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Phases and condensates in QCD at finite μ_I and μ_S

XVth Quark Confinement and the Hadron Spectrum
Stavanger, Norway August 1 - August 6 2022

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August 2 2022

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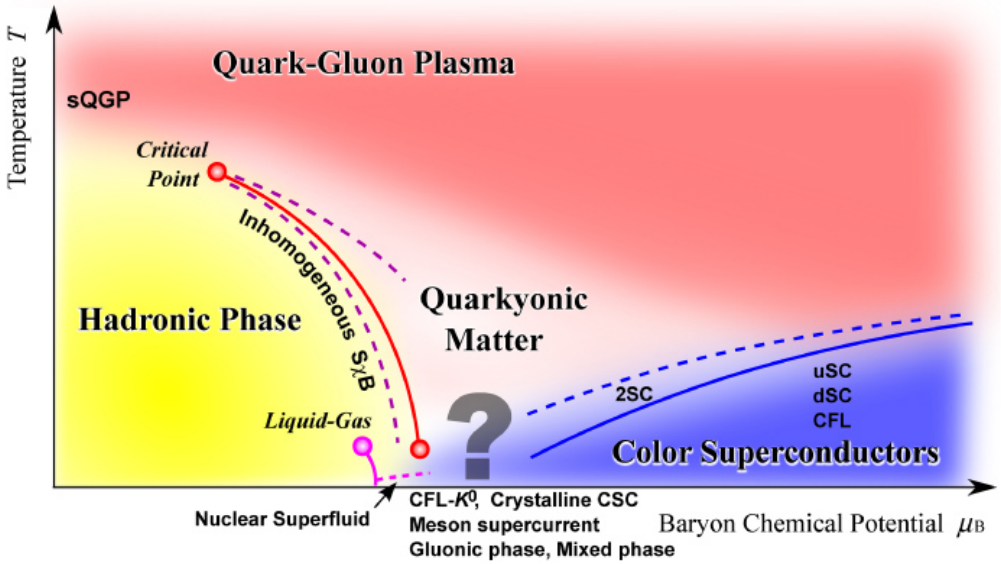
References: EPJC **79** 879 (2019), e-Print:2206.04291 [hep-ph]

Introduction

— **QCD phase diagram**

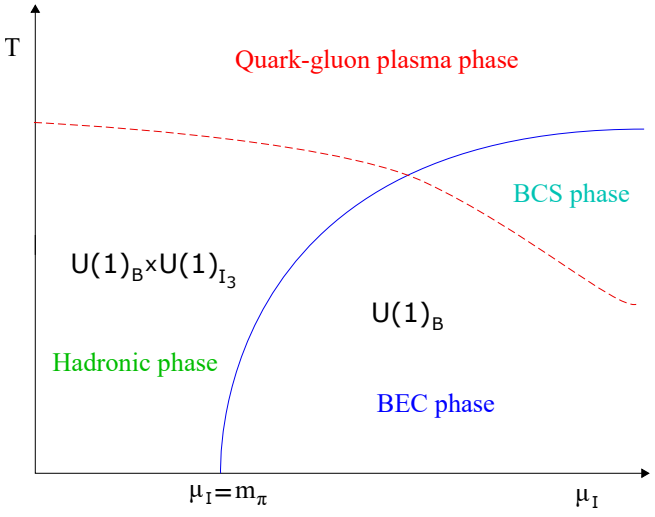


Fukushima and Hatsuda '11

— **More dimensions**

- Constant magnetic field B - no sign problem
- Finite isospin chemical potential μ_I with $\mu_B = \mu_S = 0$ - no sign problem
- Model building. Confronting lattice

Phase diagram



— **This talk**

- Pion condensation at $T = 0$ using two and three-flavor χPT
- Kaon condensation at $T = 0$ using three-flavor χPT

$$
\begin{aligned}
\mu_B &= \frac{3}{2}(\mu_u + \mu_d) \ , \\
\mu_I &= \mu_u - \mu_d \ , \\
\mu_S &= \frac{1}{2}(\mu_u + \mu_d - 2\mu_s) \ .
\end{aligned}
$$

χPT at finite isospin μ_I

— **Effective theory for QCD based on symmetries and relevant degrees of freedom** [2]

- Three-flavor QCD: pions, kaons, and η and $SU(3)_L \times SU(3)_R$
- Low-energy expansion

$$\mathcal{L} = \mathcal{L}_2 + \mathcal{L}_4 + \ldots \text{Expansion parameter } \frac{M}{4\pi f} \ .$$

