Two-photons processes at NNLO

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Quark Confinement and Hadron Spectrum, Stavanger, 02 August 2022

Outline

- Motivation
- Conformal QCD
- Evolution kernels and Coefficient functions at NNLO
- Numerics
- Conclusions

High-precision experiments

- Belle II: $\gamma^*\gamma \longrightarrow \pi$ transition formfactor
- JLAB 12GeV upgrade, Deeply Virtual Compton Scattering
  COMPASS (CERN), EIC: $\gamma^* N \longrightarrow \gamma N'$

Theory framework: Collinear Factorization

$$\text{Amplitude} = \text{Coefficient Function} \otimes \text{Matrix Element}$$

Theory predictions have to match the accuracy of experimental data.

Aims:

- Two-loop coefficient functions
- Three-loop evolution equations

- **Deeply Virtual Compton Scattering,** $\gamma^* \; N \longrightarrow \gamma \; N'$
  Müller 94, Ji, Radyushkin 96

$$\mathcal{A}_{\mu\nu}(q,q',p) = i \int d^4x \, e^{-iqx} \langle p'|T\{j^{\rm em}_\mu(x) j^{\rm em}_\nu(0)\}|p\rangle$$
$$= -g^\perp_{\mu\nu} V_+ + \epsilon^\perp_{\mu\nu} V_- + \ldots$$

Leading twist approximation

$$V_\pm(\xi,t,Q^2) = \sum_q e_q^2 \int_{-1}^1 \frac{dx}{\xi} C_\pm(x/\xi, Q^2/\mu^2) \, F_q^\pm(x,\xi,t,\mu) \, .$$

$C_\pm$ — **the coefficient functions,**
$F_q^\pm$ — **the generalized parton distributions (GPD)**

$$\langle p'|[\bar q(z_1 n)\gamma_+(\gamma_5) q(z_2 n)]|p\rangle \sim \int_{-1}^1 dx \, e^{-iP_+\xi(z_1+z_2)+iP_+x(z_1-z_2)} \, F_q^\pm(x,\xi).$$

At the classical level QCD is a conformal theory

Müller 1991, Belitsky and Müller 1998

Evolution kernels and CFs at NLO from CFs and anomalous dimensions in DIS by analysing symmetry breaking effects

Braun, A.M. 2015, QCD in $d = 4 - 2\epsilon$ dimensions at the critical point, $\beta(a_*) = 0$,

$a_* = a_*(\epsilon) \sim -\epsilon/\beta_0 + \ldots,\qquad \epsilon = \epsilon(a) \sim -\beta_0 a + \ldots.\qquad$ **($a = g^2/(4\pi)^2$)**

Scale invariance $\mapsto$ Conformal invariance

$\mapsto$ **Constraints on the correlation functions, Conformal OPE, etc**

$$C(a,\epsilon) = C_0(\epsilon) \;+\; aC_1(\epsilon) \;+\; a^2C_2(\epsilon) \;+\; O(a^3)$$

$$C_k(\epsilon) = C_k(0) + \epsilon C_k'(0) + \ldots$$

$$(C^{phys}(a,0) - C^{conf}(a,\epsilon(a)))^{L-loop} = O(a^{2(L-1)})$$

Twist-2 light ray operator

$$\mathcal{O}(z_1, z_2) = [\bar{q}(z_1 n)\gamma_+ q(z_2 n)]$$

satisfies the RG equation:

$$\Big(\mu\partial_\mu + \beta(a)\partial_a + \mathbb{H}(a)\Big)\mathcal{O}(z_1, z_2) = 0\,.$$

Evolution kernel (integral operator):

$\mathbb{H}(a) = a\mathbb{H}^{(1)} + a^2\mathbb{H}^{(2)} + \ldots$ – **does not depend on ϵ at all**

Going to the critical point $a \mapsto a_*(\epsilon)$ one does not lose any information on the kernel

