

Compositeness problem in hadronic states

José Antonio Oller

Departamento de Física
Universidad de Murcia (Spain)

Related references [1] JAO, ANP396,429(2018);
[2]Guo,JAO,PRD103,054021(2021); [3]Guo,JAO,PRD93,096001(2016);
[4]Wang,Kang,JAO,Zhang,PRD105,074016(2022)

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Basic formalism on
elementari-
ness/compositeness

Interpretation
based on the
number operator

Explicit formulas

Resonances

Explicit formulas

Application to the
 $f_0(980)$ and
 $a_0(980)$

Conclusions

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Outline

Compositeness
problem in
hadronic states

José Antonio Oller

- 1 Basic formalism on elementariness/compositeness
- 2 Interpretation based on the number operator
- 3 Explicit formulas
- 4 Resonances
- 5 Explicit formulas
- 6 Application to the $f_0(980)$ and $a_0(980)$
- 7 Conclusions

Basic formalism on
elementariness/compositeness

Interpretation
based on the
number operator

Explicit formulas

Resonances

Explicit formulas

Application to the
 $f_0(980)$ and
 $a_0(980)$

Conclusions

1.- Basic formalism

Shallow bound state. Non-relativistic dynamics

S. Weinberg, PR130,776(1963); PR131,440(1963); PR137,B672(1964)

$$H = H_0 + V$$

H_0 Kinetic energy

Bare states. The same spectrum as H

$$H_0|\varphi_\alpha\rangle = E_\alpha|\varphi_\alpha\rangle, \text{ Continuum spectrum}$$

$$H_0|\phi_n\rangle = E_{B_n}|\phi_n\rangle, \text{ Discrete spectrum}$$

$|\varphi_\alpha\rangle$ is made up by free particles of the continuum spectrum

$|\phi_n\rangle$ bare “elementary” states by one particle

Physical spectrum of H :

$$H|\psi_\alpha^\pm\rangle = E_\alpha|\psi_\alpha^\pm\rangle, |\psi_\alpha^\pm\rangle \text{ in/out states. Continuum spectrum}$$

$$H|\psi_{B_i}\rangle = E_{B_i}|\psi_{B_i}\rangle, E_{B_i} < 0. \text{ Discrete spectrum}$$

Compositeness
problem in
hadronic states

José Antonio Oller

Basic formalism on
elementari-
ness/compositeness

Interpretation
based on the
number operator

Explicit formulas

Resonances

Explicit formulas

Application to the
 $f_0(980)$ and
 $a_0(980)$

Conclusions

Elementariness: Z

Composition: X

Compositeness
problem in
hadronic states

$$\langle \psi_B | \psi_B \rangle = 1 = \underbrace{\sum_n |\langle \phi_n | \psi_B \rangle|^2}_Z + \underbrace{\int d\alpha |\langle \varphi_\alpha | \psi_B \rangle|^2}_X$$

$$1 = Z + X$$

$$X = 1 - Z = \int d\alpha \underbrace{\frac{|\langle \varphi_\alpha | V | \psi_B \rangle|^2}{(E_\alpha - E_B)^2}}_{\text{Wave function squared}}$$

$$H|\psi_B\rangle = E_B|\psi_B\rangle = (H_0 + V)|\psi_B\rangle$$

$$E_B \langle \varphi_\alpha | \psi_B \rangle = E_\alpha \langle \varphi_\alpha | \psi_B \rangle + \langle \varphi_\alpha | V | \psi_B \rangle$$

$$\langle \varphi_\alpha | \psi_B \rangle = \frac{\langle \varphi_\alpha | V | \psi_B \rangle}{E_B - E_\alpha}$$

Wave-function renormalization: $Z^{1/2}$

If there is only one bare “elementary” state $\langle \phi_1 | \psi_B \rangle = Z^{1/2}$

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Basic formalism on
elementari-
ness/compositeness

Interpretation
based on the
number operator

Explicit formulas

Resonances

Explicit formulas

Application to the
 $f_0(980)$ and
 $a_0(980)$

Conclusions

2.- Interpretation based on the number operator

Original developments in JAO, Ann.Phys. 396, 429 (2018)

Introducing bare “elementary” states as intermediate states are not needed. Asymptotic states are the only ones required

The continuum spectrum is common to H y H_0

Take two free particles of types A y B , $H_0|AB\gamma\rangle = E_\gamma|AB\gamma\rangle$

Creation and annihilation operators $a_\alpha^\dagger a_\alpha$, $b_\beta^\dagger b_\beta$

Number Operators:

$$\begin{aligned} N_D &= \int d\alpha a_\alpha^\dagger a_\alpha + \int d\beta b_\beta^\dagger b_\beta = N_D^A + N_D^B \\ &= \int d^3x \left[\psi_A^\dagger(x) \psi_A(x) + \psi_B^\dagger(x) \psi_B(x) \right] \end{aligned}$$

Compositeness problem in hadronic states

José Antonio Oller

Basic formalism on elementarieness/compositeness

Interpretation based on the number operator

Explicit formulas

Resonances

Explicit formulas

Application to the $f_0(980)$ and $a_0(980)$

Conclusions

New definition of X

Compositeness
problem in
hadronic states

$$H|\psi_B\rangle = E_B|\psi_B\rangle$$

$$X = \frac{1}{2}\langle\psi_B|N_D|\psi_B\rangle$$

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Basic formalism on
elementarity-
ness/compositeness

Interpretation
based on the
number operator

Explicit formulas

Resonances

Explicit formulas

Application to the
 $f_0(980)$ and
 $a_0(980)$

Conclusions

$$|\psi_B\rangle = \int d\gamma C_\gamma |AB_\gamma\rangle + \sum_n C_n |\phi_n\rangle$$

$$X = \frac{1}{2}\langle\psi_B|N_D^A + N_D^B|\psi_B\rangle = \int d\gamma |C_\gamma|^2$$

This definition is specially suitable for Effective Field Theories (EFTs) (e.g. ChPT)

It is very usual not to have explicit bare “elementary” states (fields)

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problem in
hadronic states

José Antonio Oller

Basic formalism on
elementari-
ness/compositeness

Interpretation
based on the
number operator

Explicit formulas

Resonances

Explicit formulas

Application to the
 $f_0(980)$ and
 $a_0(980)$

Conclusions

Calculation in QFT

This new definition is adequate for the treatment in QFT

Dirac or interaction image $[H_0, N_D] = 0 \rightarrow N_D(t) = N_D(0)$

$$V \rightarrow V e^{-\varepsilon|t|}, \quad \varepsilon \rightarrow 0^+$$

$$|\psi_B\rangle = |\varphi_B(0)\rangle = U_D(0, -\infty)|\varphi_B\rangle$$

$$|\psi_B\rangle = |\varphi_B(0)\rangle = U_D(0, +\infty)|\varphi_B\rangle$$

$$\begin{aligned} X &= \frac{1}{2}\langle\psi_B|N_D|\psi_B\rangle \\ &= \frac{1}{2}\langle\varphi_B|U_D(+\infty, 0)N_DU_D(0, -\infty)|\varphi_B\rangle \\ &= \frac{1}{2}\lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{+T/2} dt \langle\varphi_B|U_D(+\infty, t)N_D(t)U_D(t, -\infty)|\varphi_B\rangle \\ &\quad U_D(t, -\infty)|\varphi_B\rangle = e^{iH_0t}e^{-iE_Bt}U_D(0, -\infty)|\varphi_B\rangle \end{aligned}$$

$$X = \frac{1}{2} \lim_{T \rightarrow +\infty} \frac{1}{T} \int d^4x \langle\varphi_B|P \left[e^{-i \int_{-\infty}^{+\infty} dt' V_D(t')} \sum_i \psi_{A_i}^\dagger(x) \psi_{A_i}(x) \right] |\varphi_B\rangle$$