— **Leading order Lagrangian and addition of quark chemical potentials**

$$\begin{aligned}
\mathcal{L}_2 &= \frac{1}{4} f^2 \,\mathrm{Tr}\left[\nabla^\mu \Sigma^\dagger \nabla_\mu \Sigma\right] + \frac{1}{4} f^2 \,\mathrm{Tr}\left[\chi^\dagger \Sigma + \Sigma^\dagger \chi\right] \ , \\
\Sigma &= e^{i\frac{\phi_a \tau_a}{f}} \ , \qquad \nabla_\mu \Sigma \equiv \partial_\mu \Sigma - i\left[v_\mu, \Sigma\right] \ , \\
\chi &= 2B_0 \mathrm{diag}(m_u, m_d, m_s) \qquad v_0 = \tfrac{1}{3}\mu_B \mathbb{1} + \tfrac{1}{2}\mu_I \tau_3 \ , \\
f &\sim f_\pi \ , \qquad m \sim m_\pi
\end{aligned}$$

- Results independent of μ_B

[2] Weinberg '79, Gasser and Leutwyler '85

— **Rotated ground state and pion condensation** [3]

$$\Sigma_\alpha = \mathbb{1}\cos\alpha + i\tau_2 \sin\alpha = e^{i\alpha\tau_2} \ .$$

— **Thermodynamic potential**

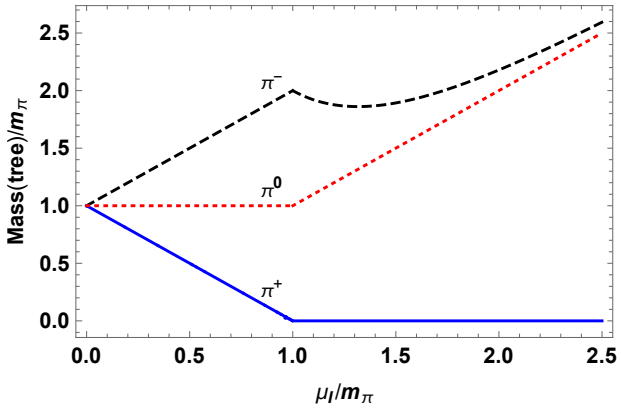
$$\Omega_0 = -f^2 m^2 \cos\alpha - \frac{1}{2} f^2 \mu_I^2 \sin^2\alpha \ .$$

— **Rotation angle α and phase diagram**

$$\begin{aligned} \cos\alpha &= \frac{m^2}{\mu_I^2} & \mu_I \geq m \\ \alpha &= 0 \ , & \mu_I < m \ . \end{aligned}$$

— **Silver Blaze propery and second order transition at $\mu_I = m$. Mean-field exponents**

— **Spectrum**



[3] Son and Stephanov '01

Dilute Bose gas

— **Small chemical potential** $\mu_I = m_\pi + \mu_{\rm NR}$

$$
\begin{aligned}
n_I &= 4 f_\pi^2 \mu_{\rm NR} \ , \\
p &= 2 f_\pi^2 \mu_{\rm NR}^2 \ , \\
\epsilon &= m_\pi n_I + \frac{1}{8 f_\pi^2} n_I^2 + \ldots
\end{aligned}
$$

— **Dilute Bose gas with** s**-wave scattering length** [4]

$$
\epsilon = \frac{2\pi a}{m} n_I^2 \left[1 + \frac{128}{15\sqrt{\pi}} \sqrt{n_I a^3} + (A \log n_I a^3 + C) n_I a^3 \right] \ , \text{Expansion parameter } \sqrt{n_I a^3} \ll 1
$$

— s**-wave scattering length** [5]

$$
a = \frac{m}{16\pi f_\pi^2} + \ldots
$$

[4] Bogoliubov '47, Lee and Yang '58, Wu, Hugenholtz and Pines, Sawada '59, Braaten and Nieto '97

[5] Gasser and Leutwyler '83

Phase diagram three flavors

— **Leading-order Lagrangian**

$$\mathcal{L}_2 = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{4}f^2\langle\nabla_\mu\Sigma\nabla^\mu\Sigma^\dagger\rangle + \frac{1}{4}f^2\langle\chi^\dagger\Sigma + \Sigma^\dagger\chi\rangle + C\langle Q\Sigma Q\Sigma^\dagger\rangle + \mathcal{L}_{\rm gf} + \mathcal{L}_{\rm ghost} \ ,$$