In conformal theory:

$$[S_{\pm,0}(a_*), \mathbb{H}(a_*)] = 0$$

$S_{\pm,0}(a_*)$ — **symmetry generators (the $\mathrm{SL}(2,\mathbb{R})$ subgroup of conformal group)**

$$S_- = -\partial_{z_1} - \partial_{z_2}$$

$$S_0 = z_1\partial_{z_1} + z_2\partial_{z_2} + 2 + \left(\beta(a) + \frac{1}{2}\mathbb{H}(a)\right)$$

$$S_+ = z_1^2\partial_{z_1} + z_2^2\partial_{z_2} + 2(z_1+z_2) + \underbrace{(z_1+z_2)\left(\beta(a) + \frac{1}{2}\mathbb{H}(a)\right) + (z_1-z_2)\Delta(a)}_{\text{Quantum corrections}}$$

$\Delta(a)$ — **conformal anomaly (absent in lowest orders in scalar theories)**

In QCD,

- **Belitsky and Müller 1998, $\Delta(a)$ at one loop**
- **Braun, A.M., Moch, Strohmaier 2016, $\Delta(a)$ at two loop (nonsinglet)**

$$\Delta^{(1)} f(z_1, z_2) = -2C_F \int_0^1 d\alpha \left(\frac{\bar{\alpha}}{\alpha} + \ln\alpha\right)\left(f(z_{12}^\alpha, z_2) - f(z_1, z_{21}^\alpha)\right)$$

$z_{12}^\alpha = z_1(1-\alpha) + z_2\alpha$

- $\Delta^{\text{NLO}} + \gamma^{\text{NNLO}}_{\text{DIS}}(a) \Longrightarrow$ **off-forward evolution kernels (DVCS) at NNLO**
- $\Delta^{\text{NLO}} + \mathrm{CFs}^{\text{NNLO}}_{\text{DIS}}(a) \Longrightarrow$ **DVCS coefficient functions at NNLO**

$\mathrm{SL}(2)$ **algebra:** $[S_0, S_\pm] = \pm S_\pm, \quad [S_+, S_-] = 2S_0.$

$\mathbb{H}(a)$ **is an invariant operator,** $[S_\alpha, \mathbb{H}(a)] = 0$**:**

$$\mathbb{H}(a) = a\, f(\mathbf{J}), \qquad \mathbf{J}(\mathbf{J}-1) = S_+S_- + S_0(S_0-1)$$

Eigenvalues of $\mathbb{H}(a)$ **are the forward anomalous dimensions** $\gamma_N(a)$**,**

(known at NNLO, Moch, Vermaseren, Vogt 2004)

In order to find $\mathbb{H}(a)$ **at** L **loop it is sufficient to know** J **(and** $\Delta(a)$ **) at one loop less.**

$$\mathbb{H}(a) = a\, f(\mathbf{J}) \Longrightarrow \gamma_N(a) = a\, f(J_N)) = a\, f\left(N + \beta(a) + \frac{1}{2}\gamma_N(a)\right).$$

Reciprocity, $f(J) \sim f(1-J)$**,** Dokshitzer, Marchesini, Salam, **2006,** Basso, Korchemsky **2007**

Practical approach:

$\mathbb{H}(a) = \mathbb{H}_{inv} + \Delta\mathbb{H}$, $\mathbb{H}_{inv}$ **commutes with the canonical generators**

$$\mathbb{H}_{inv} f(z_1, z_2) = \int_0^1 d\alpha \int_0^{\bar{\alpha}} d\beta\, \omega(\tau) f(z_{12}^{\alpha}, z_{21}^{\beta}), \qquad \tau = \frac{\alpha\beta}{\bar{\alpha}\bar{\beta}}$$

Braum, A.M., Moch, Strohmaier 2017

- **Explicit expression for $\Delta\mathbb{H}$ at NNLO**
- **Parametrization for $\omega(\tau)$**
- **Off-diagonal part of the anomalous dimension matrix**

DIS $\longleftrightarrow$ DVCS

Amplitudes of both processes are derived from the OPE of two electromagnetic currents:

$$T\{j_\mu^{\rm em}(x) j_\nu^{\rm em}(0)\} = \sum_{N,k} C_{Nk}(x) \partial^k \mathcal{O}_N(0),$$

DIS: **Only the operators with $k=0$ are relevant, C_{N0} – moments of the coefficient function.**

DVCS: **All operators contribute to the amplitude. One need to know C_{Nk} for all k.**

In a conformal theory C_{Nk} for $k>0$ are completely determined by C_{N0}

$$T\{j_\mu^{\rm em}(x) j_\nu^{\rm em}(0)\} = \sum_{N} C_{N}(x,\partial) \mathcal{O}_N(0),$$

