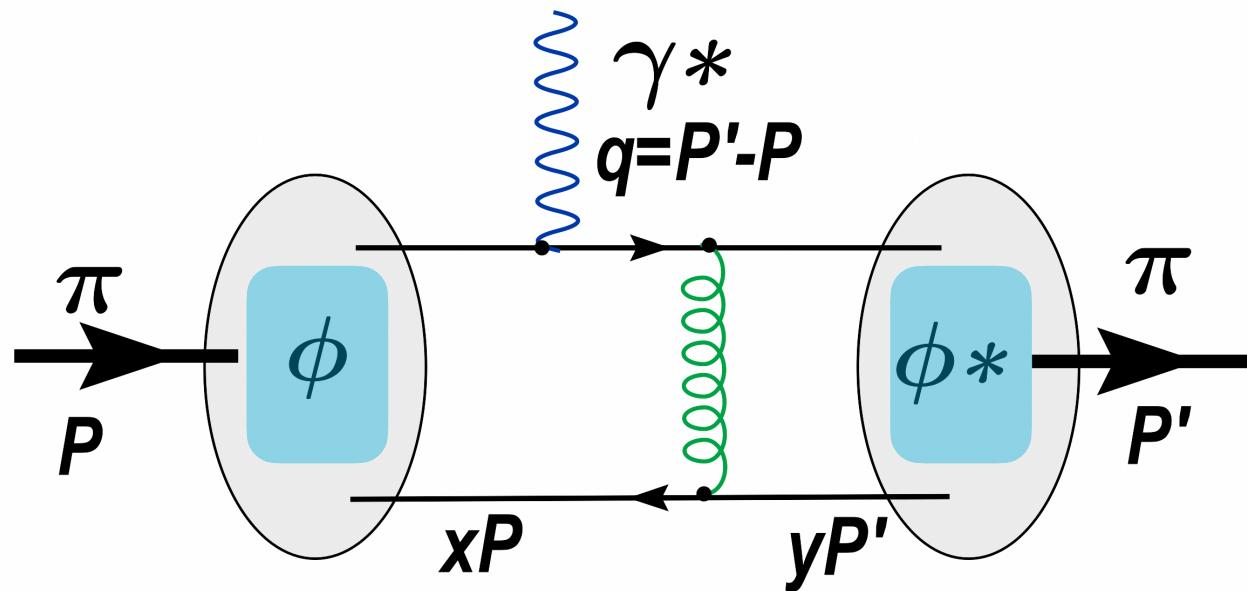


Partonic Pictures of Goldstone Mesons from Lattice QCD

meson EM form factors (FF)



very large
momentum
transfer
 $Q^2 \rightarrow \infty$

$$F(Q^2) = \int \int dx dy \phi^*(x, \mu^2) T(x, y, Q^2, \mu) \phi(y, \mu)$$

parton distribution amplitude (PDA) of meson

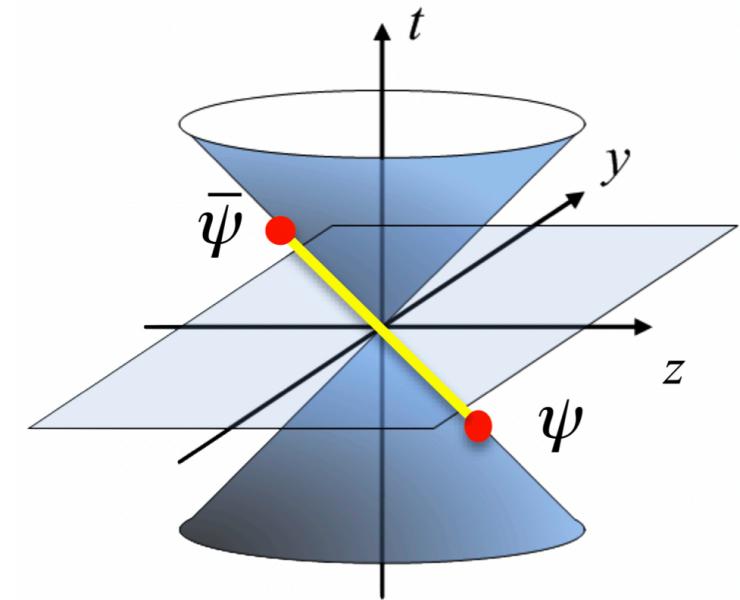
pion PDA

light-cone correlation function

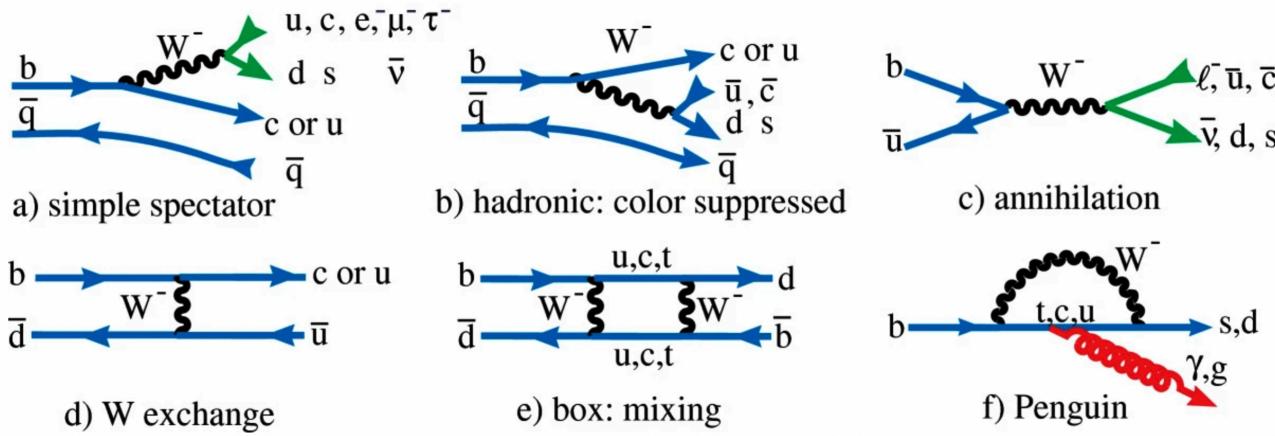
$$\phi(x, \mu) = i f_\pi P^+ \int d(P^+ z^-) e^{-ix(P^+ z^-)} \langle 0 | \bar{d}(-z^-/2) \gamma^+ \gamma_5 W_+ u(z^-/2) | \pi^+; P \rangle$$

'partonic picture'

- ➊ effective description of QCD as observed from an infinite momentum frame
 - infinite momentum limit first, regularize the theory later

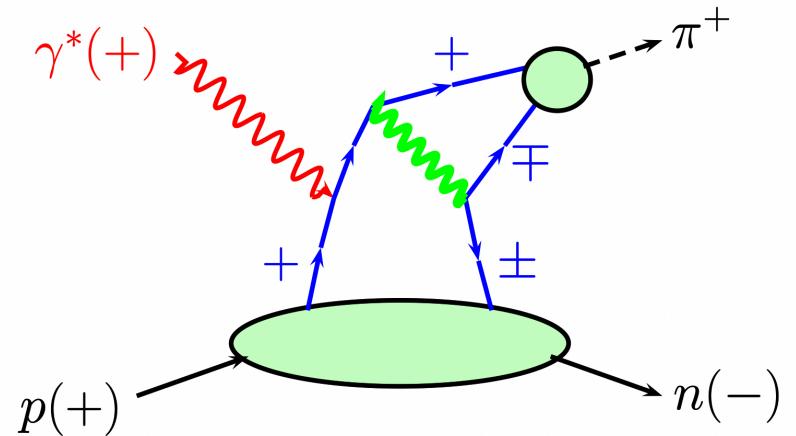


pion PDA in many high-energy process



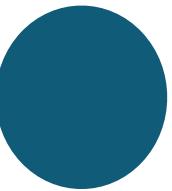
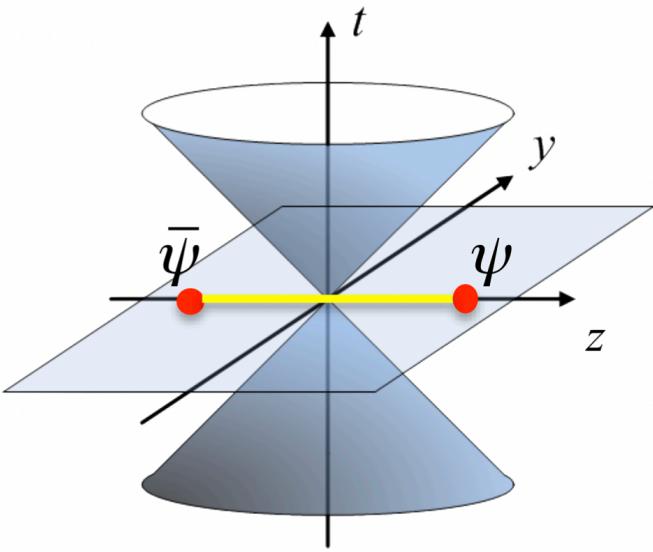
B-decays

DVMP

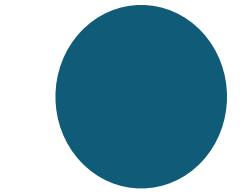
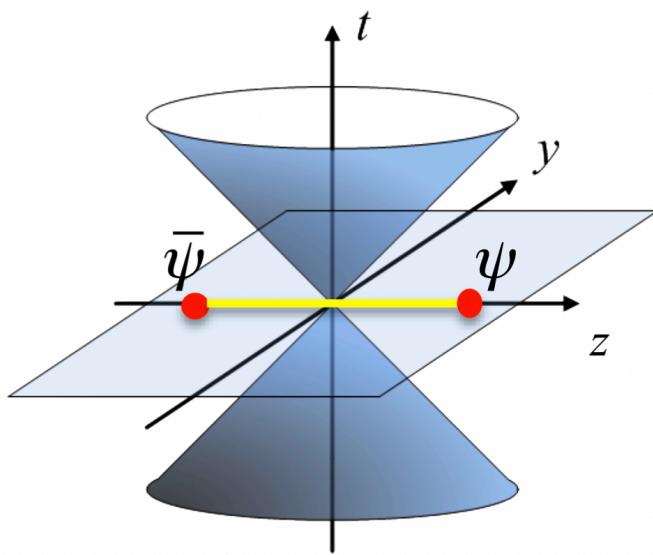


