High-accuracy calculation of charge radii of light nuclei

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in collaboration with

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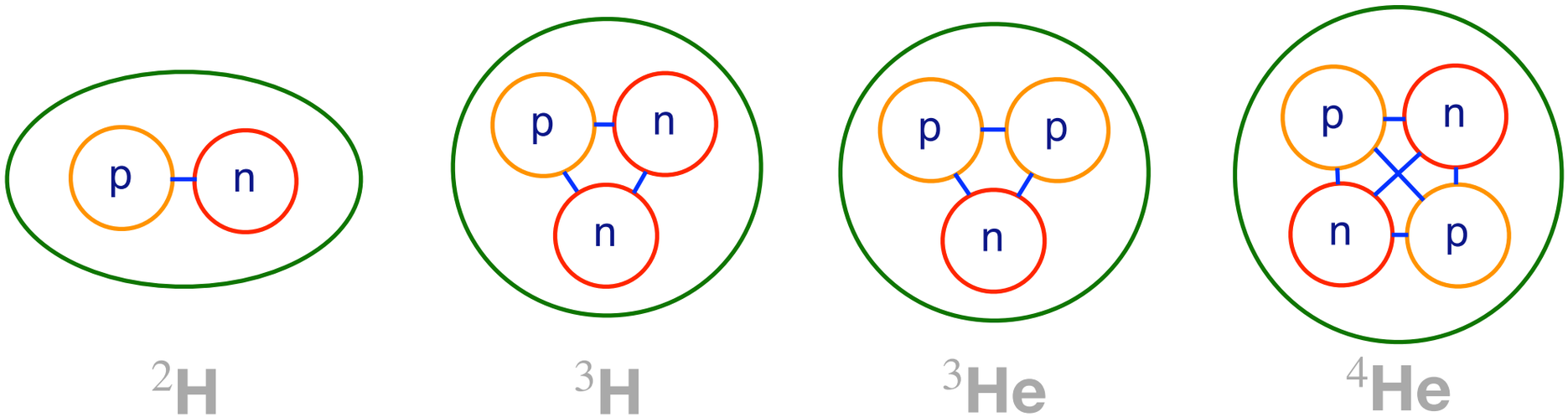
PRL 124 082501 (2020)
Phys.Rev.C 103 024313 (2021)

High-accuracy chiral EFT calculation of charge radii



Motivation:

- Precision **tests of nuclear chiral effective field theory (EFT)**

- A new way to **extract the neutron and the proton charge radii** from few-nucleon data

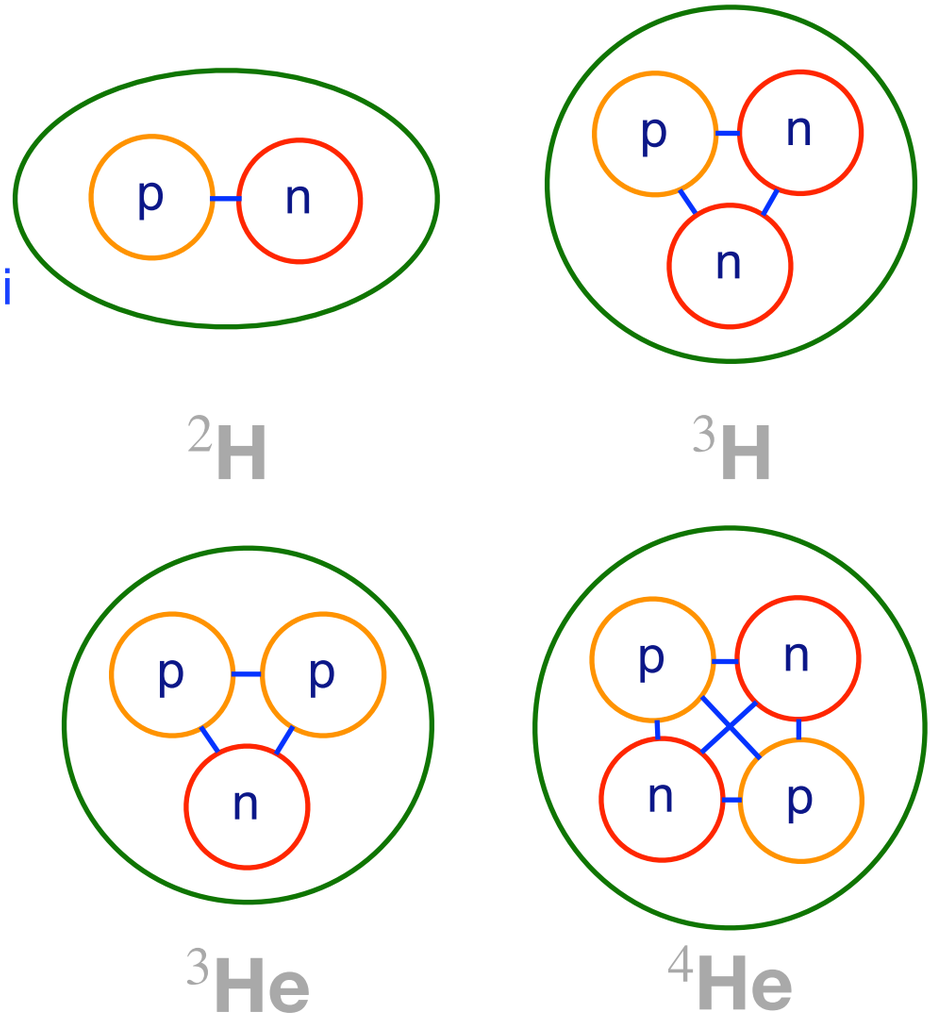
- Help to resolve long-standing issue with **underpredicted radii of medium-mass and heavy nuclei**

- Search for **Beyond-Standard-Model** physics

High-accuracy chiral EFT calculation of charge radii

Charge distribution of light nuclei depends on:

- intrinsic charge distributions of proton and neutron
- distribution of matter (proton and neutrons) inside the nuclei
- many-body electromagnetic currents



Charge radius characterises the charge distribution and consists of structure, proton and neutron radii

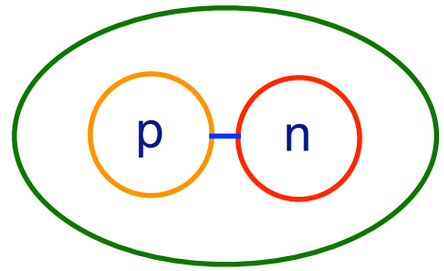
$$r_C^2 = r_{str}^2 + \left(r_p^2 + \frac{3}{4m_p^2} \right) + \frac{A-Z}{Z} r_n^2$$

Goals of this study:

- consistent χEFT calculation of isoscalar structure radii of A = 2, 3, 4 nuclei
- aim at N^{4}LO level of accuracy even in the incomplete calculation
- careful estimation of uncertainties (truncation, statistical, numerical and other)

Neutron charge radius from high-accuracy χEFT calculation of deuteron structure radius

^{2}H



$$r_d^2 = r_{str}^2(^2\mathrm{H}) + \left(r_p^2 + \frac{3}{4m_p^2} \right) + r_n^2$$

Deuteron-proton radius difference

$$(r_d^2 - r_p^2) = 3.82070(31) fm^2$$

Atomic spectroscopy

Hydrogen-deuterium 1S-2S isotope shift

+ QED corrections

Pachucki et al., PRA 97, 062511 (2018)
Jentschura et al. PRA 83 (2011)

