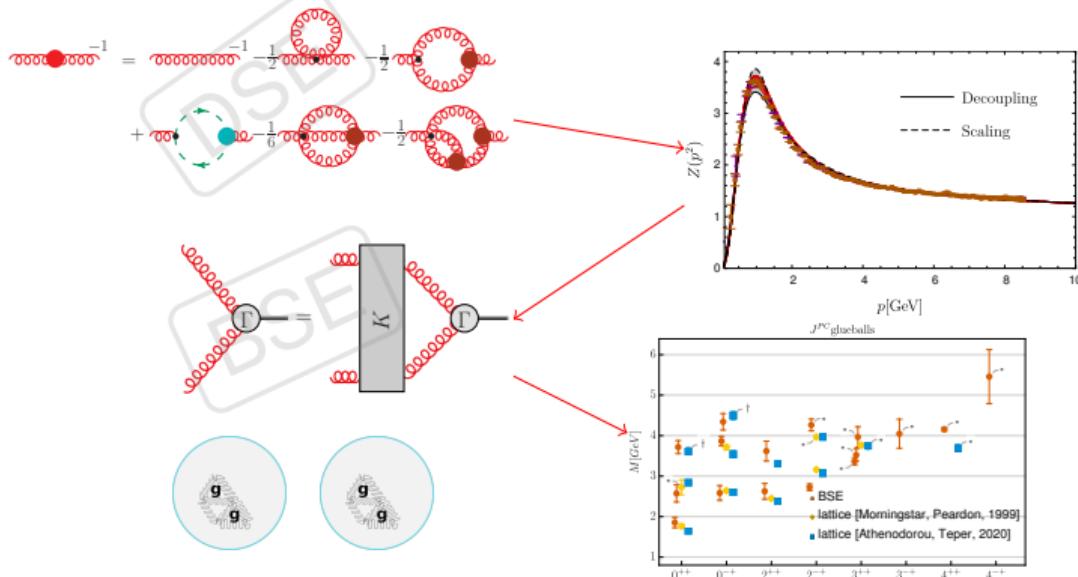


Glueballs from bound state equations



Markus Q. Huber

Institute of Theoretical Physics
Giessen University

In collaboration with
Christian S. Fischer, Hèlios Sanchis-Alepuz:
[Eur.Phys.J.C 80, arXiv:2004.00415](#)
[Eur.Phys.J.C 80, arXiv:2110.09180](#)
[vConf21, arXiv:2111.10197](#)
[HADRON2021, arXiv:2201.05163](#)

XVth Quark
confinement and the
hadron spectrum
conference

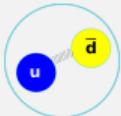
Stavanger, Norway,
Aug. 5, 2022

JUSTUS-LIEBIG-
UNIVERSITÄT
GIESSEN

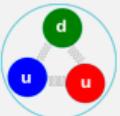
DFG Deutsche
Forschungsgemeinschaft

Bound states and multiplets

Meson



Baryon



Tetraquark



Pentaquark



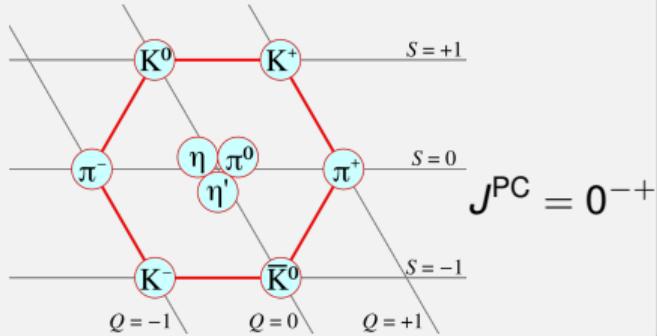
Hybrid



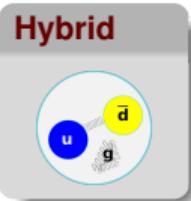
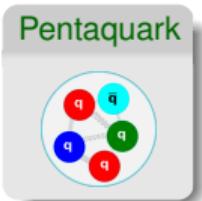
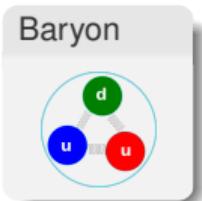
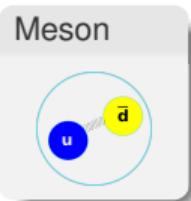
Quark model

Classification in terms of mesons or baryons → multiplets

Outside this classification
→ **exotics**



Bound states and multiplets



Quark model

Classification in terms of mesons or baryons → multiplets

Outside this classification
→ **exotics**

Classification not always easy, e.g., **scalar sector $J^{PC} = 0^{++}$** : → Talk by J. R. Peláez, Tuesday.

tetraquarks [Jaffe, PRD15 (1977)]?

glueball candidates

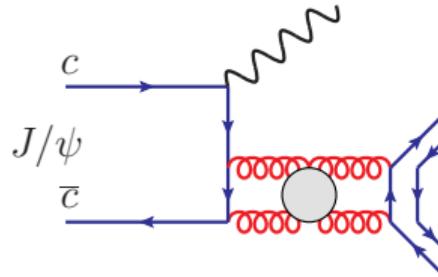
$f_0(500)$
$f_0(980)$
$f_0(1370)$
$f_0(1500)$
$f_0(1710)$

+ more states not considered established

Reviews on glueballs: [Klempt, Zaitsev, Phys.Rept.454 (2007); Mathieu, Kochelev, Vento, Int.J.Mod.Phys.18 (2009); Crede, Meyer, Prog.Part.Nucl.Phys.63 (2009); Ochs, J.Phys.G40 (2013); Llanes-Estrada, EPJST 230 (2021)]

Scalar glueballs from J/ψ decay

→ Talk by E. Klempt, Friday, 16:05.

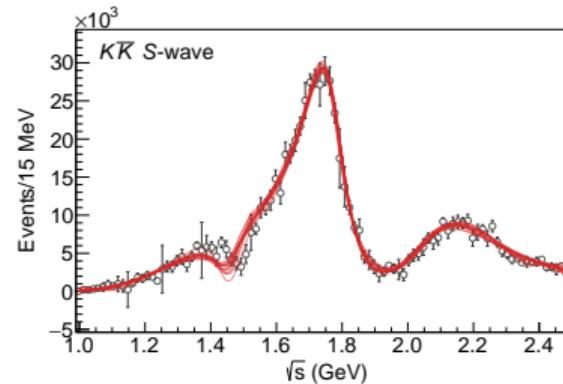
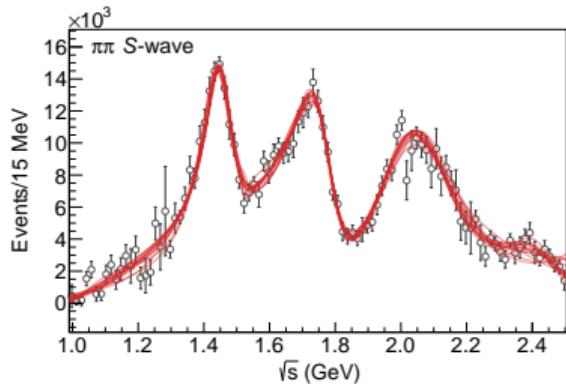


Coupled-channel analyses of exp. data (BESIII):

- +add. data, largest overlap with $f_0(1770)$
- largest overlap with $f_0(1710)$

[Sarantsev, Denisenko, Thoma, Klempt, Phys. Lett. B 816 (2021)]

[Rodas et al., Eur.Phys.J.C 82 (2022)]



Glueball calculations: Lattice

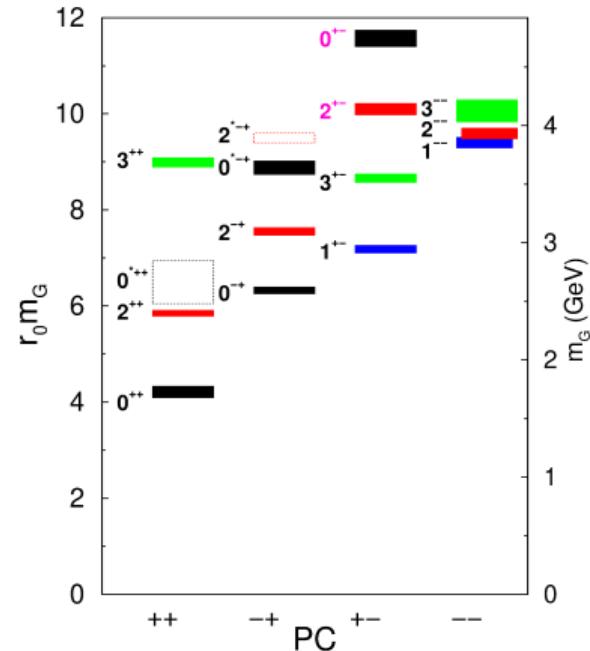
Lattice methods

Pure gauge theory:

No dynamic quarks.