'partonic picture' from lattice QCD

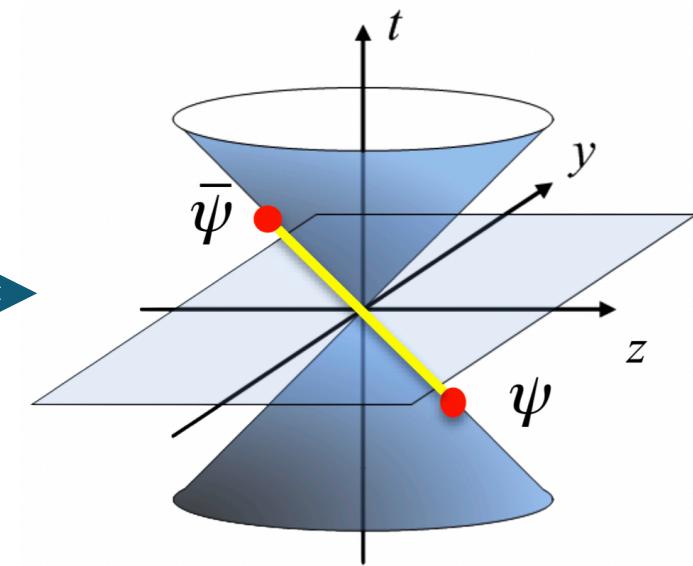
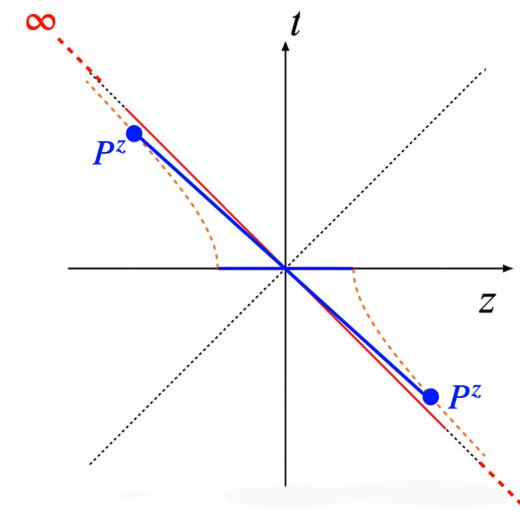
hadron at rest

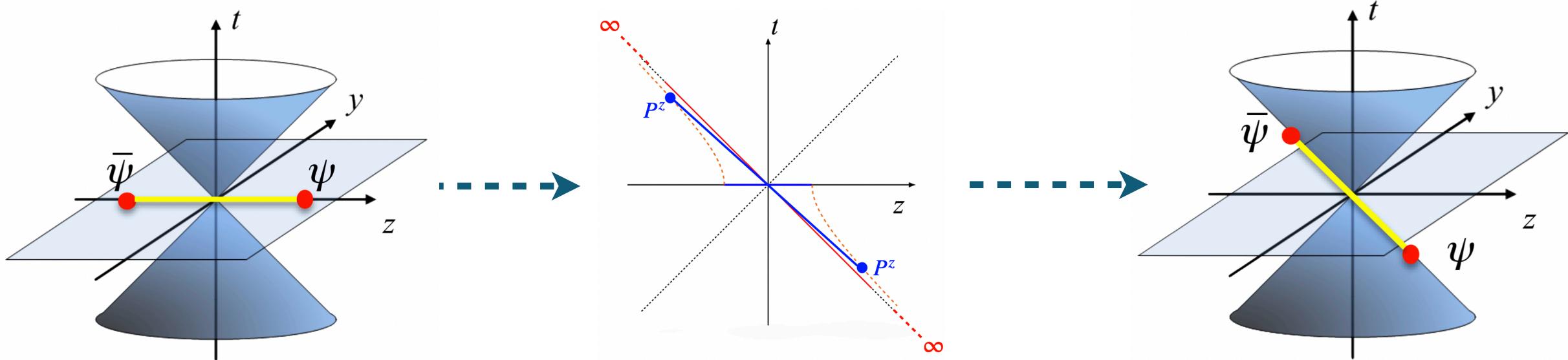


fast-moving hadron

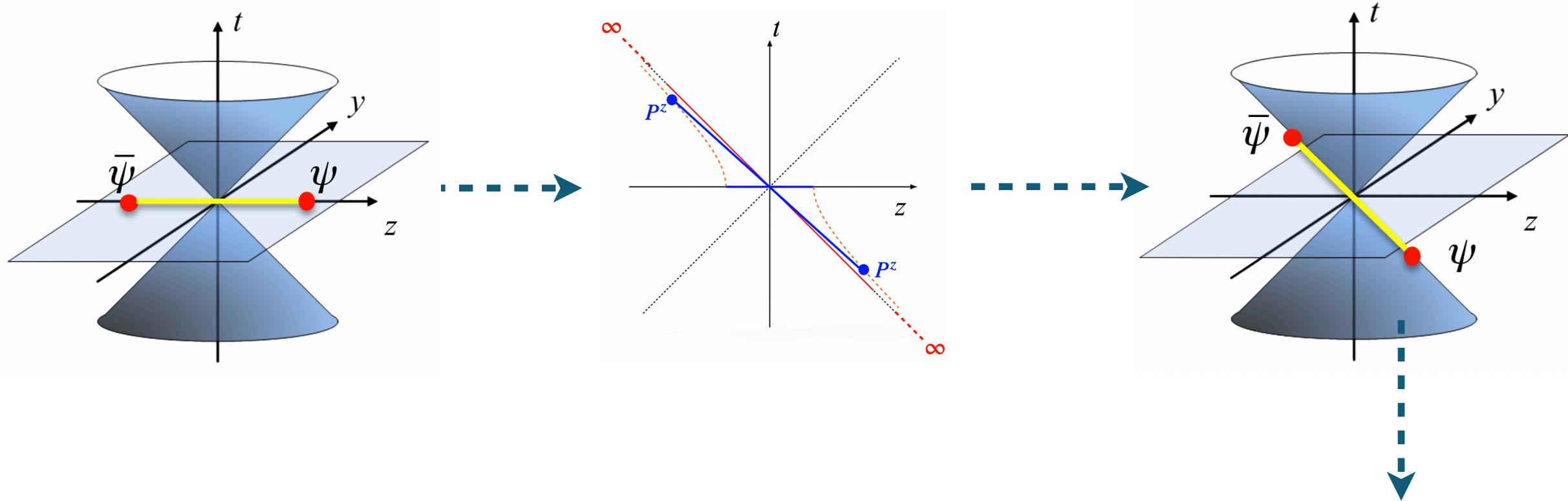


$$P_z \approx E$$





- regularize QCD first, infinite momentum limit later — opposite order of limits while seeing the ‘partonic picture’
- the 2 limits don’t commute; but it’s UV physics and can be ‘corrected’ through pQCD



'partonic picture'



$$+ \mathcal{O} \left(\frac{\Lambda_{\text{QCD}}^2}{x^2 P_z^2}, \frac{\Lambda_{\text{QCD}}^2}{(1-x)^2 P_z^2}, \frac{m^2}{P_z^2} \right)$$

$C(\omega, P_z, \mu) \otimes$

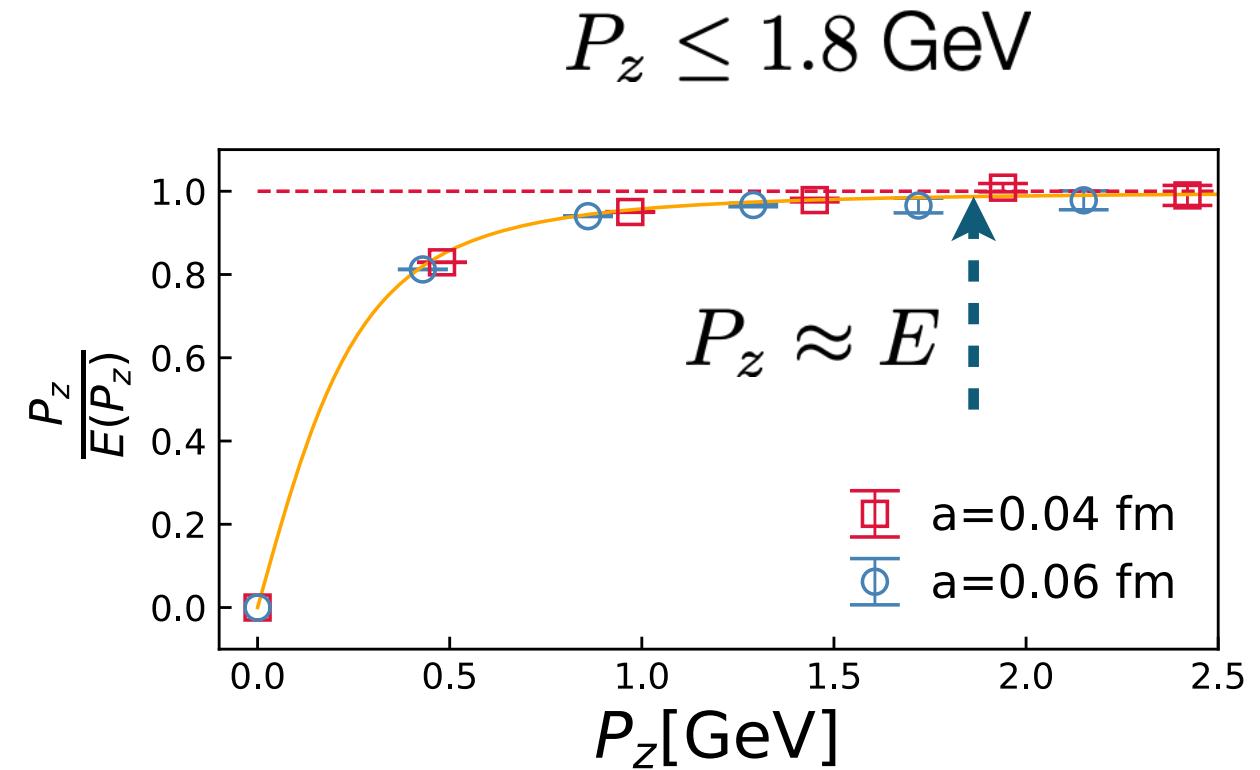
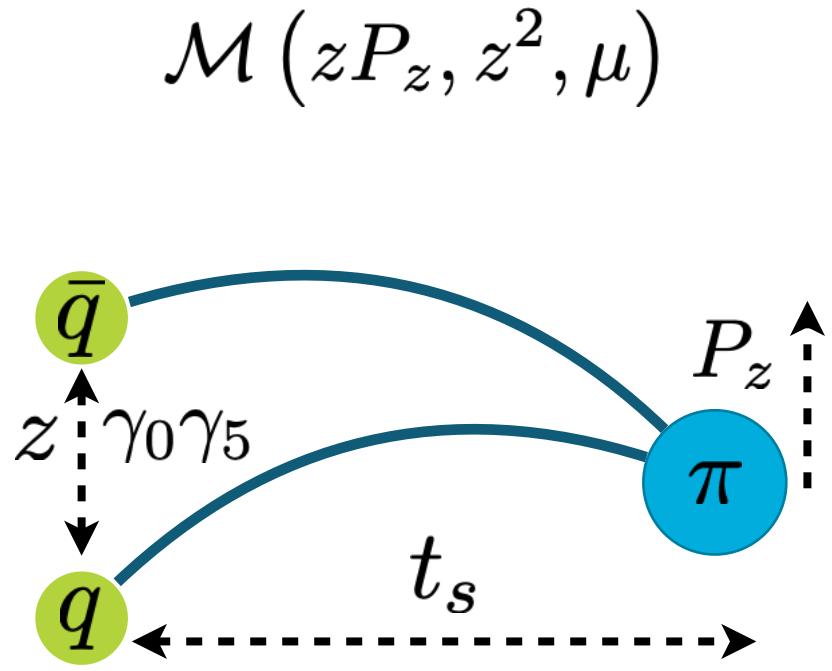
pQCD