Calculation in QFT

S-matrix elements, with in/out states

$$|\psi_\alpha^+\rangle = U_D(0, -\infty) |\varphi_\alpha\rangle$$

$$|\psi_\alpha^-\rangle = |\varphi_B(0)\rangle = U_D(0, +\infty) |\varphi_\alpha\rangle$$

LSZ formalism

$$X = \frac{1}{2} \lim_{E \rightarrow E_B} \frac{(E - E_B)^2}{g_\alpha(k_B)^2} \langle \psi_\alpha^- | N_D | \psi_\alpha^+ \rangle$$

$$\langle \psi_\alpha^- | N_D | \psi_\alpha^+ \rangle = \frac{1}{2} \langle \varphi_\alpha | U_D(+\infty, 0) N_D U_D(0, -\infty) | \varphi_\alpha \rangle$$

$$= \frac{1}{2} \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{+T/2} dt \langle \varphi_\alpha | U_D(+\infty, t) N_D(t) U_D(t, -\infty) | \varphi_\alpha \rangle$$

$$X = \frac{1}{2} \lim_{E \rightarrow E_B} \frac{(E - E_B)^2}{g_\alpha(k_B)^2}$$

$$\times \lim_{T \rightarrow +\infty} \frac{1}{T} \int d^4x \langle \varphi_\alpha | P \left[e^{-i \int_{-\infty}^{+\infty} dt' V_D(t')} \sum_i \psi_{A_i}^\dagger(x) \psi_{A_i}(x) \right] | \varphi_\alpha \rangle$$

Compositeness
problem in
hadronic states

José Antonio Oller

Basic formalism on
elementarity-
ness/compositeness

Interpretation
based on the
number operator

Explicit formulas

Resonances

Explicit formulas

Application to the
 $f_0(980)$ and
 $a_0(980)$

Conclusions

3.- Explicit formulas

Compositeness
problem in
hadronic states

José Antonio Oller

Basic formalism on
elementari-
ness/compositeness

Interpretation
based on the
number operator

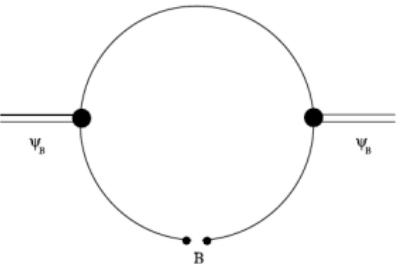
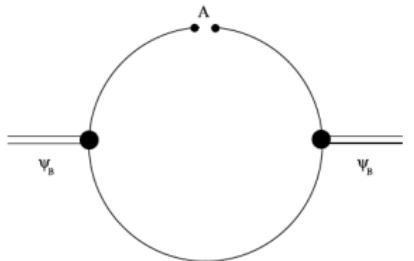
Explicit formulas

Resonances

Explicit formulas

Application to the
 $f_0(980)$ and
 $a_0(980)$

Conclusions



$$X_{\ell S} = \int \frac{d^3 k}{(2\pi)^3} \frac{g_{\ell S}^2(k^2)}{(k^2/2\mu - E_B)^2}$$

Weinberg's expression for
 $1 - Z$

$$X = \sum_{\ell S} X_{\ell S}$$

Equation for $g_{\ell S}(k)$ from $T = V + VGT$

$$g_{\ell S}(k) = \frac{1}{2\pi^2} \int_0^\infty k'^2 dk' V_{\ell S}(k, k') \frac{1}{k'^2/2\mu - E_B} g_{\ell S}(k')$$

$$g_{\ell S}(-k) = (-1)^\ell g_{\ell S}(k)$$

$|k_{n,B}| \ll \Lambda$ –Shallow bound state

Guo,JAO,PRD103,054021(2021)

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$$X = \frac{2\mu^2}{\pi^2} \int_0^\infty k^2 \frac{g(k^2)^2}{(k^2 - k_B^2)^2} dk , \quad k_B^2 = 2\mu E_B$$

- Expansion of $g(k^2)^2$ in powers of $k^2 - k_B^2$

$$g(k^2)^2 = g(k_B^2)^2 + c_1(k^2 - k_B^2) + c_2(k^2 - k_B^2)^2 + \dots$$

- Dimensional regularization → power-like divergences vanish

$$\begin{aligned} X &= -g(k_B^2)^2 \left. \frac{\partial G(E)}{\partial E} \right|_{E_B} - \frac{m^2 |k_B|}{\pi} \left. \frac{\partial g(k^2)^2}{\partial k^2} \right|_{E_B} \\ &= -g(k_B^2)^2 \frac{i\mu^2}{2\pi k_B} + \mathcal{O}\left(\frac{k_B^2}{\Lambda^2}\right) \end{aligned}$$

If k_B^2 dependence of $g(k_B^2)$ is neglected → Weinberg's analysis PR137,B672(1965) –See also Li,Guo,Pang,Wu, PRD105,L071502(2022)

Basic formalism on
elementari-
ness/compositenessInterpretation
based on the
number operator

Explicit formulas

Resonances

Explicit formulas

Application to the
 $f_0(980)$ and
 $a_0(980)$

Conclusions

Energy-independent potentials: Pure potential scattering

JAO, ANP396,429(2018)

Compositeness
problem in
hadronic states

José Antonio Oller

If the *finite-range* potential fulfills

$$\int_0^\infty \int_0^\infty dk dp |v_{\alpha\beta}(k, p)|^2 < \infty$$

Regular potential

$$\int_0^\infty dp |v_{\alpha\beta}(k, p)|^2 < M$$

If this is not fulfilled (singular potential)

Introduce some cut-off method to regularize V , e.g.

$$\omega_{\alpha\beta}(k, p) = v_{\alpha\beta}(k, p)\theta(\Lambda - k)\theta(\Lambda - p), \quad \Lambda \rightarrow \infty$$

Expansion in a complete set of linearly independent orthonormal real functions $\{f_s(k)\}$ in $[0, \infty)$:

$$\omega_{\alpha\beta}^{(N)}(k_\alpha, p_\beta) = \sum_{s,s'=1}^N f_s(k_\alpha) \omega_{\alpha\beta;ss'} f_{s'}(p_\beta)$$

$$\omega_{\alpha\beta;ss'} = \int_0^\infty \int_0^\infty dk dp f_s(k) \omega_{\alpha\beta}(k, p) f_{s'}(p)$$

$$\omega_{\alpha\beta}(k_\alpha, p_\beta) = \lim_{N \rightarrow \infty} \omega_{\alpha\beta}^{(N)}(k_\alpha, p_\beta)$$