Ferrara, Gatto, Grillo, Parisi, 1970

$$\mathbf{T}\left\{j^\mu(x)j^\nu(0)\right\} = \sum_N \frac{\mu^{\gamma_N}}{(-x^2)^{\tau_N}} \int_0^1 du(u\bar{u})^{j_N-1}\left\{a_N\left(g^{\mu\nu} - \frac{2x^\mu x^\nu}{x^2}\right) + b_N g^{\mu\nu} + \ldots\right\}\mathcal{O}_N^{(x)}(xu)\,,$$

where

$$\mathcal{O}_N^{(x)}(y) = x_{\mu_1}\ldots x_{\mu_N}\mathcal{O}_N^{\mu_1\cdots\mu_N}(y)\,, \quad \textbf{conformal operator}$$

$$\Delta_N = d - 2 + N + \gamma_N \qquad (\textbf{scaling dimension})$$

$$j_N = \frac{1}{2}(\Delta_N + N) \qquad (\textbf{conformal spin})$$

$$\tau_N = d - 1 - t_N/2 \qquad t_N = \Delta_N - N \qquad (\textbf{twist})$$

The coefficients a_N and b_N are related to the DIS coefficient functions $C_2(N)$ and $C_L(N)$

Conformal moments: ($\xi \to 1$)

$$\int_{-1}^{1} dx\, C(x)\, P_N(x) = C_1(N) \times \frac{\Gamma(d/2-1)\Gamma(2j_N)}{\Gamma(j_N)\Gamma(j_N+d/2-1)} \times \textbf{(known coefficient)} \tag{1}$$

P_N **are the eigenfunctions of the evolution kernel.**

$$P_N(x) = (1-x^2)^{\lambda_N - 1/2} C_{N-1}^{(\lambda_N)}(x), \qquad \lambda_N = \frac{3}{2} - \beta(a) + \frac{1}{2}\gamma_N(a)$$

$C(x)$ **can be effectively restored:**

$$C(x) = \int dx' C_0(x') K(x', x) \tag{2}$$

K **is an invariant kernel,** $C_0(x) = 1/(1-x) - 1/(1+x)$**.**

Equation. (1) $\mapsto$ **an equation on the eigenvalues of** K $\mapsto$ **restore** K $\mapsto$ **Equation** (2)**.**

Braun, A.M., Moch, Schönleber, 2020-21

$$C_{\pm}^{(2)}(x/\xi) = \beta_0 C_F\, C_{\pm}^{(\beta)}(x/\xi) + C_F^2 C_{\pm}^{(P)}(x/\xi) + \frac{C_F}{N_c} C_{\pm}^{(NP)}(x/\xi)\,,$$

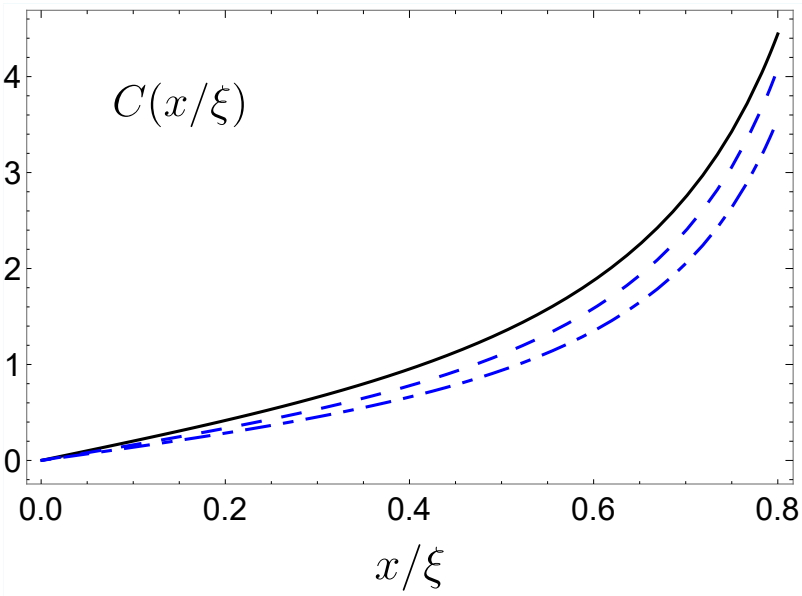
$$\begin{aligned}
C_{\pm}^{(\beta)}(x/\xi) = \Bigg\{ & \frac{2}{z}\Big(\mathrm{H}_{100} - \frac{1}{2}\mathrm{H}_{110} - \mathrm{H}_{000}\Big) + \left(\frac{10}{3z} \pm \frac{1}{\bar z}\right)\mathrm{H}_{00} - \left(\frac{1\pm 1}{\bar z} + \frac{14}{3z}\right)\mathrm{H}_{10} - \frac{\zeta_2}{z}\mathrm{H}_1 \\
& - \left(\frac{19}{9z} + \frac{10\pm 13}{6\bar z}\right)\mathrm{H}_0 - \frac{1}{z}\left(\frac{457}{24} + \frac{11\mp 3}{3}\zeta_2 + \zeta_3\right)\Bigg\} \mp (z\to\bar z),
\end{aligned}$$