Accurate **χEFT calculation**

of the **deuteron structure** radius

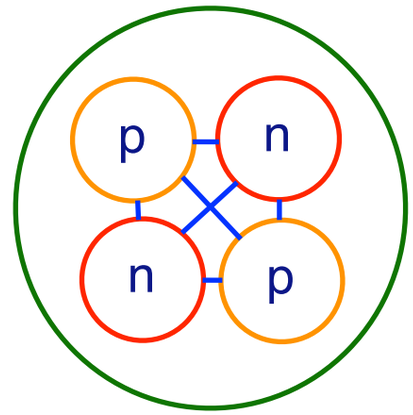
New method of determination

of the neutron charge radius

$$r_n^2 = (r_d^2 - r_p^2) - \frac{3}{4m_p^2} - r_{str}^2(^2\mathrm{H})$$

Proton charge radius from high-accuracy χEFT calculation of ^{4}He structure radius

^{4}He



$$r_C(^4\mathbf{He}) = r_{str}^2(^4\mathbf{He}) + \left(r_p^2 + \frac{3}{4m_p^2} \right) + r_n^2$$

^{4}He charge radius data

$r_C(^4\mathbf{He}) = (1.67824 \pm 0.00083)\, fm$

Muonic spectroscopy

Krauth et al., Nature 589 (2021)

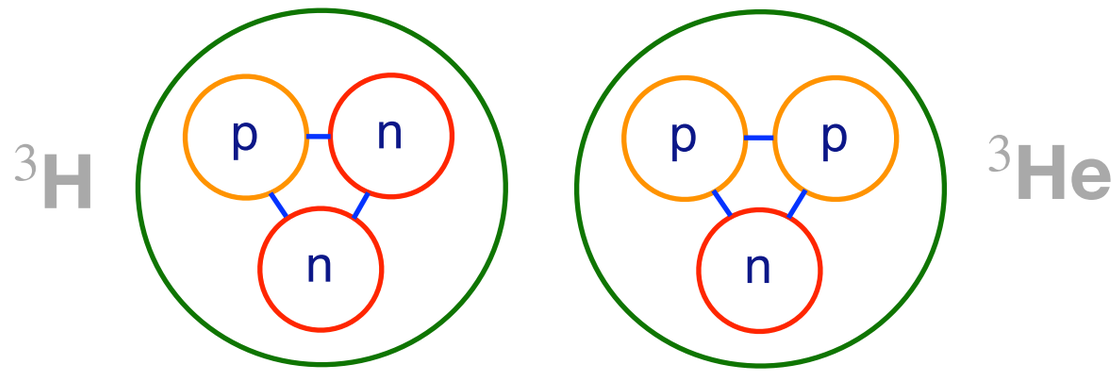
Accurate **χEFT calculation** of the **^{4}He structure** radius

New method of determination of the isoscalar nucleon charge radius

$$(r_n^2 + r_p^2) = r_C(^4\mathbf{He}) - r_{str}^2(^4\mathbf{He}) - \frac{3}{4m_p^2}$$

Extraction of the proton charge radius using r_n determined from the deuteron

Prediction for isoscalar 3N charge radius



$$\frac{r_C^2(^3\mathbf{H}) + 2r_C^2(^3\mathbf{He})}{3} = \frac{r_{str}^2(^3\mathbf{H}) + 2r_{str}^2(^3\mathbf{He})}{3} + \left(r_p^2 + \frac{3}{4m_p^2} \right) + r_n^2$$

Accurate **prediction**

for isoscalar 3N charge radius

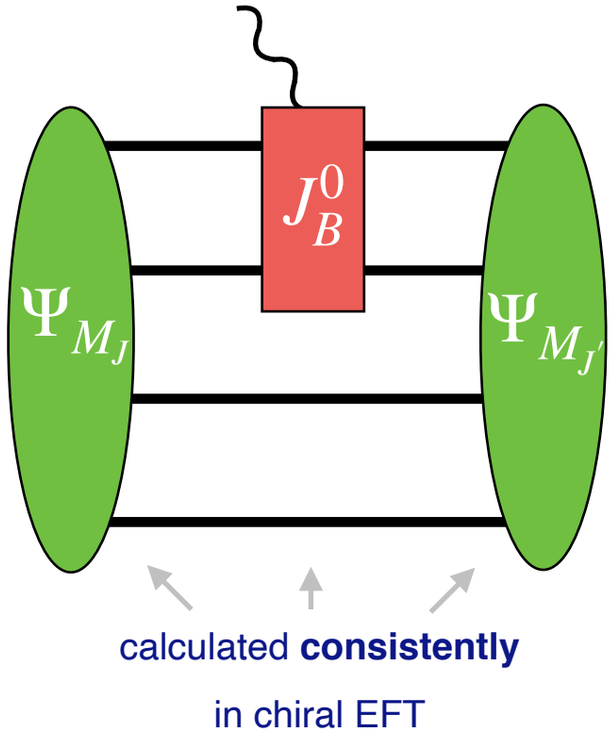
Precision test of chiral EFT

Accurate **χEFT calculation**

of the isoscalar **3N structure** radius

Previously extracted

Chiral EFT calculation of the nuclear charge radius



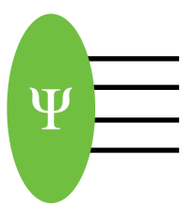
Charge radius r_C is related to the charge form factor $F_C(Q)$

$$r_C^2 = (-6)\frac{\partial}{\partial Q^2} F_C(Q^2)\Big|_{Q=0}$$

Charge form factor F_C can be computed (in the Breit frame) as

$$F_C(Q^2) = \frac{1}{2J+1}\sum_{M_J} < P', M_J | J_B^0 | P, M_J >$$

The matrix element is a convolution of nuclear wave function and charge density operator



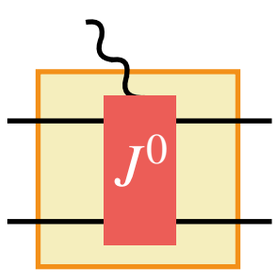
Nuclear wave function - based on high-precision chiral EFT interactions

- New high-precision chiral NN forces (N^4LO^+) Reinert et al. PRL 126, 092501 (2021)

  Nearly perfect description of pp and pn scattering data up to pion production threshold

- Chiral 3N forces (general N^2LO; selected terms at N^4LO) Epelbaum:2019kcf

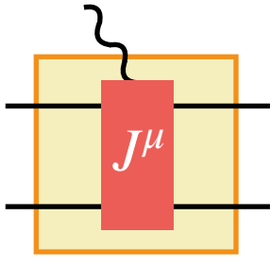
  Charge radii of 3N and ^{4}He are not sensitive to N^3LO 3N forces as soon as the binding energy is reproduced