→ “Pure” glueballs

- [Morningstar, Peardon, Phys. Rev. D60 (1999)]: standard reference
- [Athenodorou, Teper, JHEP11 (2020)]: improved statistics, more states



[Morningstar, Peardon, Phys. Rev. D60 (1999)]

Glueball calculations: Lattice

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“Real QCD”:

- [Gregory et al., JHEP10 (2012)]

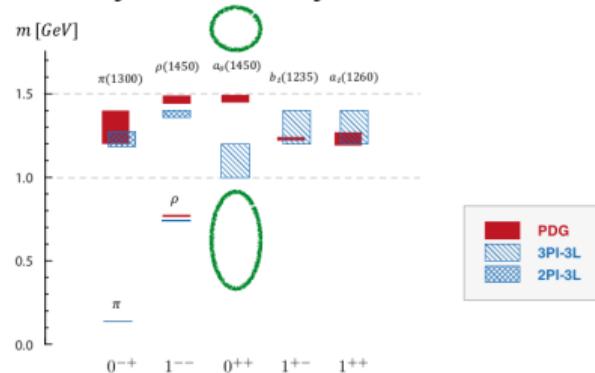
Challenging:

- Much higher statistics required (poor signal-to-noise ratio)
- Continuum extrapolation and inclusion of fermionic operators still to be done
- Mixing with $\bar{q}q$ challenging
- $m_\pi = 360 \text{ MeV}$
- Tiny (e.g., $0^{++}, 2^{++}$) to moderate unquenching effects (e.g., 0^{-+}) found

No quantitative results yet.

Functional spectrum calculations

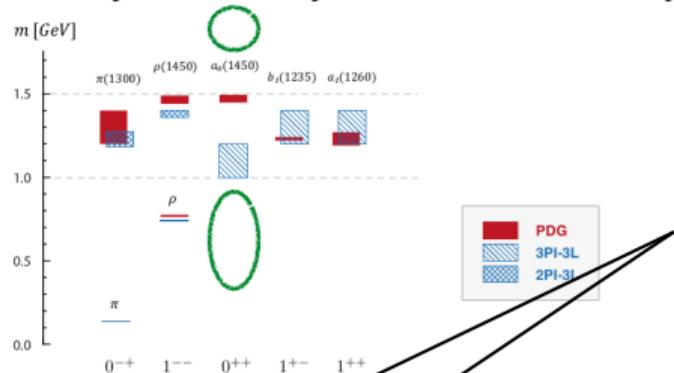
Functional methods successful in describing many aspects of the hadron spectrum qualitatively and quantitatively! → Talk by C. Fischer, Monday.



1. Conventional mesons
2. Pure Yang-Mills: glueballs
3. Light four-quark states: the f0(500)
4. Heavy-light four-quark states

Functional spectrum calculations

Functional methods successful in describing many aspects of the hadron spectrum qualitatively and quantitatively! → Talk by C. Fischer, Monday.



Transition to first-principles calculations.

1. Conventional mesons

2. Pure Yang-Mills: glueballs

3. Light four-quark states: the $f_0(500)$

4. Heavy-light four-quark states

Rainbow-ladder with Maris-Tandy (or similar) has been the workhorse for more than 20 years.

Functional glueball calculations

Functional methods successful in describing many aspects of the hadron spectrum qualitatively and quantitatively!

Glueballs? Rainbow-ladder?

Functional glueball calculations

Functional methods successful in describing many aspects of the hadron spectrum qualitatively and quantitatively!

Glueballs? Rainbow-ladder?

There is no rainbow for gluons!

Model based BSE calculations ($J = 0$):

- [Meyers, Swanson, Phys.Rev.D87 (2013)]
- [Sanchis-Alepuz, Fischer, Kellermann, von Smekal, Phys.Rev.D92, (2015)]
- [Souza et al., Eur.Phys.J.A56 (2020)]
- [Kaptari, Kämpfer, Few Body Syst.61 (2020)]

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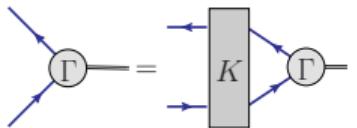
- [Meyers, Swanson, Phys.Rev.D87 (2013)]
- [Sanchis-Alepuz, Fischer, Kellermann, von Smekal, Phys.Rev.D92, (2015)]
- [Souza et al., Eur.Phys.J.A56 (2020)]
- [Kaptari, Kämpfer, Few Body Syst.61 (2020)]

Alternative: Calculated input

- $J = 0$: [MQH, Fischer, Sanchis-Alepuz, Eur.Phys.J.C80 (2020)]
- $J = 0, 2, 3, 4$: [MQH, Fischer, Sanchis-Alepuz, Eur.Phys.J.C81 (2021)]

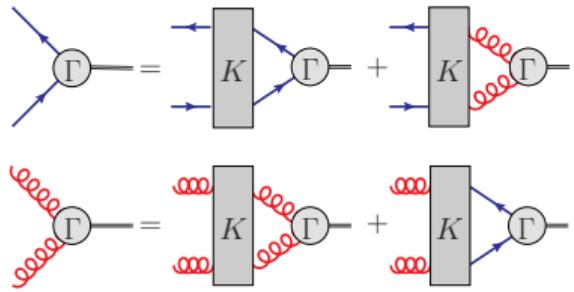
Extreme sensitivity on input!

Bound state equations for QCD



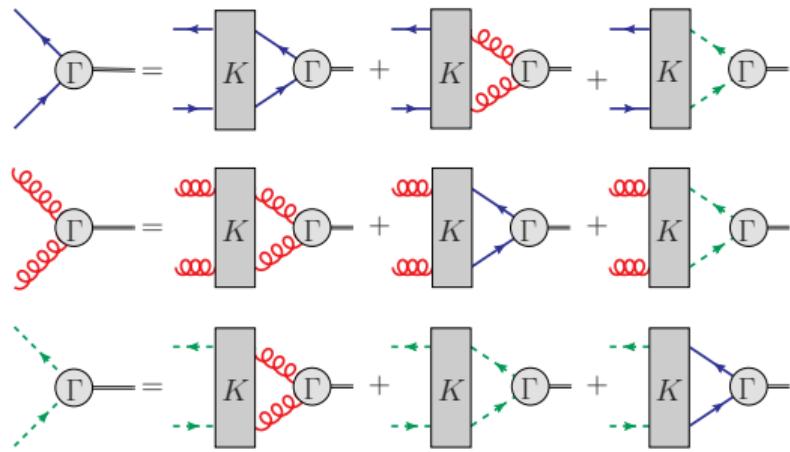
- Require scattering kernel K and propagator.

Bound state equations for QCD



- Require scattering kernels K and propagators.
- Quantum numbers determine which amplitudes Γ couple.

Bound state equations for QCD



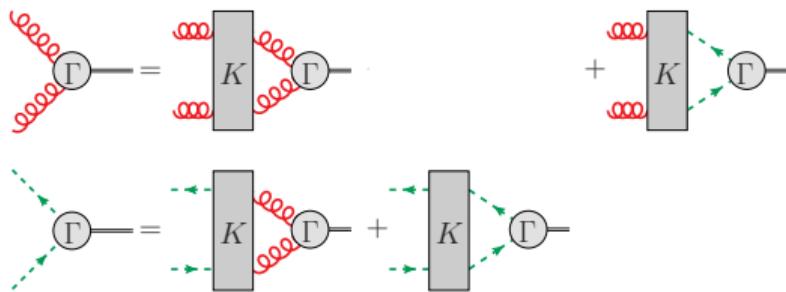
- Require scattering kernels K and propagators.
- Quantum numbers determine which amplitudes Γ couple.
- **Ghosts** from gauge fixing

One framework

- Natural description of **mixing**.
- Similar equations for hadrons with more than two constituents

Bound state equations for QCD

Focus on pure glueballs.



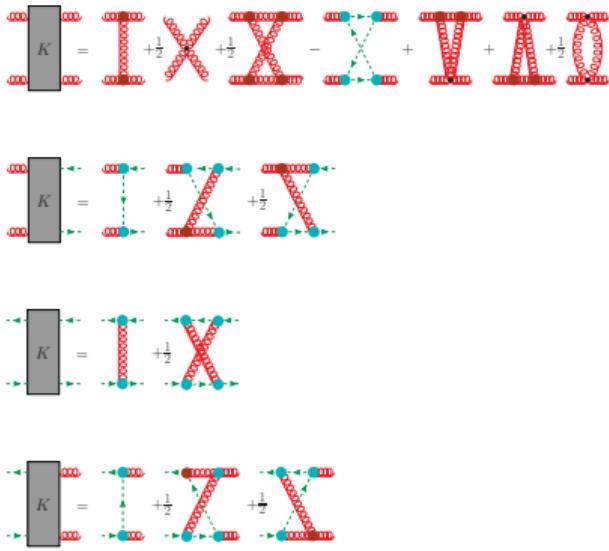
- Require scattering kernels K and propagators.
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One framework

- Natural description of **mixing**.
- Similar equations for hadrons with more than two constituents

Kernels

Systematic derivation from 3PI effective action: [Berges, PRD70 (2004); Carrington, Gao, PRD83 (2011)]
 Self-consistent treatment of 3-point functions requires 3-loop expansion.