Basic formalism on
elementarity-
ness/compositeness

Interpretation
based on the
number operator

Explicit formulas

Resonances

Explicit formulas

Application to the
 $f_0(980)$ and
 $a_0(980)$

Conclusions

$$t_{\alpha\beta}^{(N)}(k, p; E) = \omega_{\alpha\beta}^{(N)}(k, p) + \sum_{\gamma} \frac{m_{\gamma}}{\pi^2} \int_0^{\infty} \frac{dq q^2}{q^2 - 2m_{\gamma}E} \omega_{\alpha\gamma}^{(N)}(k, q) t_{\gamma\beta}^{(N)}(q, p; E)$$

$$t_{\alpha\beta;ss'}(E) = \int_0^{\infty} \int_0^{\infty} dk dp f_s(k) t_{\alpha\beta}(k, p; E) f_{s'}(p) .$$

$$[\omega] = \begin{pmatrix} [\omega_{11}] & [\omega_{12}] & \dots & [\omega_{1n}] \\ [\omega_{21}] & [\omega_{22}] & \dots & [\omega_{2n}] \\ \dots & \dots & \dots & \dots \\ [\omega_{n1}] & [\omega_{n2}] & \dots & [\omega_{nn}] \end{pmatrix}, \quad [\omega_{\alpha\beta}] \text{ } N \times N \text{ matrix}$$

$$[f(k_{\alpha})]^T = (\underbrace{0, \dots, 0}_{N(\alpha-1) \text{ places}}, f_1(k_{\alpha}), f_2(k_{\alpha}), \dots, f_N(k_{\alpha}), 0, \dots, 0)$$

$$\omega_{\alpha\beta}(k_{\alpha}, p_{\beta}) = [f(k_{\alpha})]^T \cdot [\omega] \cdot [f(p_{\beta})]$$

$$t_{\alpha\beta}(k_{\alpha}, p_{\beta}; E) = [f(k_{\alpha})]^T \cdot [t(E)] \cdot [f(p_{\beta})]$$

$$[G(E)] = \sum_{\alpha=1}^n [G_{\alpha}(E)], \quad [G_{\alpha}(E)] = \frac{m_{\alpha}}{\pi^2} \int_0^{\infty} dq \frac{q^2}{q^2 - 2m_{\alpha}E} [f_{\alpha}(q)] \cdot [f_{\alpha}(q)]^T$$

Basic formalism on
elementari-
ness/compositeness

Interpretation
based on the
number operator

Explicit formulas

Resonances

Explicit formulas

Application to the
 $f_0(980)$ and
 $a_0(980)$

Conclusions

$$[t(E)] = [\omega(E)] - [\omega(E)] \cdot [G(E)] \cdot [t(E)]$$

$$[t(E)] = [D(E)]^{-1} = [d]/\det[D]$$

$$[D(E)] = [\omega(E)]^{-1} + [G(E)]$$

$$X_\alpha = \frac{m_\alpha/\pi^2}{[f(p_\alpha)]^T \cdot [d] \cdot \frac{\partial[D]}{\partial E} \cdot [d] \cdot [f(p_\alpha)]} \frac{\partial}{\partial E} \int_0^\infty dk \frac{k^2}{k^2 - 2m_\alpha E} \\ \times [f(p_\alpha)]^T \cdot [d] \cdot [f(k_\alpha)] [f(k_\alpha)]^T \cdot [d] \cdot [f(p_\alpha)] \Big|_{E=E_B, p_\alpha=i\gamma_\alpha}$$

- For $\partial[\omega]/\partial E = 0$ then

$$\frac{\partial[D]}{\partial E} = \frac{\partial[G]}{\partial E}$$

$$1 = \sum_{\alpha=1}^n X_\alpha$$

Basic formalism on
elementari-
ness/compositeness

Interpretation
based on the
number operator

Explicit formulas

Resonances

Explicit formulas

Application to the
 $f_0(980)$ and
 $a_0(980)$

Conclusions

Deuteron

Compositeness
problem in
hadronic states

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ChPT Energy-independent potentials worked up to N⁴LO
Error $\lesssim 4\%$ for Deuteron properties

Rodríguez-Entem, Machleidt, Nosyk, Front.Phys.8:57(2022)

$$X = 0.96 - 1.0$$

$X = 1$ for pure potential scattering

Basic formalism on
elementarity-
ness/compositeness

Interpretation
based on the
number operator

Explicit formulas

Resonances

Explicit formulas

Application to the
 $f_0(980)$ and
 $a_0(980)$

Conclusions

4.- Gamow states

Compositeness
problem in
hadronic states

Confusion: Often one finds written that resonances are not normalizable

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Outgoing spherical wave in a scattering process: e^{ikr}/r

Bound state, $k \rightarrow ik_i$, $k_i > 0 \Rightarrow e^{-k_i r}$ $k_i = \sqrt{2m|E_B|}$

Basic formalism on
elementarity-
ness/compositeness

Interpretation
based on the
number operator

Resonance

Explicit formulas

$$k_n = \sqrt{2mE_n} \equiv k_r - ik_i$$

Resonances

$$k \rightarrow k_r - ik_i, k_r, k_i > 0 \Rightarrow e^{ik_r r} e^{\color{red}k_i r}$$

Explicit formulas

Resonance wave function is not square integrable

Application to the
 $f_0(980)$ and
 $a_0(980)$

Conclusions

$$\int_0^\infty dr e^{2k_i r} \rightarrow \infty$$

Gamow wave functions

Gamow function $\psi(r)$ in position representation

$$-\frac{\hbar^2}{2m} \nabla^2 \psi + V(r) \psi = E_n \psi$$

Boundary condition $r \rightarrow \infty$ outgoing spherical waves

$$E = M_R - i\Gamma/2 \in \mathbb{C}$$

Hernández, Mondragón PRC29, 722 (1984) The problem is set first in
momentum representation

$$\frac{p^2}{2m} \psi_{E_n}(p) + \int V(p, p'') \psi_{E_n}(p'') d^3 p'' = E_n \psi_{E_n}(p)$$

The solution is

$$\psi_{E_n}(p) = \left[\int \int \frac{\delta(p - p')}{E^{(+)} - \frac{p'^2}{2m}} V(p', p'') \psi_E(p'') dp' dp'' \right]_{E_n}$$

$E^{(+)} = E + i\varepsilon, \varepsilon \rightarrow 0^+$ then $E + i\varepsilon \rightarrow E_n$

Analytic continuation of the free particle Green function

Partial-wave decomposition

$$\psi_{E_n}(p) = \sum_{l=0}^{\infty} \sum_{m=-l}^l u_{nl}(p; k_n) \frac{Y_{lm}(\Omega_p)}{p}$$

$$V(p, p') = \sum_{l=0}^{\infty} \sum_{m=-l}^l \frac{Y_{lm}(\hat{p})}{p} V_l(p, p') \frac{Y_{lm}(\hat{p}')^*}{p'}$$

Complete set of orthonormal functions in L^2

$$u_{nl}(p; k_n) = \sum_s u_{nls}(k_n) f_s^{(l)}(p)$$

$$V_l(q, q') = \sum_{s,s'=1}^N f_s^{(l)}(q) v_{ss'} f_s^{(l)}(q')^*$$

Compositeness
problem in
hadronic states

José Antonio Oller

Basic formalism on
elementarity-
ness/compositeness

Interpretation
based on the
number operator

Explicit formulas

Resonances

Explicit formulas

Application to the
 $f_0(980)$ and
 $a_0(980)$

Conclusions

Analytic continuation: $k + i\varepsilon \rightarrow k_n$, $\text{Im}k_n < 0$

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Basic formalism on
elementari-
ness/compositeness

