

# Oriented event shapes for massive quarks



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In collaboration with: N.G. Gracia & A. Bris

to appear in arXiv this fall

builds on: C. Lepenik & VM: JHEP 01 (2018) 122

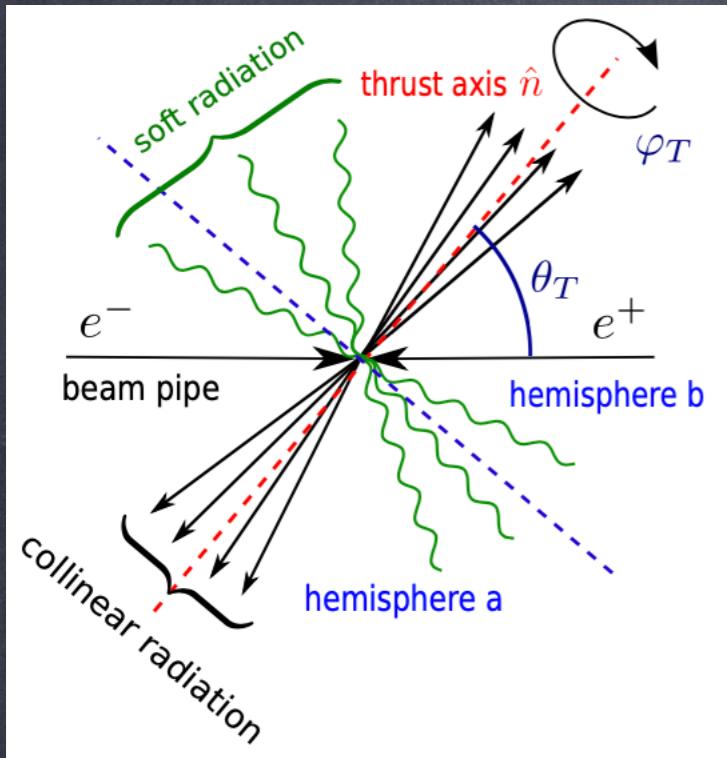
VM & G. Rodrigo: JHEP 11 (2013) 030

XV Confinement Stavanger, 02-08-2022

# Introduction

# Thrust axis and event shapes

Event shapes: study jets without jet algorithms (global variables)

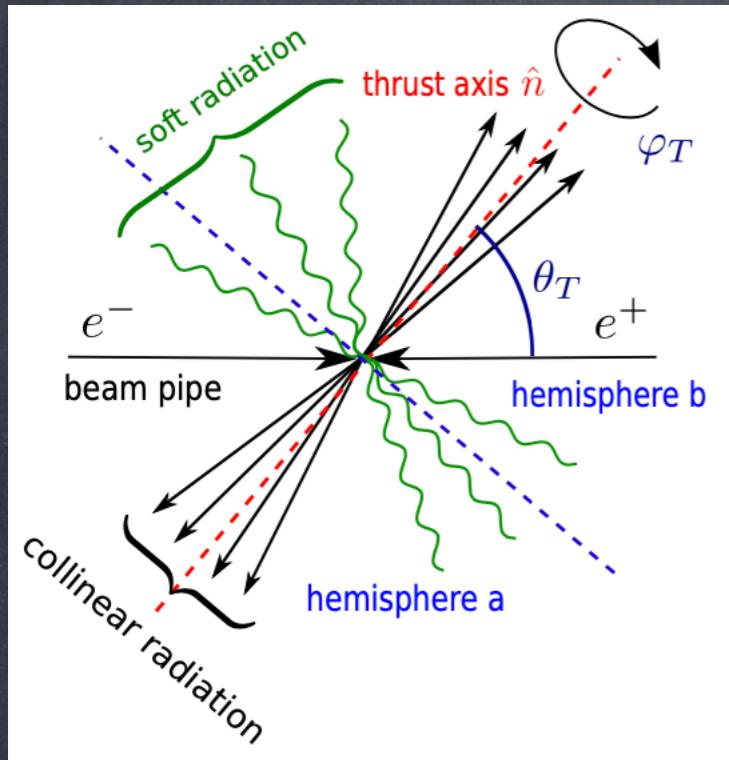


With momenta of final-state particles one builds kinematic variables encoding information on geometry of the event

Designed to be small for dijet configurations and  $O(1)$  for multijets

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Thrust: most common event shape

$$\tau = \min_{\vec{n}} \left( 1 - \frac{\sum_i |\vec{n} \cdot \vec{p}_i|}{\sum_j |\vec{p}_j|} \right)$$

Thrust axis, essential to measure the event's orientation globally

Only relevant parameter: angle between thrust and beam axes

# State of the art: massless oriented distributions

[VM & G. Rodrigo: JHEP 11 (2013) 030]