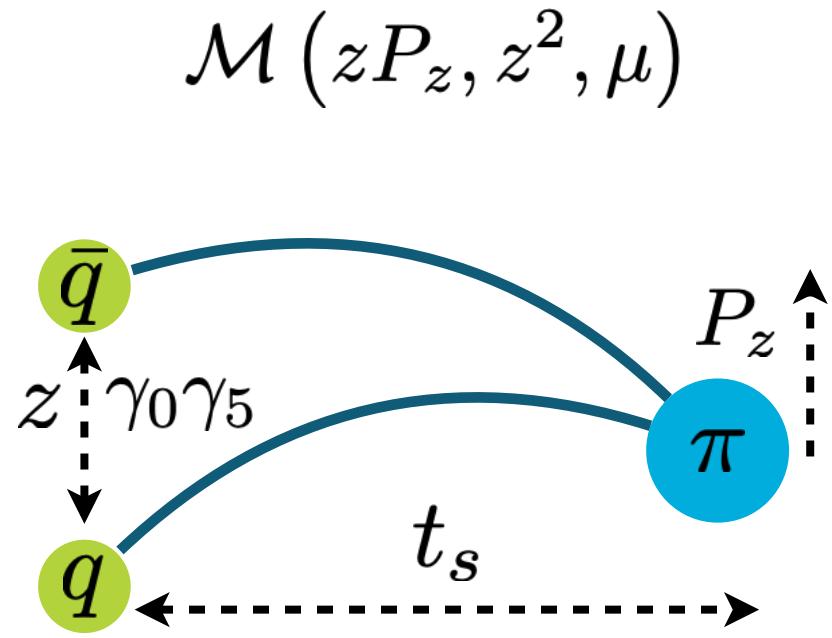


F.T. wrt z

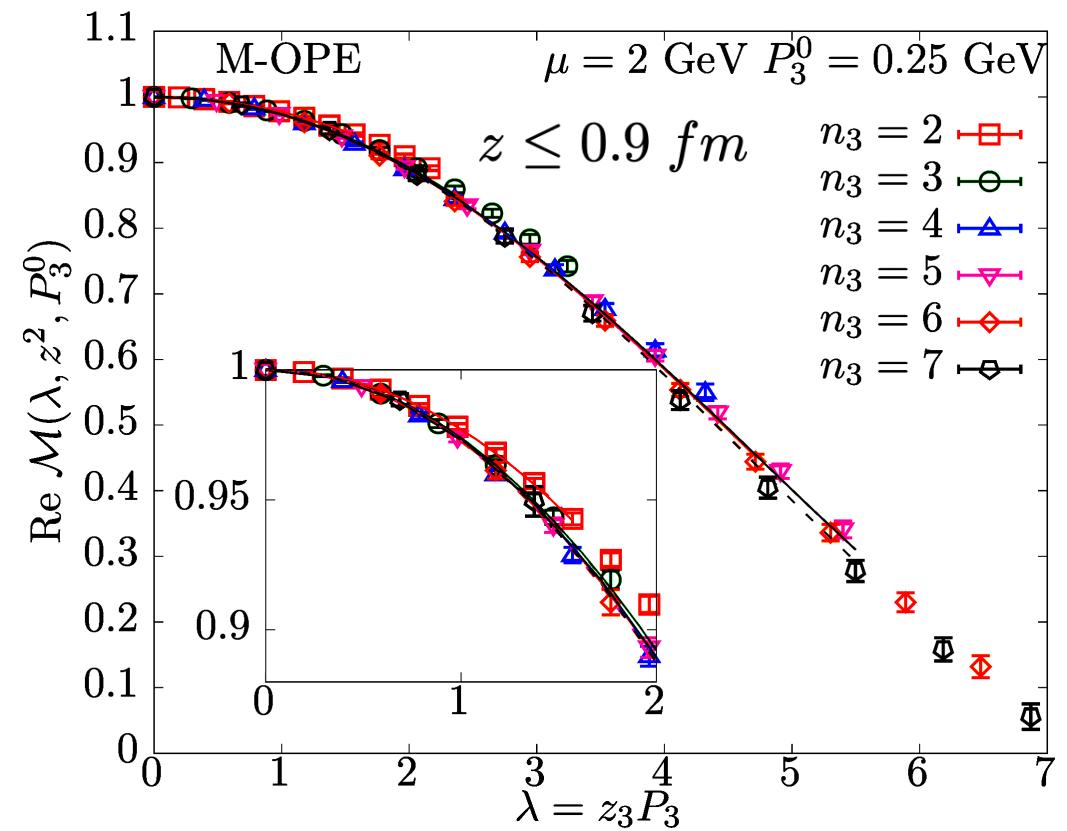
renormalize

pion PDA from lattice QCD





$$\mathcal{M}(zP_z, z^2, \mu) = \int d(zP_z) e^{-ix(zP_z)} C(\omega, zP_z, z^2\mu^2) \otimes \phi(\omega, x, \mu)$$



Mellin moments of PDA

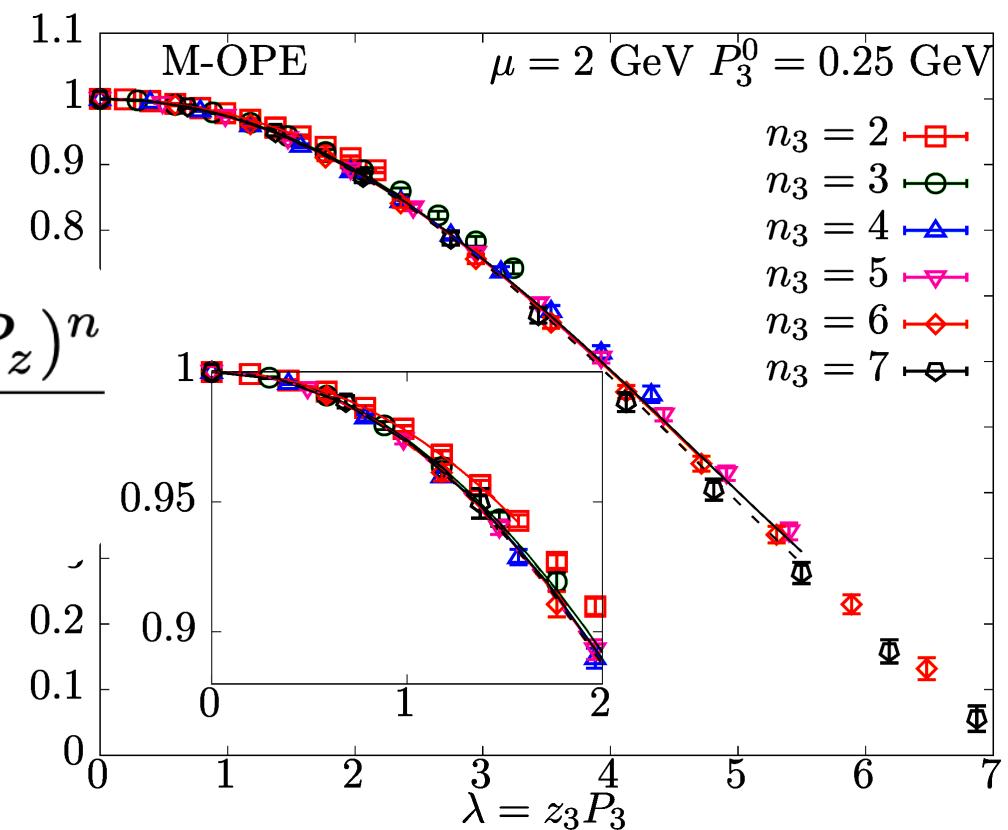
$$\langle x^n \rangle(\mu) = \int_{-1}^1 x^n \phi(x, \mu) dx$$

$$\mathcal{M}(zP_z, z^2, \mu) = \sum_n C_n (z^2 \mu^2) \langle x^n \rangle(\mu) \frac{(-izP_z)^n}{n!}$$

\downarrow

NLO pQCD

Radyushkin: Phys. Rev. D96, 034025 (2017), Phys. Lett. B781, 433 (2018)