Interpretation
based on the
number operator

Explicit formulas

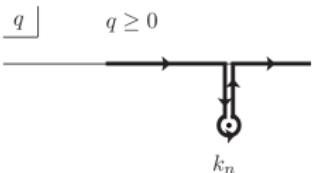
Resonances

Explicit formulas

Application to the
 $f_0(980)$ and
 $a_0(980)$

Conclusions

$$G_{lss'}^{(+)}(k) = \int_0^\infty \frac{f_s^{(I)}(q)^* f_{s'}^{(I)}(q)}{k^2 - q^2} dq - i\pi \underbrace{\frac{f_s^{(I)}(k)^* f_{s'}^{(I)}(k)}{k}}_{\text{Second Riemann sheet}}$$



$$n_{nl s}(\textcolor{red}{k_n}) = \sum_{s,s'} G_{lss'}^{(+)}(\textcolor{red}{k_n}) \underbrace{V_{s's''}}_{\text{Potential}} n_{nl s''}(\textcolor{red}{k_n})$$

Normalization and orthogonality relations

$$\left[\int_0^\infty u_{nl}(q; k) u_{n'l'}(q; k') dq \right] \begin{matrix} k \rightarrow k_n \\ k' \rightarrow k_{n'} \end{matrix} = 0$$

$$\left[\int_0^\infty u_{nl}(q; k)^2 dq \right]_{k_n} = \left[\int_0^\infty u_{nl}(r; k)^2 dr \right]_{k_n} = 1$$

$$\left[\int |u_{nl}(q; k)|^2 dq \right]_{k \rightarrow k_n} = 0$$

Example: $V(r) = 0$ for $r > R$

For $r > R$ we have the free case of outgoing spherical waves

$$u_{n0}^{(as)}(r; k_n) = C_{n0} e^{ik_n r}, \quad r > R$$

Normalization

$$\left[\int_0^\infty u_{n0}(r; k)^2 dr \right]_{k_n} = \left[\int_0^R u_{n0}(r; k)^2 dr + C_{n0}^2 \frac{e^{2ik_n R}}{2ik_n} \Big|_R^\infty \right]_{k_n}$$

$$\int_0^R u_{n0}(r; k_n)^2 dr - C_{n0}^2 \frac{e^{ik_n R}}{2ik_n} = 1$$

Analysis based on Scattering Theory. Number operators

JAO, Ann.Phys.396,429(2018)

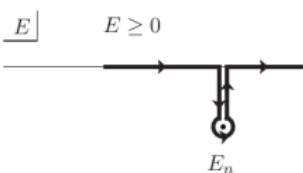
A resonance stems from the analytic continuation in energy of the in states with energy $E + i\varepsilon$, just above the real E axis

- Let $|\psi_\alpha^+\rangle$ be an in two-body scattering state

$$\begin{aligned} |\psi_\alpha^+\rangle &= U_D(0, -\infty) |\varphi_\alpha\rangle \\ &= |\varphi_\alpha\rangle + \int d\gamma \frac{T_{\gamma\alpha}(E + i\varepsilon)}{E + i\varepsilon - E_\gamma} |\varphi_\gamma\rangle + \sum_n \frac{T_{n\alpha}(E)}{E - E_n} |\varphi_n\rangle \end{aligned}$$

S. Weinberg, QFT, Vol.1

- Then $E + i\varepsilon \rightarrow E_n$



When crossing the real positive energy axis $E > 0$ one has to deform the contour of integration: change of Riemann sheet

$$T(E + i\varepsilon) \rightarrow T''(E - i\varepsilon)$$

Compositeness problem in hadronic states

José Antonio Oller

Basic formalism on elementarieness/compositeness

Interpretation based on the number operator

Explicit formulas

Resonances

Explicit formulas

Application to the $f_0(980)$ and $a_0(980)$

Conclusions

However ...

$$\langle \psi_\alpha^+ | = \langle \varphi_\alpha | + \int d\gamma \frac{T_{\alpha\gamma}(E - i\varepsilon)}{E - i\varepsilon - E_\gamma} \langle \varphi_\gamma | + \sum_n \frac{T_{\alpha n}(E)}{E - i\varepsilon - E_n} \langle \varphi_n |$$

$$T(E \pm i\varepsilon)^\dagger = T(E \mp i\varepsilon)$$

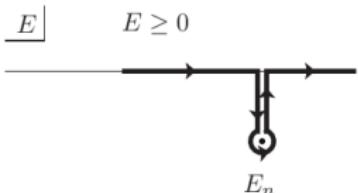
The analytic continuation to $E_n \equiv M_R - i\Gamma/2$ is in the 1st RS, but **there is no resonance pole there**

The out scattering states must be used $|\psi_\alpha^- \rangle$, $E - i\varepsilon$

$$|\psi_\alpha^- \rangle = |\varphi_\alpha \rangle + \int d\gamma \frac{T_{\alpha\gamma}(E - i\varepsilon)}{E - i\varepsilon - E_\gamma} |\varphi_\gamma \rangle + \sum_n \frac{T_{\alpha n}(E)}{E - E_n} |\varphi_n \rangle$$

$$\langle \psi_\alpha^- | = \langle \varphi_\alpha | + \int d\gamma \frac{T_{\alpha\gamma}(E + i\varepsilon)}{E + i\varepsilon - E_\gamma} \langle \varphi_\gamma | + \sum_n \frac{T_{\alpha n}(E)}{E - E_n} \langle \varphi_n |$$

Crossing the right-hand cut (RC) moving to E_n



$$T(E + i\varepsilon) \rightarrow T''(E - i\varepsilon)$$

The resonance pole is reached
both by $|\psi_\alpha^+ \rangle$ and $\langle \psi_\alpha^- |$

Calculation in QFT

S-matrix element with an external source $\sum_i \psi_{A_i}^\dagger(x) \psi_{A_i}(x)$

$$\frac{1}{2} \langle \psi_\alpha^- | N_D^A + N_D^B | \psi_\alpha^+ \rangle = \frac{1}{2} \langle \psi_\alpha^- | \int d^3x \left(\psi_A^\dagger(x) \psi_A(x) + \psi_B^\dagger(x) \psi_B(x) \right) | \psi_\alpha^+ \rangle$$

