Trace anomaly in neutron stars

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Reference:

Y. Fujimoto, K. Fukushima, L. McLerran, M. Praszalowicz, arXiv:2207.06753

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Outline of this talk

Q. Dense (conformal) quark matter in neutron stars (NSs)?

→ Trace anomaly can be a useful measure

1. Rapid approach to the conformal limit of the trace anomaly, giving rise to the sound velocity peak

2. Strongly-interacting conformal matter inside NS

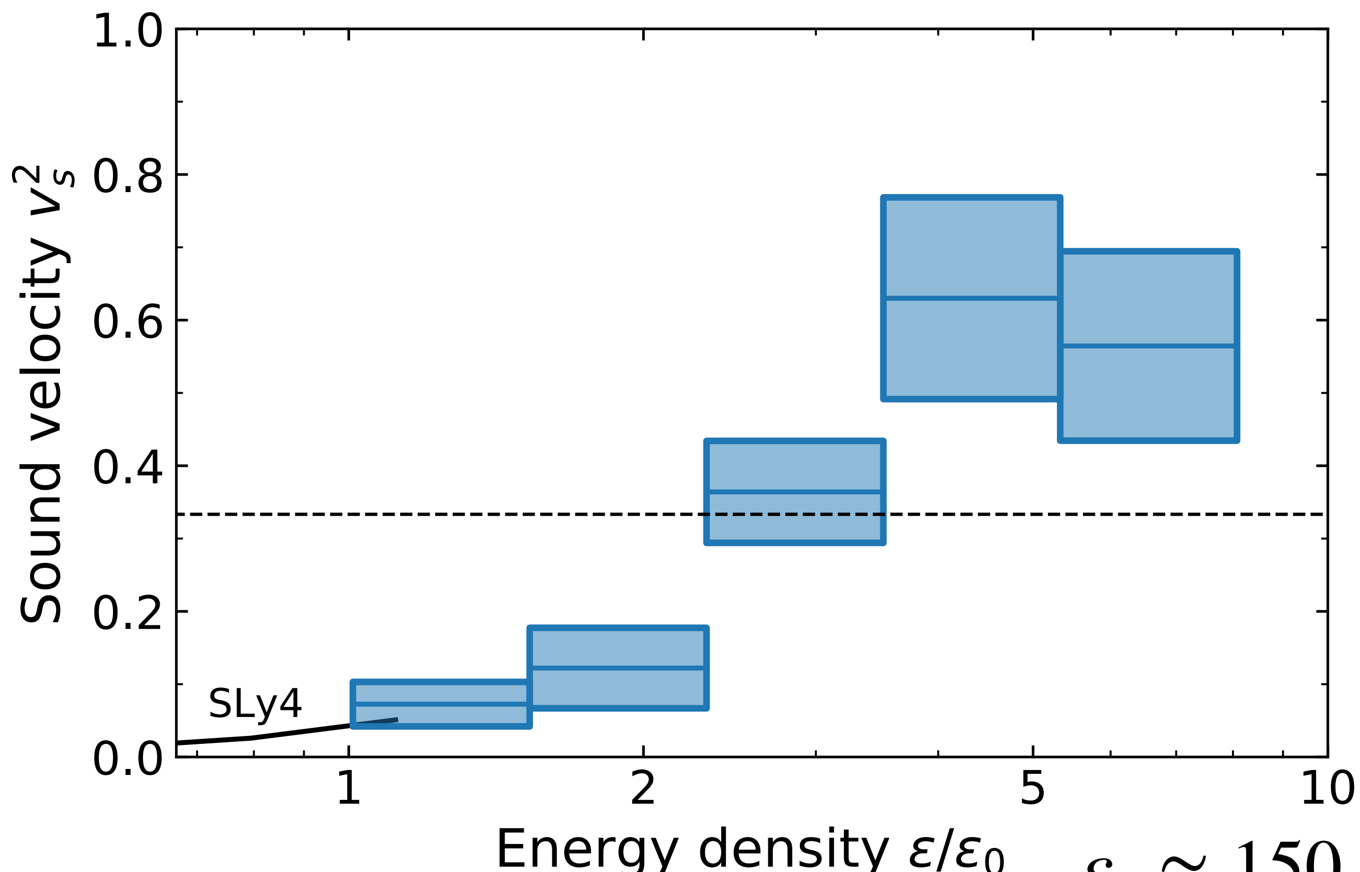
3. Positivity of the trace anomaly and its observational consequences