— **Ansatz for the ground state $K^\pm$ condensation**

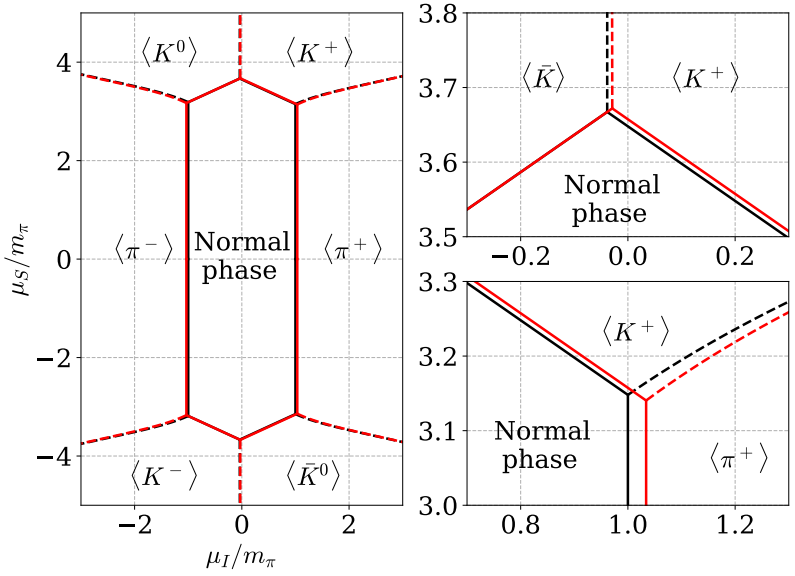
$$\begin{aligned}
\Sigma_\beta^{K^\pm} &= e^{i\beta\lambda_5} \ , \\
v_0 &= \tfrac{1}{3}(\mu_B - \mu_S)\mathbb{1} + \tfrac{1}{2}\mu_{K^\pm}\lambda_Q + \frac{1}{2}\mu_{K^0}\lambda_K \ , \\
\mu_{K^\pm} &= \frac{1}{2}\mu_I + \mu_S \ , \quad \mu_{K^0} = -\frac{1}{2}\mu_I + \mu_S \ , \\
U(1)_Q \times U(1)_K &\to U(1)_K \ ,
\end{aligned}$$

— **Leading-order thermodynamic potential in $K^\pm$-condensed phase**

$$\Omega_0 = -f^2 m_{K^0}^2 \cos\alpha - \frac{1}{2}f^2\left[\mu_{K^\pm}^2 - \Delta m_{\rm EM}^2\right]\sin^2\alpha \ .$$

— **With electromagnetic interactions, the condensed phases with a charged meson is a Higgs phase**

Phase diagram three flavors [6]



— **Thermodynamic potential independent of μ_i in the normal phase (Silver-Blaze property) etc**

— **Second-order phase transition from the vacuum for $\mu_I = m_{\pi^\pm}$, $\mu_{K^\pm} = \frac{1}{2}\mu_I + \mu_S = m_{K^\pm}$ etc**

— **First-order transitions between the condensed phases (Order parameters and densities jump discontinously)**

[6] Kogut and Toublan '01

NLO calculation

— **Thermodynamic potential**

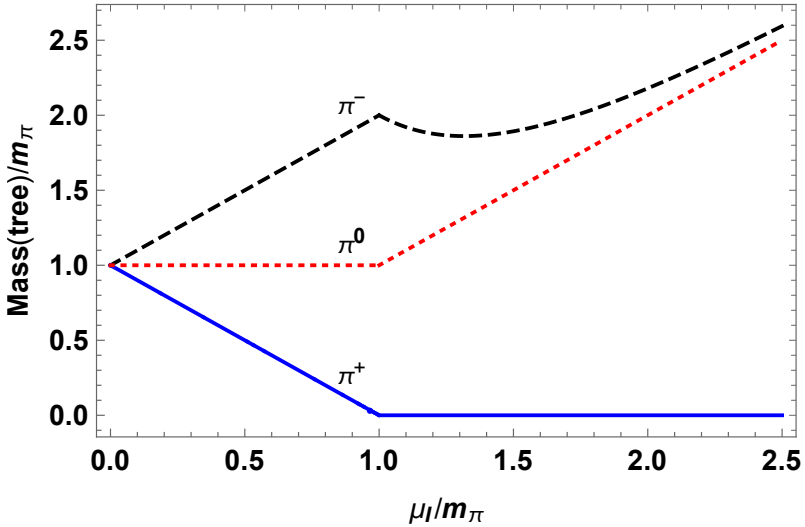
$$\Omega = \Omega_0 + \Omega_1 + \ldots$$