José Antonio Oller

Basic formalism on
elementari-
ness/compositeness

Interpretation
based on the
number operator

Explicit formulas

Resonances

Explicit formulas

Application to the
 $f_0(980)$ and
 $a_0(980)$

Conclusions

$$X_\alpha = \frac{1}{g_\alpha(k_n)^2} \lim_{E \rightarrow E_n} (E - E_n)^2 \frac{1}{2} \langle \psi_\alpha^- | N_D^A + N_D^B | \psi_\alpha^+ \rangle$$

$$X_\alpha = \frac{1}{2} \lim_{E \rightarrow E_n} \frac{(E - E_n)^2}{g_\alpha(k_n)^2}$$

$$\times \lim_{T \rightarrow +\infty} \frac{1}{T} \int d^4x \langle \varphi_\alpha | P \left[e^{-i \int_{-\infty}^{+\infty} dt' V_D(t')} \sum_i \psi_{A_i}^\dagger(x) \psi_{A_i}(x) \right] | \varphi_\alpha \rangle$$



$$g_\alpha(k_n)^2 = \lim_{E \rightarrow E_n} (E - E_n) T_{\alpha\alpha}(E)$$

5.- Explicit formulas

Expansion in a complete set of orthonormal linearly independent real functions $\{f_s(k)\}$ in $[0, \infty)$:

$$\omega_{\alpha\beta;ss'} = \int_0^\infty \int_0^\infty dk dp f_s(k) \omega_{\alpha\beta}(k, p) f_{s'}(p)$$

$$\omega_{\alpha\beta}(k_\alpha, p_\beta; E) = [f_\alpha(k_\alpha)]^T \cdot [\omega] \cdot [f_\beta(p_\beta)]$$

$$t_{\alpha\beta}^{II}(k_\alpha, p_\beta; E) = [f_\alpha(k_\alpha)]^T \cdot [t^{II}(E)] \cdot [f_\beta(p_\beta)]$$

$$\begin{aligned} [G_\alpha^{II}(E)] &= \frac{m_\alpha}{\pi^2} \int_0^\infty dq \frac{q^2}{q^2 - 2m_\alpha E} [f_\alpha(q_\alpha)] \cdot [f_\alpha(q_\alpha)]^T \\ &\quad + \frac{im_\alpha}{\pi} \sqrt[4]{2m_\alpha E} \left[f_\alpha(\sqrt[4]{2m_\alpha E}) \right] \cdot \left[f_\alpha(\sqrt[4]{2m_\alpha E}) \right]^T \end{aligned}$$

$$[t^{II}(E)] = [D^{II}(E)]^{-1}$$

$$[D^{II}(E)] = [\omega(E)]^{-1} + [G^{II}(E)]$$

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Basic formalism on
elementarity-
ness/compositeness

Interpretation
based on the
number operator

Explicit formulas

Resonances

Explicit formulas

Application to the
 $f_0(980)$ and
 $a_0(980)$

Conclusions

$$X_\alpha = \left([p_\alpha]^T \cdot [d^{II}] \cdot \frac{\partial[D^{II}]}{\partial E} \cdot [d^{II}] \cdot [p_\alpha] \right)^{-1}$$

$$\times \frac{\partial}{\partial E} \left(\frac{m_\alpha}{\pi^2} \int_0^\infty dk \frac{k^2}{k^2 - 2m_\alpha E} [k_\alpha]^T \cdot [d^{II}] \cdot [p_\alpha] [k_\alpha]^T \cdot [d^{II}] \cdot [p_\alpha] \right.$$

$$\left. + i \frac{m_\alpha}{\pi} \sqrt[\"]{2m_\alpha E} [\sqrt[\"]{2m_\alpha E}]^T \cdot [d^{II}] \cdot [p_\alpha] [\sqrt[\"]{2m_\alpha E}]^T \cdot [d^{II}] \cdot [p_\alpha] \right) \Big|_{E=E_R, p_\alpha=\nu_\alpha}$$

- For $\partial[\omega]/\partial E = 0$. Pure potential scattering

$$1 = X = \sum_{\alpha=1}^n X_\alpha$$

For instance, virtual state in the 1S_0 NN scattering

Basic formalism on
elementari-
ness/compositeness

Interpretation
based on the
number operator

Explicit formulas
 $E=E_R, p_\alpha=\nu_\alpha$
Resonances

Explicit formulas

Application to the
 $f_0(980)$ and
 $a_0(980)$

Conclusions

Basic formalism on
elementari-
ness/compositeness

Interpretation
based on the
number operator

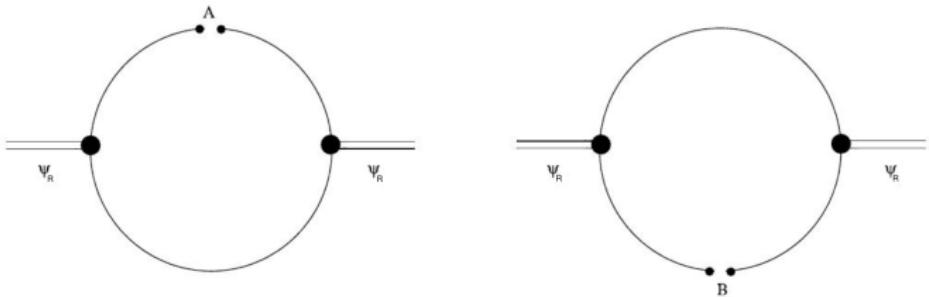
Explicit formulas

Resonances

Explicit formulas

Application to the
 $f_0(980)$ and
 $a_0(980)$

Conclusions



$$X_{\ell S} = \int \frac{d^3 k}{(2\pi)^3} \frac{g_{\ell S}^2(k^2)}{(k^2/2\mu - E_n^2)^2} + \frac{i\mu^2}{\pi k_n} \left[\frac{\partial}{\partial k} k g_{\ell S}^2(k^2) \right]_{k=k_n}$$

$$X = \sum_{\ell S} X_{\ell S}$$

For energy-independent potentials

$$\frac{\partial V}{\partial E} = 0$$

$$X = \sum_{\alpha=1}^n X_\alpha = 1 \quad \text{normalization to 1}$$

Sum rule

From two-body unitarity $\text{Im } T^{-1}|_{ij} = \delta_{ij}\rho_i$ along the RC

$$\rho_i = p_i / 8\pi\sqrt{s}$$

General expression for a PWA in coupled channel in matrix notation

$$T(s) = [\mathcal{K}(s)^{-1} + G(s)]^{-1}$$

$$G(s)_i = a(s_0)_i - \frac{s - s_0}{\pi} \int_0^\infty \frac{\rho_i(s')ds'}{(s' - s)(s' - s_0)}$$



Derivative with respect to s , $s \rightarrow s_R$, double pole

$$1 = \underbrace{- \sum_i g_i^2 \left. \frac{dG^{II}(s)_i}{ds} \right|_{s_R}}_{X_i} + \underbrace{g^T G^{II}(s_R) \left. \frac{dK(s)}{ds} \right|_{s_R} G^{II}(s_R) g}_Z$$

This is the same expression as for near-threshold states or separable potentials

$X_i \rightarrow |X_i|$ for resonances

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Basic formalism on
elementarity-
ness/compositeness

Interpretation
based on the
number operator

Explicit formulas

Resonances

Explicit formulas

Application to the
 $f_0(980)$ and
 $a_0(980)$

Conclusions

Transformation of the S matrix: Phase redefinition of the couplings

Z.H.Guo, JAO, PRD93,096001(2016)

Let us consider a narrow resonance $\Gamma \ll M_R - m_{th}$

Laurent series around the resonance pole $s_P = (M_R - i\Gamma/2)^2$

$$S(s) = \frac{R}{s - s_P} + S_0(s)$$

$$S(s)S(s)^\dagger = I$$

$S_0(s) \rightarrow S_0$, constant

$$\begin{aligned} & (s - s_P)(s - s_P^*)S_0S_0^\dagger + (s - s_P)S_0R^\dagger + (s - s_P^*)RS_0^\dagger + RR^\dagger \\ &= (s - s_P)(s - s_P^*) \end{aligned}$$

$$S_0S_0^\dagger = I, \quad S_0R^\dagger + RS_0^\dagger = 0$$

$$-s_P S_0 R^\dagger - s_P^* R S_0^\dagger + R R^\dagger = 0$$

Time reversal invariance $\rightarrow S(s)$ and S_0 are symmetric

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Basic formalism on
elementarity-
ness/compositeness