Prelude: sound velocity peak in the EoS

Fujimoto,Fukushima,Murase (2019)

- NS data favors rapid increase in sound velocity, accompanied by a peak structure

$$v_s^2 \equiv \frac{dP}{d\varepsilon} = \frac{P'(\mu_B)}{\varepsilon'(\mu_B)}$$



$\varepsilon_0 \approx 150$ MeV/fm^3

- The upper bound put by the conformal limit, $v_s^2 \leq 1/3$, is in tension with the existence of $2M_\odot$ pulsars

Bedaque,Steiner (2015); Tews,Carlson,Gandolfi,Reddy (2018); Altiparmak,Ecker,Rezzolla (2022); Gorda,Komoltsev,Kurkela (2022) & many others

Trace anomaly equation

- The trace of the QCD energy-momentum tensor $T^\mu_{\ \mu}$ is a measure of scale invariance, or conformality:

$$T^\mu_{\ \mu} = \frac{\beta}{2g} F^a_{\mu\nu} F^{\mu\nu}_a + (1 + \gamma_m) \sum_f m_f \bar{q}_f q_f$$

- Finite-μ_B part of the trace anomaly (interaction measure):

$$\langle T^\mu_{\ \mu} \rangle_{\mu_B} = \varepsilon - 3P$$

$$\langle T^\mu_{\ \mu} \rangle = \langle T^\mu_{\ \mu} \rangle_{\mu_B} + \langle T^\mu_{\ \mu} \rangle_0$$

- We consider the normalized trace anomaly:

$$\Delta \equiv \frac{\langle T^\mu_{\ \mu} \rangle_{\mu_B}}{3\varepsilon} = \frac{1}{3} - \frac{P}{\varepsilon}$$

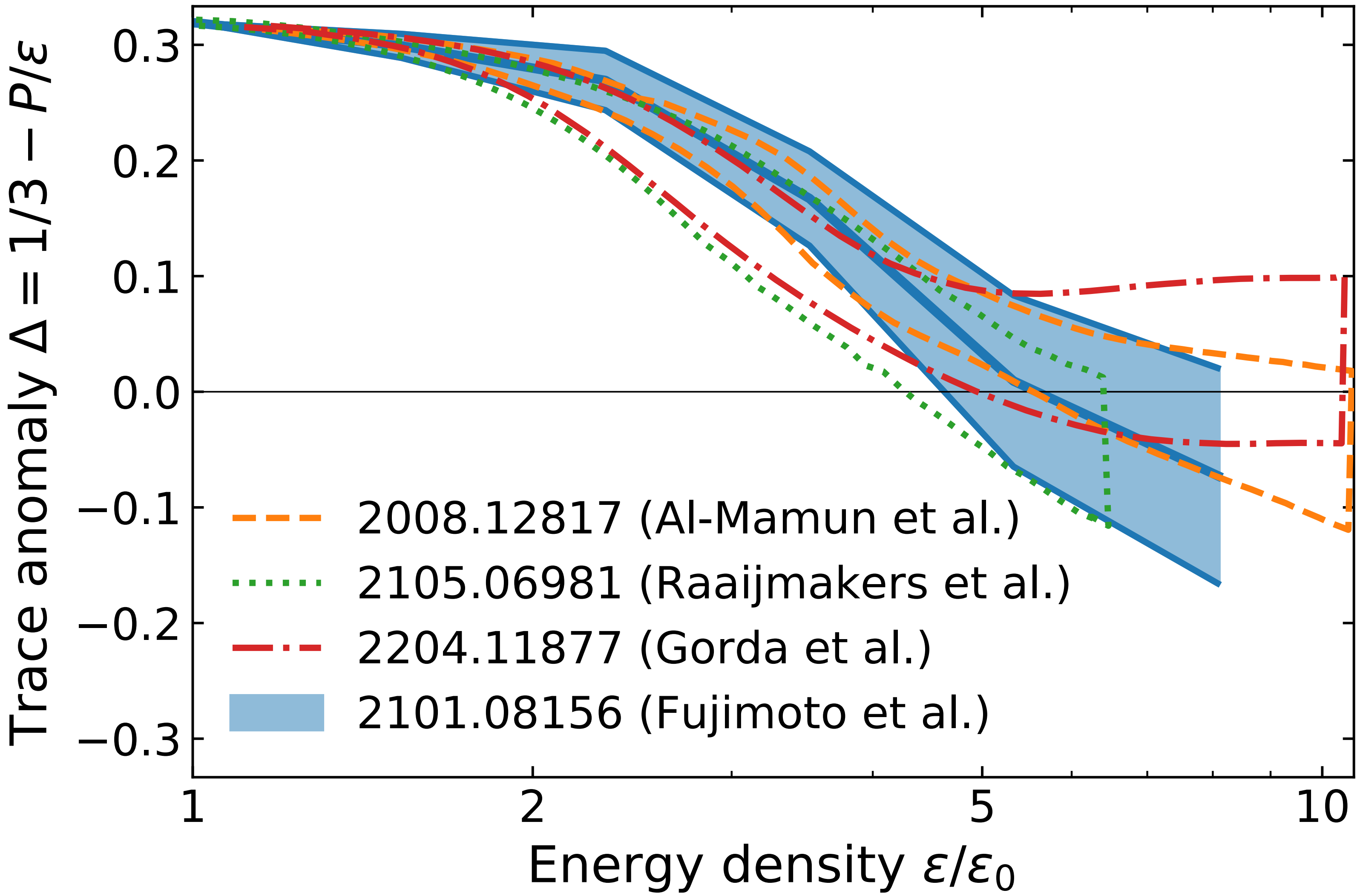
$$-\frac{2}{3} \lesssim \Delta \leq \frac{1}{3}$$