— Ω_1 **receives contributions from** $\mathcal{L}_2^{\rm quadratic}$ **and** $\mathcal{L}_4^{\rm static}$

— **Leading-order Lagrangian for two-flavor** χ**PT**

$$\begin{aligned}
\mathcal{L}_2^{\rm quadratic} &= \tfrac{1}{2}(\partial_\mu \phi_a)(\partial^\mu \phi_a) + \mu_I \cos\alpha(\phi_1 \partial_0 \phi_2 - \phi_2 \partial_0 \phi_1) \\
&\quad - \tfrac{1}{2}\Big[(m^2 \cos\alpha - \mu_I^2 \cos 2\alpha)\phi_1^2 + (m^2 \cos\alpha - \mu_I^2 \cos^2\alpha)\phi_2^2 \\
&\quad + (m^2 \cos\alpha + \mu_I^2 \sin^2\alpha)\phi_3^2\Big] \ , \\
\mathcal{L}_4^{\rm static} &= (l_1 + l_2)\mu_I^4 \sin^4\alpha + l_4 m^2 \mu_I^2 \cos\alpha \sin^2\alpha + (l_3 + l_4) m^4 \cos^2\alpha \ .
\end{aligned}$$

— **Spectrum**



— **Renormalized effective potential**

$$
\begin{aligned}
V_{\text{eff}} \quad = \quad & -f^2 m^2 \cos\alpha - \frac{1}{2} f^2 \mu_I^2 \sin^2\alpha \\
& - \frac{1}{4(4\pi)^2} \left[\frac{3}{2} - \bar{l}_3 + 4\bar{l}_4 + \log\left(\frac{m^2}{\tilde{m}_2^2}\right) + 2\log\left(\frac{m^2}{m_3^2}\right) \right] m^4 \cos^2\alpha \\
& - \frac{1}{(4\pi)^2} \left[\frac{1}{2} + \bar{l}_4 + \log\left(\frac{m^2}{m_3^2}\right) \right] m^2 \mu_I^2 \cos\alpha \sin^2\alpha \\
& - \frac{1}{4(4\pi)^2} \left[1 + \frac{2}{3}\bar{l}_1 + \frac{4}{3}\bar{l}_2 + 2\log\left(\frac{m^2}{m_3^2}\right) \right] \mu_I^4 \sin^4\alpha + V^{\text{fin}}_{1,\pi+} + V^{\text{fin}}_{1,\pi-} \ .
\end{aligned}
$$

— **Parameter fixing in two-flavor χPT**

$$
\begin{aligned}
m_\pi^2 \quad &= \quad m^2 \left[1 - \frac{m^2}{2(4\pi)^2 f^2} \bar{l}_3 \right] = 131 \pm 3\mathrm{MeV} \ , \\
f_\pi^2 \quad &= \quad f^2 \left[1 + \frac{2m^2}{(4\pi)^2 f^2} \bar{l}_4 \right] = \frac{128 \pm 3}{\sqrt{2}} \mathrm{MeV} \ .
\end{aligned}
$$

— **$\bar{l}_i$ from experiment**

Quark and pion condensates

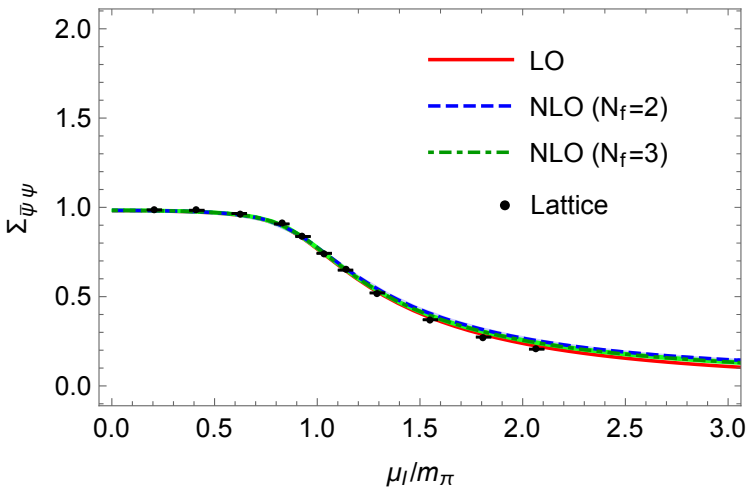
- **Calculate thermodynamic potential with sources m and j**
- **Quark and pion condensates**