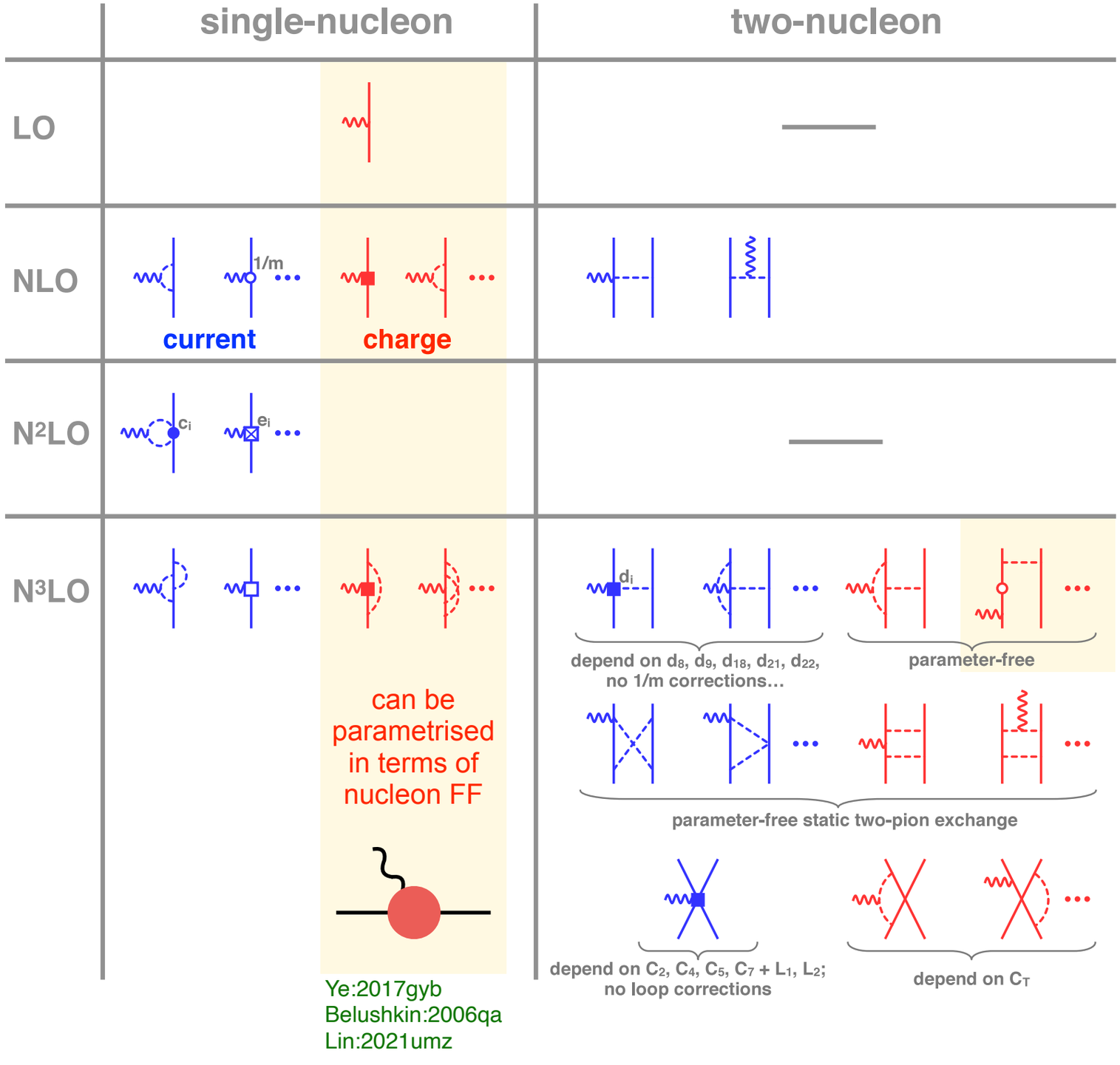


Charge density operator - consistent with chiral nuclear forces

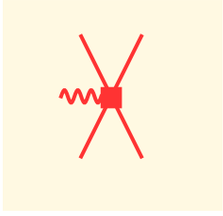
Nuclear electromagnetic currents

Kolling:2009iq, Kolling:2012cs, Krebs:2019aka
Review: H. Krebs, EPJA 56 (2020) 240





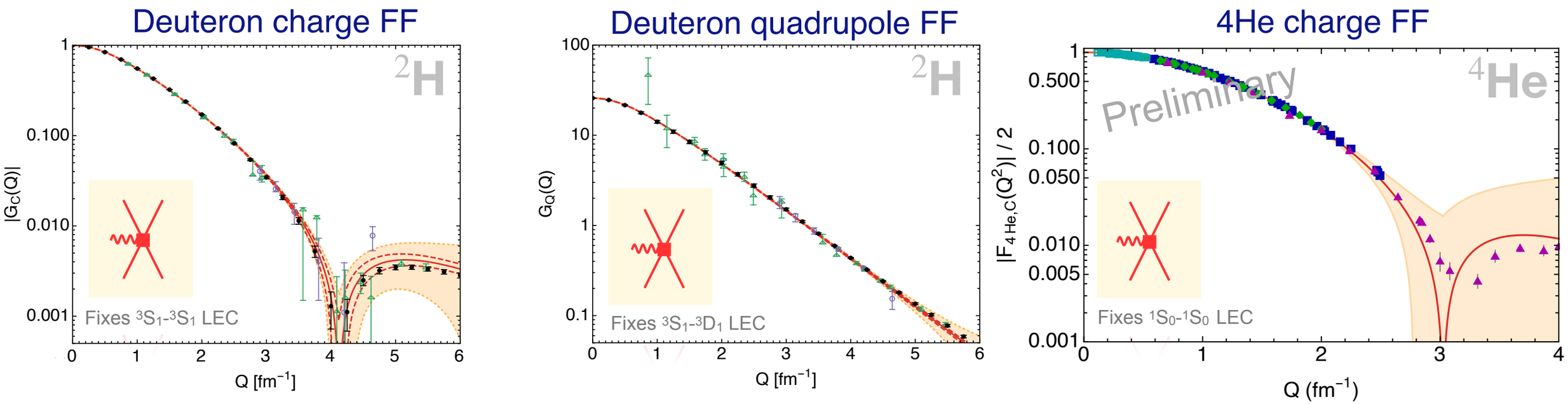
Ye:2017gyb
Belushkin:2006qa
Lin:2021umz



depend on **3 LECs**
$^3S_1$-$^3S_1$ - can be fitted to deuteron FF data
$^3S_1$-$^3D_1$ - this one too
$^1S_0$-$^1S_0$ - can be fitted to ^{4}He FF data