Trace anomaly from neutron star data

Fujimoto, Fukushima, McLerran, Praszalowicz (2022)

$$\Delta \equiv \frac{\langle T^{\mu}_{\mu} \rangle_{\mu_B}}{3\varepsilon} = \frac{1}{3} - \frac{P}{\varepsilon}$$

- Inferred from neutron star data:



$\Delta \sim 0$ already at $\sim 5\varepsilon_0$

→ **rapid approach to conformality**

Suggests strongly-coupled conformal matter with $P \approx \varepsilon/3$

Decomposition of sound velocity

- Sound velocity can be decomposed into Δ and its derivative

$$v_s^2 = \varepsilon \frac{d}{d\varepsilon}\left(\frac{P}{\varepsilon}\right) + \frac{P}{\varepsilon}$$

$$= \varepsilon \frac{d\Delta}{d\varepsilon} + \left(\frac{1}{3} - \Delta\right)$$

Derivative component

Non-derivative component

- Two bounds put by conformal limit: $\Delta \geq 0$ and $v_s^2 \leq 1/3$

This decomposition explains why $\Delta \geq 0$ and $v_s^2 > 1/3$ are possible simultaneously

Decomposition of sound velocity

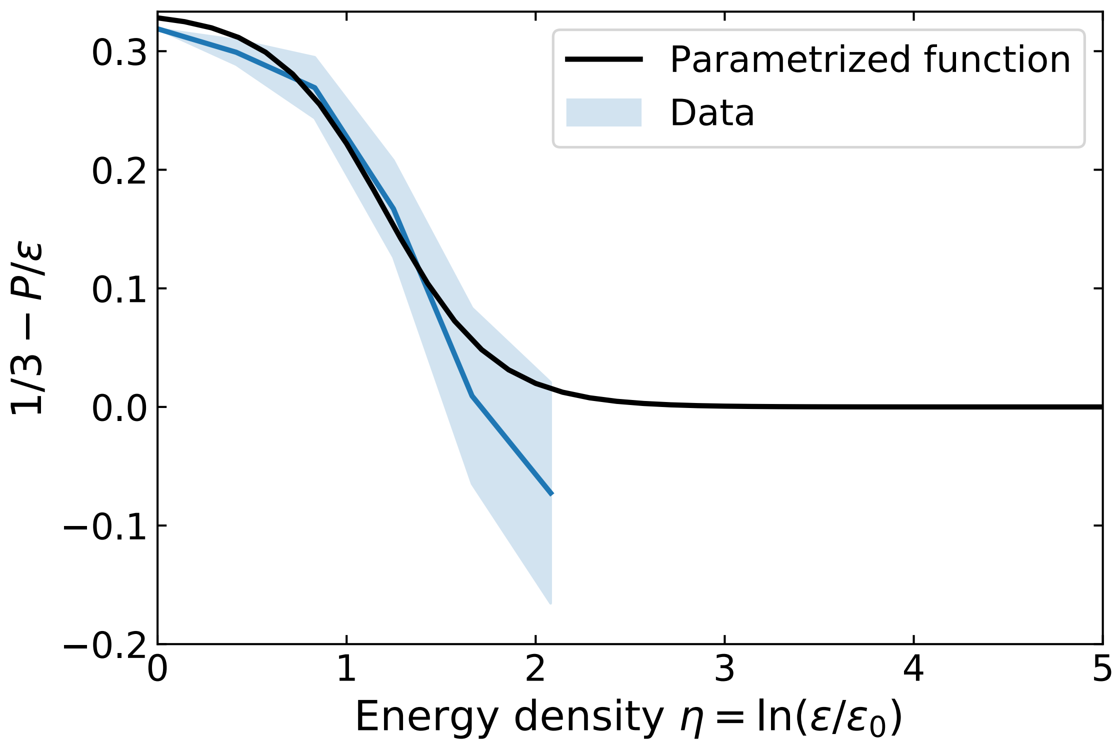
Fujimoto, Fukushima, McLerran, Praszalowicz (2022)