[Fukuda, Prog. Theor. Phys 78 (1987); McKay, Munczek, Phys. Rev. D 40 (1989); Sanchis-Alepuz, Williams, J. Phys: Conf. Ser. 631 (2015); MQH, Fischer, Sanchis-Alepuz, Eur.Phys.J.C80 (2020)]

Correlation functions of quarks and gluons

Equations of motion: 3-loop 3PI effective action

→ [Review: MQH, Phys.Rept. 879 (2020)]

$$\text{Diagram}_1^{-1} = \text{Diagram}_2^{-1} - \frac{1}{2} \text{Diagram}_3^{-1} - \frac{1}{2} \text{Diagram}_4^{-1} + \text{Diagram}_5^{-1}$$

$$+ \text{Diagram}_6^{-1} - \frac{1}{6} \text{Diagram}_7^{-1} - \frac{1}{2} \text{Diagram}_8^{-1}$$

$$\text{Diagram}_9^{-1} = \text{Diagram}_{10}^{-1} - 2 \text{Diagram}_{11}^{-1} - 2 \text{Diagram}_{12}^{-1} + \text{Diagram}_{13}^{-1}$$

$$+ \frac{1}{2} \text{Diagram}_{14}^{-1} + \frac{1}{2} \text{Diagram}_{15}^{-1} + \frac{1}{2} \text{Diagram}_{16}^{-1}$$

$$\text{Diagram}_{17}^{-1} = \text{Diagram}_{18}^{-1} + \text{Diagram}_{19}^{-1} + \text{Diagram}_{20}^{-1}$$

$$= \text{Diagram}_{21}^{-1} + \text{Diagram}_{22}^{-1} + \text{Diagram}_{23}^{-1}$$

$$\text{Diagram}_1^{-1} = \text{Diagram}_2^{-1} - \text{Diagram}_3^{-1}$$

$$\text{Diagram}_4^{-1} = \text{Diagram}_5^{-1} - \text{Diagram}_6^{-1}$$

- Conceptual and technical challenges: nonperturbative renormalization, two-loop diagrams, convergence, size of kernels, ...
- Self-contained: Only parameters are the **strong coupling and the quark masses!**
- Long way, e.g., ghost-gluon vertex [MQH, von Smekal, JHEP 04 (2013)], three-gluon vertex [Blum, MQH, Mitter, von Smekal, PRD89 (2014)], four-gluon vertex [Cyrol, MQH, von Smekal, EOJC 75 (2015)], ...

Correlation functions of quarks and gluons

Equations of motion: 3-loop 3PI effective action

→ [Review: MQH, Phys.Rept. 879 (2020)]

$$\text{Diagram 1: } \text{Diagram with a red loop and a crossed-out diagram.}$$

$$+ \text{Diagram with a green dashed loop and a red loop.}$$

$$\text{Diagram 2: } \text{Diagram with a red loop and a crossed-out diagram.}$$

$$+\frac{1}{2} \text{Diagram with a red loop and a red loop.}$$

$$\text{Diagram 3: } \text{Diagram with a red loop and a crossed-out diagram.}$$

$$= \text{Diagram with a green dashed loop and a red loop.}$$

$$\text{Diagram 4: } \text{Diagram with a blue loop and a crossed-out diagram.}$$

$$-\text{Diagram with a green dashed loop and a red loop.}$$

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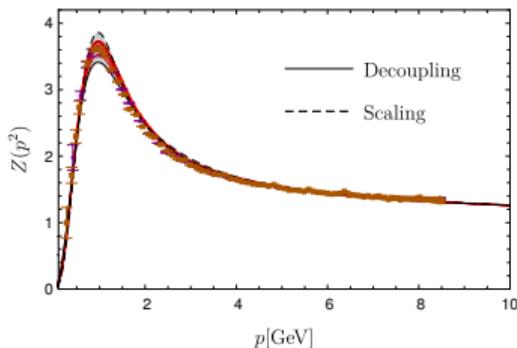
Start with **pure gauge theory**.

Landau gauge propagators

Self-contained: Only external input is the coupling!

[MQH, Phys.Rev.D 101 (2020)]

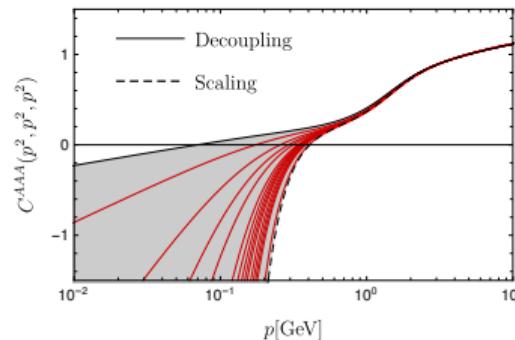
Gluon dressing function:



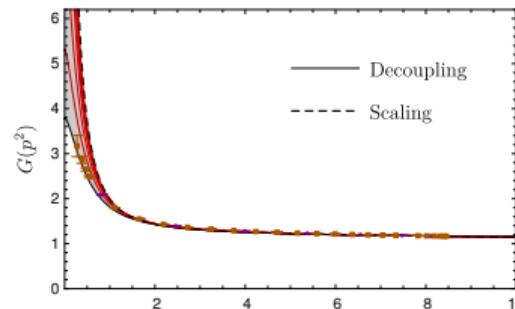
Family of solutions [von Smekal, Alkofer, Hauck, PRL79 (1997); Aguilar, Binosi, Papavassiliou, Phys.Rev.D 78 (2008); Boucaud et al., JHEP06 (2008); Fischer, Maas, Pawłowski, Ann.Phys. 324 (2008); Alkofer, MQH, Schwenzer, Phys. Rev. D 81 (2010)]

Nonperturbative completions of Landau gauge [Maas, Phys. Lett. B 689 (2010)]?

Three-gluon vertex:



Ghost dressing function:



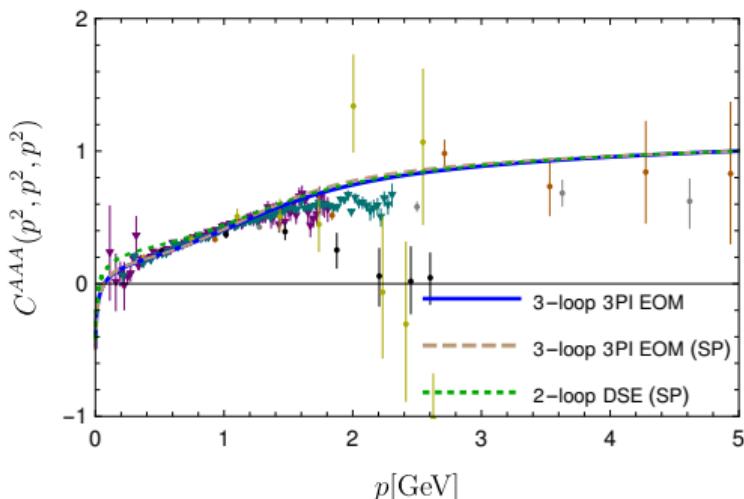
Stability of the solution

- Agreement with lattice results. ✓

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- Concurrence between functional methods:

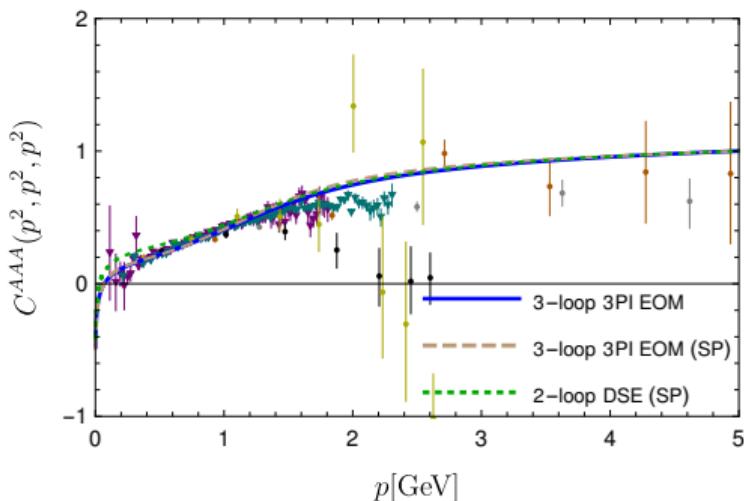
3PI vs. 2-loop DSE:



Stability of the solution

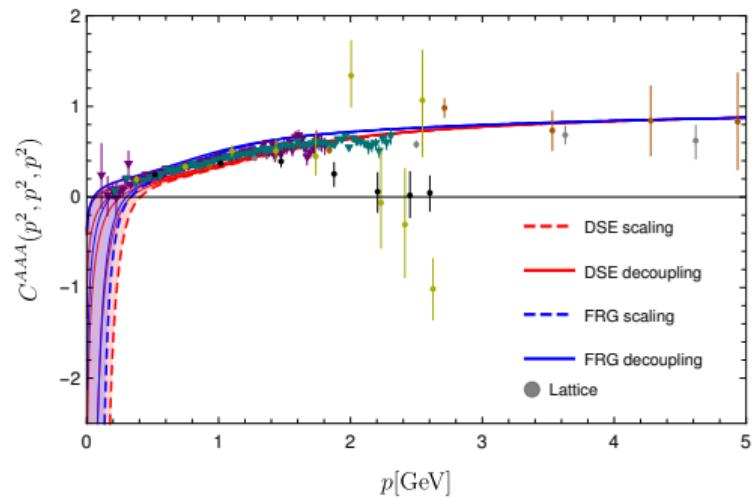
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3PI vs. 2-loop DSE:



[Cucchieri, Maas, Mendes, Phys.Rev.D77 (2008); Sternbeck et al., Proc.Sci. LATTICE2016 (2017); Cyrol et al., Phys.Rev.D 94 (2016); MQH, Phys.Rev.D101 (2020)]

DSE vs. FRG:

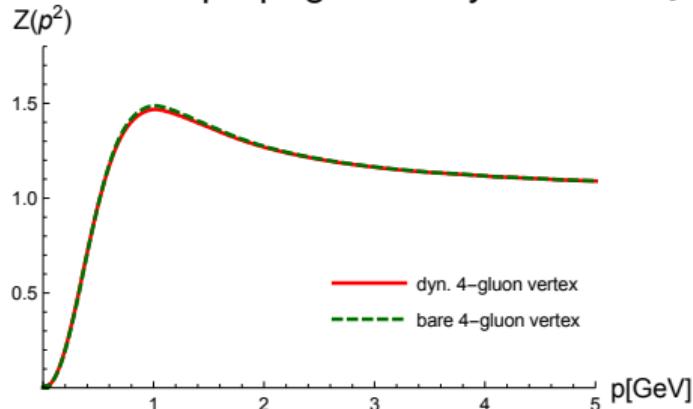


Stability of the solution: Extensions

- Three-gluon vertex: Tree-level dressing dominant, others subleading [Eichmann, Williams, Alkofer, Vujinovic, Phys.Rev.D89 (2014); Pinto-Gómez et al., 2208.01020] ✓ → Poster by F. de Soto.