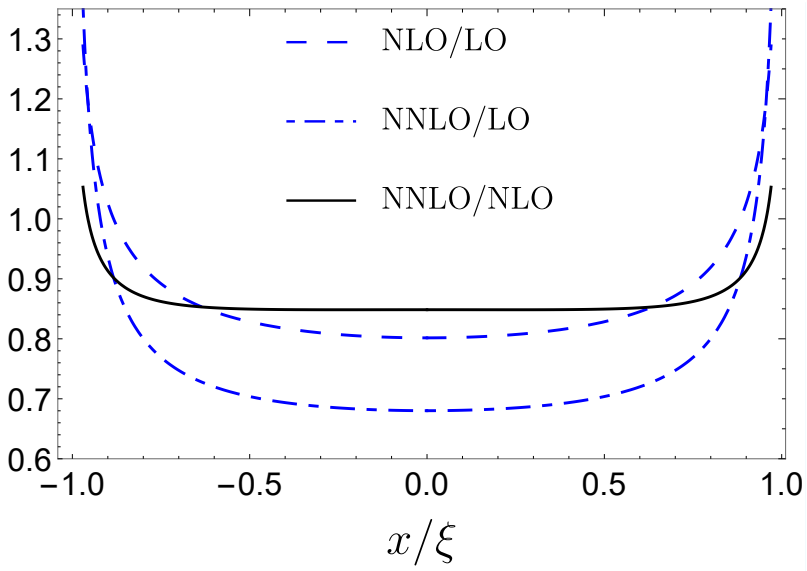
$$\begin{aligned}
C_{\pm}^{(P)}(x/\xi) = \Bigg\{ & \frac{2}{z}\Big(6\mathrm{H}_{0000} - \mathrm{H}_{1000} - 2\mathrm{H}_{200} - \mathrm{H}_{1100} - \mathrm{H}_{120} - \mathrm{H}_{210} + \mathrm{H}_{1110}\Big) \mp \frac{2}{\bar z}\mathrm{H}_{000} + \frac{2}{\bar z}\mathrm{H}_{20} \\
& + \frac{4}{z}\mathrm{H}_{110} - \left(\frac{8}{z} - \frac{(2\pm 2)}{\bar z}\right)\mathrm{H}_{100} - \left(\frac{12\pm 1}{\bar z} + \frac{38}{3z}\right)\mathrm{H}_{00} + \left(\frac{3\pm 3}{\bar z} + \frac{28\mp 6}{3z}\right)\mathrm{H}_{10} \\
& + \frac{2}{z}\zeta_2\Big(\mathrm{H}_{11} - \mathrm{H}_2 - \mathrm{H}_{10} - 4\mathrm{H}_{00}\Big) + \frac{2}{\bar z}\left(\frac{218\pm 5}{12} \pm (3\pm 2)\zeta_2 \mp 2\zeta_3\right)\mathrm{H}_0 \\
& + \frac{2}{z}\left(3\zeta_2 + 16\zeta_3 - \frac{32}{9}\right)\mathrm{H}_0 + \frac{1}{z}\Big(\frac{701}{24} + \frac{25\mp 9}{3}\zeta_2 + (41\mp 2)\zeta_3 + 3\zeta_2^2\Big)\Bigg\} \mp (z\leftrightarrow\bar z),
\end{aligned}$$