Rapid approach to $\Delta \to 0$ naturally spikes v_s^2

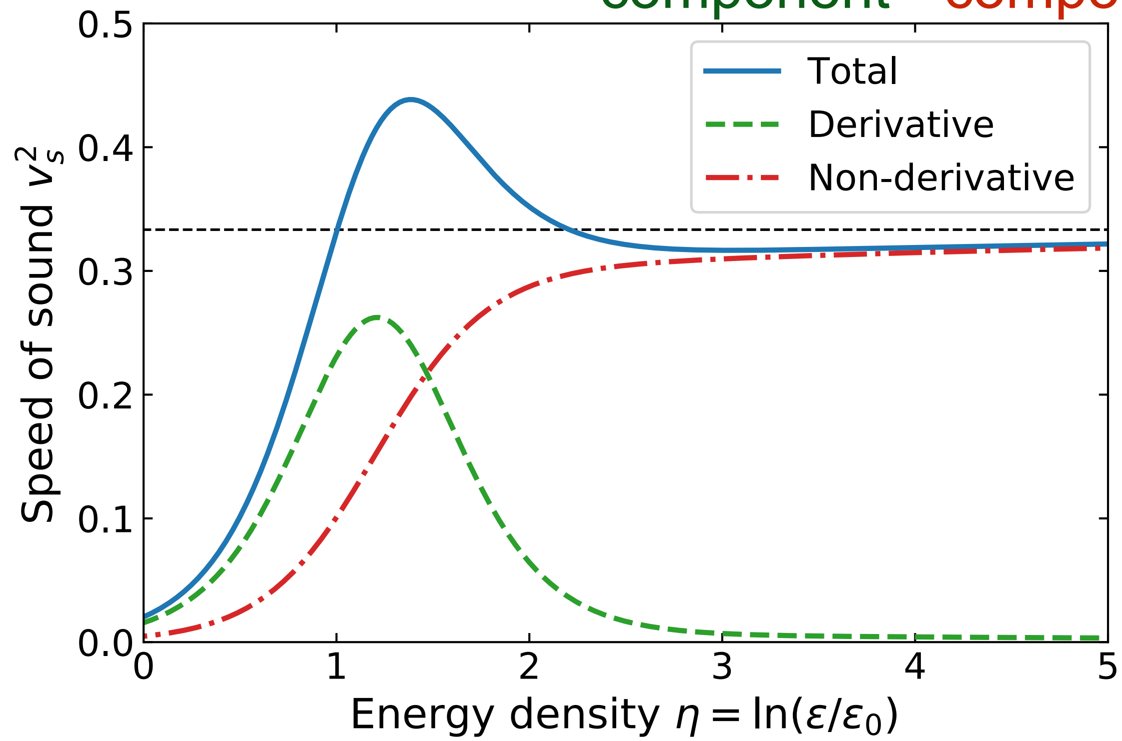
Sound velocity $v_s^2 = \varepsilon \dfrac{d\Delta}{d\varepsilon} + \left(\dfrac{1}{3} - \Delta\right)$