There are only two angular structures

$$\frac{1}{\sigma_0} \frac{d\sigma}{d \cos \theta_T de} = \frac{3}{8} (1 + \cos^2 \theta_T) \frac{1}{\sigma_0} \frac{d\sigma}{de} + (1 - 3 \cos^2 \theta_T) \frac{1}{\sigma_0} \frac{d\sigma_{\text{ang}}}{de}$$
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For massless quarks

$$\left\{ \begin{array}{l} \text{orientation starts at } \mathcal{O}(\alpha_s) \\ \text{oriented distribution is non-singular} \\ \text{(regular at event-shape threshold)} \end{array} \right.$$

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[Hagiwara & Kirilin: JHEP 10 (2010) 093]

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[Lampe: Phys. Lett. B 301 (1993) 435-439]

Computation of  $R_{\text{ang}}$  up to  $\mathcal{O}(\alpha_s^2)$ : disagreement with VM & Rodrigo

# Motivation

$R_{\text{ang}} \propto \alpha_s$  for massless quarks. Measured with good precision

Tiny hadronization corrections

Great to determine  $\alpha_s$ , but need to include bottom mass corrections

Might offer good opportunity to measure top quark mass

boosted tops or threshold scan

Computations

# General strategy

Project out angular distribution right-away in d-dimensions

$$\frac{1}{\sigma_0} \frac{d\sigma_{\text{ang}}}{de} = \frac{3}{8} \int_{-1}^1 d\cos\theta_T (2 - 5\cos^2\theta_T) \frac{1}{\sigma_0} \frac{d\sigma}{d\cos\theta_T de}$$

for 2-particle final states need the following integral:

$$\int_{-1}^1 d\cos(\theta) \sin^{-2\varepsilon}(\theta) \cos^{2k}(\theta) = \frac{2^{1-2\varepsilon} \Gamma(1+2k) \Gamma(1-\varepsilon) \Gamma(1+k-\varepsilon)}{\Gamma(1+k) \Gamma(2+2k-2\varepsilon)}$$

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Expanding in  $\varepsilon$  first generates "fake" angular structures

Artefact of regularisation procedure, disappear when adding real radiation (we will encounter others)

Another strategy: expand in  $\varepsilon$  and identify angular structures  
(used as a cross check)

# Lowest order

IR divergences @ NLO  $\longrightarrow$  use dim-reg:

d-dimensional differential phase space

d-dimensional Born normalisation

$$\frac{1}{2s} \frac{d\Phi_2}{d\cos(\theta)} = \frac{\beta^{1-2\varepsilon} \sin^{-2\varepsilon}(\theta)}{2^{5-4\varepsilon} s^{1+\varepsilon} \Gamma(1-\varepsilon) \pi^{1-\varepsilon}}$$

$$\sigma_B = \frac{(4\pi)^{1+\varepsilon} (1-\varepsilon) \Gamma(2-\varepsilon) \alpha_{\text{em}}^2}{(3-2\varepsilon) s^{1+\varepsilon} \Gamma(2-2\varepsilon)}$$

(useful to have nicer intermediate expressions)

$$s = Q^2 \quad \text{center of mass energy}$$

$$\beta = \sqrt{1 - 4\hat{m}^2} \quad \text{quark velocity}$$

$$\hat{m} = \frac{m}{Q} \quad \text{reduced mass}$$

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At this order thrust axis coincides with quark direction

$$R_{\text{ang}}^A = -\frac{3\beta^{3-2\varepsilon}}{8} \frac{\varepsilon(9-4\varepsilon)}{(1-\varepsilon)(5-2\varepsilon)} \rightarrow 0$$

same as massless

$$R_{\text{ang}}^V = \frac{3\beta^{1-2\varepsilon}}{16(1-\varepsilon)} \frac{[5 + 8\varepsilon^2 - 22\varepsilon - \beta^2(5-4\varepsilon)]}{5-2\varepsilon} \rightarrow \frac{3\beta}{16}(1-\beta^2)$$

vanishes for  $m=0$  but non-zero otherwise

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Important implications for vector current at  $\mathcal{O}(\alpha_s)$ :

- Presence of IR singularities
- Contribution from virtual diagram
- Singular structures at event-shape threshold

# Virtual radiation

Massive form factors

$$V^\mu = \left[ 1 + C_F \frac{\alpha_s}{\pi} A(\hat{m}) \right] \gamma^\mu + C_F \frac{\alpha_s}{\pi} \frac{B(\hat{m})}{2m} (q_1 - q_2)^\mu$$

$$A^\mu = \left[ 1 + C_F \frac{\alpha_s}{\pi} C(\hat{m}) \right] \gamma^\mu \gamma_5 + C_F \frac{\alpha_s}{\pi} \frac{D(\hat{m})}{2m} \gamma_5 q^\mu$$