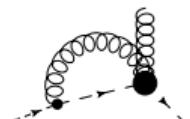
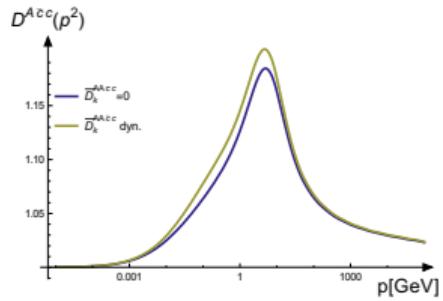
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- Four-gluon vertex: Influence on propagators tiny for $d = 3$ [MQH, Phys.Rev.D93 (2016)]

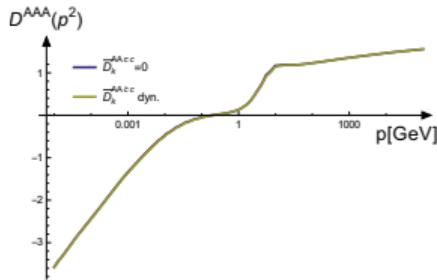


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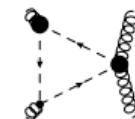
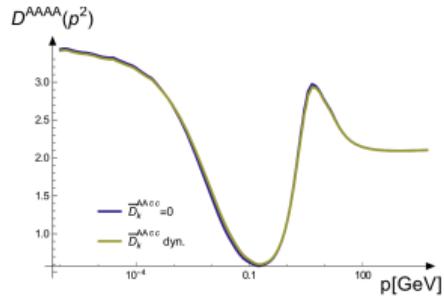
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- Two-ghost-two-gluon vertex [MQH, Eur. Phys.J.C77 (2017)]: ✓
(FRG: [Corell, SciPost Phys. 5 (2018)])



Markus Q. Huber (Giessen University)

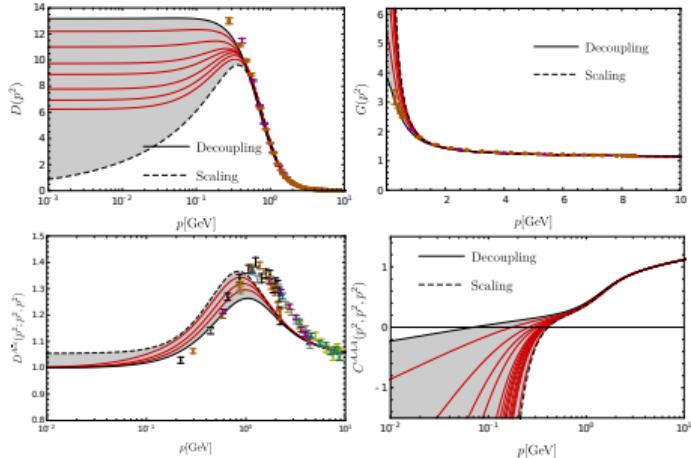


Glueball spectrum



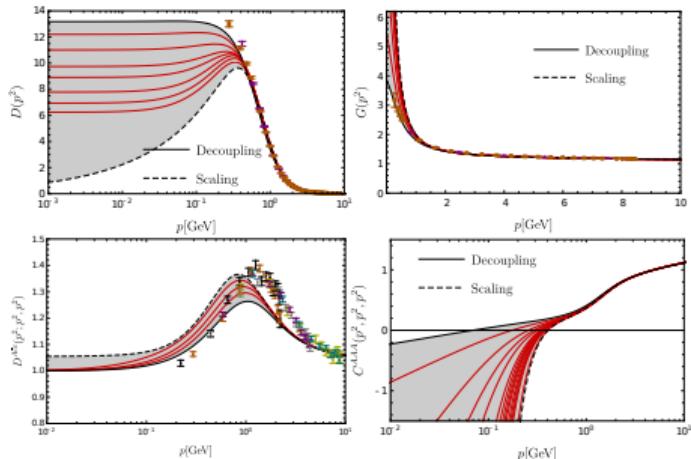
Glueball results J=0

Family of solutions:



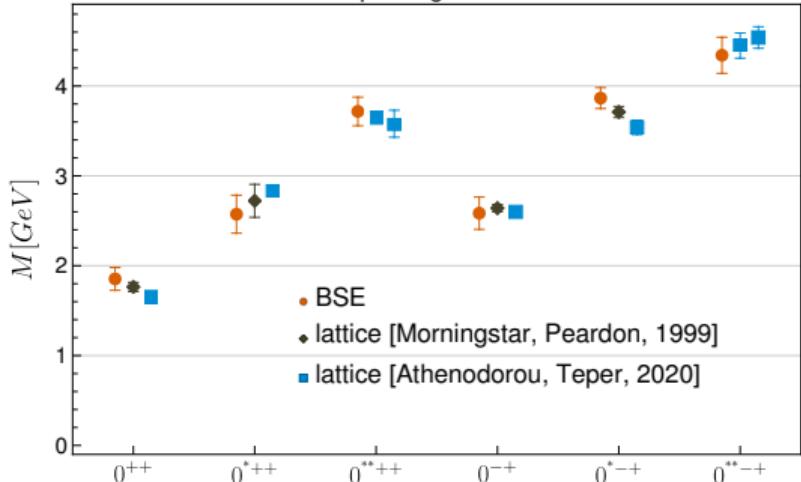
Glueball results J=0

Family of solutions:



Unique physical spectrum:

Spin-0 glueballs

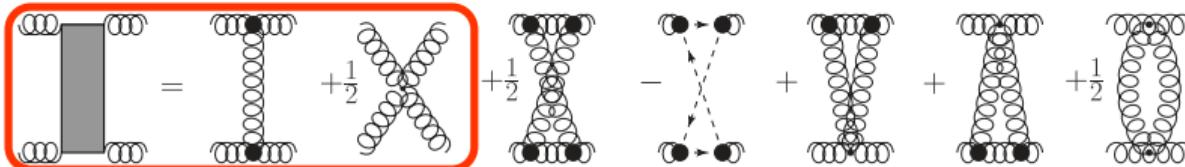


Spectrum independent! → Family of solutions yields the same physics.

All results for $r_0 = 1/418(5)$ MeV.

[MQH, Fischer, Sanchis-Alepuz, Eur.Phys.J.C80 (2020)]

Higher order diagrams



One-loop diagrams only:

[MQH, Fischer, Sanchis-Alepuz, Eur.Phys.J.C80 (2020); MQH, Fischer, Sanchis-Alepuz, Eur.Phys.J.C81 (2021)]

Two-loop diagrams: subleading effects

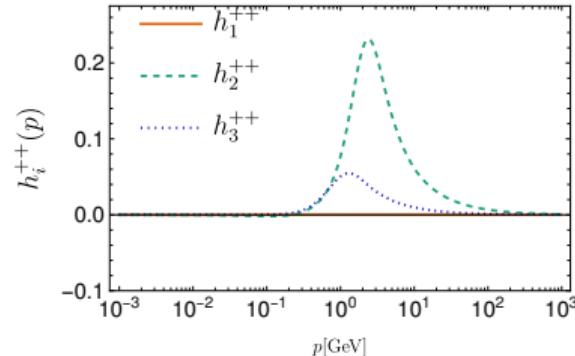
- 0^{-+} : none [MQH, Fischer, Sanchis-Alepuz, EPJ Web Conf. 258 (2022)]
- 0^{++} : < 2% [MQH, Fischer, Sanchis-Alepuz, HADRON2021, arXiv:2201.05163]

Amplitudes

Information about significance of single parts.