Chen, Rupak, Savage '99;
Phillips '07
AF et al. '20

Low-energy constants from a fit to charge and quadrupole form factors



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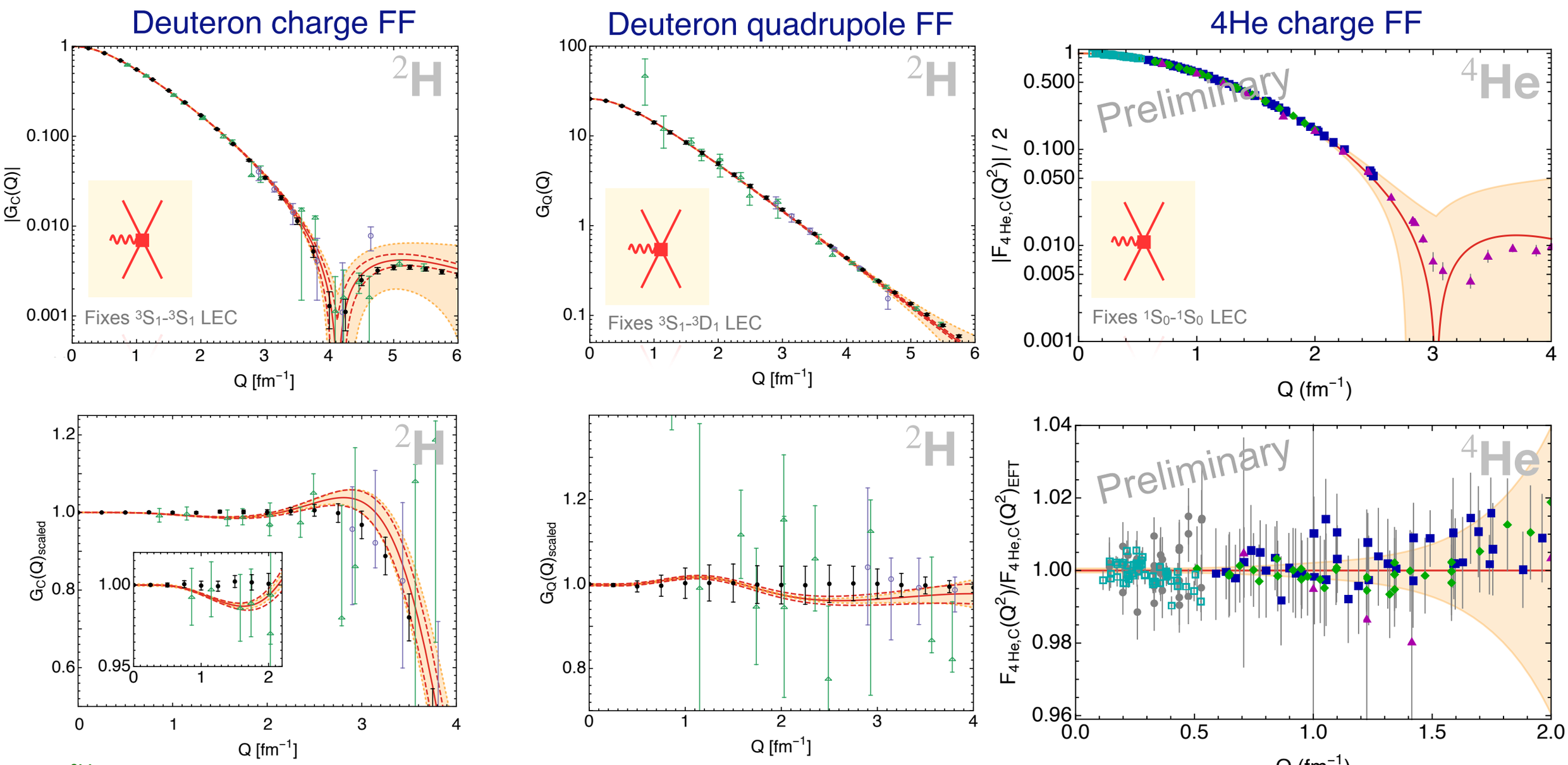

^{2}H:
AF, Möller, Baru, Epelbaum, Krebs, Reinert,
PRL 124 (2020) 082501; PRC 103 (2021) 024313

^{4}He: Preliminary

best fit + N^{4}LO truncation uncertainty

3 LECs in J^0 are fixed from the form factor data of deuteron and ^{4}He

Parameter-free prediction of structure radii

After all three LECs in charge density operators are fixed we get predictions for the structure radii

$r_{str}(^2\mathbf{H}) = 1.9729 \pm 0.0006_{\text{trunc}}\,^{+0.0012}_{-0.0008\,\text{stat}}\,fm$

AF, Möller, Baru, Epelbaum, Krebs, Reinert,
PRL 124 (2020) 082501; PRC 103 (2021) 024313

$r_{str}(^4\mathbf{He}) = 1.4784 \pm 0.0030_{\text{trunc}} \pm 0.0013_{\text{stat}} \pm 0.0007_{\text{num}}\ fm$ (Preliminary)

$r_{str}(\mathbf{Isoscalar\ 3N}) = 1.7309 \pm 0.0020_{\text{trunc}} \pm 0.0006_{\text{stat}} \pm 0.0002_{\text{iso-v}} \pm 0.0003_{\text{num}}$ (Preliminary)

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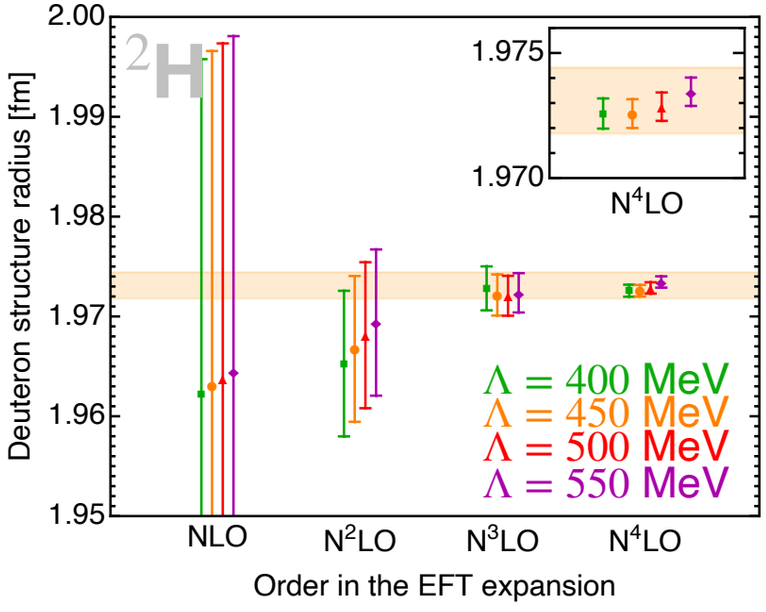
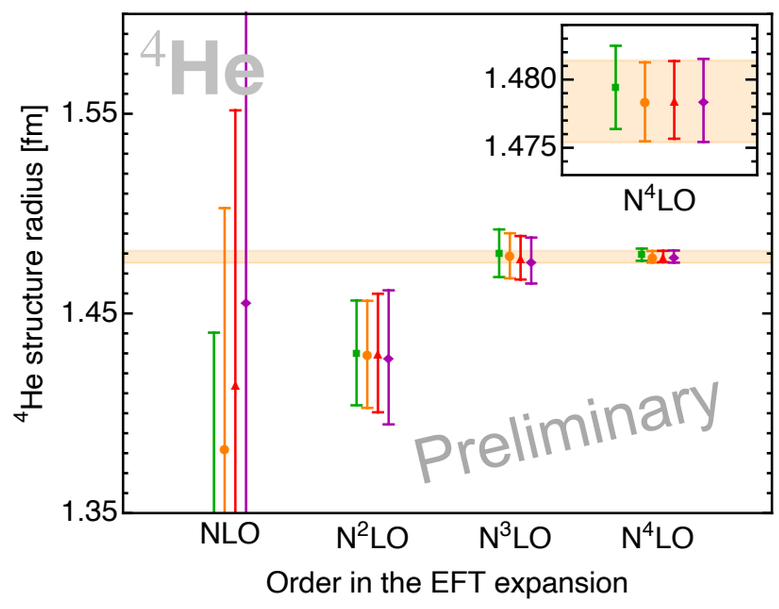
Using Bayesian model to estimate truncation uncertainty at each order Epelbaum et al. EPJA 56, 92 (2020)