Derivative component / Non-derivative component

Trace anomaly $\Delta = \dfrac{1}{3} - \dfrac{P}{\varepsilon}$



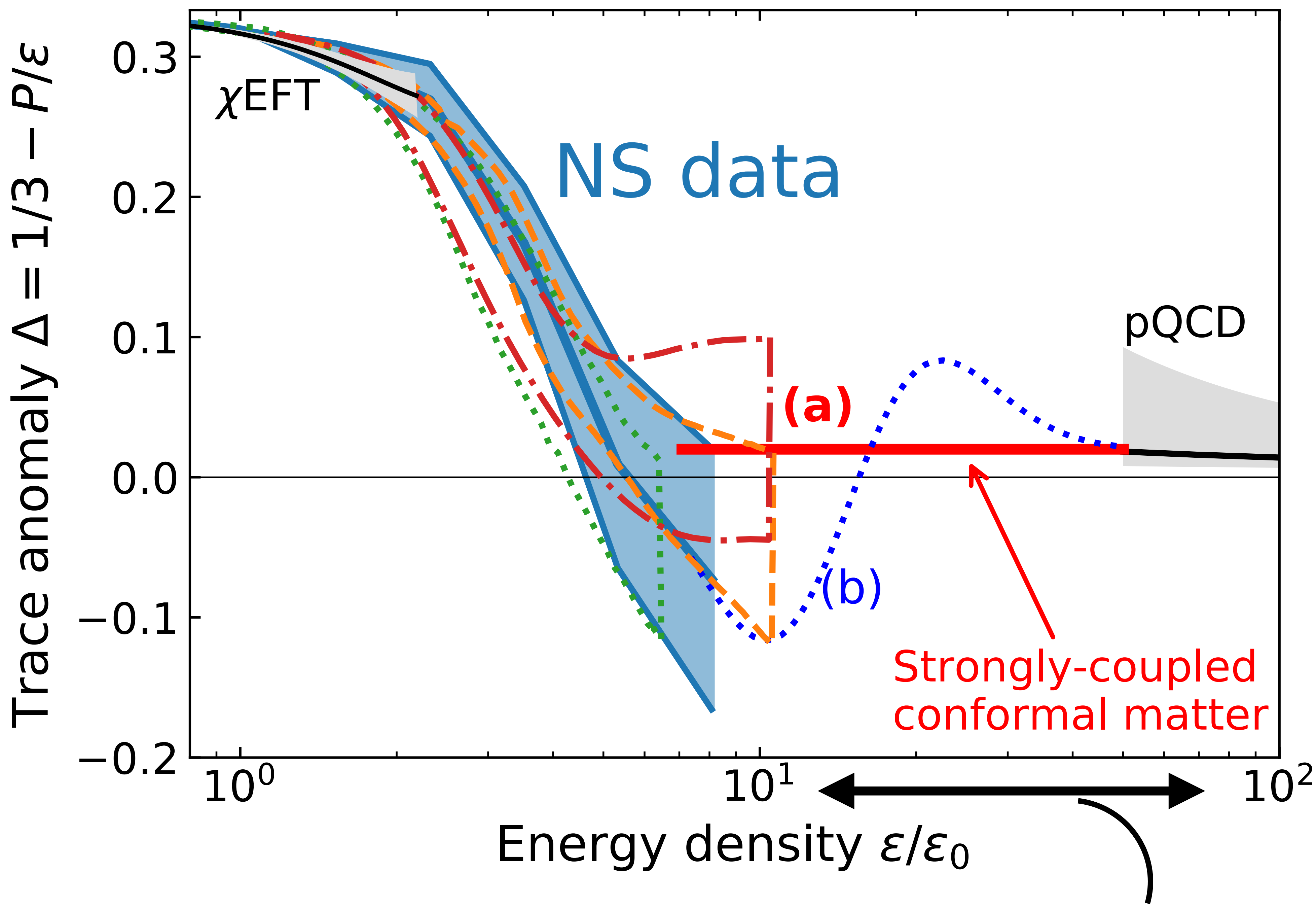
$\Delta \geq 0$



$v_s^2 > 1/3$

Monotonic Δ gives rise to non-monotonic v_s^2 and violation of $v_s^2 \leq 1/3$

Strongly-coupled conformal matter



not well-constrained by data
→ QCD tells us something?

Is the trace anomaly positive?