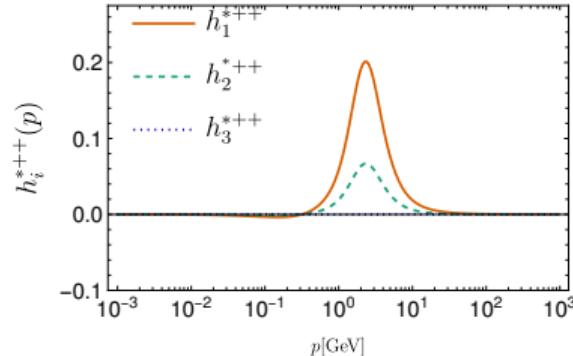
Ground state scalar glueball:

Amplitudes 0^{++}



Excited scalar glueball:

Amplitudes 0^{*++}

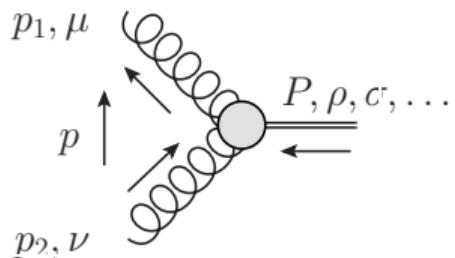


- Amplitudes have different behavior for ground state and excited state. Useful guide for future developments.
- Meson/glueball amplitudes: **Information about mixing.**

Glueball amplitudes for spin J

[MQH, Fischer, Sanchis-Alepuz, Eur.Phys.J.C81 (2021)]

$$\Gamma_{\mu\nu\rho\sigma\dots}(p_1, p_2) = \sum \tau^i_{\mu\nu\rho\sigma\dots}(p_1, p_2) h_i(p_1, p_2)$$



Increase in complexity:

- 2 gluon indices (transverse)
- J spin indices (symmetric, traceless, transverse to P)

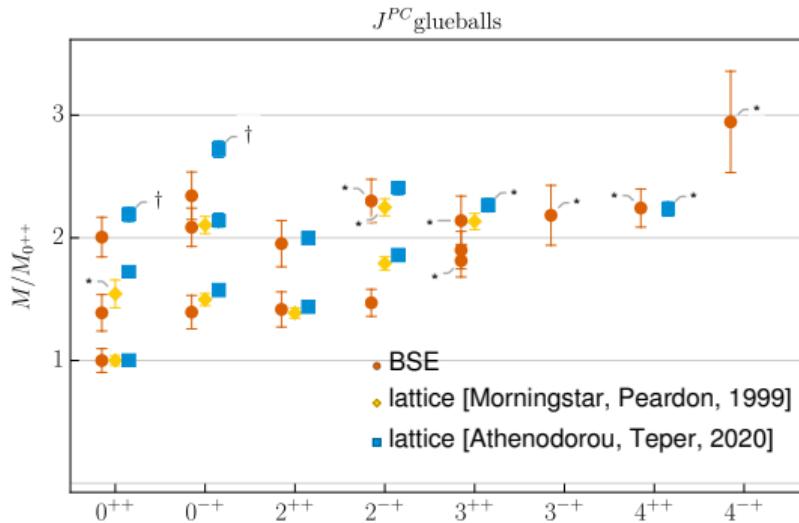
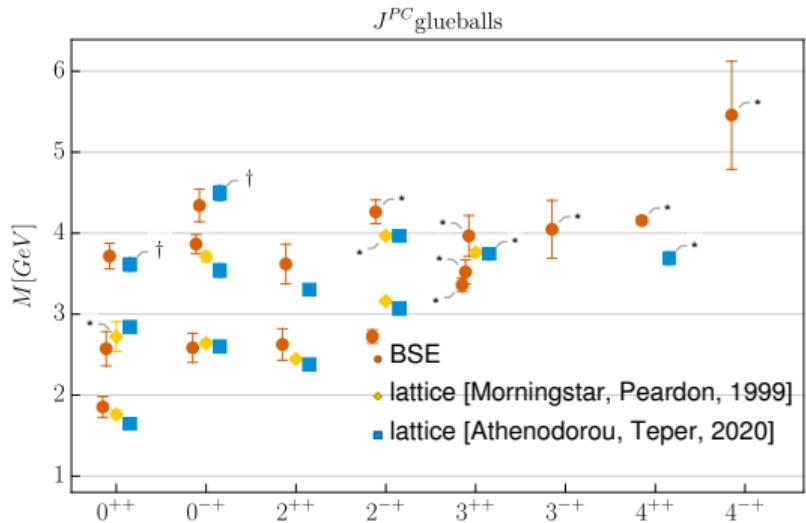
Numbers of tensors:

J	$P = +$	$P = -$
0	2	1
1	4	3
>2	5	4

Low number of tensors, but high-dimensional tensors!

→ Computational cost increases with J .

Glueball results



[MQH, Fischer, Sanchis-Alepuz, Eur.Phys.J.C81 (2021)]

*: identification with some uncertainty

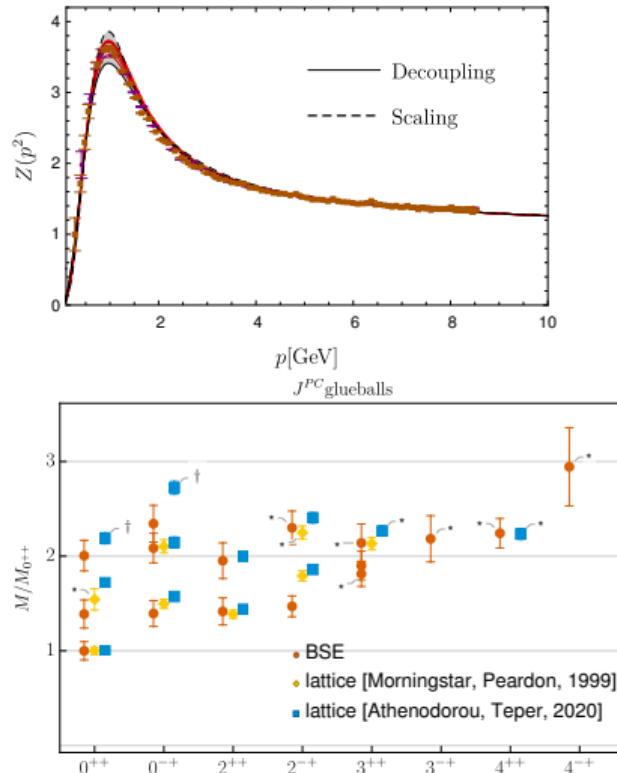
†: conjecture based on irred. rep of octahedral group

- Agreement with lattice results
- (New states: 0^{***++} , 0^{**-+} , 3^{-+} , 4^{-+})

Summary and outlook

- Alternative to models in bound state equations: **Direct calculation** of input.
- Large system of equations may be necessary.
- **Independent tests:**
 - Agreement with other methods:
lattice + continuum
 - Extensions

Spectrum from **first principles** for pure glueballs.



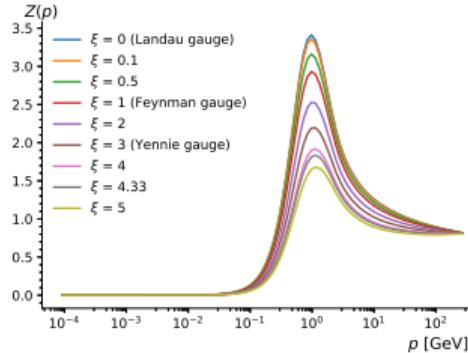
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Extensions:

- Real QCD
- Beyond Landau gauge: Linear covariant gauges [Napetschnig, Alkofer, MQH, Pawlowski, Phys.Rev.D 104 (2021)]



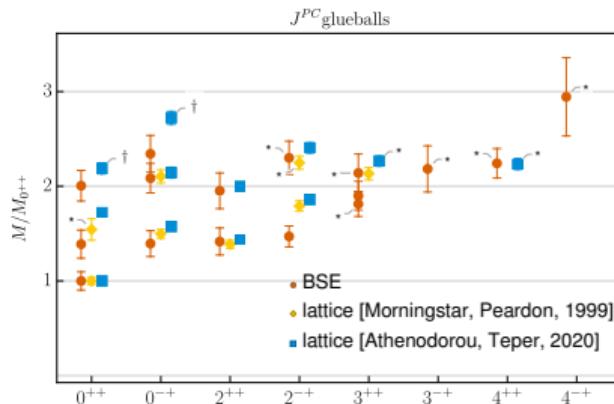
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Thank you for your attention.

$J = 1$ glueballs

Landau-Yang theorem

Two-photon states cannot couple to $J^P = \mathbf{1}^\pm$ or $(2n+1)^-$

[Landau, Dokl.Akad.Nauk SSSR 60 (1948); Yang, Phys. Rev. 77 (1950)].

(→ Exclusion of $J = 1$ for Higgs because of $h \rightarrow \gamma\gamma$.)

Applicable to glueballs?

- Not in this framework, since gluons are not on-shell.
- Presence of $J = 1$ states is a dynamical question.

$J = 1$ not found here.

Glueballs as bound states

Hadron masses from correlation functions of color singlet operators.

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Example: For $J^{PC} = 0^{++}$ glueball take $O(x) = F_{\mu\nu}(x)F^{\mu\nu}(x)$:

$$D(\mathbf{x} - \mathbf{y}) = \langle O(x)O(y) \rangle$$

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Lattice: Mass exponential Euclidean time decay:

$$\lim_{t \rightarrow \infty} \langle O(x)O(0) \rangle \sim e^{-tM}$$

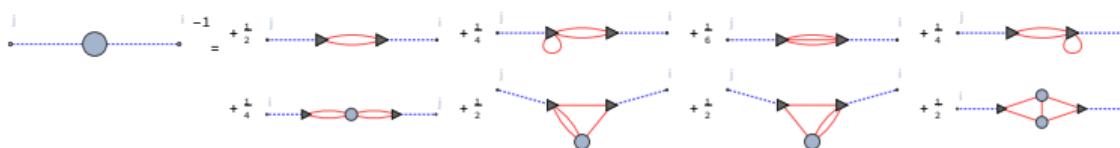
Glueballs as bound states

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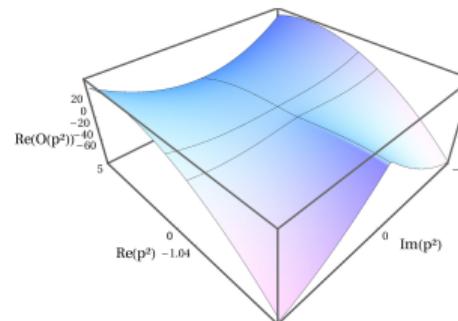
Functional approach: Complicated object in a diagrammatic language, 2-, 3- and 4-gluon contributions [MQH, Cyrol, Pawlowski, Comput.Phys.Commun. 248 (2020)]



+ 3-loop diagrams

Leading order:

[Windisch, MQH, Alkofer, Phys.Rev.D87 (2013)]



Glueballs as bound states

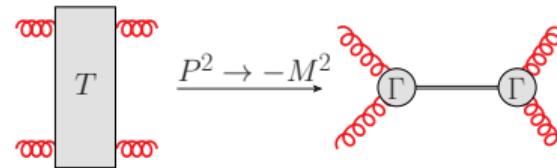
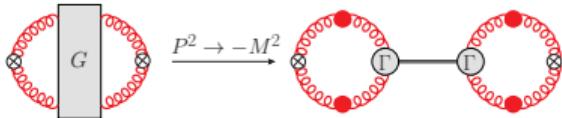
Hadron masses from correlation functions of color singlet operators.