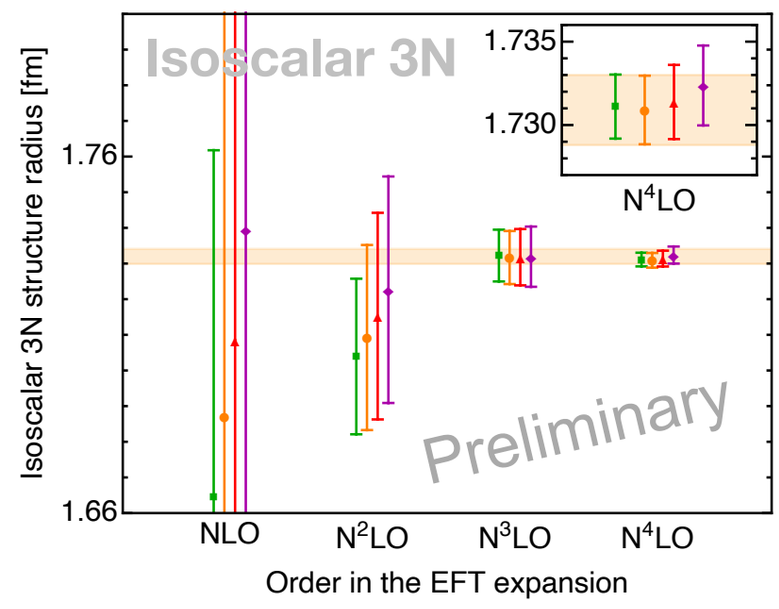
Deuteron structure radius



4He structure radius



Isoscalar 3N structure radius



error bands = χEFT truncation uncertainty

orange band = our prediction ± total uncertainty

Chiral EFT expansion converges well

Regulator dependence is smaller than the truncation uncertainty

Extraction of the neutron charge radius

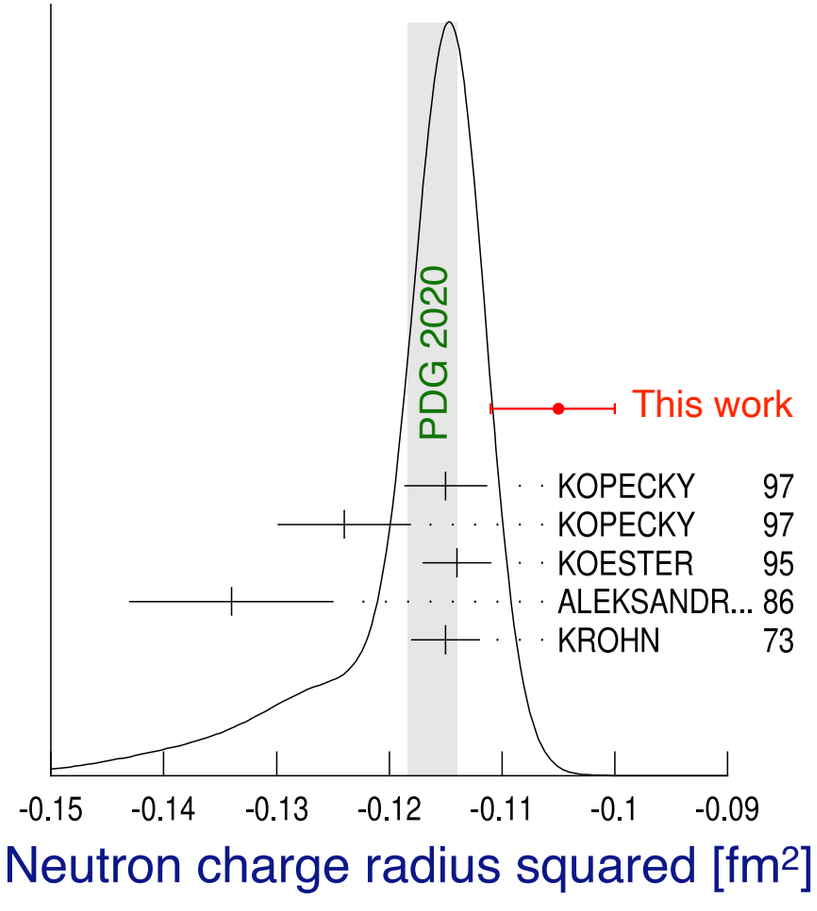
Prediction for the deuteron structure radius: $r_{str} = 1.9729^{+0.0015}_{-0.0012} fm$

Extraction of the neutron radius from $(r_d^2 - r_p^2) = 3.82070(31) fm^2$ (atomic spectroscopy + QED corrections)

$$r_n^2 = -0.105^{+0.005}_{-0.006} fm^2$$

$$r_n^2 = (r_d^2 - r_p^2) - \frac{3}{4m_p^2} - r_{str}^2$$

~2σ deviation from the PDG (2020) weighted average $r_n^2 = -0.1161(22) fm^2$



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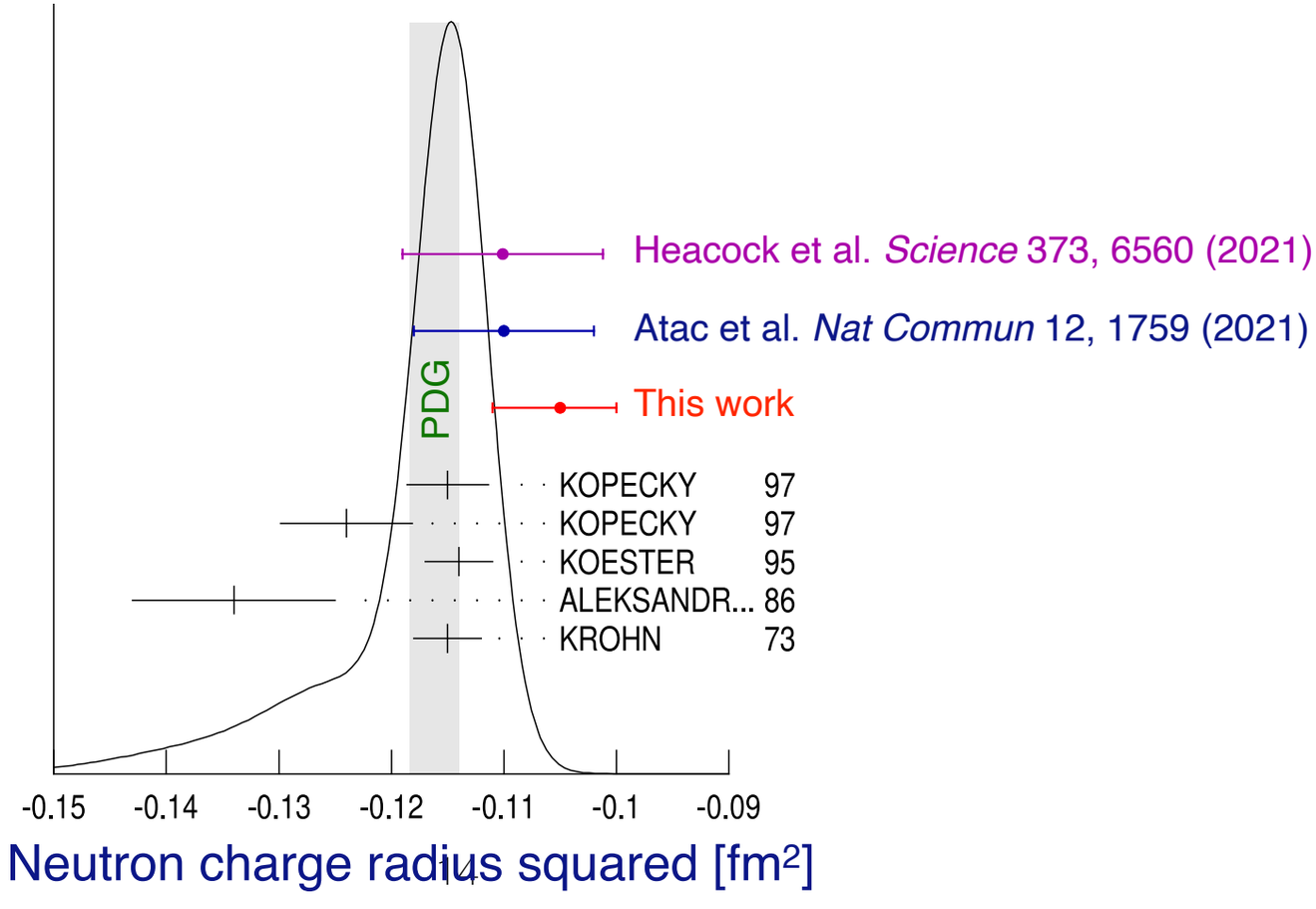