No contribution to the axial current

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Defining  $L_\beta = \log\left(\frac{1+\beta}{2\hat{m}}\right)$  we find (vector current)

$$\begin{aligned} \frac{\sigma_{\text{ang,V}}^{\alpha_s}}{\sigma_0} &= C_F \frac{\alpha_s}{\pi} \frac{3\beta}{8} \left\{ \left( 1 - \frac{2 - 4\hat{m}^2}{\beta} L_\beta \right) \left[ -\frac{2\hat{m}^2}{\varepsilon} + 4\hat{m}^2 \log\left(\frac{m}{\mu}\right) + \frac{3(3 - 2\hat{m}^2)}{5} + 4\hat{m}^2 \log(\beta) \right] \right. \\ &\quad \left. + 4\hat{m}^2 \text{Re}[A^{\text{fin}}(\hat{m})] - 2\hat{m}^2 \beta L_\beta \right\} \end{aligned}$$

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does not vanish for  $m=0$ ! Artefact of regularisation

non-trivial check: when adding real contribution should vanish

# 3-particle phase space

Differential in 2 angles and 2 energies (harder than usual)

We choose 1 & 2 for  $q$  and  $\bar{q}$  (massive) and 3 for (massless) gluon

$$\frac{d\Phi_3}{2Q^2} = \frac{4^\varepsilon Q^{-4\varepsilon}}{2(4\pi)^{4-2\varepsilon} \Gamma(1-2\varepsilon)} \int dx_1 dx_2 d\cos(\theta_i) d\cos(\theta_j) \frac{[(x_1^2 - 4\hat{m}^2)(x_2^2 - 4\hat{m}^2)]^{-\varepsilon} \theta[\sin^2(\tilde{\theta}_{ij})] \theta(h_{12})}{h_{ij}^{1/2+\varepsilon}}$$

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momentum conservation  
in beam direction

$$\sqrt{x_1^2 - 4\hat{m}^2} \cos(\theta_1) + \sqrt{x_2^2 - 4\hat{m}^2} \cos(\theta_2) + x_3 \cos(\theta_3) = 0$$

useful to write  $\theta_3$  in terms of  $\theta_{1,2}$

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relative angles  $\tilde{\theta}_{ij}$   
independent of orientation

$$\sin^2(\tilde{\theta}_{ij}) = \frac{4[(1-x_1)(1-x_2)(1-x_3) - \hat{m}^2 x_3^2]}{(x_i^2 - 4\hat{m}_i^2)(x_j^2 - 4\hat{m}_j^2)}$$

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$$h_{ij} = \sin^2(\tilde{\theta}_{ij}) - \cos^2(\theta_i) - \cos^2(\theta_j) + 2\cos(\tilde{\theta}_{ij})\cos(\theta_i)\cos(\theta_j)$$

depends on  
angles only

# Real radiation

Differential cross section

$$\frac{1}{\sigma_0} \frac{d^4\sigma}{dx_1 dx_2 d\cos(\theta_i) d\cos(\theta_j)} = \frac{4^\varepsilon \alpha_s C_F}{16\pi^2} \frac{(3 - 2\varepsilon)(1 - 2\varepsilon)}{(1 - \varepsilon)\Gamma(2 - \varepsilon)} \left(\frac{4\pi\tilde{\mu}^2}{s}\right)^\varepsilon \frac{[(x_1^2 - 4\hat{m}^2)(x_2^2 - 4\hat{m}^2)]^{-\varepsilon}}{h_{ij}^{1/2+\varepsilon}} \\ \times [A + B\beta_1^2 \cos^2(\theta_1) + C\beta_2^2 \cos^2(\theta_2) + D\beta_1\beta_2 \cos(\theta_1) \cos(\theta_2)]$$

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with  $\beta_i = \sqrt{x_i^2 - 4\hat{m}^2}$  and A, B, C and D depending on  $x_{1,2}$

For 3 particles, thrust axis coincides with 3-momentum of particle with largest  $|\vec{p}_i|$ , therefore  $\theta_T = \theta_i$

Strategy: project out angular structure right away

$$\frac{3}{8} \int d\cos(\theta_1) d\cos(\theta_2) \{ \theta(x_1 - x_2) \theta(|\vec{p}_1| - E_3) [2 - 5 \cos^2(\theta_1)] \\ + \theta(x_2 - x_1) \theta(|\vec{p}_2| - x_3) [2 - 5 \cos^2(\theta_2)] + \theta(E_3 - |\vec{p}_1|) \theta(E_3 - |\vec{p}_2|) [2 - 5 \cos^2(\theta_3)] \}$$