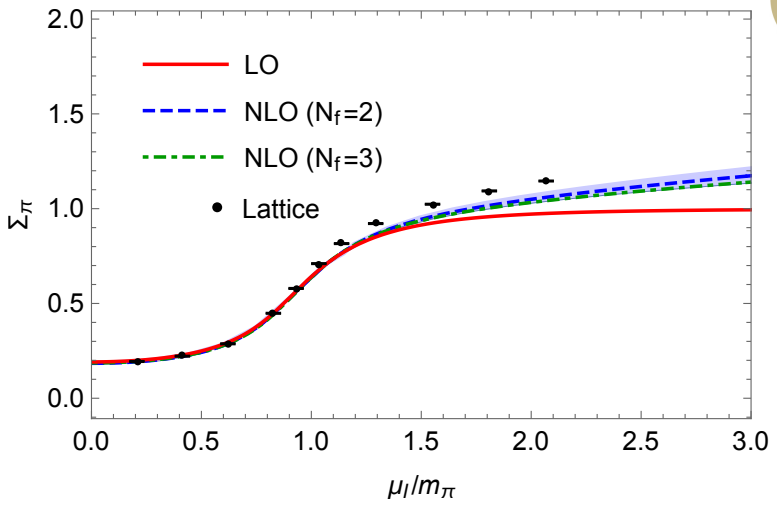
$$
\begin{aligned}
\langle \bar{\psi}\psi \rangle_{\mu_I} &= \frac{1}{2}\frac{\partial \Omega}{\partial m} = -f^2 B_0 \cos\alpha + \dots \,, \\
\langle \pi^+ \rangle_{\mu_I} &= \frac{1}{2}\frac{\partial \Omega}{\partial j} = -f^2 B_0 \sin\alpha + \dots \,. \\
\langle \bar{\psi}\psi \rangle^2_{\mathrm{tree},\mu_\mathrm{I}} + \langle \pi^+ \rangle^2_{\mathrm{tree},\mu_\mathrm{I}} &= \langle \bar{\psi}\psi \rangle^2_{\mathrm{tree},0} = -f^2 B_0 \,.
\end{aligned}
$$

- **Rotation of quark condensate (only at LO)**

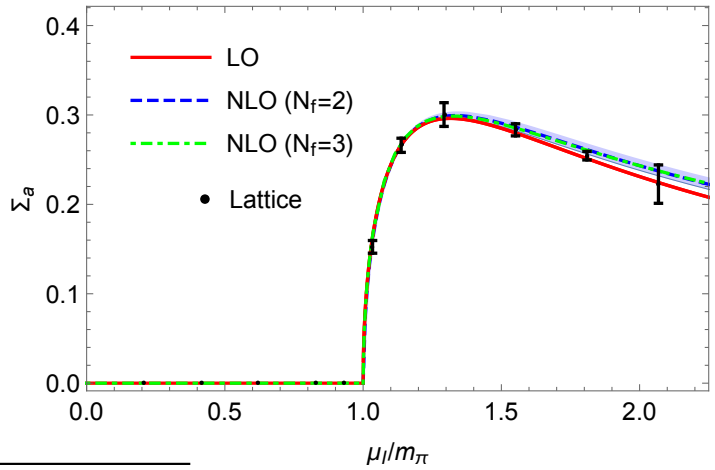
Condensates

— **Quark condensate and pion condensates** [7]





— **Axial vector condensate**



[7] Lattice data by Brandt, Endrodi, Schmalzbauer '18

Conclusions and Outlook

— Conclusions

- First calculation of thermodynamic functions at next-to-leading order in the pion-condensed phase in two -and three-flavor χPT
- Good agreement with lattice data at $T = 0$, also for pressure and EoS. First precision test of χPT at NLO with nonzero μ_I
- Good agreement with lattice data at $T \neq 0$ only for very low temperatures (phase diagram not shown)

— Outlook

- Effective theory for the Goldstone boson valid at scales $p \ll \mu_I$
- Masses of photons in Higgs phase?
- Construction of NR effective field theory near the transition (non-universal effects...)