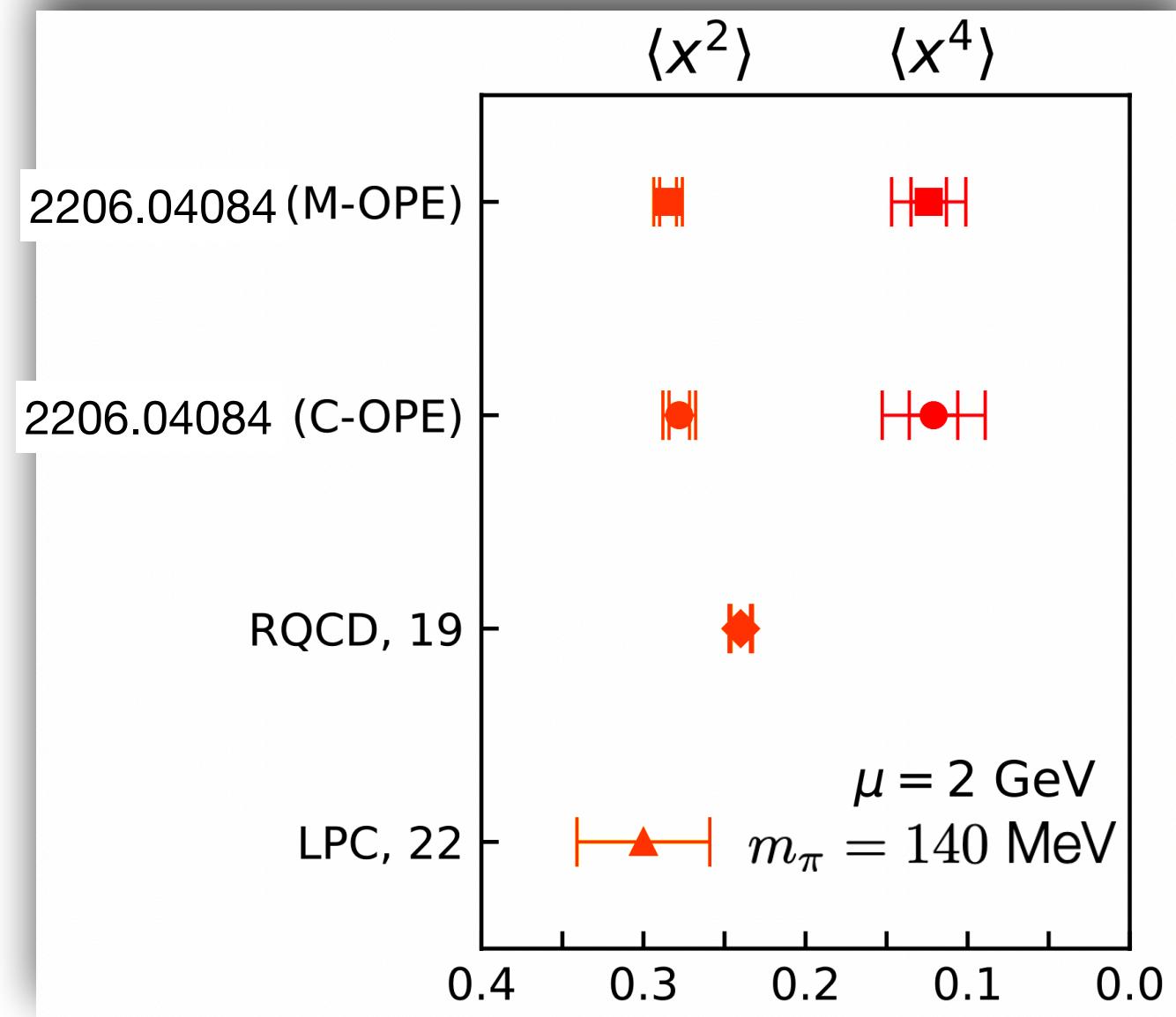


$$\langle x^2 \rangle = 0.287(6)(7)$$

$$\langle x^4 \rangle = 0.14(3)(3)$$

$$\langle x^6 \rangle = 0.1(1)$$

$$\langle x^8 \rangle = 0.3(4)$$

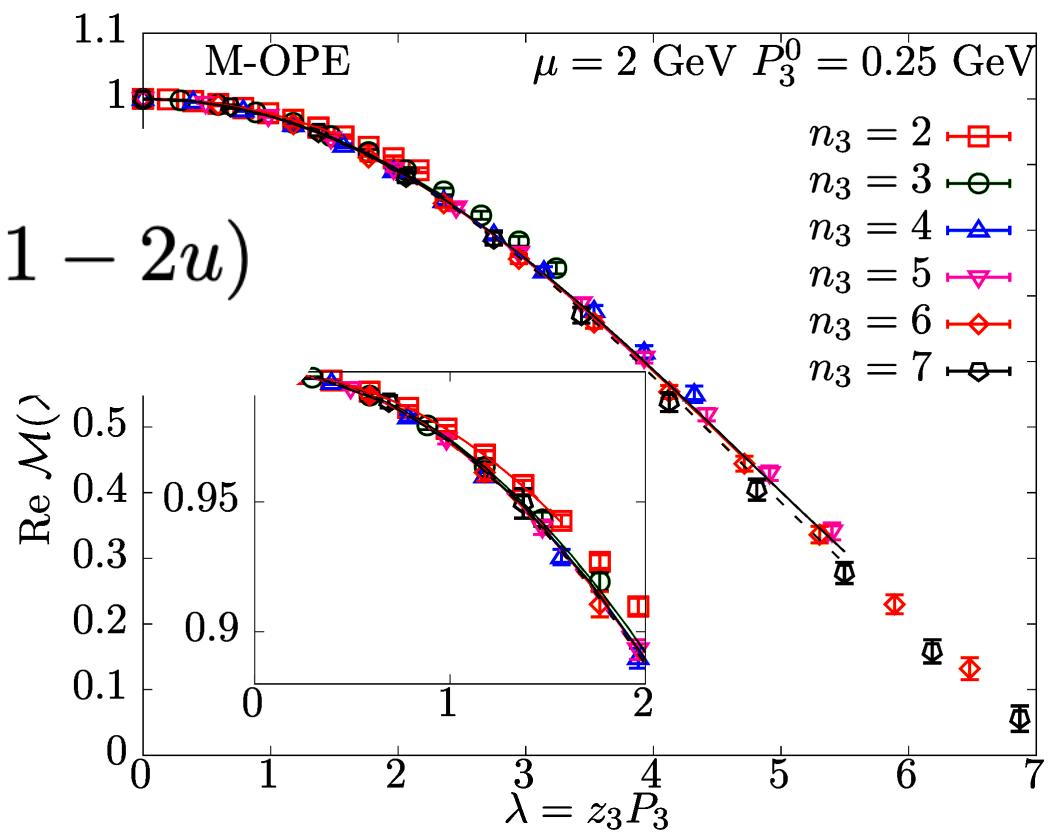


PDA fit function:

$$\phi(u) = \mathcal{N} u^\alpha (1-u)^\alpha \sum_{n=0}^{N_G+1} s_n C_{2n}^{\frac{1}{2}+\alpha} (1-2u)$$

$$|s_n| \lesssim \delta$$

$$u = (x+1)/2$$

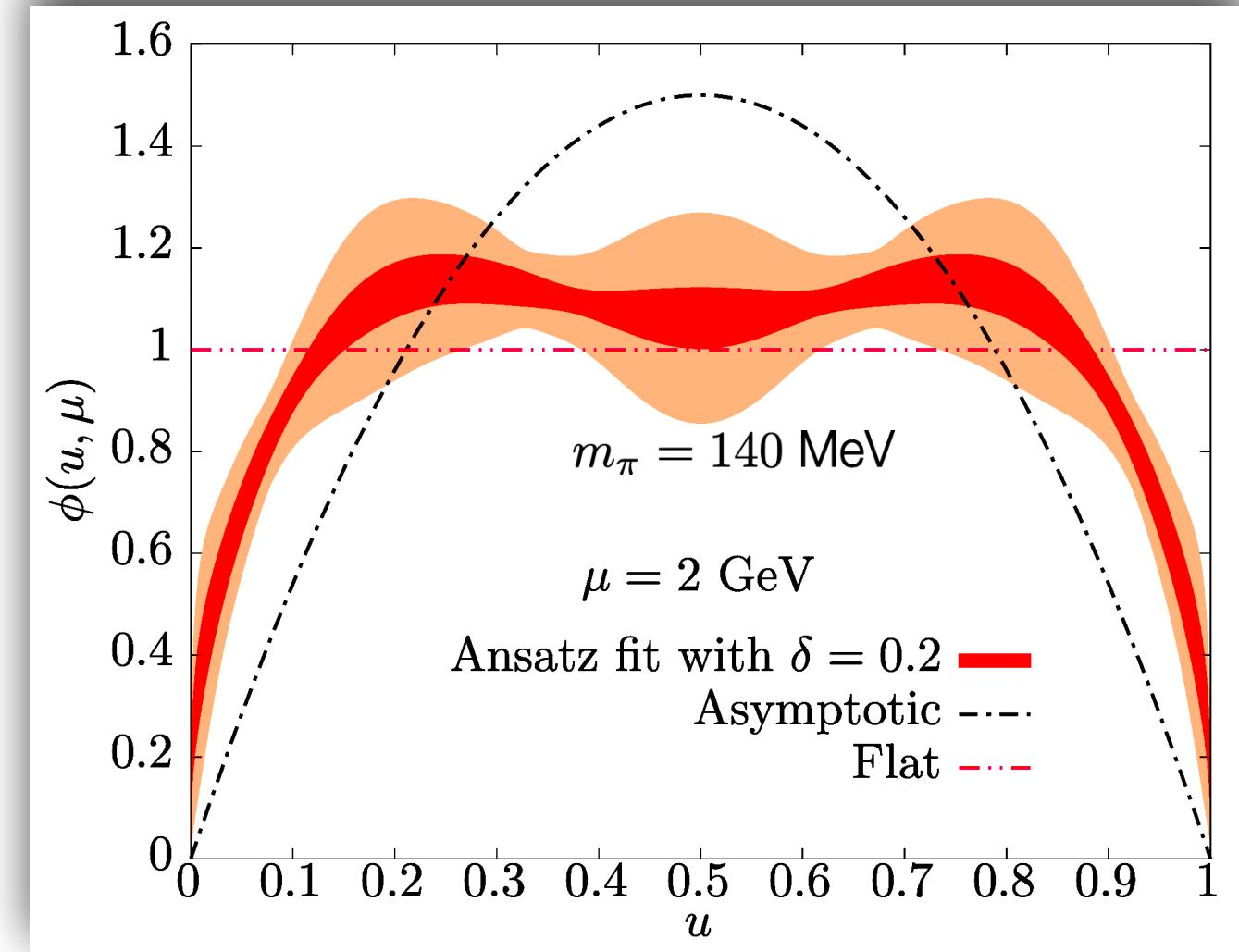


$$\mathcal{M}(zP_z, z^2, \mu) = \int d(zP_z) e^{-ix(zP_z)} C(\omega, zP_z, z^2\mu^2) \otimes \phi(\omega, x, \mu)$$

