^i}\gamma_\mu Q^j$	ρ	1^-	10	1
T	$\overline{Q^i}\gamma_0\gamma_\mu Q^j$	ρ	1^-	$10(+5+1)$	1
AV	$\overline{Q^i}\gamma_5\gamma_\mu Q^j$	a_1	1^+	$5(+1)$	1
AT	$\overline{Q^i}\gamma_5\gamma_0\gamma_\mu Q^j$	b_1	1^+	$10(+5+1)$	1
ps	$\overline{\Psi^k}\gamma_5\Psi^m$	π	0^-	1	$20'(+1)$
s	$\overline{\Psi^k}\Psi^m$	a_0	0^+	1	$20'(+1)$
v	$\overline{\Psi^k}\gamma_\mu\Psi^m$	ρ	1^-	1	15
t	$\overline{\Psi^k}\gamma_0\gamma_\mu\Psi^m$	ρ	1^-	1	$15(+20'+1)$
av	$\overline{\Psi^k}\gamma_5\gamma_\mu\Psi^m$	a_1	1^+	1	$20'(+1)$
at	$\overline{\Psi^k}\gamma_5\gamma_0\gamma_\mu\Psi^m$	b_1	1^+	1	$15(+20'+1)$

Observables III: Chimera baryon in *Sp(4)*



- Recall the global symmetry and its spontaneous breaking

$$SU(4)/Sp(4) \otimes SU(6)/SO(6)$$

where $SO(4)$ subgroup of $Sp(4)$ ~ $SU(2)_L$ gauge group in SM &

$SU(3)$ subgroup of $SO(6)$ ~ $SU(3)_c$ gauge group in SM

- Then, the top partner can be sourced by the operators (similar to Λ baryon in QCD)

$$\begin{aligned}
\mathcal{O}^{L,R}_{\mathrm{CB},1} &= (\overline{Q^{1\,a}}\gamma^5 Q^{2\,b} + \overline{Q^{2\,a}}\gamma^5 Q^{1\,b})\Omega_{bc} P_{L,R}\Psi^{k\,ca}, \\
\mathcal{O}^{L,R}_{\mathrm{CB},2} &= i(-\overline{Q^{1\,a}}\gamma^5 Q^{2\,b} + \overline{Q^{2\,a}}\gamma^5 Q^{1\,b})\Omega_{bc} P_{L,R}\Psi^{k\,ca}, \\
\mathcal{O}^{L,R}_{\mathrm{CB},3} &= (\overline{Q^{1\,a}}\gamma^5 Q^{1\,b} - \overline{Q^{2\,a}}\gamma^5 Q^{2\,b})\Omega_{bc} P_{L,R}\Psi^{k\,ca}, \\
\mathcal{O}^{L,R}_{\mathrm{CB},4} &= -i(\overline{Q^{1\,a}} Q^{2\,b}_C + \overline{Q^{2\,a}_C} Q^{1\,b})\Omega_{bc} P_{L,R}\Psi^{k\,ca}, \\
\mathcal{O}^{L,R}_{\mathrm{CB},5} &= i(-i\overline{Q^{1\,a}} Q^{2\,b}_C + i\overline{Q^{2\,a}_C} Q^{1\,b})\Omega_{bc} P_{L,R}\Psi^{k\,ca}.
\end{aligned}$$

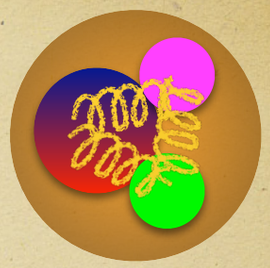
E. Bennett et al (2022), arXiv:2202.05516

where 4 of these operators transform 3 of $SU(3)_c$ and 4 of $SO(4)$.

- We also consider the parity projections in the nonrelativistic limit.

$$\mathcal{O}^{\pm}_{\mathrm{CB}}(x) = P_{\pm}\,\mathcal{O}_{\mathrm{CB}}(x) \quad \text{with} \quad P_{\pm} = \frac{1}{2}(\mathbb{1} \pm \gamma_0)$$

Observables III: Chimera baryon in *Sp(4)*



- More chimera baryons with different spin and/or irreps. of fundamental flavors

$$
\begin{aligned}
\mathcal{O}_{\mathrm{CB},6} &= -\frac{i}{2}\left(\overline{Q^{1\,a}}\gamma^\mu\gamma^5 Q_C^{1\,b} + \overline{Q^{2\,a}}\gamma^\mu\gamma^5 Q_C^{2\,b} - \overline{Q_C^{1\,a}}\gamma^\mu\gamma^5 Q^{1\,b} - \overline{Q_C^{2\,a}}\gamma^\mu\gamma^5 Q^{2\,b}\right)\Omega_{bc}\Psi^{k\,ca}\,,\\
\mathcal{O}_{\mathrm{CB},9} &= -\frac{i}{2}\left(\overline{Q^{1\,a}}\gamma^\mu\gamma^5 Q_C^{1\,b} - \overline{Q^{2\,a}}\gamma^\mu\gamma^5 Q_C^{2\,b} - \overline{Q_C^{1\,a}}\gamma^\mu\gamma^5 Q^{1\,b} + \overline{Q_C^{2\,a}}\gamma^\mu\gamma^5 Q^{2\,b}\right)\Omega_{bc}\Psi^{k\,ca}\,,\\
\mathcal{O}_{\mathrm{CB},10} &= \frac{\sqrt{2}}{2}\left(\overline{Q^{1\,a}}\gamma^\mu\gamma^5 Q_C^{1\,b} + \overline{Q_C^{1\,a}}\gamma^\mu\gamma^5 Q^{1\,b}\right)\Omega_{bc}\Psi^{k\,ca}\,,\\
\mathcal{O}_{\mathrm{CB},12} &= \frac{\sqrt{2}}{2}\left(\overline{Q^{2\,a}}\gamma^\mu\gamma^5 Q_C^{2\,b} + \overline{Q_C^{2\,a}}\gamma^\mu\gamma^5 Q^{2\,b}\right)\Omega_{bc}\Psi^{k\,ca}\,.
\end{aligned}
$$

- Spin projections in the non relativistic limit

$$
\begin{aligned}
P_{ij}^{3/2} &= \delta_{ij} - \frac{1}{3}\gamma_i\gamma_j,\\
P_{ij}^{1/2} &= \frac{1}{3}\gamma_i\gamma_j.
\end{aligned}
$$

$\Sigma^*\,(\frac{3}{2},\,10)$

$\Sigma\,(\frac{1}{2},\,10)$