$$C_{\pm}^{(NP)}(x/\xi) = \cdots$$

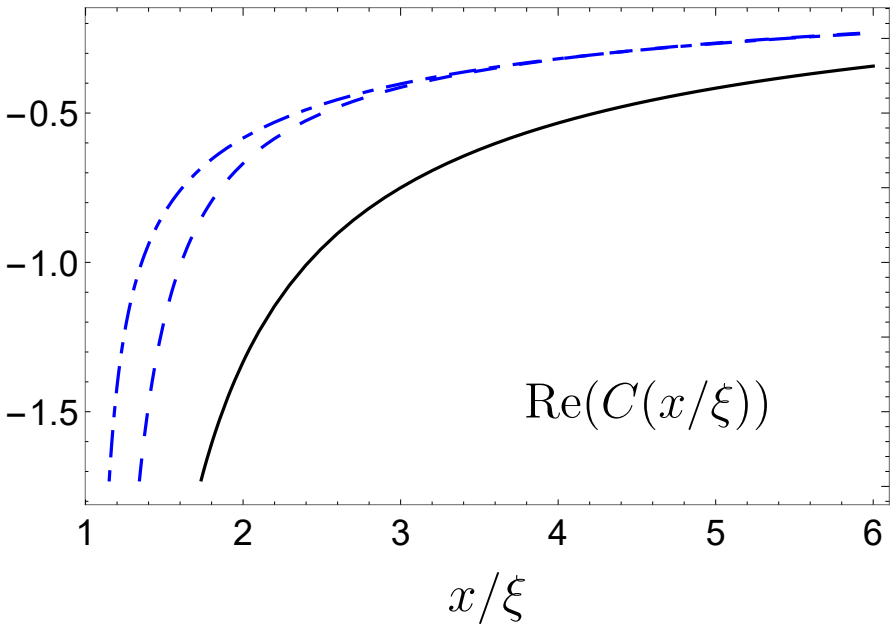
$z = \frac{1-x}{2}$ **and** $\mathrm{H}_n = \mathrm{H}_n\,(z)$ **HPL functions,** β_0 **— agrees with Melic, Nizic, Passek, 2002**

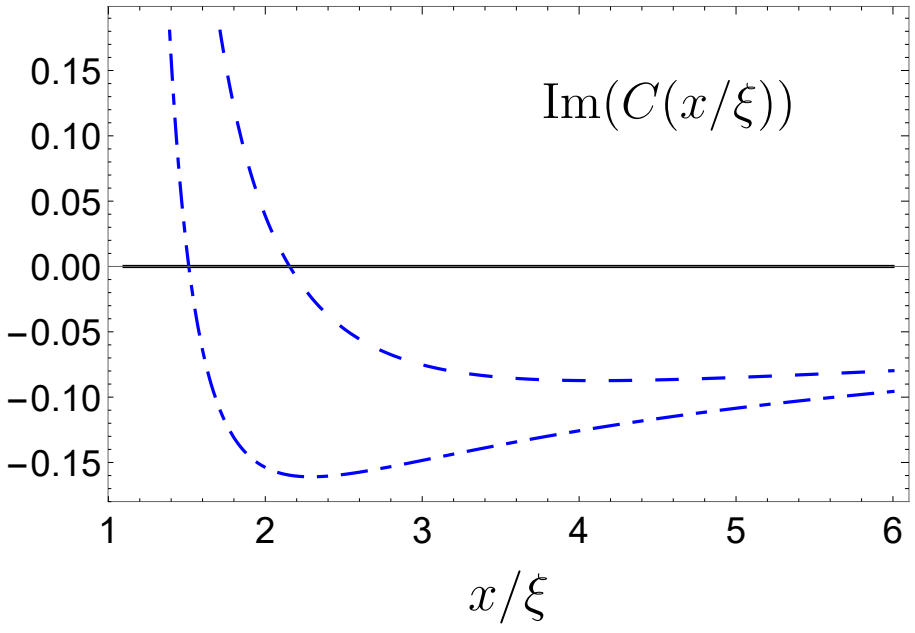
C_- **(axial) agrees with Gao, Huber, Ji, Wang 2021**