x -dependence of pion PDA

Nikhil Karthik et.al., arXiv:2206.04084

$$\phi(u, 2 \text{ GeV}) \approx u^{0.3}(1 - u)^{0.3}$$



$$u = (x + 1)/2$$

pion EM FF from PDA

Nikhil Karthik et.al., arXiv:2206.04084

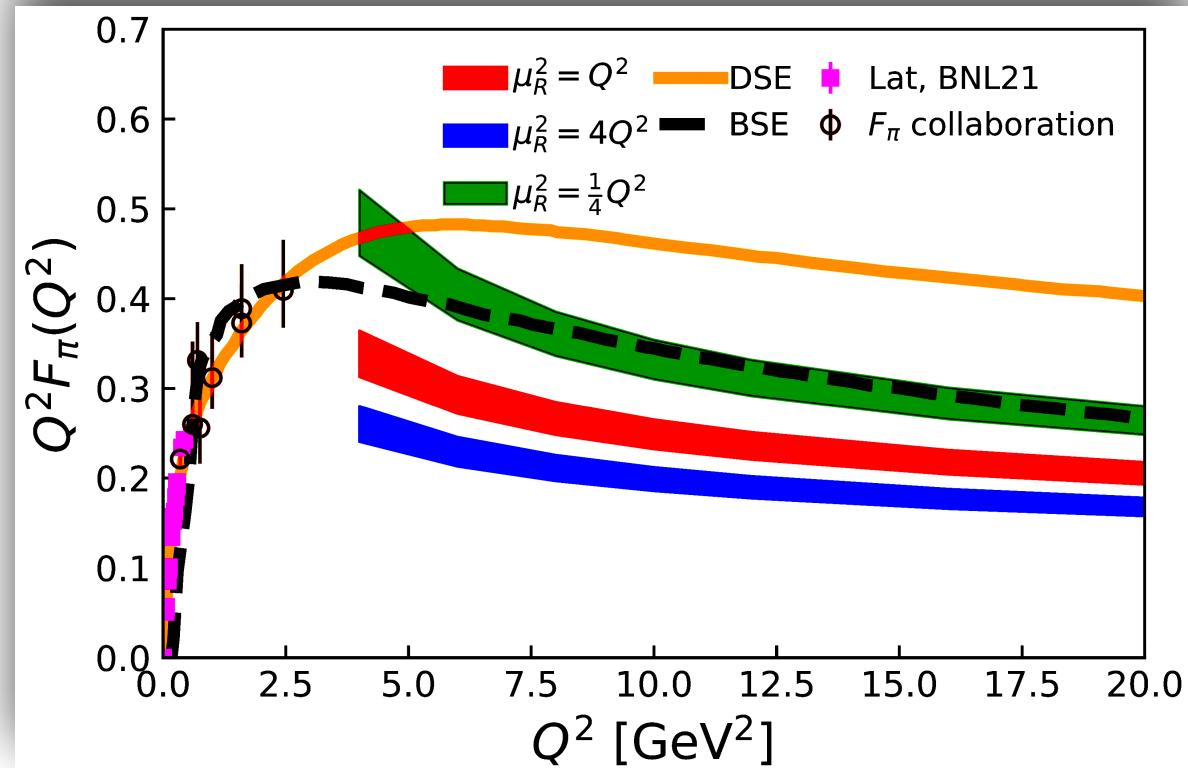
$$F_\pi(Q^2) = \frac{8\alpha_s(\mu_R^2)\pi f_\pi^2}{9Q^2} \int_0^1 \int_0^1 du dv \frac{\phi^*(v, \mu_F^2)\phi(u, \mu_F^2)}{(1-u)(1-v)}$$

+ 1-loop evolution of DA

BNL21: Gao et.al., Phys. Rev. D104, 11, 11 (2021)

DSE: Chang et.al., Phys. Rev. Lett. 111, 141802 (2013)

BSE: Ydrefors et.al., Phys. Lett. B 820, 136494 (2021)

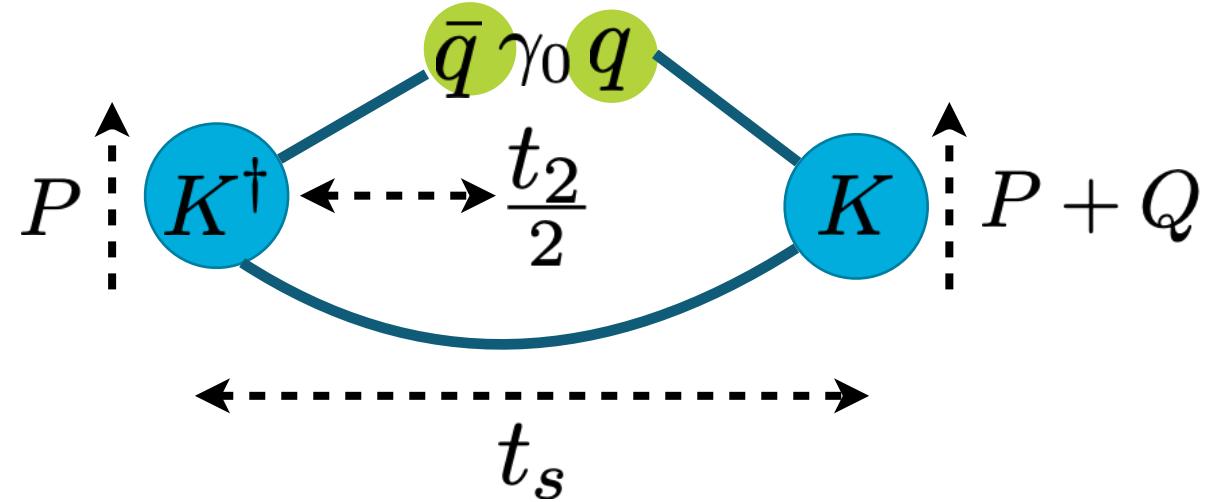


JLAB12

EIC

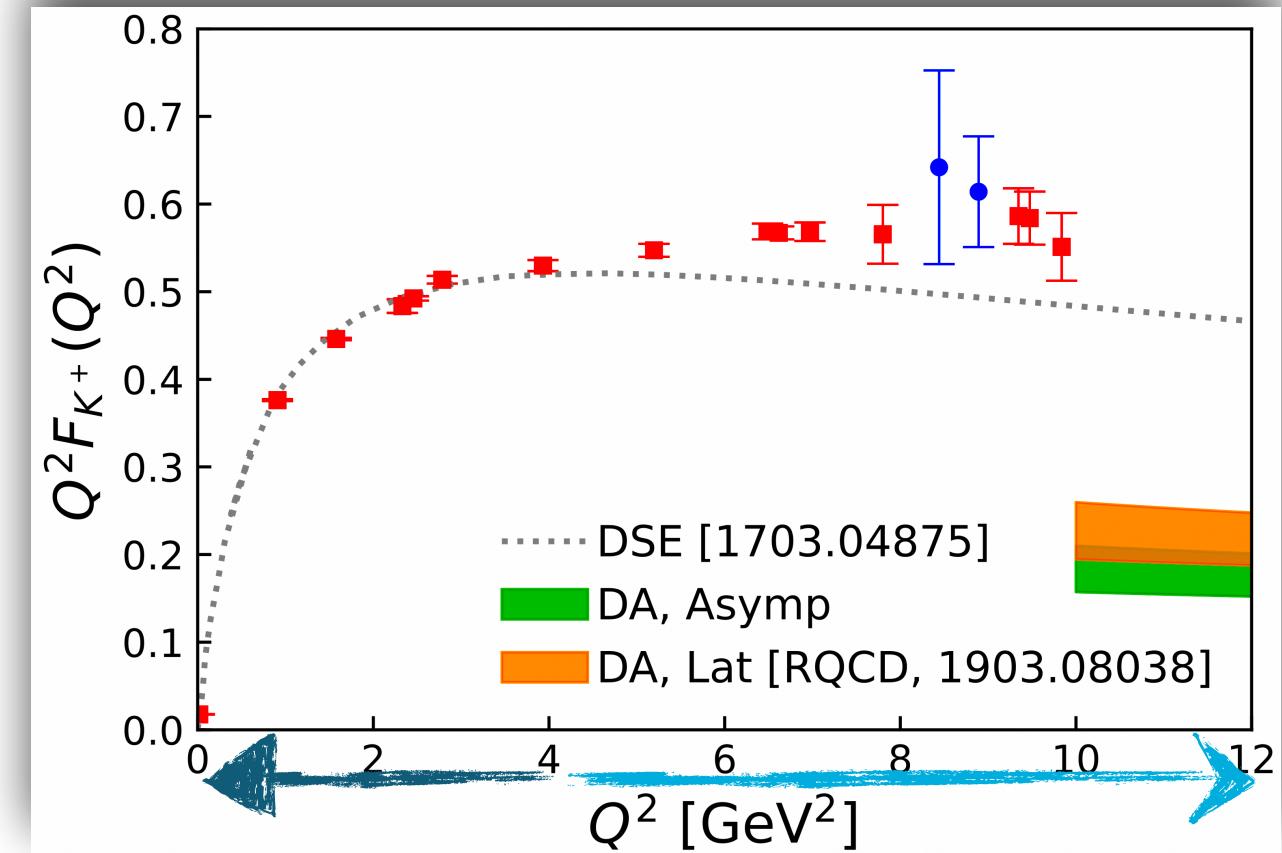
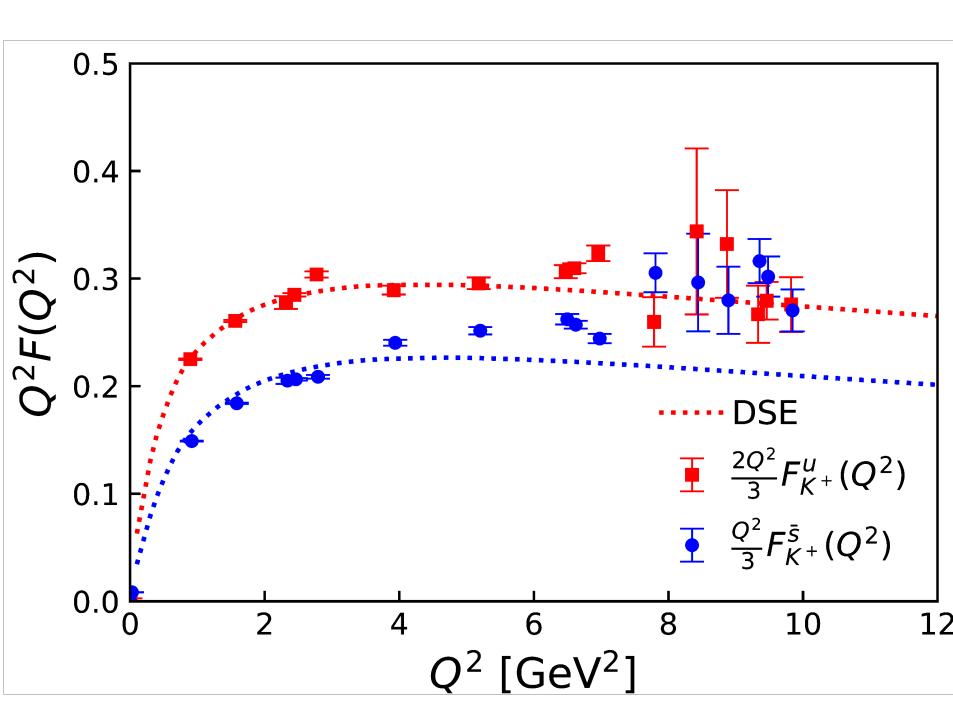
kaon EM FF