Neutron charge radius in PDG 2022

Citation: R.L. Workman *et al.* (Particle Data Group), to be published (2022)

n MEAN-SQUARE CHARGE RADIUS

VALUE (fm^2)	*DOCUMENT ID*		*COMMENT*
−0.1155±0.0017 OUR AVERAGE			
−0.115 ±0.002 ±0.003	KOPECKY	97	*ne* scattering (Pb)
−0.124 ±0.003 ±0.005	KOPECKY	97	*ne* scattering (Bi)
−0.114 ±0.003	KOESTER	95	*ne* scattering (Pb, Bi)
−0.115 ±0.003	[1] KROHN	73	*ne* scattering (Ne, Ar, Kr, Xe)
• • • We do not use the following data for averages, fits, limits, etc. • • •			
−0.1101±0.0089	[2] HEACOCK	21	*n* interferometry
$-0.106\ ^{+0.007}_{-0.005}$	[3] FILIN	20	chiral EFT analysis
$-0.117\ ^{+0.007}_{-0.011}$	BELUSHKIN	07	Dispersion analysis
−0.113 ±0.003 ±0.004	KOPECKY	95	*ne* scattering (Pb)
−0.134 ±0.009	ALEKSANDR...	86	*ne* scattering (Bi)
−0.114 ±0.003	KOESTER	86	*ne* scattering (Pb, Bi)
−0.118 ±0.002	KOESTER	76	*ne* scattering (Pb)
−0.120 ±0.002	KOESTER	76	*ne* scattering (Bi)
−0.116 ±0.003	KROHN	66	*ne* scattering (Ne, Ar, Kr, Xe)

[1] KROHN 73 measured -0.112 ± 0.003 fm^2. This value is as corrected by KOESTER 76.

[2] HEACOCK 21 extract the value from Pendelloesung interferometry to measure the neutron structure factors of silicon. This value is strongly anti-correlated with the mean-square thermal atomic displacement.

[3] FILIN 20 extract the value based on their chiral-EFT calculation of the deuteron structure radius and use as input the atomic data for the difference of the deuteron and proton charge radii.

^{4}He charge radius

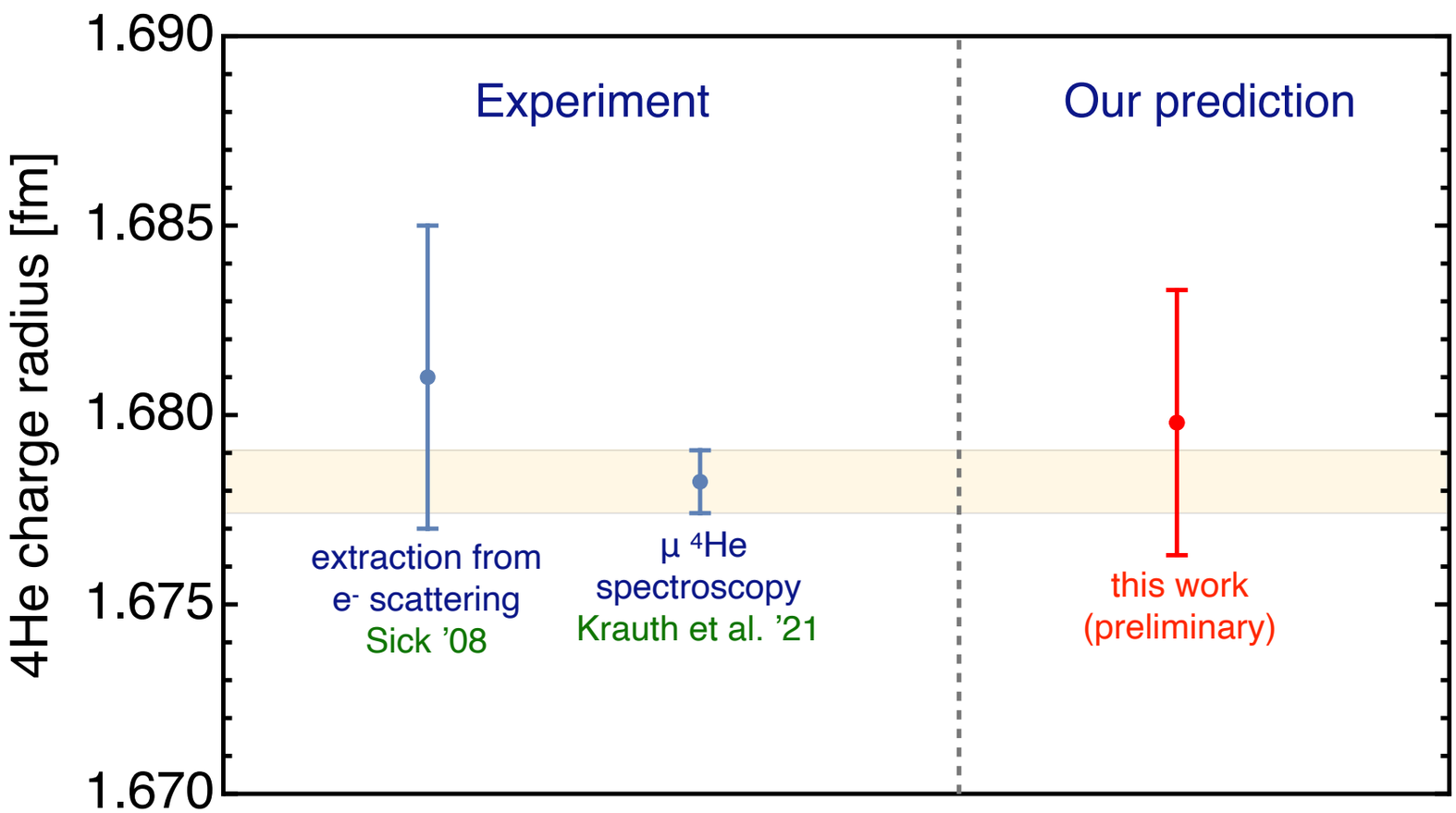
$$r_{str}(^4\mathbf{He}) = 1.4784 \pm 0.0030_{\text{trunc}} \pm 0.0013_{\text{stat}} \pm 0.0007_{\text{num}}\ fm$$

Our prediction for ^{4}He **charge** radius

$$r_C(^4\mathbf{He}) = (1.6798 \pm 0.0035)\, fm$$

preliminary, using CODATA 2018 r_p and own determination of r_n

$$r_C(^4\mathbf{He}) = r_{str}^2(^4\mathbf{He}) + \left(r_p^2 + \frac{3}{4m_p^2} \right) + r_n^2$$



Our prediction for ^{4}He charge radius is fully consistent with the muonic-atom spectroscopy

Indications of BSM physics?