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Put total momentum **on-shell** and consider individual 2-, 3- and 4-gluon contributions. →
Each can have a pole at the glueball mass.

A^4 -part of $D(x - y)$, total momentum on-shell:



Kernel construction

From 3PI effective action truncated to three-loops:

[Fukuda, Prog. Theor. Phys 78 (1987); McKay, Munczek, Phys. Rev. D 40 (1989); Sanchis-Alepuz, Williams, J. Phys: Conf. Ser. 631 (2015); MQH, Fischer, Sanchis-Alepuz, Eur.Phys.J.C80 (2020)]

$$\Gamma^{3l}[\Phi, D, \Gamma^{(3)}] = \Gamma^{0,3l}[\Phi, D, \Gamma^{(3)}] + \Gamma^{\text{int},3l}[\Phi, D, \Gamma^{(3)}]$$

$$\Gamma^{0,3l}[\Phi, D, \Gamma^{(3)}] = \frac{1}{8} \text{ (two loops)} + \frac{1}{6} \text{ (one loop)} - \frac{1}{48} \text{ (one loop)}$$

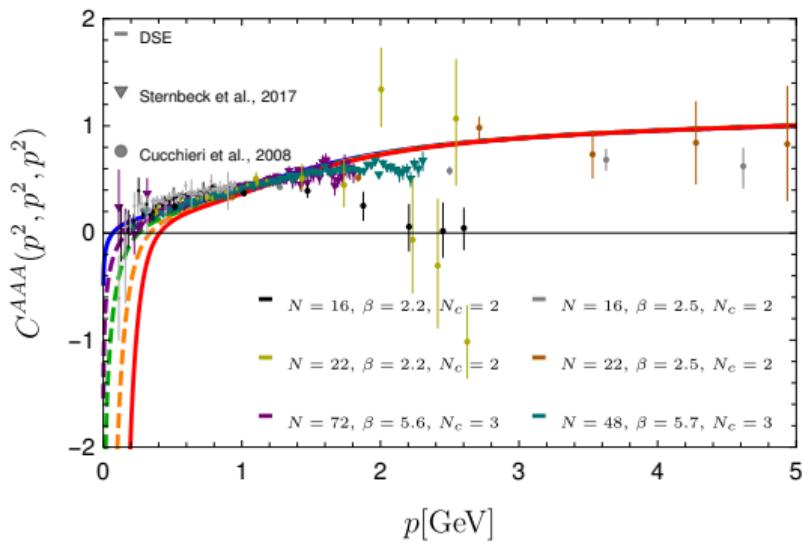
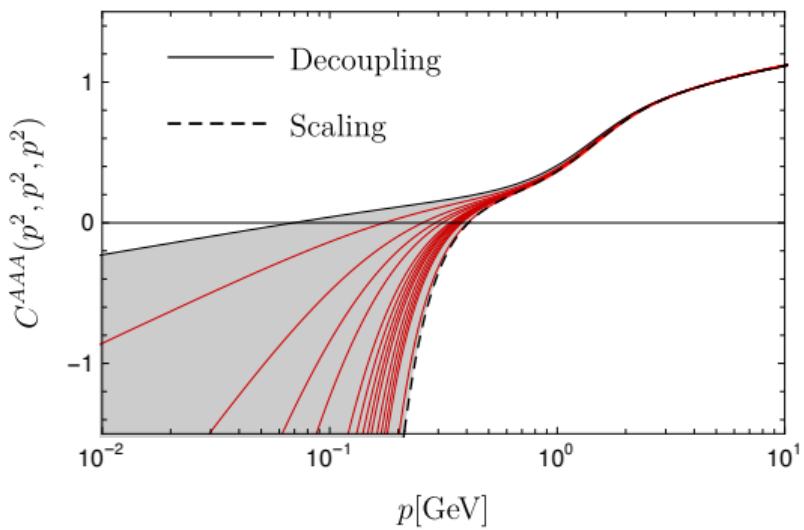
$$\Gamma^{\text{int},3l}[D, \Gamma^{(3)}] = -\frac{1}{12} \text{ (one loop)} + \frac{1}{2} \text{ (one loop)} + \frac{1}{24} \text{ (one loop)} - \frac{1}{3} \text{ (one loop)} - \frac{1}{4} \text{ (one loop)}$$

Kernels constructed by cutting two legs:

gluon/gluon, ghost/gluon, gluon/ghost, ghost/ghost

Three-gluon vertex

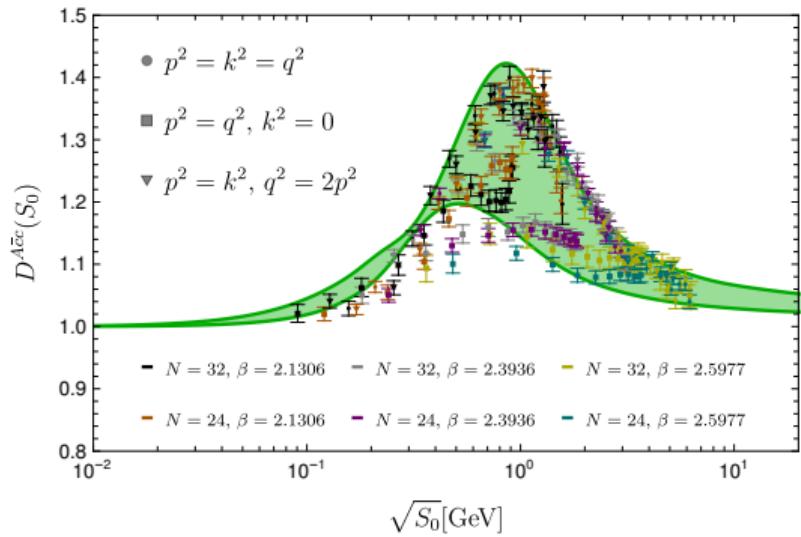
[Cucchieri, Maas, Mendes, Phys. Rev. D 77 (2008); Sternbeck et al., 1702.00612; MQH, Phys. Rev. D 101 (2020)]



- Simple kinematic dependence of three-gluon vertex (only singlet variable of S_3)
- Large cancellations between diagrams

Ghost-gluon vertex

Ghost-gluon vertex:



[Maas, SciPost Phys. 8 (2019);
MQH, Phys. Rev. D 101 (2020)]

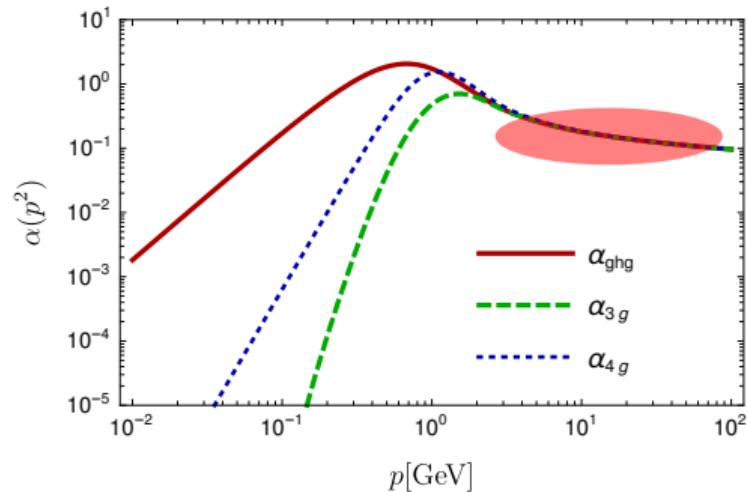
- Nontrivial kinematic dependence of ghost-gluon vertex
- Qualitative agreement with lattice results, though some quantitative differences (position of peak!).

Gauge invariance

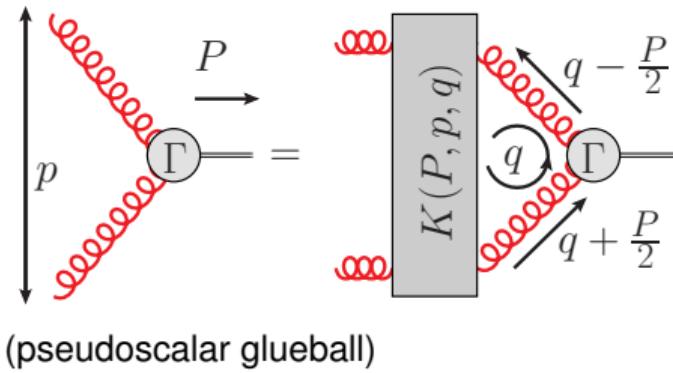
[MQH, Phys. Rev. D 101 (2020)]

Couplings can be extracted from each vertex.