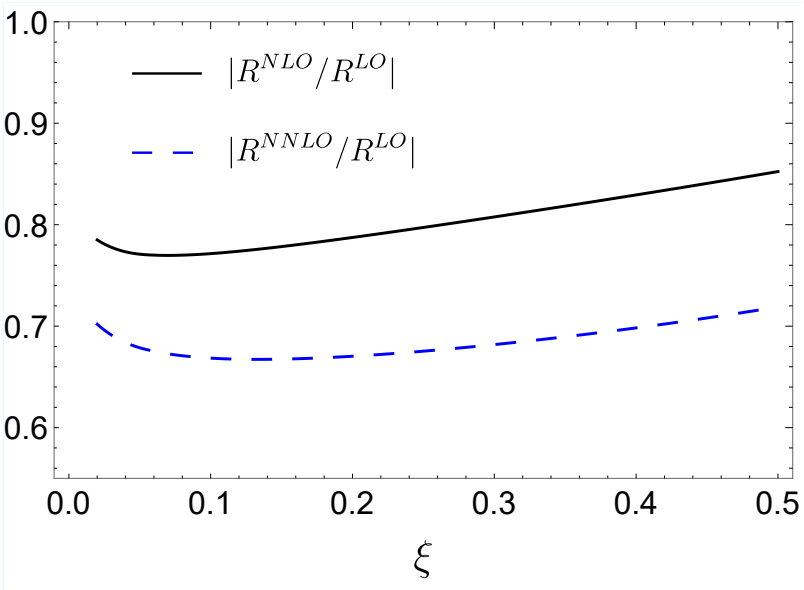
The DVCS CF $C(x/\xi)$ at $\mu = Q = 2$ GeV in the ERBL region $x < \xi$. The LO (tree-level), NLO (one-loop) and NNLO (two-loop) CFs are shown by the black solid, blue dashed and blue dash-dotted curves on the left panel, respectively. The right panel shows the ratios NLO/LO (dashed), NNLO/LO (dash-dotted) and NNLO/NLO (solid).

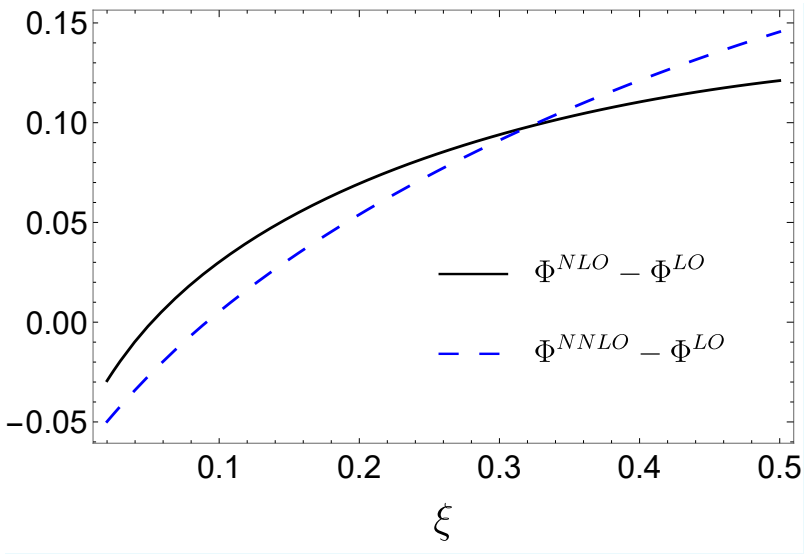




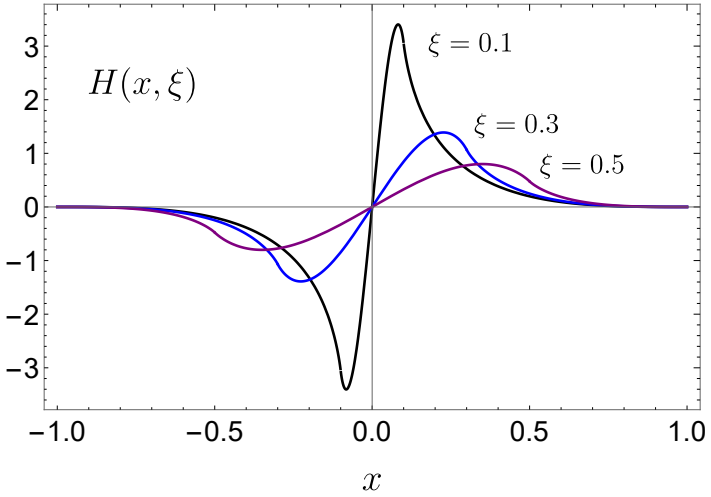
The DVCS CF $C(x/\xi)$ at $\mu = Q = 2$ GeV analytically continued into the DGLAP region $x > \xi$: real part on the left and imaginary part on the right panel. The LO (tree-level), NLO (one-loop) and NNLO (two-loop) CFs are shown by the black solid, blue dashed and blue dash-dotted curves. Note, that imaginary part of the LO CF contains a local term $\sim \delta(x - \xi)$ (not shown).