$$F(Q^2)$$



kaon EM FF for very large momentum transfer $Q^2 \leq 10 \text{ GeV}^2$

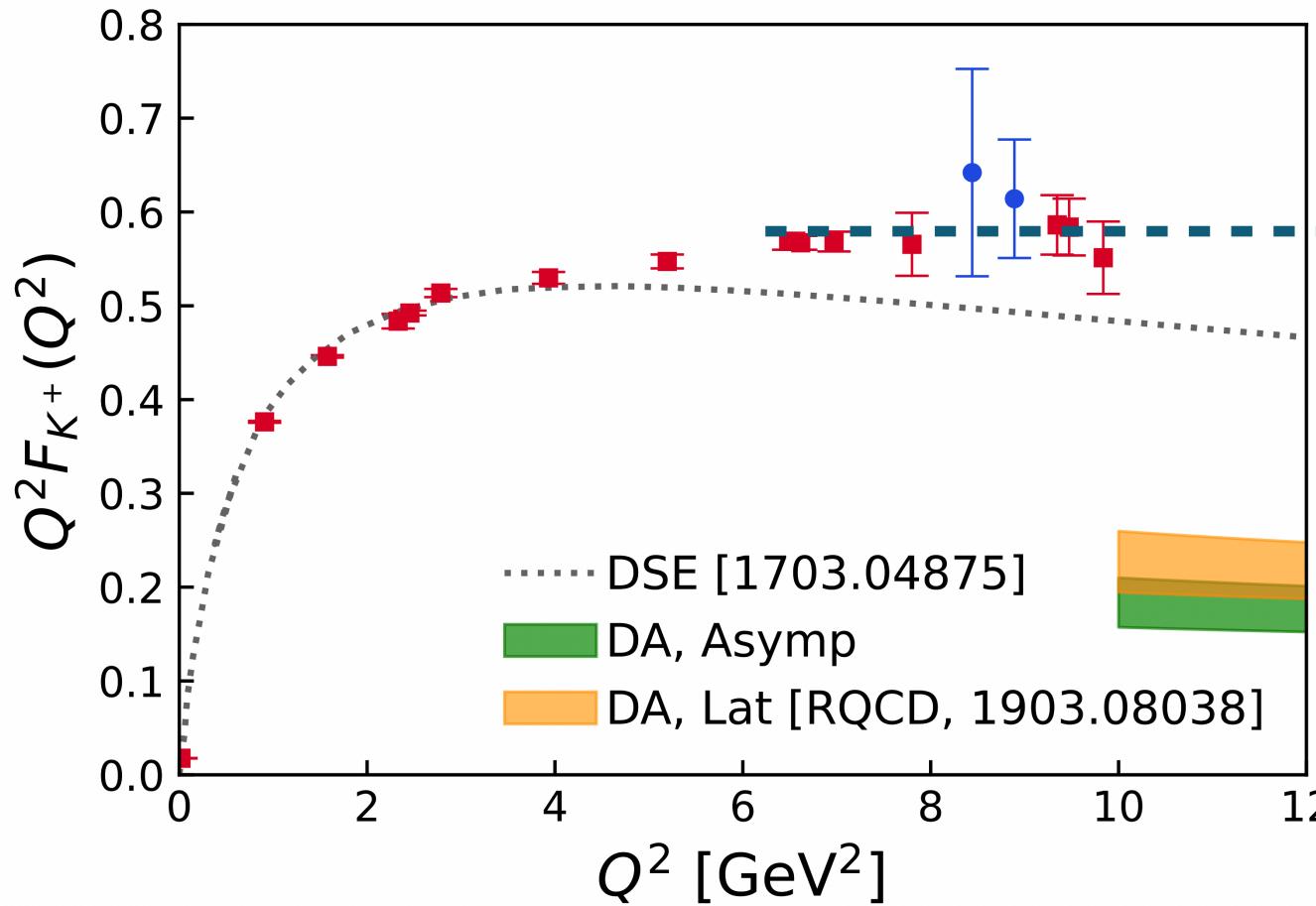
Qi Shi et.al., in preparation



JLAB12

EIC

FF and valance parton distribution function (PDF)



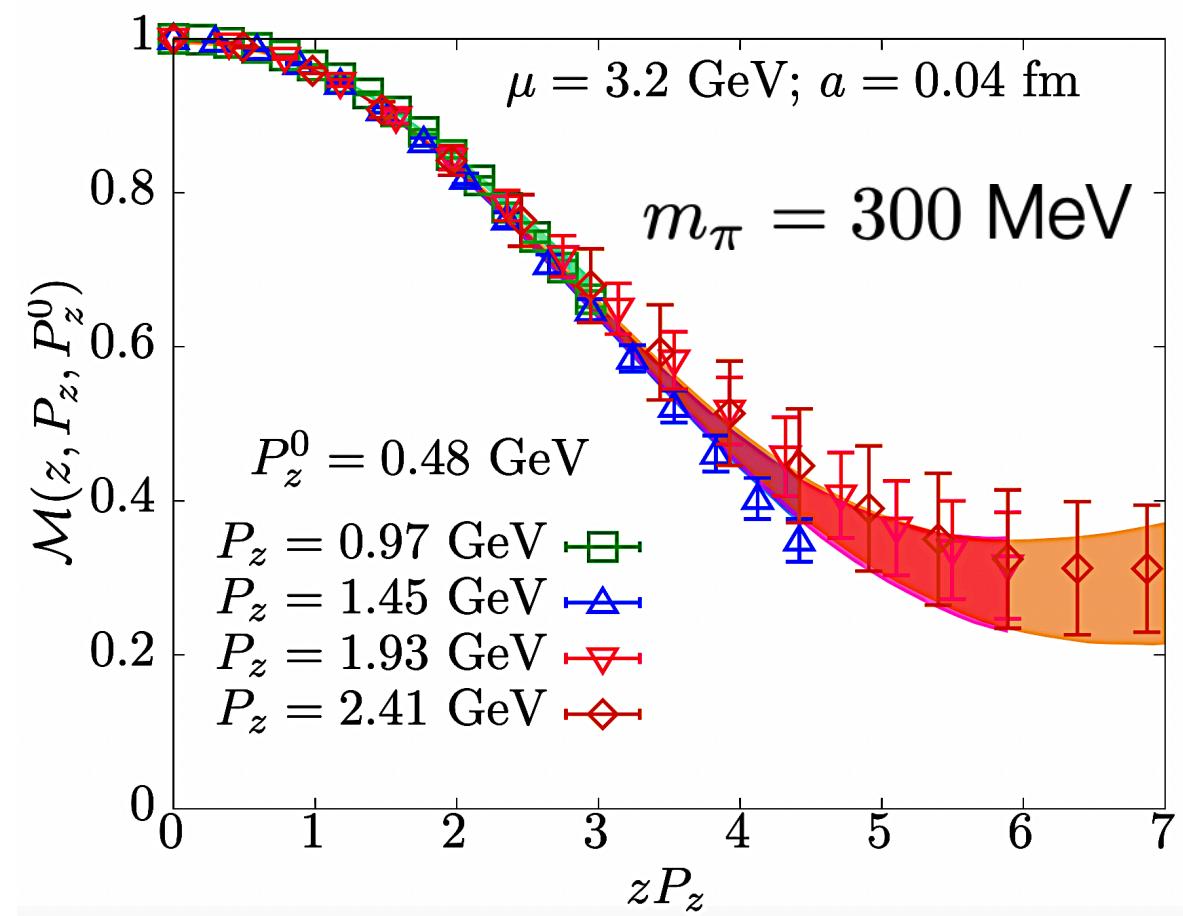
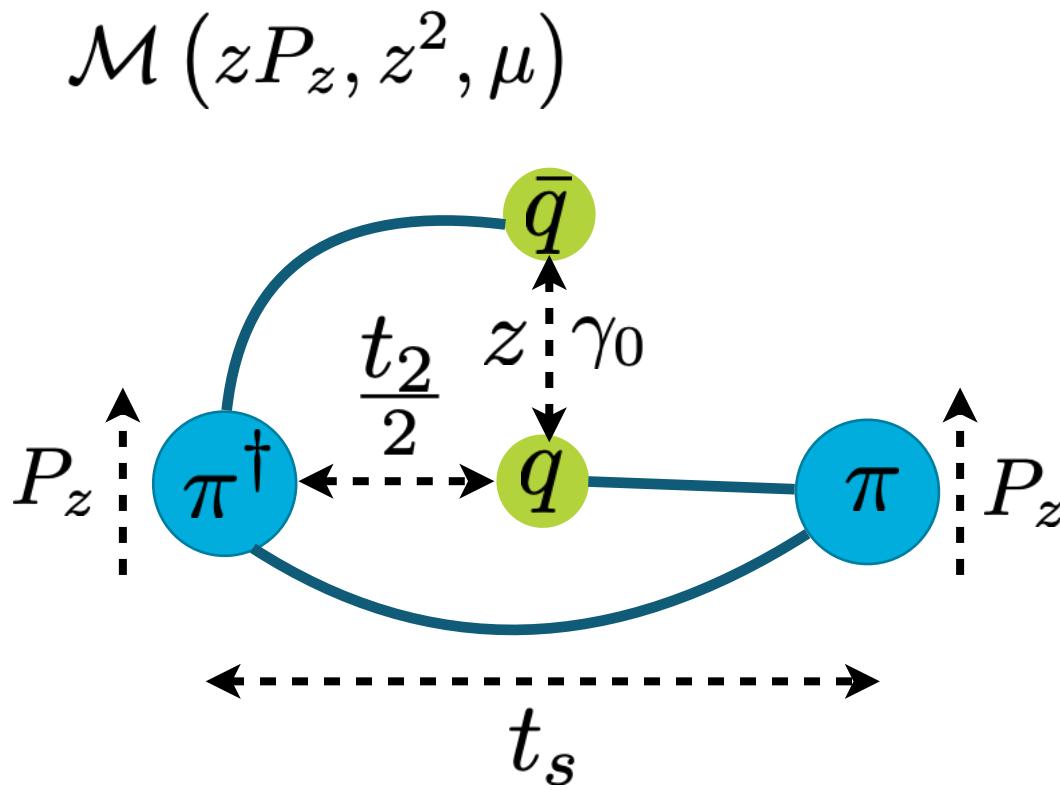
$$F(Q^2) \sim Q^{-2} ?$$

Drell-Yan-West relation —
valance PDF at large x ?