Interpretation
based on the
number operator

Explicit formulas

Resonances

Explicit formulas

Application to the
 $f_0(980)$ and
 $a_0(980)$

Conclusions

Solution

Compositeness
problem in
hadronic states

$$S_0 = \mathcal{O} \mathcal{O}^T, \text{ [Jost functions } \mathcal{O}^T = \mathcal{O}^{*-1}]$$

$$\mathcal{O} \mathcal{O}^\dagger = I$$

Matrix of residues $R = i\lambda \mathcal{O} \mathcal{A} \mathcal{O}^T ; \lambda = 2\text{Im } s_P = -2M_R \Gamma_R$

\mathcal{A} is a rank 1 symmetric projector operator

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Basic formalism on
elementarity-
ness/compositeness

Interpretation
based on the
number operator

Explicit formulas

Resonances

Explicit formulas

Application to the
 $f_0(980)$ and
 $a_0(980)$

Conclusions

$$\mathcal{A}^\dagger = \mathcal{A}, \mathcal{A}^2 = \mathcal{A}, \mathcal{A}^T = \mathcal{A}$$

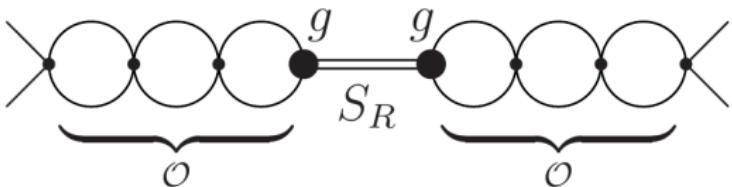
$$\mathcal{A} = -\frac{1}{\lambda} \omega \omega^T, \text{ Tr}\mathcal{A} = 1, \text{ con } \omega \text{ pure real or imaginary}$$

$S_R(s)$ is a purely resonant S matrix

$$S(s) = \mathcal{O} \underbrace{\left(I + \frac{i\lambda \mathcal{A}}{s - s_R} \right)}_{S_R(s)} \mathcal{O}^T$$

$$S_{\alpha\beta}(s) = \mathcal{O}_{\alpha\mu}\mathcal{O}_{\beta\nu} \left(I + \frac{i\lambda A}{s - s_R} \right)_{\mu\nu}$$

The nonresonant S matrix, S_0 , dresses in the same way the initial and final resonant couplings



Corrections due to initial- and final-state interactions from S_0

They typically modify the phases of the resonance couplings

Basic formalism on
elementari-
ness/compositeness

Interpretation
based on the
number operator

Explicit formulas

Resonances

Explicit formulas

Application to the
 $f_0(980)$ and
 $a_0(980)$

Conclusions

Example: $\pi\pi - K\bar{K}$ scattering

JAO, Oset, NPA620,438(1997)

Compositeness
problem in
hadronic states

José Antonio Oller

Basic formalism on
elementari-
ness/compositeness

Interpretation
based on the
number operator

Explicit formulas

Resonances

Explicit formulas

Application to the
 $f_0(980)$ and
 $a_0(980)$

Conclusions

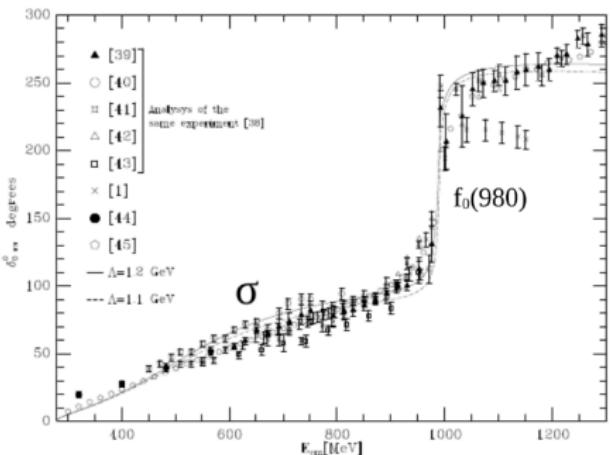


Figure: Isoscalar scalar $\pi\pi$ phase shifts. $J^{PC} = 0^{++}$

$$S = \begin{pmatrix} \eta e^{2i\delta_{11}} & i(1-\eta^2)^{1/2} e^{i(\delta_1+\delta_2)} \\ i(1-\eta^2)^{1/2} e^{i(\delta_1+\delta_2)} & \eta e^{2i\delta_2} \end{pmatrix}$$

The coupling between channels implies a phase $\delta_1 \approx \pi/2$ just at the $f_0(980)$ rise \rightarrow phase of the $f_0(980)$ coupling to $\pi\pi$

The moduli of the couplings g_α have physical meaning

Compositeness
problem in
hadronic states

$$\Gamma_\alpha = \frac{|g_\alpha|^2}{8\pi M_R^2}$$

The S -matrix phase transformation only change the phases of the resonance couplings

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Basic formalism on
elementarity-
ness/compositeness

Interpretation
based on the
number operator

Explicit formulas

Resonances

Explicit formulas

Application to the
 $f_0(980)$ and
 $a_0(980)$

Conclusions

$$S_{\mathcal{O}}(s) \equiv \mathcal{O} S(s) \mathcal{O}^T$$

$$\mathcal{O} = \text{diag}(e^{i\phi_1}, \dots, e^{i\phi_n})$$

$$g_i^2 \rightarrow g_i^2 e^{2i\phi_i}$$

There is a scattering matrix $T_{\mathcal{O}}(s)$ associated to the new $S_{\mathcal{O}}$

$$\langle \psi_R^- | N_D | \psi_R^+ \rangle \rightarrow |\langle \psi_R^- | N_D | \psi_R^+ \rangle|$$

$$|X_\alpha| = \frac{1}{2} |\langle \psi_R^- | N_D | \psi_R^+ \rangle|$$

Condition $|X_\alpha|$

Compositeness
problem in
hadronic states

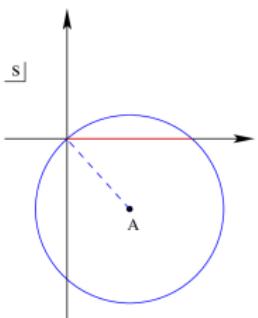
It makes sense if:

- ▷ The Laurent expansion around s_p is valid in some interval of physical (real values above threshold) for s

$S(s)S(s)^\dagger = I$ is meaningful

Condition A: $s_n < \text{Res}_p < s_{n+1}$

s_n is the threshold of channel n



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Basic formalism on
elementarity-
ness/compositeness

Interpretation
based on the
number operator

Explicit formulas

Resonances

Explicit formulas

Application to the
 $f_0(980)$ and
 $a_0(980)$

Conclusions

6.- Application to the $f_0(980)$ and $a_0(980)$

Wang,Kang,JAO,Zhang,PRD105,074016(2022)