All data used to constrain chiral EFT LECs are from strong interaction / electron-based experiments:

- π N Roy-Steiner analysis Hoferichter:2015tha, Hoferichter:2015hva
- NN pn and pp scattering data, deuteron BE Reinert:2020mcu
- Deuteron charge and quadrupole FF data JLABt20:2000qyq, Nikolenko:2003zq
- Deuteron-proton radii difference from atomic spectroscopy Pachucki:2018yxe, Jentschura et al. PRA 83 (2011)
- Proton charge radius CODATA2018
- ^{4}He form factor data Erich:1971rhg, Mccarthy:1977vd, VonGunten:1982yna, Ottermann:1985km, Frosch:1967pz, Arnold:1978qs, Camsonne:2013df
- Binding energies of ^{3}He and ^{4}He
- Nd DCS minimum @ 70 MeV RIKEN data

No muonic data is used in our chiral EFT predictions

Our prediction for ^{4}He charge radius is consistent with the muonic experiment

No indication of BSM physics at this accuracy level

Isoscalar nucleon charge radius from data on ^{4}He

Our prediction for ^{4}He **structure** radius:

$$r_{str}(^4\mathbf{He}) = 1.4784 \pm 0.0030_{\text{trunc}} \pm 0.0013_{\text{stat}} \pm 0.0007_{\text{num}}\ fm$$ preliminary

^{4}He charge radius experimental data

$$r_C(^4\mathbf{He}) = (1.67824 \pm 0.00083)\, fm$$

Krauth et al., Nature 589 (2021) 7843, 527-531

$$r_C(^4\mathbf{He}) = r_{str}^2(^4\mathbf{He}) + \left(r_p^2 + \frac{3}{4m_p^2} \right) + r_n^2$$

New method of determination

of the isoscalar nucleon charge radius

$$(r_n^2 + r_p^2) = (0.597 \pm 0.009)\, fm$$

preliminary

Proton charge radius from isoscalar nucleon radius

Our determination of the

isoscalar nucleon charge radius from ^{4}He

$(r_n^2 + r_p^2) = (0.597 \pm 0.009)\,fm$ preliminary

Our determination of the

neutron charge radius from ^{2}H

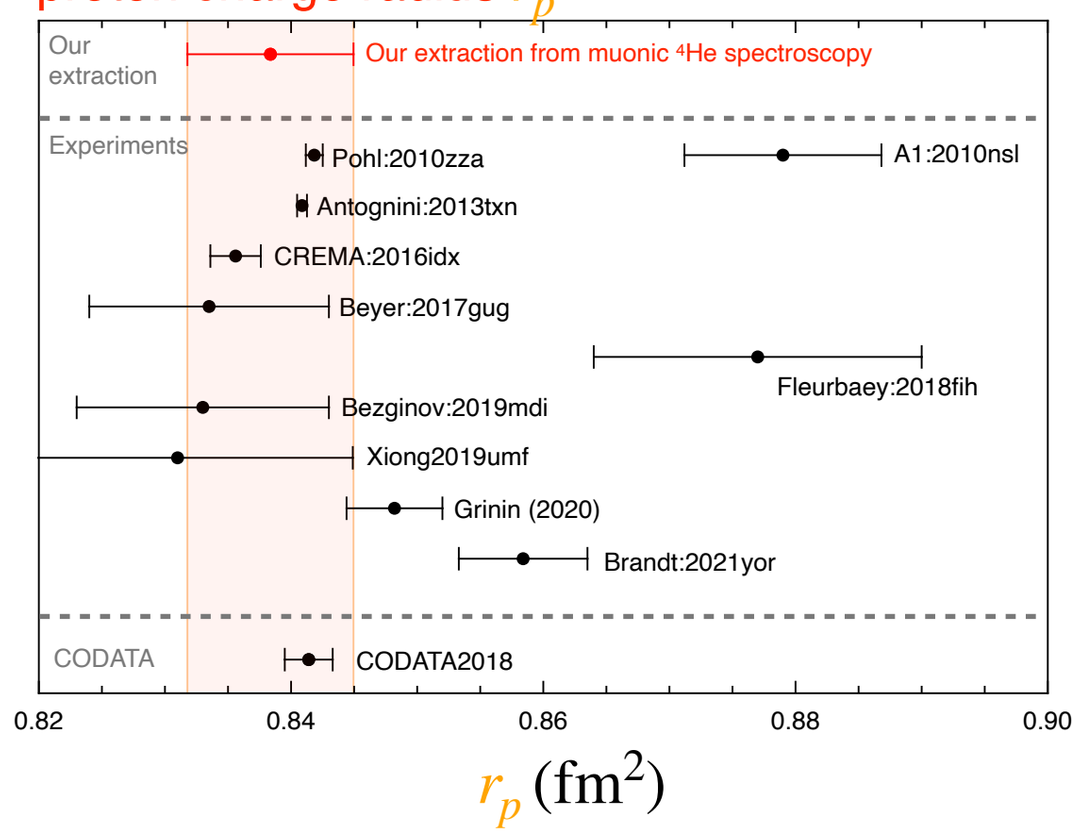
$r_n^2 = -0.105^{+0.005}_{-0.006}\,fm^2$

AF, Möller, Baru, Epelbaum, Krebs, Reinert,
PRL 124 (2020) 082501; PRC 103 (2021) 024313

New determination of the proton charge radius: $r_p = (0.838 \pm 0.007)\,fm$

preliminary

proton charge radius r_p



Our extraction supports the „small“ proton radius

Prediction for isoscalar 3N charge radius

With all LECs being fixed, we can predict the isoscalar 3N charge radius: $r_C^{isoscalar3N} = \sqrt{\frac{1}{3}(r_C^{3H})^2 + \frac{2}{3}(r_C^{3He})^2}$

$$r_C^{isoscalar3N} = (1.9058 \pm 0.0026)\,fm$$

preliminary, using CODATA 2018 r_p and own determination of r_n

Our result is 10x more precise than current experimental data:

the ^{3}H charge radius from e$^-$ scattering experiments:	$r_C^{3H} = (1.7550 \pm 0.0860)\,fm$ (5%)	Amroun et al. '94 (world average)
the ^{3}He charge radius from muonic ^{3}He (preliminary):	$r_C^{3He} = (1.9687 \pm 0.0013)\,fm$	Pohl '20 (preliminary)

Exp. **3N isoscalar charge radius**: (using muonic ^{3}He and old ^{3}H) $r_{C,\,exp.}^{isoscalar3N} = (1.9030 \pm 0.0290)\,fm$

T-REX experiment in Mainz [Pohl et al.] aims at measuring r_C^{3H} within ±0.0002 fm (400x more precise)

The isoscalar 3N radius will be then known within ±0.0009 fm

⇒ **precision tests of nuclear chiral EFT!**

Summary

Precise calculation of A = 2, 3, 4 charge radii in chiral effective field theory

Few-body calculations with sub-percent accuracy!

Charge radii of neutron and proton from light nuclei:

- ^{2}H r_{str} combined with isotope-shift data => extracted the neutron charge radius (2σ tension with PDG)
- ^{4}He r_{str} combined with spectroscopic data => extracted isoscalar nucleon and proton charge radii preliminary

^{4}He calculation: preliminary

- calculated ^{4}He charge radius (0.2% accuracy) agrees with the new μ^{4}He measurement
- no indications of BSM physics at this accuracy level

^{3}H-^{3}He: preliminary

- predicted the isoscalar 3N charge radius r_C (0.1% accuracy)
- our r_C is in agreement with the current exp. value (which has 10x larger errors)
- the ongoing T-REX (^{3}H) exp. in Mainz will allow for a precision test of nuclear chiral EFT

Outlook

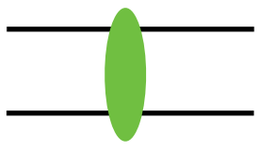
- **Consistent** inclusion of **N^3LO, N^4LO three-nucleon forces**
- **Consistent** inclusion of **isovector currents** (individual predictions for ^3H and ^3He)

- Analysis of **magnetic form factors** of ^2H, ^3H and ^3He
- Application to processes with two photons (**polarizabilities**, ...)