Valance PDF: $f(x \rightarrow 1) \sim (1 - x)^\beta \Rightarrow F(Q^2) \sim Q^{-(\beta+1)}$

pion valence PDF

Nikhil Karthik et.al.: Phys. Rev. D102, 9, 094513 (2020)

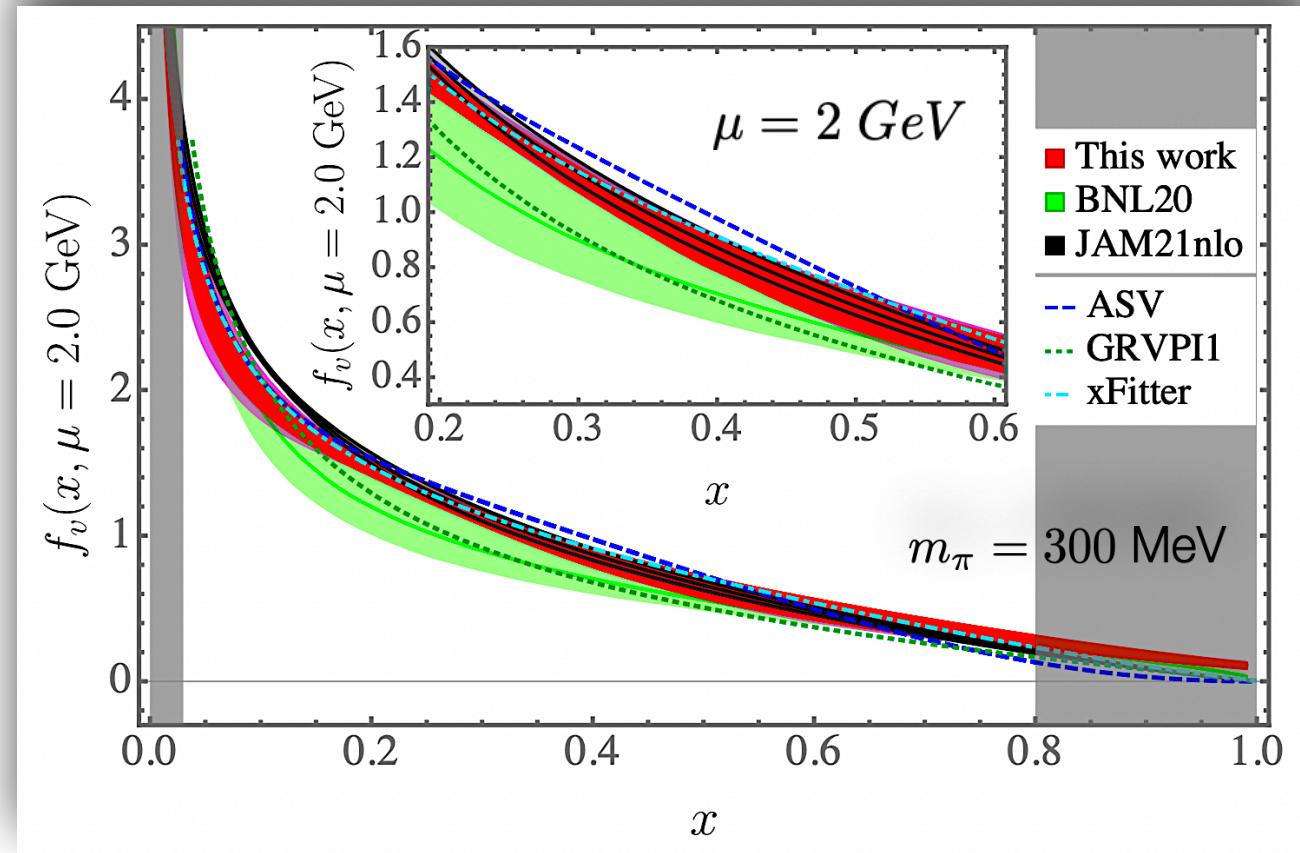


$z < 0.8 \text{ fm}$

$P_z \leq 2.5 \text{ GeV}$

pion valence PDF— first NNLO result

Yong Zhao et.al., Phys. Rev. Lett. 128 (2022) 14, 142003

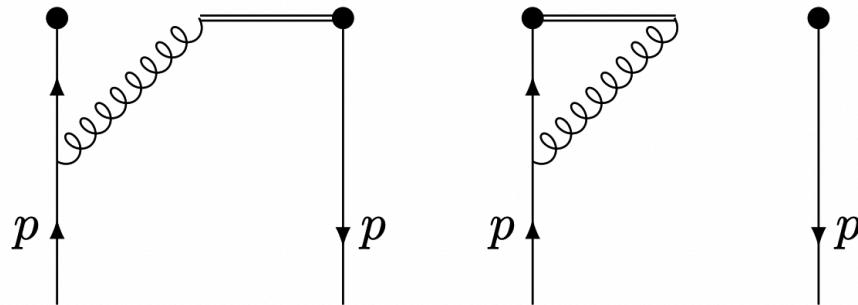


matching in x -space with
NNLO pQCD

$$\int dz e^{ixzP_z} \mathcal{M}(zP_z, z^2, \mu) = C(\omega, P_z, \mu) \otimes f(\omega, x, \mu)$$

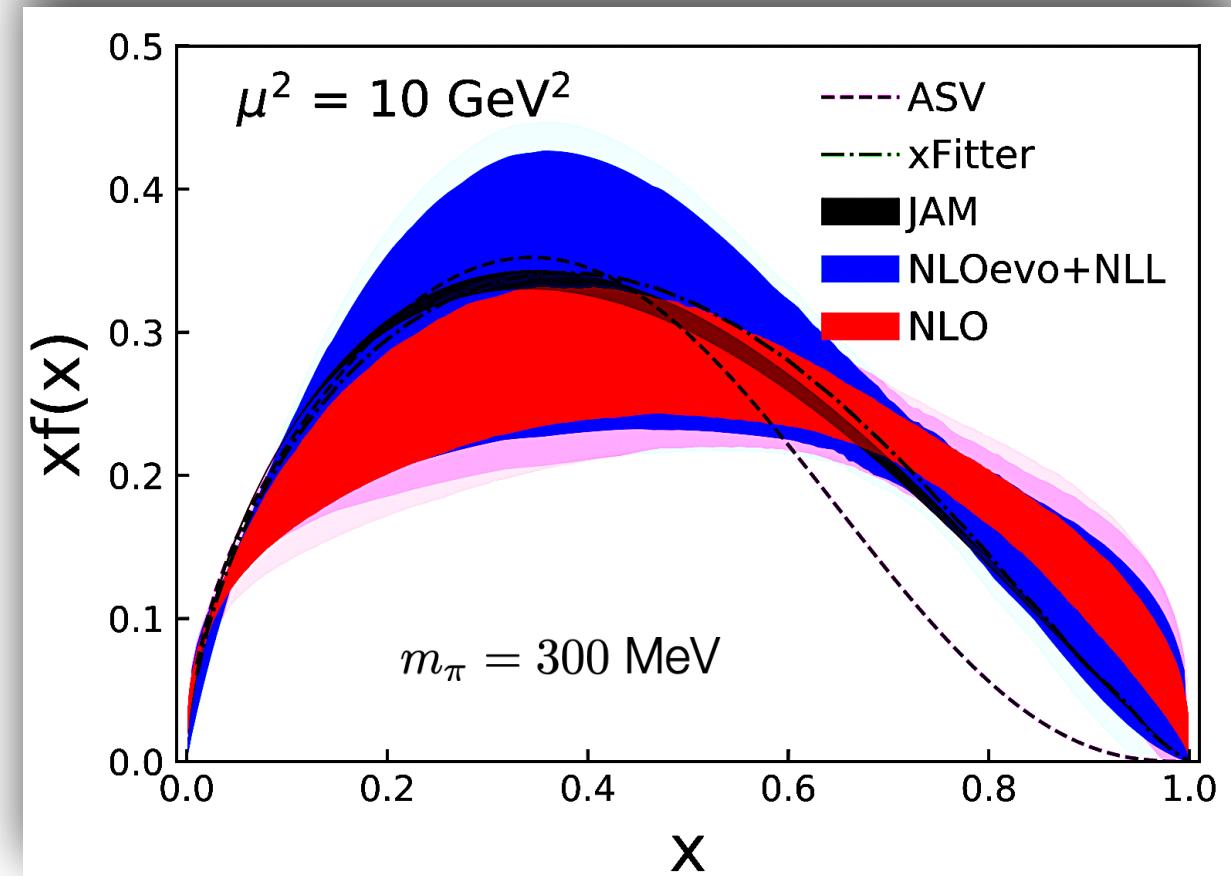
pion valence PDF: threshold resummation

Zhao, Gao et.al., Phys. Rev. D103, 9, 094504 (2021)