Compositeness
problem in
hadronic states

$f_0(980)$ Garcia-Martin,Kaminski,Pelaez, Ruiz de Elvira, PRL107, 072001
(2011)

$$f_0(980) \quad \text{RS II [MeV]} \quad (-, +) \quad m_{f_0} = 996 \pm 7 \quad \Gamma_{f_0} = 50^{+20}_{-12}$$

$$r_{\text{exp}} = \Gamma_{\pi\pi}/\Gamma_{f_0}$$

$r_{\text{exp}} = 0.52$ Aubert *et al.* (BABAR Collaboration), PRD74,032003 (2006)

$r_{\text{exp}} = 0.75$ Ablikim *et al.* (BES Collaboration), PRD72,092002 (2005)

$a_0(980)$ Albrecht et al. (Crystal Barrel Collaboration), EPJC80,453(2020)

$$\begin{array}{lll} a_0(980) & \text{RS II [MeV]} \quad (-, +) & \text{RS III [MeV]} \quad (-, -) \\ m_{a_0} & = 1004.1 \pm 6.7 & 1002.4 \pm 6.5 \\ \Gamma_{a_0} & = 97.2 \pm 6.0 & 127.0 \pm 7.1 \\ \frac{\Gamma_{K\bar{K}}}{\Gamma_{\pi\eta}} & = (13.8 \pm 3.5)\% & \frac{\Gamma_{K\bar{K}}}{\Gamma_{\pi\eta}} = (14.9 \pm 3.9)\% \end{array}$$

PDG: $\Gamma(a_0 \rightarrow K\bar{K})/\Gamma(a_0 \rightarrow \pi\eta) = (17.7 \pm 2.4)\%$

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Basic formalism on
elementarity-
ness/compositeness

Interpretation
based on the
number operator

Explicit formulas

Resonances

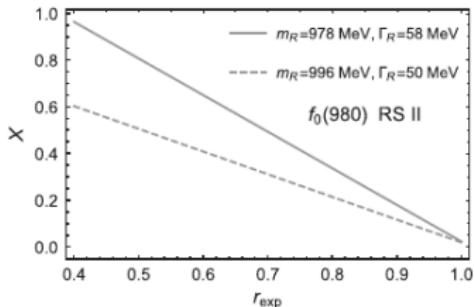
Explicit formulas

Application to the
 $f_0(980)$ and
 $a_0(980)$

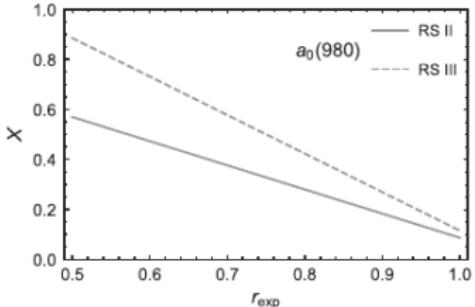
Conclusions

We make use of a Flatté parameterization and fix its 3 unknown parameters in terms of m_R , Γ_R and r_{exp}

$$T_{ij} = \frac{g_i g_j}{E - E_f + i \frac{\tilde{\Gamma}_1}{2} + \frac{i}{2} g_2 \sqrt{m_K E}}$$



(a)



(b)

$f_0(980)$ RSII

r_{exp}	$[-\Delta, \Delta]$	W_{f_0}	$(1 - W_{f_0})\Delta$	$1 - W_{f_0}$	X
0.52 [70]	[-25, 25]	0.13	0.87		
	[-50, 50]	0.21	0.79		
	[-75, 75]	0.27	0.73		
	[-100, 100]	0.31	0.69	0.76 ± 0.15	0.51 ± 0.22
0.75 [95]	[-25, 25]	0.30	0.70		
	[-50, 50]	0.45	0.55		
	[-75, 75]	0.55	0.45		
	[-100, 100]	0.61	0.39	0.50 ± 0.15	0.29 ± 0.14

Compositeness problem in hadronic states

José Antonio Oller

Basic formalism on elementarity/compositeness

Interpretation based on the number operator

Explicit formulas

Resonances

Explicit formulas

Application to the $f_0(980)$ and $a_0(980)$

Conclusions

$a_0(980)$ RSII

Compositeness
problem in
hadronic states

José Antonio Oller

Basic formalism on
elementari-
ness/compositeness

Interpretation
based on the
number operator

Explicit formulas

Resonances

Explicit formulas

Application to the
 $f_0(980)$ and
 $a_0(980)$

Conclusions

r_{exp}	$[-\Delta, \Delta]$	W_{a_0}	$(1 - W_{a_0})_{\Delta}$	$1 - W_{a_0}$	X
0.85 [49]	$[-50, 50]$	0.38	0.62		
	$[-100, 100]$	0.57	0.43		
	$[-150, 150]$	0.67	0.33		
	$[-200, 200]$	0.73	0.27	0.33–0.43	0.216 ± 0.017
0.87 [72]	$[-50, 50]$	0.39	0.61		
	$[-100, 100]$	0.59	0.41		
	$[-150, 150]$	0.68	0.32		
	$[-200, 200]$	0.74	0.26	0.32–0.41	0.198 ± 0.016

$a_0(980)$ RSIII

r_{exp}	$[-\Delta, \Delta]$	W_{a_0}	$(1 - W_{a_0})_{\Delta}$	$1 - W_{a_0}$	X
0.85 [49]	$[-50, 50]$	0.39	0.61		
	$[-100, 100]$	0.59	0.41		
	$[-150, 150]$	0.69	0.31		
	$[-200, 200]$	0.75	0.25		
	$[-250, 250]$	0.79	0.21	0.31–0.41	0.303 ± 0.030
0.87 [72]	$[-50, 50]$	0.40	0.60		
	$[-100, 100]$	0.60	0.40		
	$[-150, 150]$	0.70	0.30		
	$[-200, 200]$	0.76	0.24		
	$[-250, 250]$	0.80	0.20	0.30–0.40	0.279 ± 0.037

Spectral density function. NR Resonance

Bogdanova, Hale, Markushin, PRC44,1289(1991);

Baru,Haidenbauer,Hanhart,Kalashnikova,Kudryavtsev,PLB586,53(2004)

Spectral density of the bare state $|\psi_0\rangle$: $\omega(E)$

$$|\psi_0\rangle = \int dk c_0(k) |k\rangle$$

$$\omega(E) = 4\pi\mu k |c_0(E)|^2 \theta(E)$$

$$\int_0^\infty dE \omega(E) = \begin{cases} 1 & \text{No bound states} \\ 1 - Z & \text{With bound states} \end{cases}$$

How to implement it? Select the *resonant* region around threshold

$$W = \int_{E_-}^{E_+} dE \omega(E)$$

Conceptually, it is not *fully* settled as a quantitative estimate of compositeness for resonances

Compositeness problem in hadronic states

José Antonio Oller

Basic formalism on elementarieness/compositeness

Interpretation based on the number operator

Explicit formulas

Resonances

Explicit formulas

Application to the $f_0(980)$ and $a_0(980)$

Conclusions

It provides a smooth transition from the clear bound states and narrow resonances

It has a clear connection with the pole-counting rule of Morgan NPA543,632(1992), with the presence of nearby CDD poles Kang,Oller, EPJC77,399(2017)