Need integrals of the sort  $\int \frac{d\cos(\theta_1) d\cos(\theta_2) \cos^n(\theta_i) \cos^m(\theta_j)}{h_{12}^{1/2+\varepsilon}}$

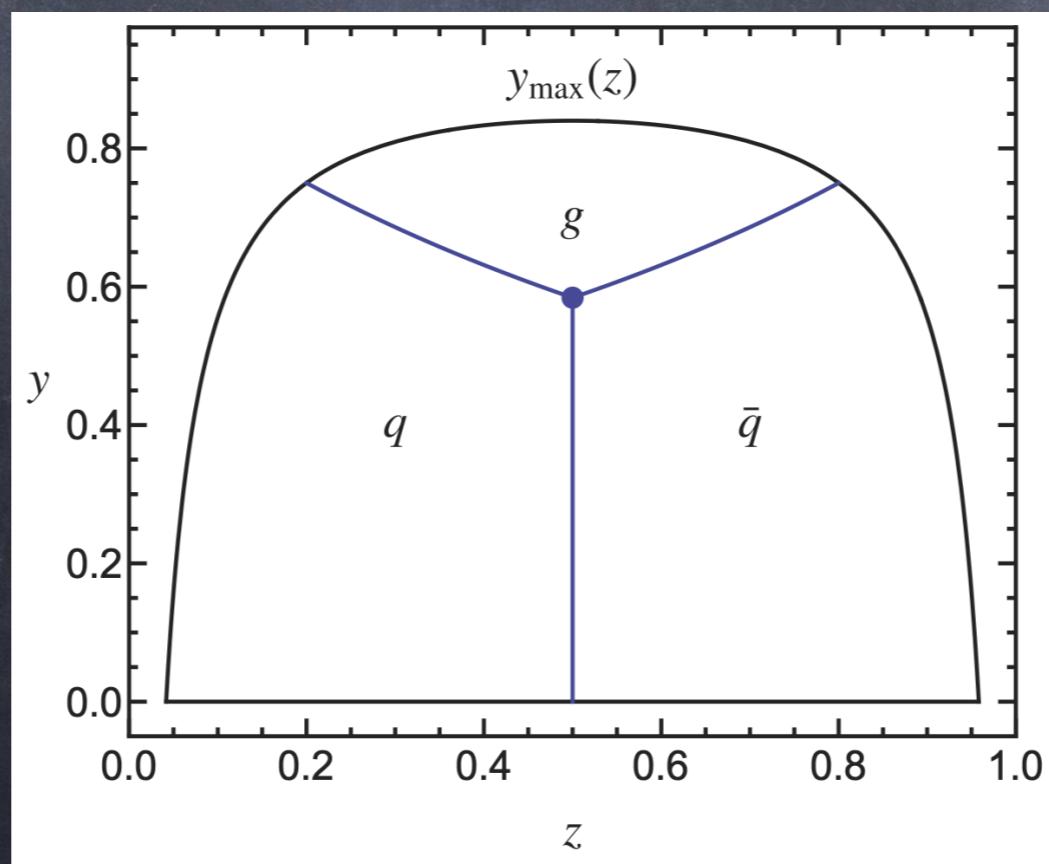
# Real radiation results

After projecting out angular distribution we find

$$\frac{1}{\sigma_0} \frac{d^2\sigma^{\text{ang}}}{dzdy} = \frac{3\alpha_s C_F}{64\pi} \frac{y^{1-2\varepsilon}}{(1-\varepsilon)\Gamma(2-\varepsilon)} \left(\frac{4\pi\tilde{\mu}^2}{s}\right)^\varepsilon [(1-y)(1-z)z - \hat{m}^2]^{-\varepsilon}$$
$$\{a\theta(x_1-x_2)\theta(\beta_1-x_3) + b\theta(x_2-x_1)\theta(\beta_2-x_3) + c\theta(x_3-\beta_1)\theta(x_3-\beta_2)\}$$

a, b and c depend on previous A, B, C, D

with  $x_1 = 1 - y(1 - z)$ ,  $x_2 = 1 - yz$  (soft limit:  $y \rightarrow 0$ )



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Agreement with VM & Rodrigo for  $m = 0$

Coefficient c does not "see" soft sector of phase space

Vector current:  $a, b \propto y^{-2} \longrightarrow$  signals IR divergence

Axial current: no IR divergence

# Final results

Axial current: only real-radiation contributes. Pure non-singular

Vector current: real and virtual contribute. Singular and non-singular

Following [Lepenik & Mateu] one can single out Dirac delta and plus function coefficients:

$$\frac{1}{\sigma_0^V} \frac{d\sigma_{V,\delta}^{\text{ang}}}{de} = \frac{3\alpha_s C_F}{8\pi} \left\{ A_{e,V}^{\text{ang}}(\hat{m})\delta(e - e_{\min}) + B_{\text{plus}}^{\text{ang}}(\hat{m}) \left[ \frac{1}{e - e_{\min}} \right]_+ + F_{\text{NS}}^{\text{ang}}(e) \right\}$$