- **It may well be positive, but it doesn't have to be.**

  cf) single-particle matrix element: $\langle p \,|\, T^{\mu}{}_{\mu} \,|\, p \rangle \sim p^2 = m^2 \geq 0$

- Several known examples of negative trace anomaly:
  - ○ Two-color QCD e.g., Cotter,Giudice,Hands,Skullerud (2012); Iida,Itou (2022)
  - ○ QCD at finite isospin chemical potential Son,Stephanov (2001)…

- Consider the following simple expression:

$$\varepsilon(n) = mn + \frac{C}{\Lambda^2} n^2$$

$\Delta \to 0$ is reached at $n = \dfrac{m\Lambda^2}{2C}$,

$m = m_N$ for finite-μ_B

$m = m_\pi$ for finite-μ_I

Is the trace anomaly positive?

- In the chiral limit, $\langle T^{\mu}_{\ \mu} \rangle_{\mu_B} = \frac{\beta}{2g} \langle F^{a}_{\mu\nu} F^{\mu\nu}_{a} \rangle_{\mu_B}$

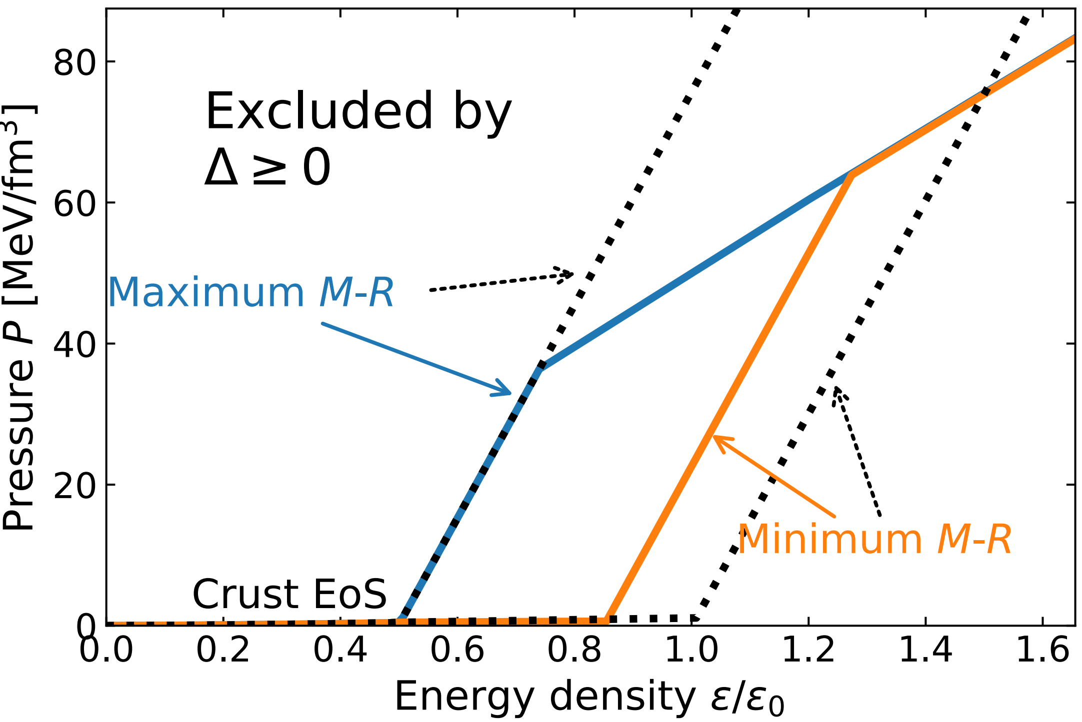
- Trace anomaly is related to the counting of the degrees of freedom in pressure, $\nu \equiv P/\mu_B^4$:

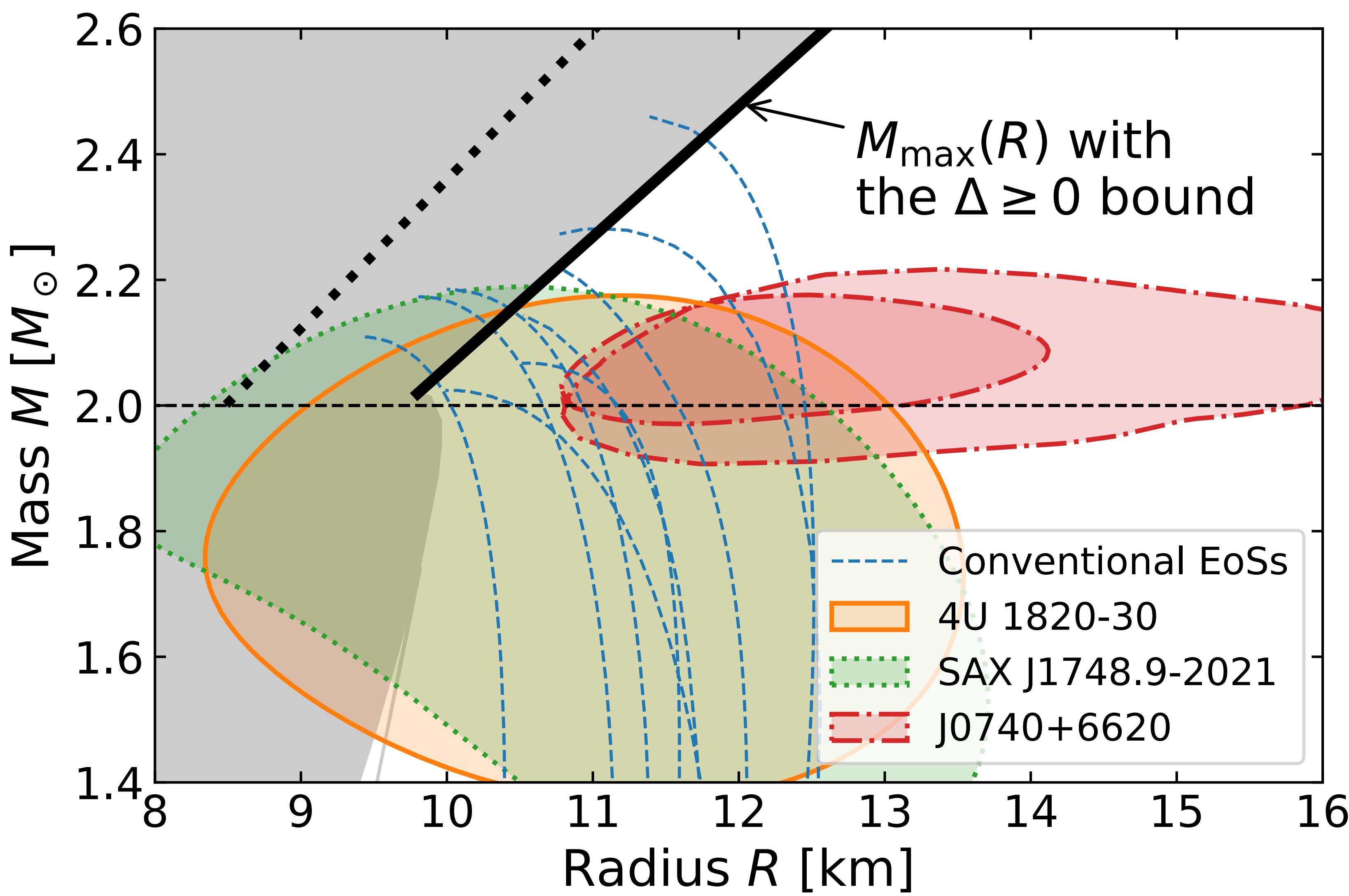
$$\frac{\langle T^{\mu}_{\ \mu} \rangle_{\mu_B}}{\mu_B^4} = \mu_B \frac{d\nu}{d\mu_B} \geq 0$$

If ν keeps increasing, we get $\Delta \geq 0$.
Open question: what if we have color superconductivity?

Testing $\Delta \geq 0$ by NS observation

- One example: $\Delta \geq 0$ put the bound on the maximum mass





The maximally large M-R: the stiffest EoS Rhoades Jr.,Ruffini (1974)

The most massive and compact M-R: soft at low density and stiff at high density

Koranda,Stergioulas,Friedman (1995)

See also: Drischler,Han,Lattimer,Prakash,Reddy,Zhao (2020)

Summary

- Trace anomaly Δ measures conformality, is is a complement to the speed of sound v_s^2

- NS data suggest Δ rapidly approach to the conformal limit

- $\Delta \to 0$ gives rise to the sound velocity peak

  Consistent with microscopic pictures, e.g., Masuda,Hatsuda,Takatsuka (2013); McLerran,Reddy (2018); Pisarski (2021); Kojo (2021) & many others

- Strongly-interacting conformal matter may be inside NSs

- The trace anomaly may be positive (not proven. QCD inequalities?). It can be tested by, e.g., the bound on the maximum mass of NSs