$$C(\omega, P_z, \mu) \sim \frac{\ln(1 - \omega)}{(1 - \omega)}$$

- DGLP evolution in z at NLO
- resum NLL in C

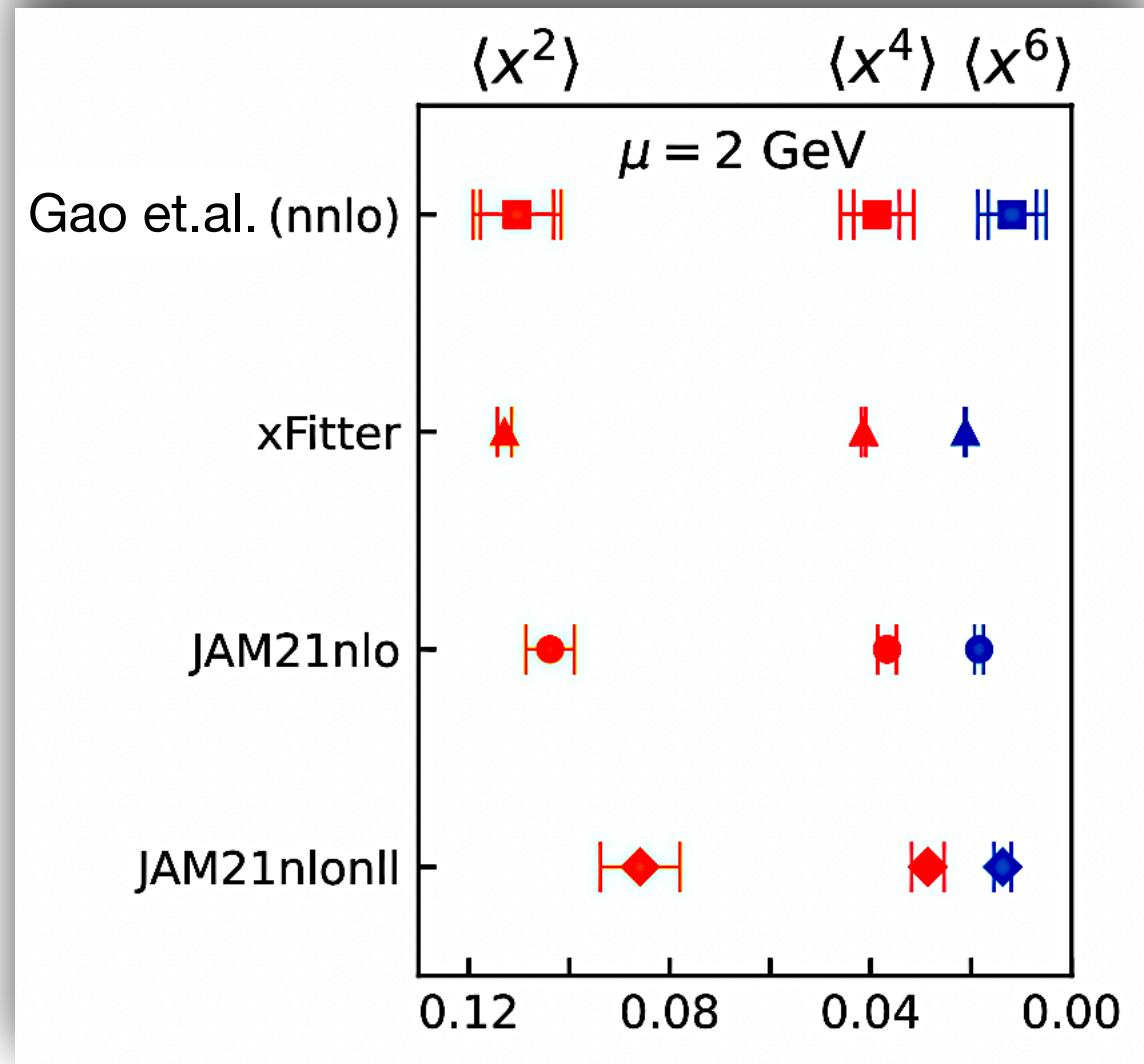


pion valence PDF: Mellin moments

Xiang Gao et.al., in preparation

- NNLO OPE
- $m_\pi = 140 \text{ MeV}$
- continuum extrapolated

$$\langle x^n \rangle(\mu) = \int_0^1 x^n f(x, \mu) dx$$



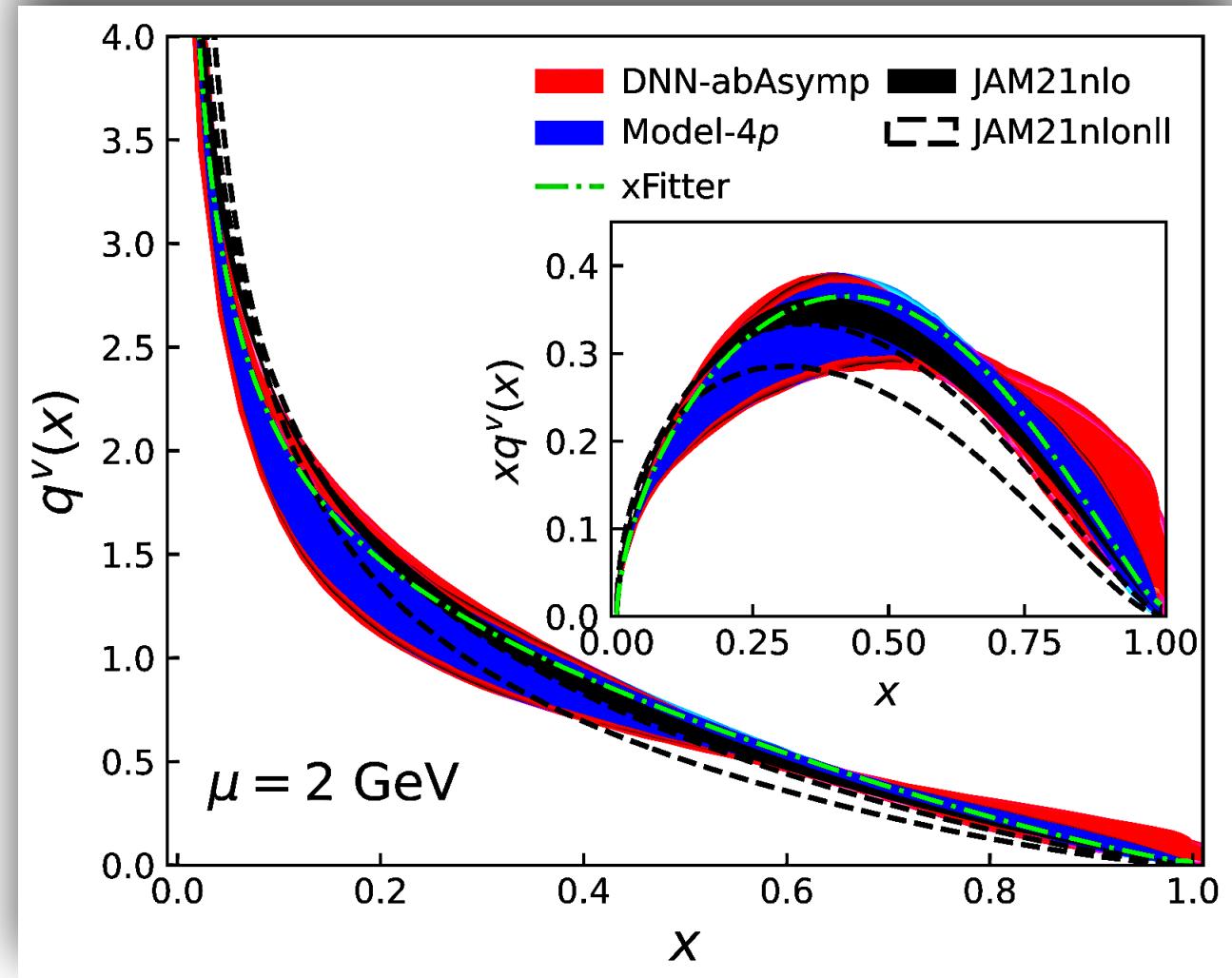
pion valence PDF: x-dependence

Xiang Gao et.al., in preparation

- deep neural network (DNN) reconstruction
- NNLO pQCD matching
- $m_\pi = 140$ MeV
- continuum extrapolated

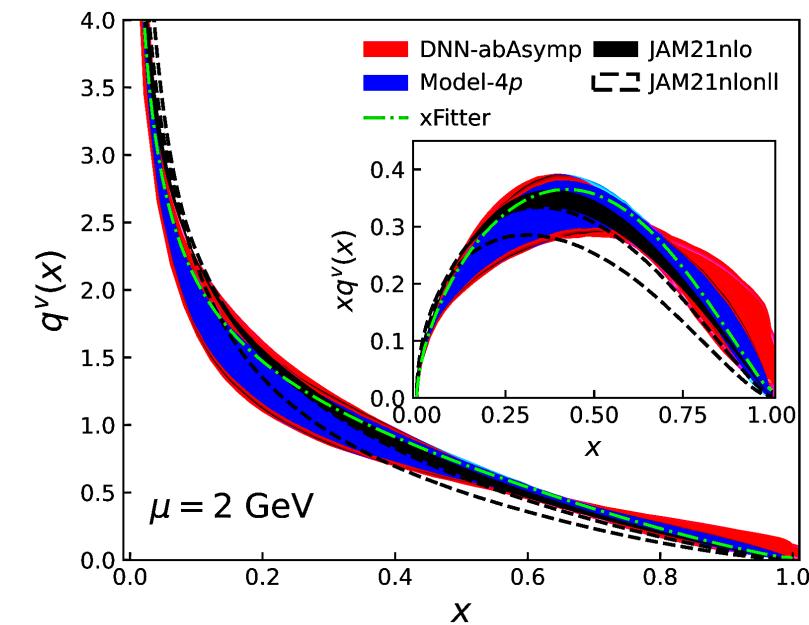
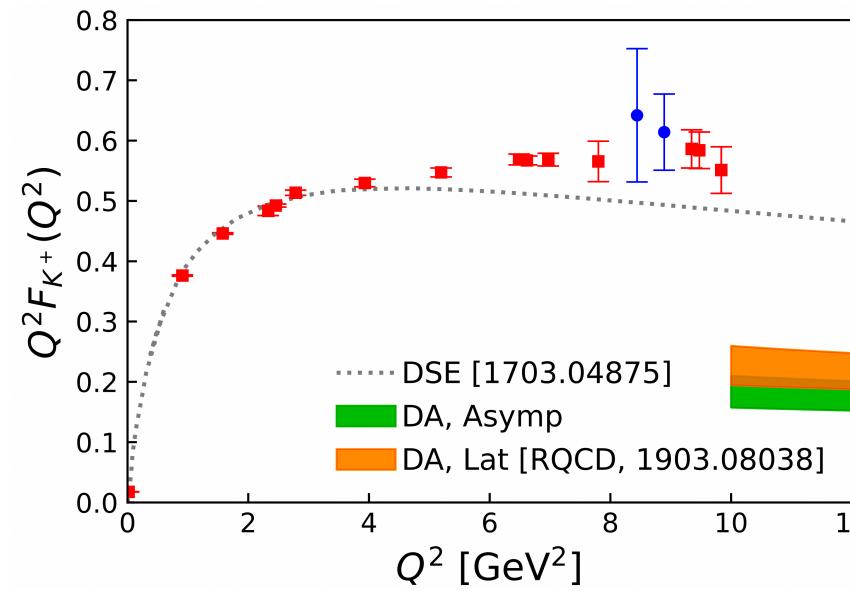
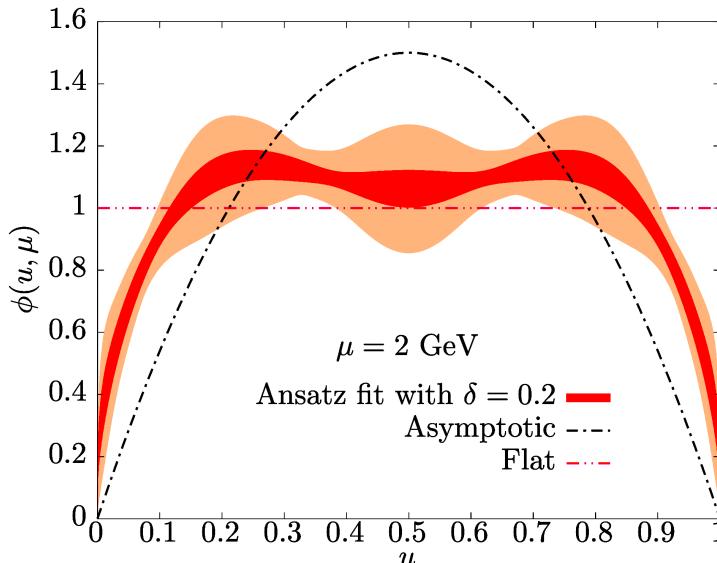
$$f(x, \mu) = \int d(zP_z) e^{-ix(xP_z)} f(zP_z, \mu)$$

$$\mathcal{M}(zP_z, z^2, \mu) = C(\omega, zP_z, z^2, \mu) \otimes \tilde{f}(\omega, zP_z, \mu)$$



summary

- rapid, robust progress from lattice QCD



- on course to facilitate/complement experiments —
JLAB12, EIC, AMBER, JPARC, EIICC ...