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cero for axial current

non-zero  
for vector  
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defining  $\hat{e}(z, y) = e_{\min} + y f_e(z) + \mathcal{O}(y^2)$  we obtain for the vector current

$$B_{\text{plus}}^{\text{ang}}(\hat{m}) = 4\hat{m}^2 [2(1 - 2\hat{m}^2)L_\beta - \beta], \quad \text{universal}$$

$$A_{e,V}^{\text{ang}}(\hat{m}) = \hat{m}^2 \left\{ (1 + \beta^2) \left[ \pi^2 - 2L_\beta^2 - 3\text{Li}_2\left(\frac{2\beta}{1 + \beta}\right) + \text{Li}_2\left(\frac{2\beta}{\beta - 1}\right) - 4L_\beta [\log(\hat{m}) - 1] \right] \right.$$

$$\left. + 4\beta[\log(\hat{m}) - 1] - 2I_e(\hat{m}) \right\}$$

$$I_e(\hat{m}) = \frac{1}{2} \int_{z_-}^{z_+} dz \frac{(1 - z)z - \hat{m}^2}{(1 - z)^2 z^2} \log[f_e(z)] \quad \text{depends on the event shape}$$

# Final results

Axial current: only real-radiation contributes. Pure non-singular

Vector current: real and virtual contribute. Singular and non-singular

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$$\frac{1}{\sigma_0^V} \frac{d\sigma_{V,\delta}^{\text{ang}}}{de} = \frac{3\alpha_s C_F}{8\pi} \left\{ A_{e,V}^{\text{ang}}(\hat{m}) \delta(e - e_{\min}) + B_{\text{plus}}^{\text{ang}}(\hat{m}) \left[ \frac{1}{e - e_{\min}} \right]_+ + F_{\text{NS}}^{\text{ang}}(e) \right\}$$