- Isoscalar 2N charge-density can be used to predict **charge radii of heavy nuclei** (LECs are fixed)

Spares

Chiral effective field theory - **precise, accurate** and **consistent**

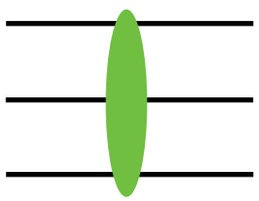


N^4LO^+

New high-precision chiral NN forces (N^4LO^+)

Reinert et al. PRL 126, 092501 (2021)

- Nearly perfect description of pp and pn scattering data up to pion production threshold



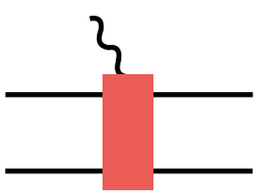
N^2LO

Chiral 3N forces (general N^2LO; selected terms at N^4LO)

Epelbaum:2019kcf

- charge radii of 3N and ^{4}He are not sensitive to N^3LO 3N forces as soon as the binding energy is reproduced

  (Strong correlations between the binding energy and charge radius)



N^2LO

Isoscalar: N^4LO^-

2N Chiral electromagnetic currents (general N^2LO; isoscalar N^4LO^-)

Kolling:2009iq
Kolling:2012cs
Krebs:2019aka
Krebs:2020pii (Review)

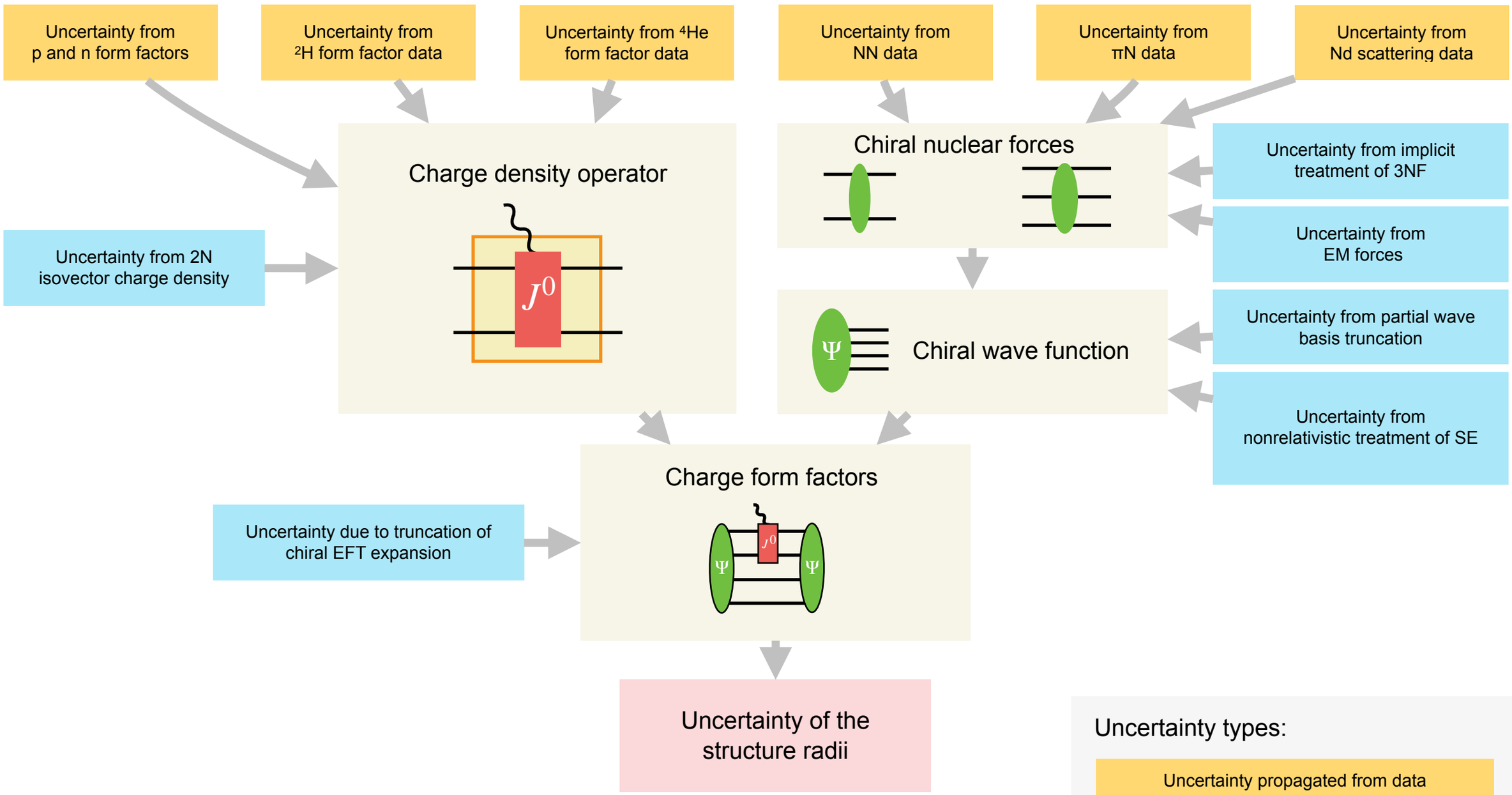
- N^2LO (**isoscalar N^4LO^-**) is derived and regularised consistently with the chiral NN forces
- Consistent regularisation of N^3LO (isovector) is in progress

Reliable methods to quantify truncation uncertainty of the EFT expansion

Epelbaum et al. EPJA 51 (2015); Furnstahl et al. PRC 92, 024005 (2015); Melendez et al. PRC 96, 024003 (2017), Wesolowski et al. J. Phys. G 46, 045102 (2019); Melendez et al. PRC 100, 044001 (2019), ...

Extensive uncertainty analysis

Propagation of uncertainties from data and theory



Estimation of ^{3}H charge radius

Our preliminary prediction for **isoscalar** 3N **charge** radius:

$$r_C^{isoscalar3N} = (1.9058 \pm 0.0026)\,fm$$

preliminary, using CODATA 2018 r_p and own determination of r_n

Isoscalar 3N charge radius definition:

$$(r_C^{isoscalar3N})^2 = \frac{(r_C^{3H})^2 + 2\,(r_C^{3He})^2}{3}$$

Preliminary ^{3}He charge radius [Pohl et al.]

$$r_C^{3He} = (1.9687 \pm 0.0013)fm$$

Expression for **^{3}H radius:**

$$(r_C^{3H})^2 = 3(r_C^{isoscalar3N})^2 - 2\,(r_C^{3He})^2$$

Coefficients 2 and 3 amplify both theoretical and experimental uncertainties

Our ^{3}H radius estimation:

$$r_C^{(3H)} = (1.7734 \pm 0.0088)fm$$

preliminary

This estimation is 10x more precise than e$^-$ data $r_C^{3H} = (1.7550 \pm 0.0860)\,fm$ Amroun et al. '94 (world average)

But it suffers from parametric amplification of uncertainties (both from theory and from ^{3}He data)

=> isoscalar 3N charge radius should be used for precision tests

^{4}He charge radius: effective field theory and experiment

Our prediction for ^{4}He **charge** radius (preliminary)

$$r_C(^4He) = (1.6798 \pm 0.0035)\,fm$$

using CODATA 2018 r_p and own determination of r_n

The μ ^{4}He exp. value is

$$r_C(^4\mathbf{He}) = (1.67824 \pm 0.00083)\,fm$$

Krauth et al., Nature 589 (2021) 7843, 527-531