The spectral density method compares well with the on-shell method to get X

Basic formalism on elementari-
ness/compositeness

Interpretation
based on the
number operator

Explicit formulas

Resonances

Explicit formulas

Application to the
 $f_0(980)$ and
 $a_0(980)$

Conclusions

Proper interpretation of the Flatté parameterization

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$$T_{ij} = \frac{g_i g_j}{E - E_f + i \frac{\tilde{\Gamma}_1}{2} + \frac{i}{2} g_2 \sqrt{m_K E}}$$

$$D(E) = E - E_f + i \frac{\tilde{\Gamma}_1}{2} + \frac{i}{2} g_2 \sqrt{m_K E}$$

Wave-function renormalization

$$\begin{aligned}\beta &= \lim_{E \rightarrow E_R} \left| \frac{E - E_R}{D(E)} \right| \\ &= \left| \frac{1}{1 + \frac{ig_2}{4} \sqrt{\frac{m_K}{E_R}}} \right| = \frac{\sqrt{8u}}{\left(g_2^2 m_K + 8u + 4\sigma g_2 \sqrt{m_K(u - 2M_R)} \right)^{1/2}}\end{aligned}$$

$$E_R = M_R - i\Gamma_R/2 , \quad u = 2|E_R| , \quad \sigma = \pm 1 \text{ (2nd, 3rd RS)}$$

$$\sigma = +1 \text{ 2nd RS } (-, +) , \quad \sigma = -1 \text{ 3rd RS } (-, -)$$

RSs are distinguished according to the signs of $\Im p_1$ and $\Im p_2$

Basic formalism on
elementarity-
ness/compositeness

Interpretation
based on the
number operator

Explicit formulas

Resonances

Explicit formulas

Application to the
 $f_0(980)$ and
 $a_0(980)$

Conclusions

Bare couplings $g_1 = 8\pi M_R^2 \tilde{\Gamma}_1 / p_1(M_R)$, and g_2

Compositeness
problem in
hadronic states

Dressed couplings

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$$|\gamma_1|^2 = g_1 \beta$$

$$|\gamma_2|^2 = 32\pi m_K^2 g_2 \beta$$

Physical widths

- 2nd RS $(-, +)$: $\ln D(E) = E - E_f + \frac{i}{2} \tilde{\Gamma}_1 - \frac{i}{2} g_2 \sqrt{m_K E}$
The $+/-$ is adjusted to 2nd RS

$$m_R - \frac{i}{2} \Gamma_1 + \frac{i}{2} \Gamma_2 = m_R - \frac{i}{2} (\Gamma_1 - 2\Gamma_2) - \frac{i}{2} \Gamma_2$$

$$\Gamma = \Gamma_1 - \Gamma_2 = \Gamma_1 - \Gamma(1 - r_{\text{exp}})$$

$$\Gamma_1 = \beta \tilde{\Gamma}_1 = (2 - r_{\text{exp}}) \Gamma \rightarrow 2\Gamma > \Gamma_1 > \Gamma \quad , \quad \Gamma_2 = \Gamma - \Gamma_1$$

- 3rd RS $(-, -)$: $\ln D(E) = E - R_f + \frac{i}{2} \tilde{\Gamma}_1 + \frac{i}{2} g_2 \sqrt{m_K E}$

$$\Gamma_1 = \beta \tilde{\Gamma}_1 = r_{\text{exp}} \Gamma \quad , \quad \Gamma_2 = \Gamma - \Gamma_1$$

Basic formalism on
elementarity-
ness/compositeness

Interpretation
based on the
number operator

Explicit formulas

Resonances

Explicit formulas

Application to the
 $f_0(980)$ and
 $a_0(980)$

Conclusions

7.- Conclusions

Compositeness
problem in
hadronic states

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- 1 We review the classic formalism for elementariness and compositeness
- 2 We discuss the new formalism based on the use of the number operators of free particles [JAO,ANP\(2018\)](#)
- 3 We have discussed based on this that the Deuteron and the antibound 1S_0 NN states have $X = 0.96 - 1$
- 4 $X = 1$ for energy-independent potentials
- 5 Gamow states are normalized to one [E. Hernández,
A. Mondragón PRC29,722\(1984\) \[1\]](#)
- 6 Application to the $f_0(980)$ and $a_0(980)$ resonances.

Basic formalism on
elementari-
ness/compositeness

Interpretation
based on the
number operator

Explicit formulas

Resonances

Explicit formulas

Application to the
 $f_0(980)$ and
 $a_0(980)$

Conclusions

Contact interactions: $|k_{n,B}| \ll \Lambda$

Guo, JAO, PRD 103, 054021 (2021)

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$$X = \frac{2m^2}{\pi^2} \int_0^\infty k^2 \frac{g(k^2)^2}{(k^2 - k_B^2)^2} dk , \quad k_B^2 = 2mE_B$$

- Expansion of $g(k^2)^2$ in powers of $k^2 - k_B^2$

$$g(k^2) = g(k_B^2) + c_1(k^2 - k_B^2) + c_2(k^2 - k_B^2)^2 + \dots$$

- Dimensional regularization → power-like divergences vanish

$$\begin{aligned} X &= -g(k_B^2)^2 \left. \frac{\partial G(E)}{\partial E} \right|_{E_B} - \frac{m^2 |k_B|}{\pi} \left. \frac{\partial g(k^2)^2}{\partial k^2} \right|_{E_B} \\ &= -g(k_B^2)^2 \frac{i\mu^2}{2\pi k_B} + \mathcal{O}\left(\frac{k_B^2}{\Lambda^2}\right) \end{aligned}$$

If k_B^2 dependence of $g(k_B^2)$ is neglected → Weinberg's formula for $1 - Z$ for a shallow bound state

Basic formalism on
elementarity-
ness/compositenessInterpretation
based on the
number operator

Explicit formulas

Resonances

Explicit formulas

Application to the
 $f_0(980)$ and
 $a_0(980)$

Conclusions

Equality of the wave functions of the Gamow state and its dual

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$$|\psi_\alpha^+\rangle = |\varphi_\alpha\rangle + \int d\gamma \frac{T_{\gamma\alpha}(E + i\varepsilon)}{E + i\varepsilon - E_\gamma} |\varphi_\gamma\rangle + \sum_n \frac{T_{n\alpha}(E)}{E - E_n} |\varphi_n\rangle$$

$$\langle\psi_\alpha^-| = \langle\varphi_\alpha| + \int d\gamma \frac{T_{\gamma\alpha}(E + i\varepsilon)}{E + i\varepsilon - E_\gamma} \langle\varphi_\gamma| + \sum_n \frac{T_{n\alpha}(E)}{E + i\varepsilon - E_n} \langle\varphi_n|$$

Therefore,

$$\langle\psi_\alpha^-|\varphi_\gamma\rangle = \langle\varphi_\gamma|\psi_\alpha^+\rangle = \frac{T_{\gamma\alpha}(E + i\varepsilon)}{E + i\varepsilon - E_\gamma} \rightarrow \frac{g_\gamma(k_n)^2}{(E_n - E_\gamma)^2}$$

Instead of $|g_\gamma(k_n)|^2$. Wave-function squared

Hernández,Mondragón (1984)

$$u_{nl}(q; k_n) = \tilde{u}_{nl}(q; k_n)$$

Basic formalism on
elementari-
ness/compositeness

Interpretation
based on the
number operator

Explicit formulas

Resonances

Explicit formulas

Application to the
 $f_0(980)$ and
 $a_0(980)$

Conclusions