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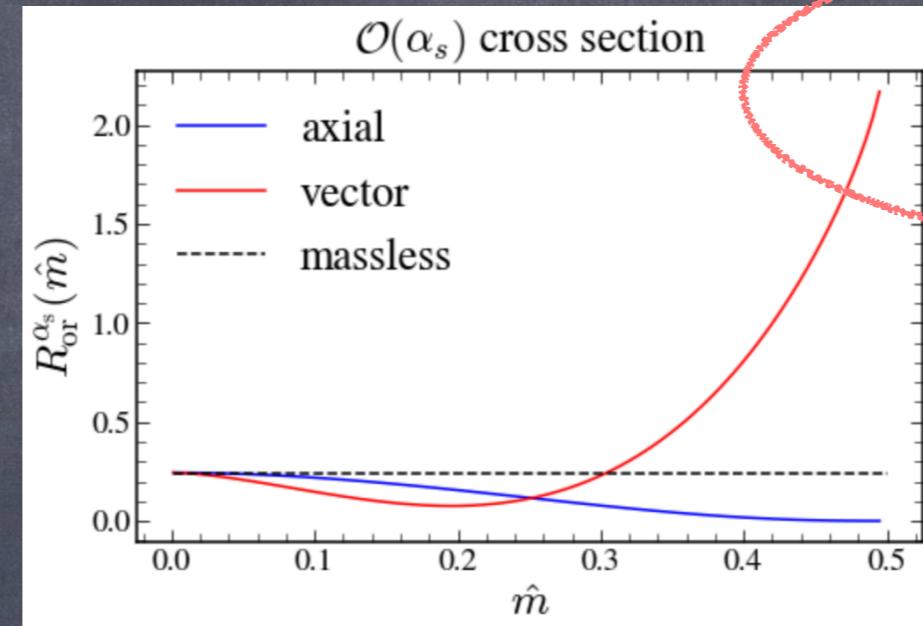
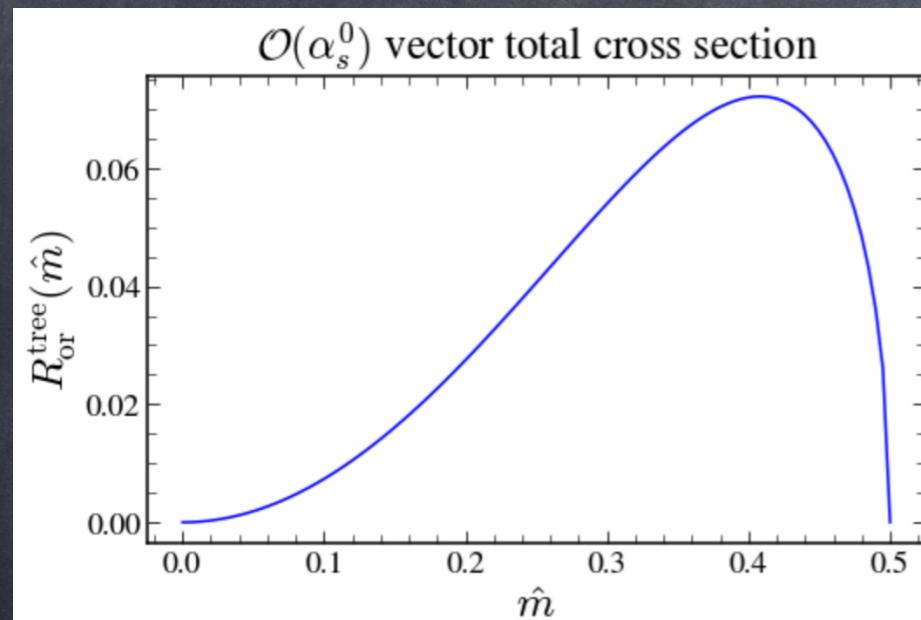
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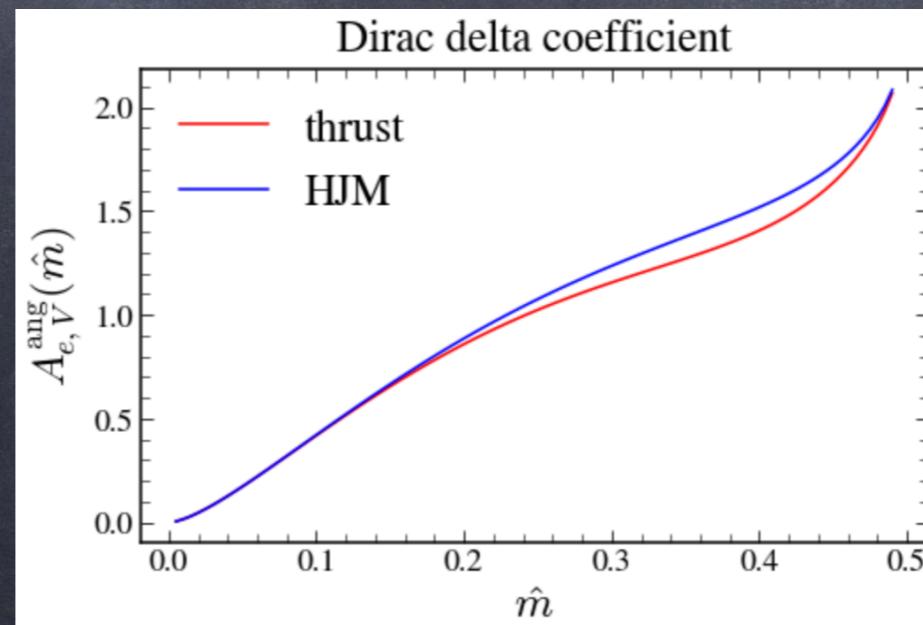
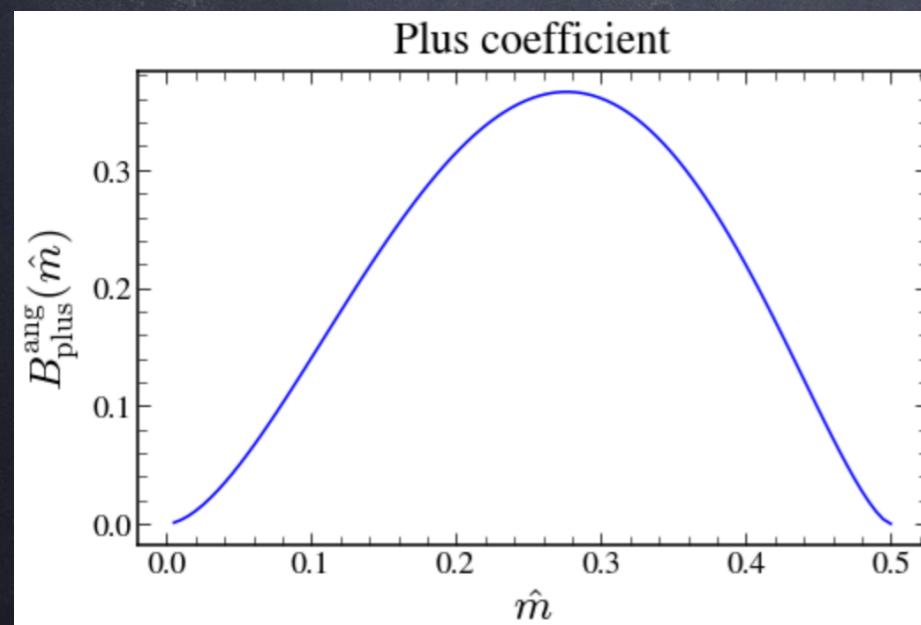
proportional to  $\hat{m}^2$

$$I_e(\hat{m}) = \frac{1}{2} \int_{z_-}^{z_+} dz \frac{(1 - z)z - \hat{m}^2}{(1 - z)^2 z^2} \log[f_e(z)]$$