Numerics

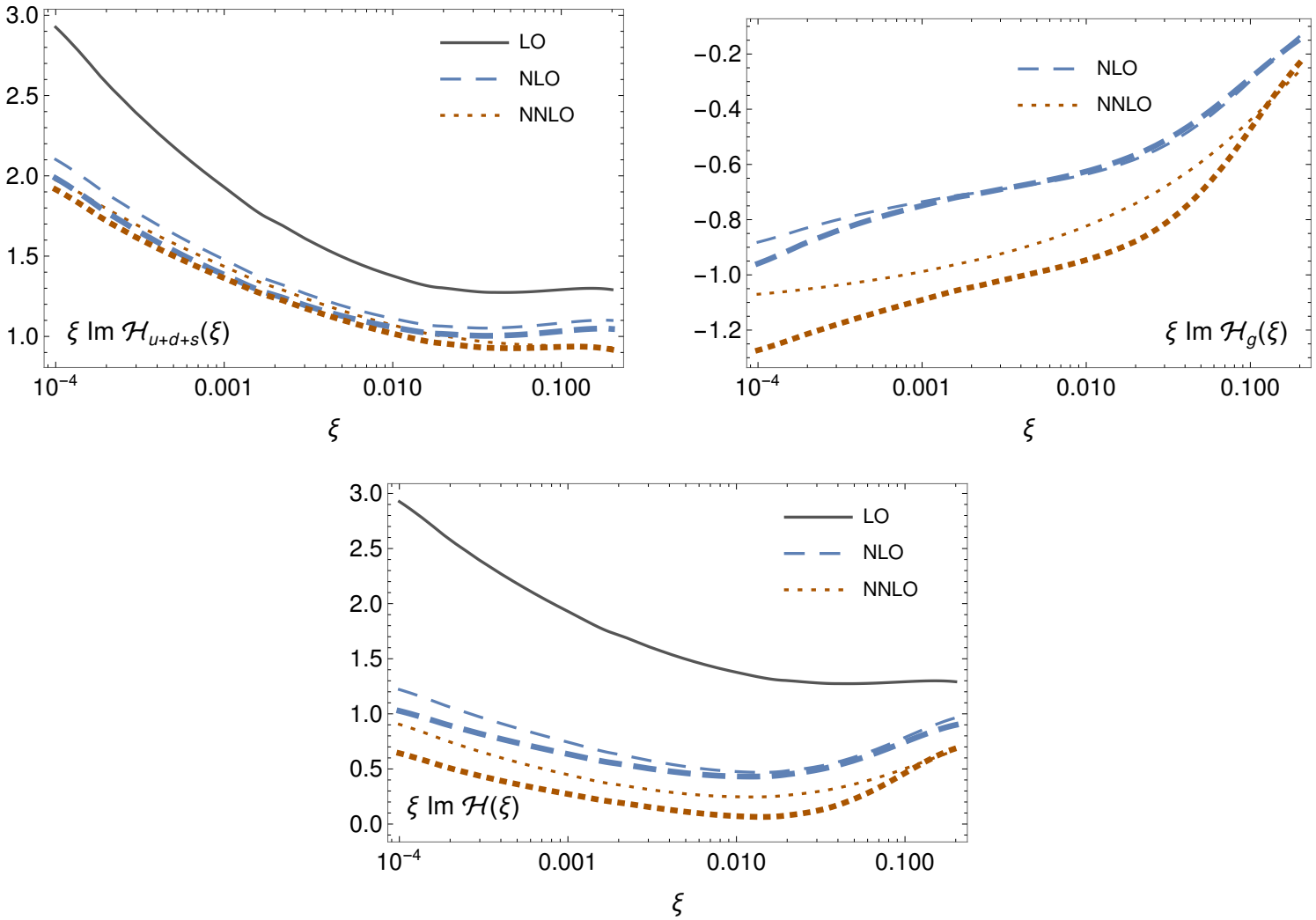




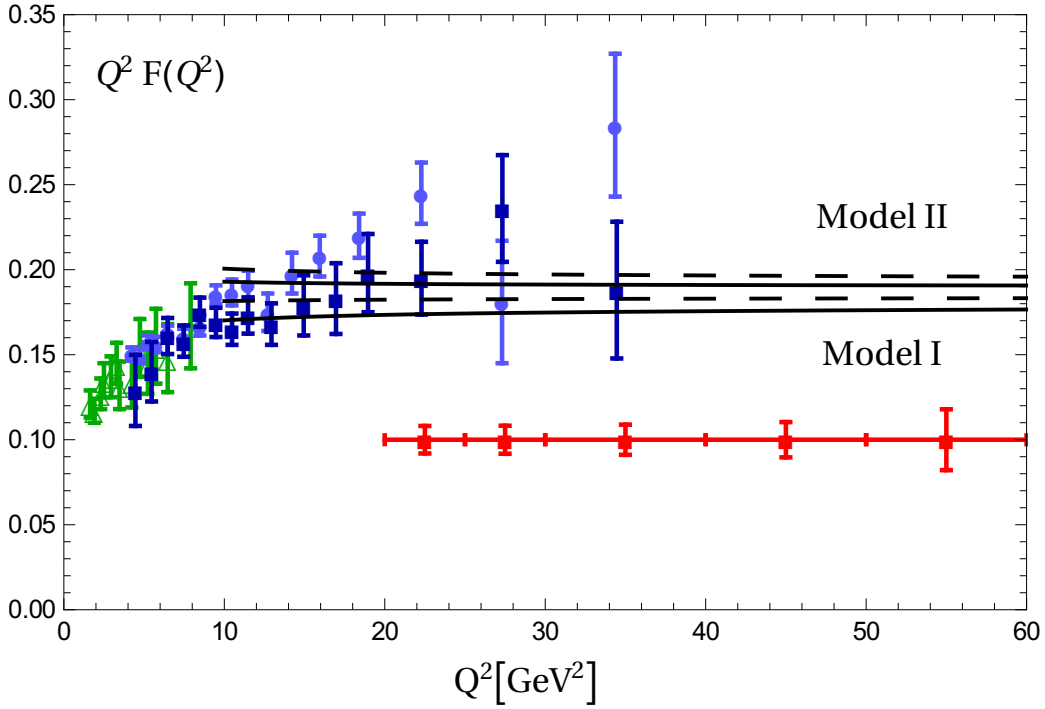
Higher-order QCD corrections to the Compton form factor $\mathcal{H}(\xi)$. The ratios of the Compton form factor calculated to the NNLO and NLO accuracy with respect to the tree-level are shown for the absolute value and the phase of $\mathcal{H}(\xi) = Re^{i\Phi}$, the left and the right panels



- **DVCS to NNLO accuracy [Braun, Yao Ji, Schönleber, 2207.06818, singlet vector CFs]**



Imaginary part of the CFF H as a function of at $\mu^2 = Q^2 = 4$GeV^2 and $t = -0.1$GeV^2



Models for the pion LCDA

$$\phi_\pi^{(I)}(z,\mu_0) = 6z(1-z)\times [1 + a_2(\mu_0)C^{3/2}(2z-1)]\,,$$

$$\phi_\pi^{(II)}(z,\mu_0) = B(1+\alpha,1+\alpha)\times z^\alpha(1-z)^\alpha\,,$$

The $\pi\gamma^*\gamma$ form factor at the NNLO (solid curves) and NLO (dashed curves) in QCD perturbation theory for the two models of the pion LCDA The experimental data are from CLEO (green, open triangles), BaBar (light blue, circles) and Belle (dark blue, squares). In addition, the expectation for the error bars achievable at Belle II is shown in red. The central value for the red boxes is arbitrary.

Outlook

- NNLO evolution kernel:
  - (axial)vector nonsinglet operators
  - singlet ??
- NNLO coefficient functions
  - (axial)vector nonsinglet
  - vector singlet, axial singlet ??