- Slavnov-Taylor identities (gauge invariance): Agreement perturbatively (UV) necessary.
[Cyrol et al., Phys. Rev. D 94 (2016)]
- Difficult to realize: Small deviations → Couplings cross and do not agree.
- Here: Vertex couplings agree down to GeV regime (IR can be different).



Solving a bound state equation

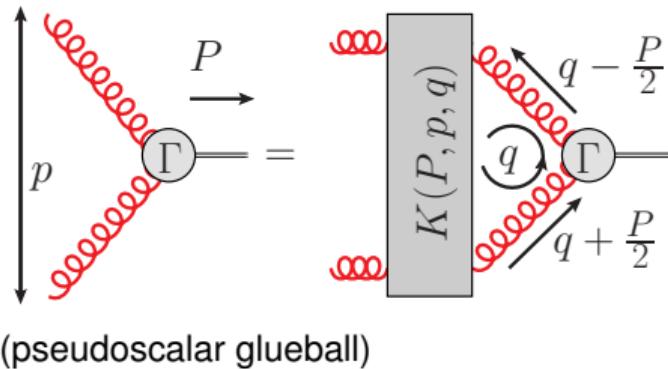


$$\lambda(\mathbf{P})\Gamma(P) = \mathcal{K} \cdot \Gamma(P)$$

→ Eigenvalue problem for $\Gamma(P)$:

- ① Solve for $\lambda(P)$.
- ② Find P with $\lambda(P) = 1$.
 $\Rightarrow M^2 = -P^2$

Solving a bound state equation



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→ Eigenvalue problem for $\Gamma(P)$:

- ① Solve for $\lambda(P)$.
- ② Find P with $\lambda(P) = 1$.
⇒ $M^2 = -P^2$

However:

Propagators are probed at $\left(q \pm \frac{P}{2}\right)^2 = \frac{P^2}{4} + q^2 \pm \sqrt{P^2 q^2} \cos \theta = -\frac{M^2}{4} + q^2 \pm i M \sqrt{q^2} \cos \theta$
→ Complex for $P^2 < 0$!

Time-like quantities ($P^2 < 0$) → Correlation functions for complex arguments.

Extrapolation of $\lambda(P^2)$

Extrapolation method

- Extrapolation to time-like P^2 using Schlessinger's continued fraction method (proven superior to default Padé approximants) [Schlessinger, Phys.Rev.167 (1968)]
- Average over extrapolations using subsets of points for error estimate

$$f(x) = \frac{f(x_1)}{1 + \frac{a_1(x-x_1)}{1 + \frac{a_2(x-x_2)}{1 + \frac{a_3(x-x_3)}{\dots}}}}$$

Coefficients a_i can
determined such that
 $f(x)$ exact at x_i .

Extrapolation of $\lambda(P^2)$

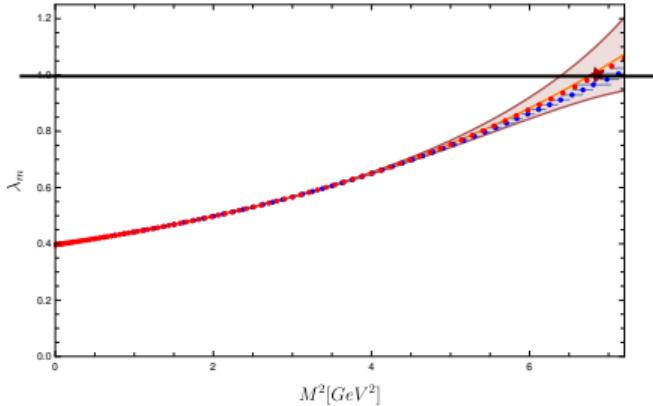
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Test extrapolation for solvable system:
Heavy meson [MQH, Sanchis-Alepuz, Fischer, Eur.Phys.J.C 80 (2020)]

$$f(x) = \frac{f(x_1)}{1 + \frac{a_1(x-x_1)}{1 + \frac{a_2(x-x_2)}{1 + \frac{a_3(x-x_3)}{\dots}}}}$$

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Landau gauge propagators in the complex plane

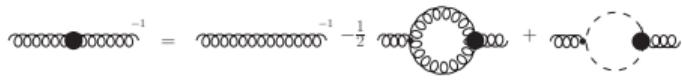
Simpler truncation:

$$\text{Diagram with two external gluons and a loop}^{-1} = \text{Diagram with three external gluons}^{-1} - \frac{1}{2} \text{Diagram with one external gluon and a loop} + \text{Diagram with one external gluon and a dashed loop}$$

[Fischer, MQH, Phys.Rev.D 102 (2020)]

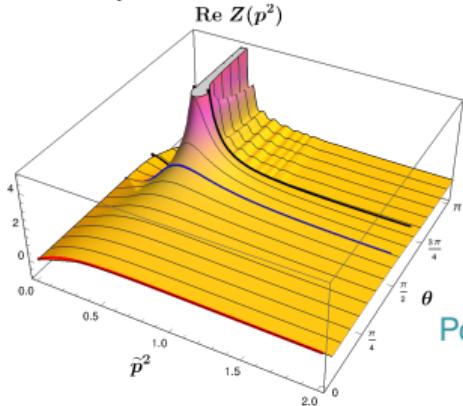
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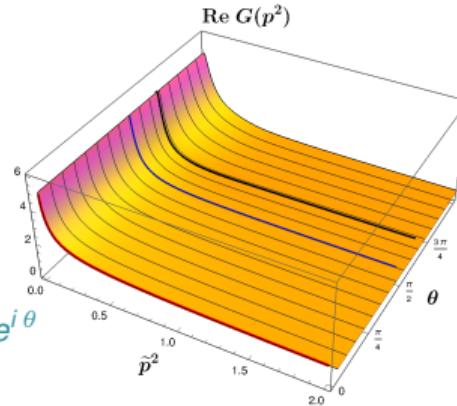


[Fischer, MQH, Phys.Rev.D 102 (2020)]

Ray technique for self-consistent solution of a DSE:



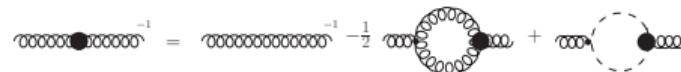
$$\text{Polar coordinates: } p^2 = \tilde{p}^2 e^{i\theta}$$



- Current truncation leads to a pole-like structure in the gluon propagator.
- Analyticity up to 'pole' confirmed by various tests (Cauchy-Riemann, Schlessinger, reconstruction)

Landau gauge propagators in the complex plane

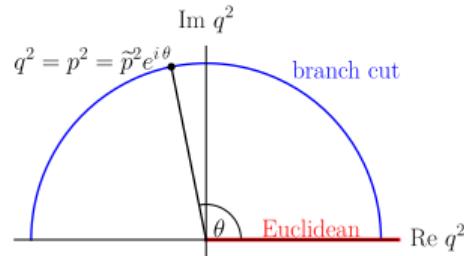
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Landau gauge propagators in the complex plane

Simpler truncation:

$$\text{propagator}^{-1} = \text{propagator}^{-1} - \frac{1}{2} \text{loop} + \text{loop}$$

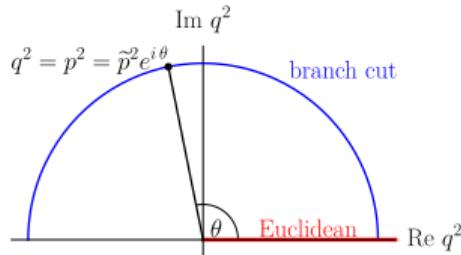


→ Opening at $q^2 = p^2$.

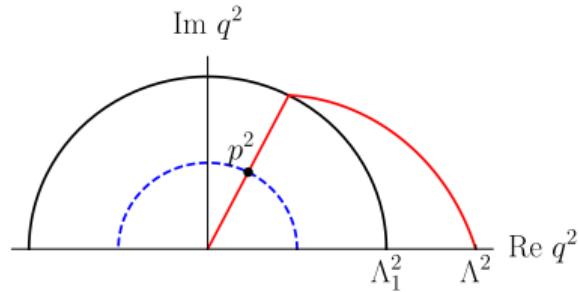
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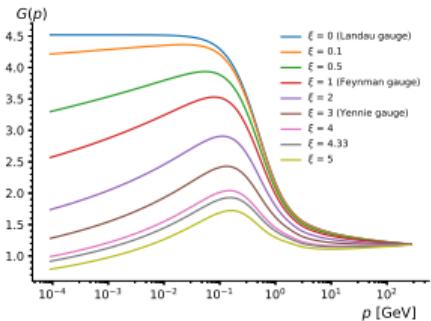
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Appearance of branch cuts for complex momenta forbids integration directly to cutoff.