# Numerics

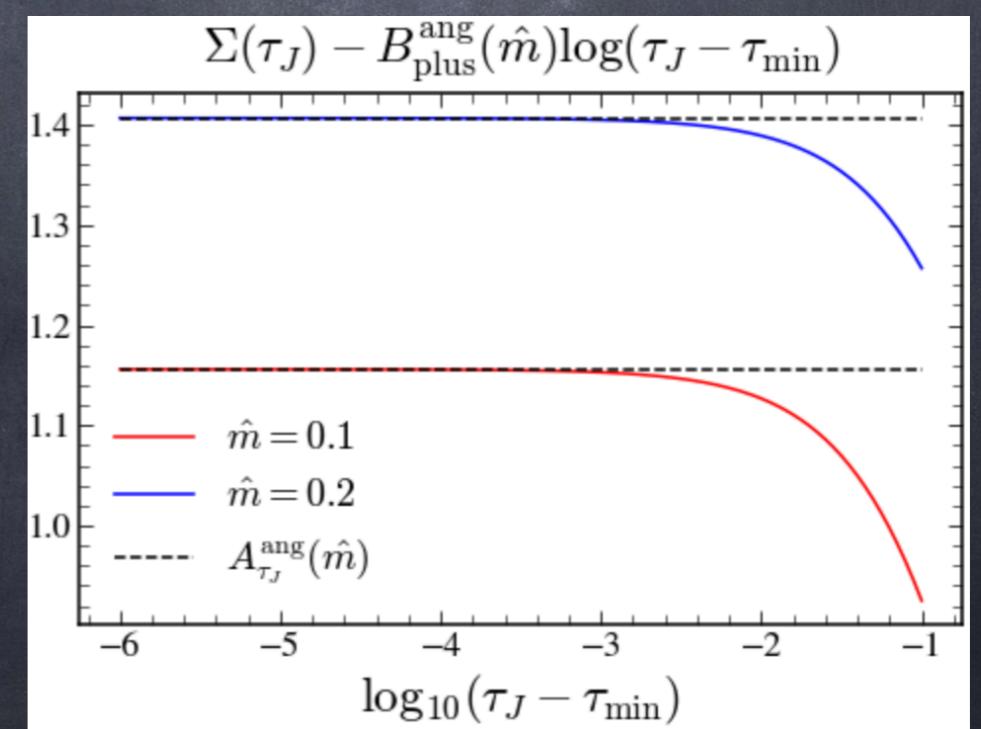
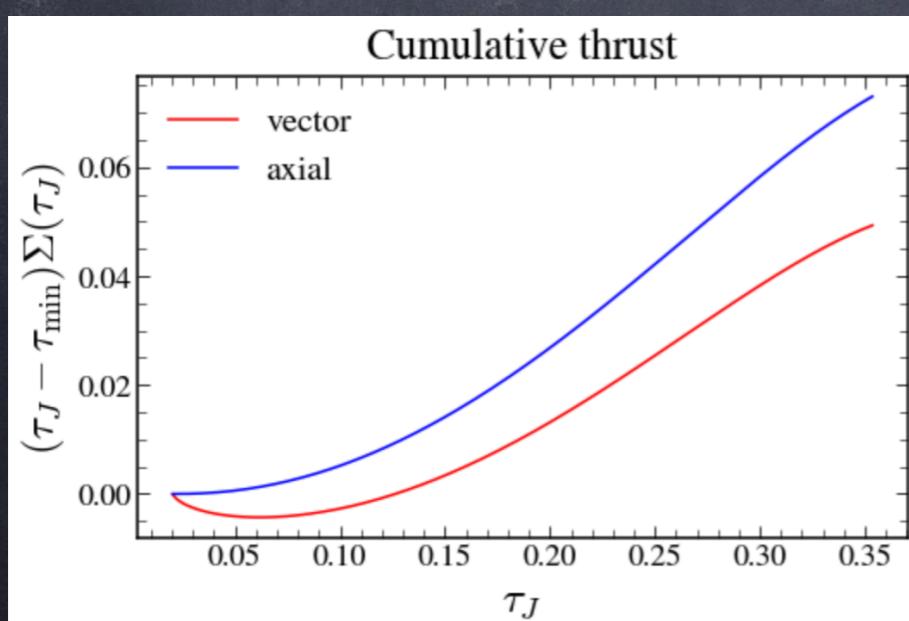
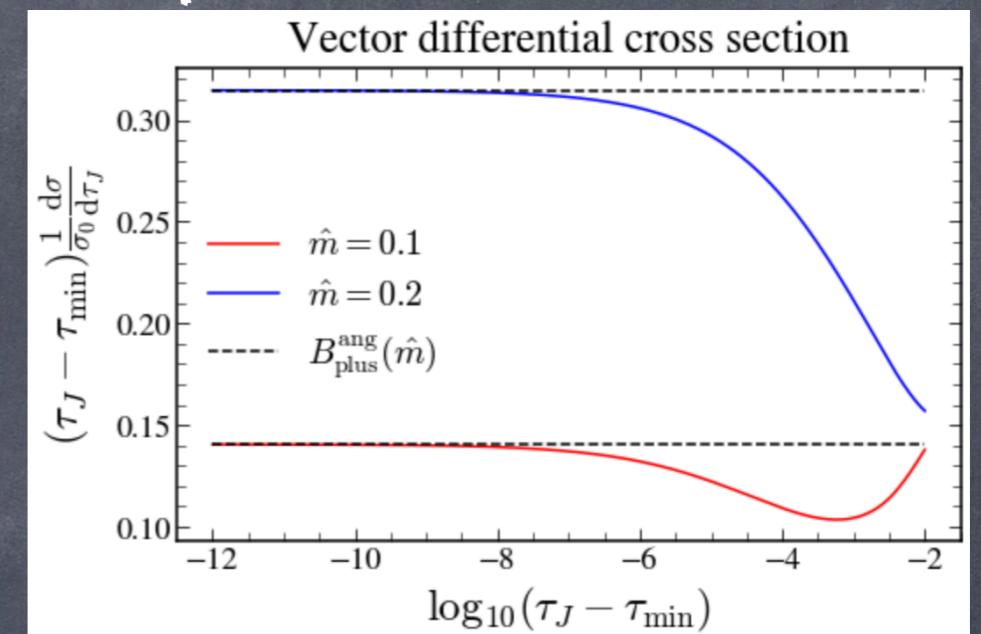
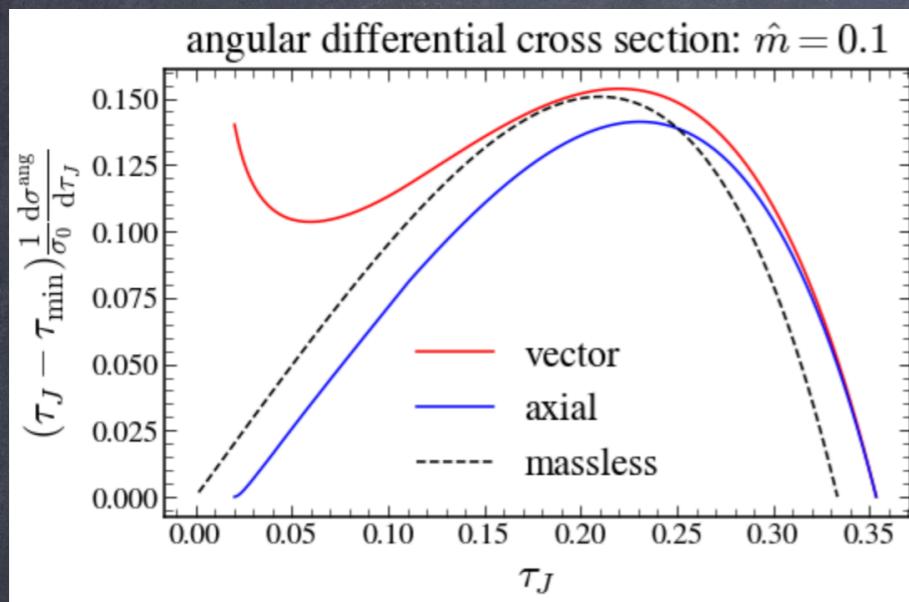


threshold enhancement!  
top quark mass determination from threshold scans



# Numerics

important checks



# Conclusions

- Determined the NLO oriented distribution for massive quarks
- Vector current starts at LO and develops singular structure
- Analytic results for delta and plus function coefficients
- Analytic or numeric results for non-singular and total cross section

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# Outlook

- Work out factorisation theorem for massive quarks
- Include effects of top-quark decay products
- Carry out fits to the strong coupling