Ghost propagator in linear covariant gauges

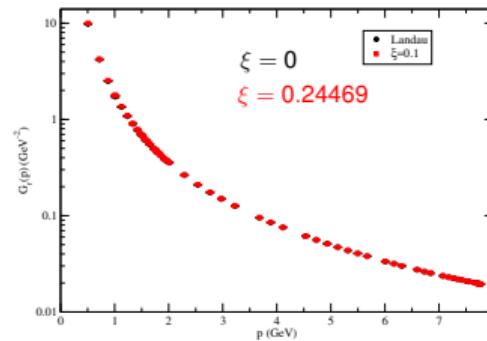
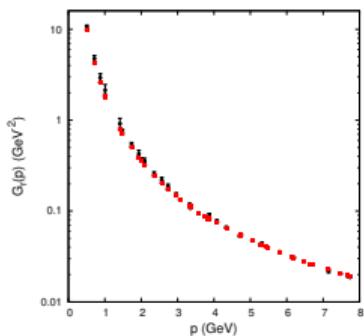
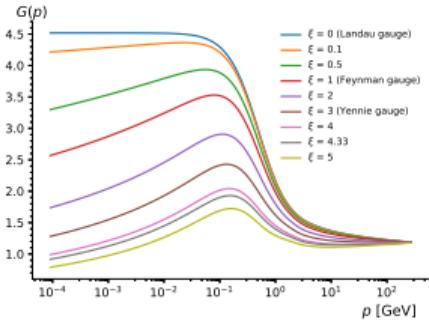
[Napetschnig, Alkofer, MQH, Pawłowski, Phys.Rev.D 104 (2021)]



- Logarithmic IR suppression for $\xi > 0$
[Aguilar, Binosi, J. Papavassiliou, Phys.Rev. D91 (2015); MQH, Phys. Rev. D91 (2015)]
- Otherwise effects small for low ξ .

Ghost propagator in linear covariant gauges

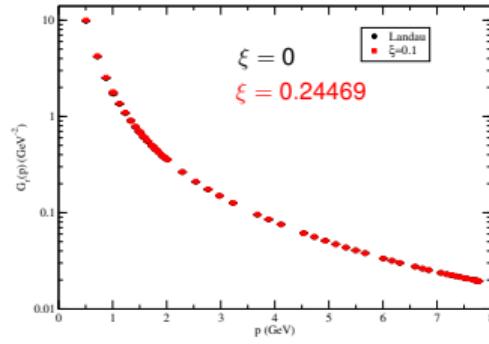
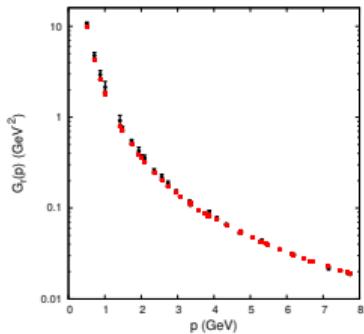
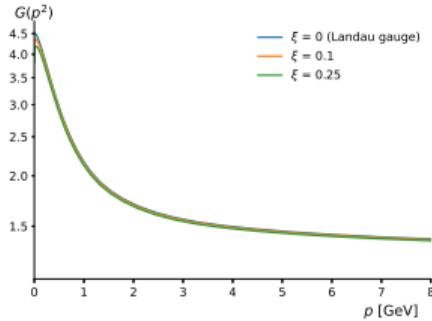
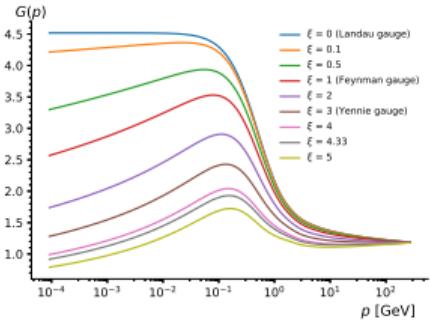
[Napetschnig, Alkofer, MQH, Pawłowski, Phys.Rev.D 104 (2021)]



[Cucchieri et al. '18]

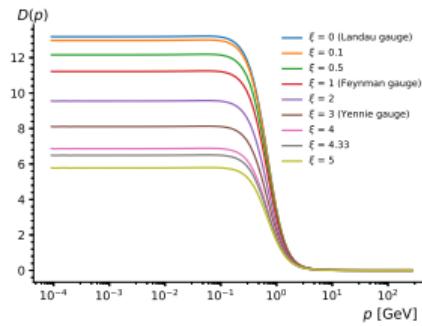
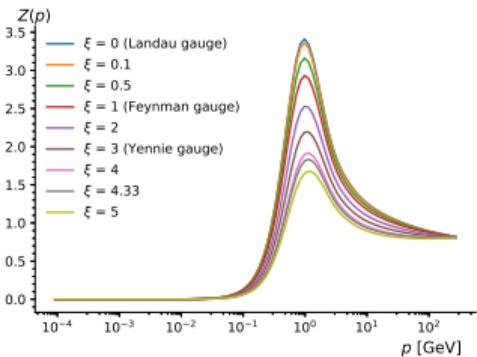
Ghost propagator in linear covariant gauges

[Napetschnig, Alkofer, MQH, Pawłowski, Phys.Rev.D 104 (2021)]



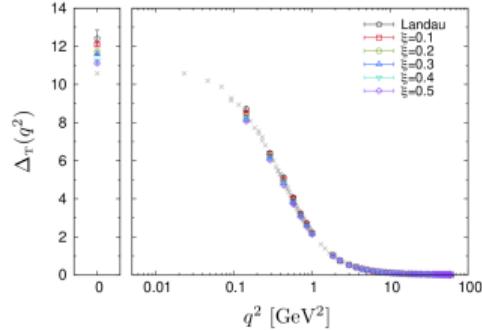
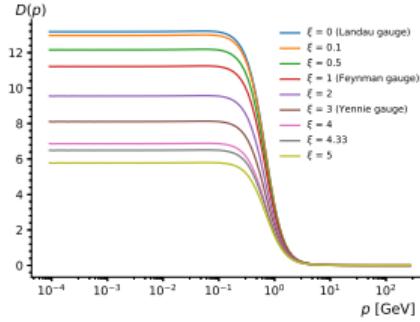
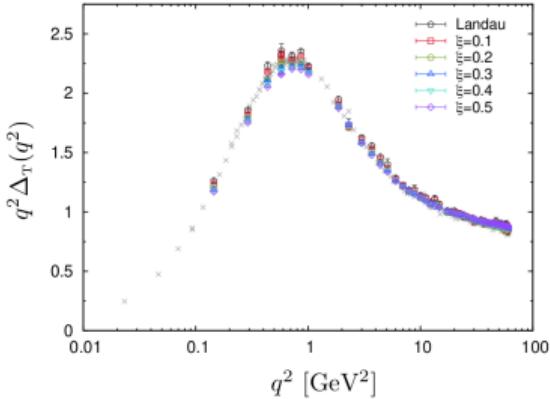
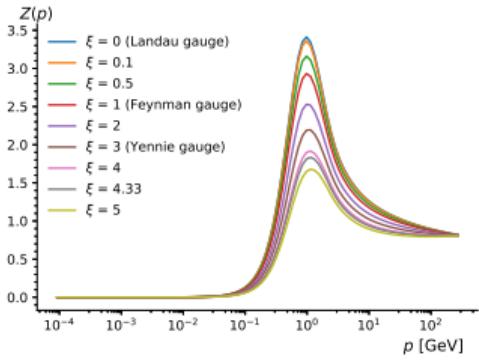
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Gluon propagator in linear covariant gauges



[Napetschnig, Alkofer, MQH,
Pawlowski, Phys.Rev.D 104 (2021)]

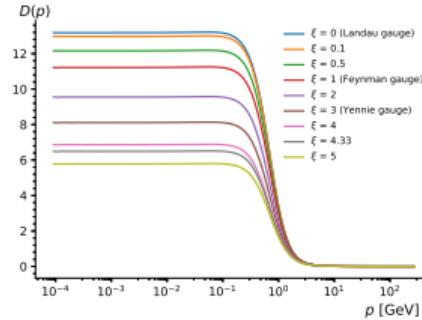
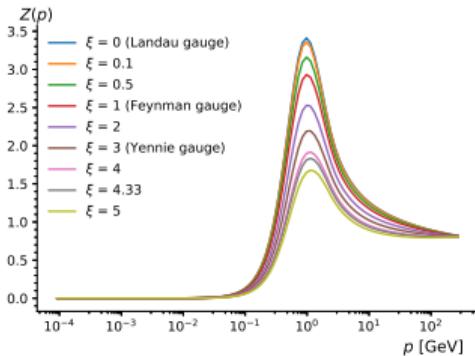
Gluon propagator in linear covariant gauges



[Napetschnig, Alkofer, MQH,
Pawlowski, Phys.Rev.D 104 (2021)]

[Bicudo et al., Phys. Rev.
D92 (2015)]

Gluon propagator in linear covariant gauges



[Napetschnig, Alkofer, MQH,
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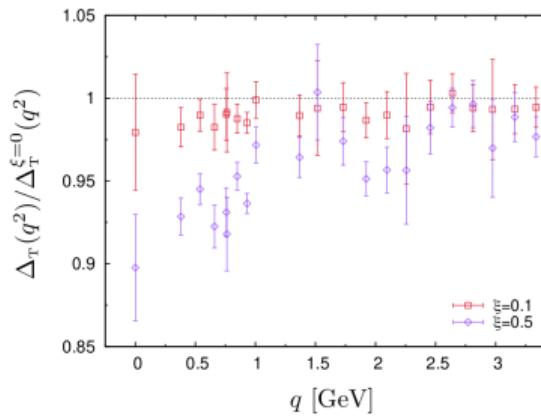
Ratios from Nielsen identities:

$\xi = 0.1$:

- 0 GeV: 0.98
- 1 GeV: 0.98

$\xi = 0.5$:

- 0 GeV: 0.92
- 1 GeV: 0.93



[Bicudo et al., Phys. Rev.
D92 (2015)]