# Finite angle effects in jet quenching

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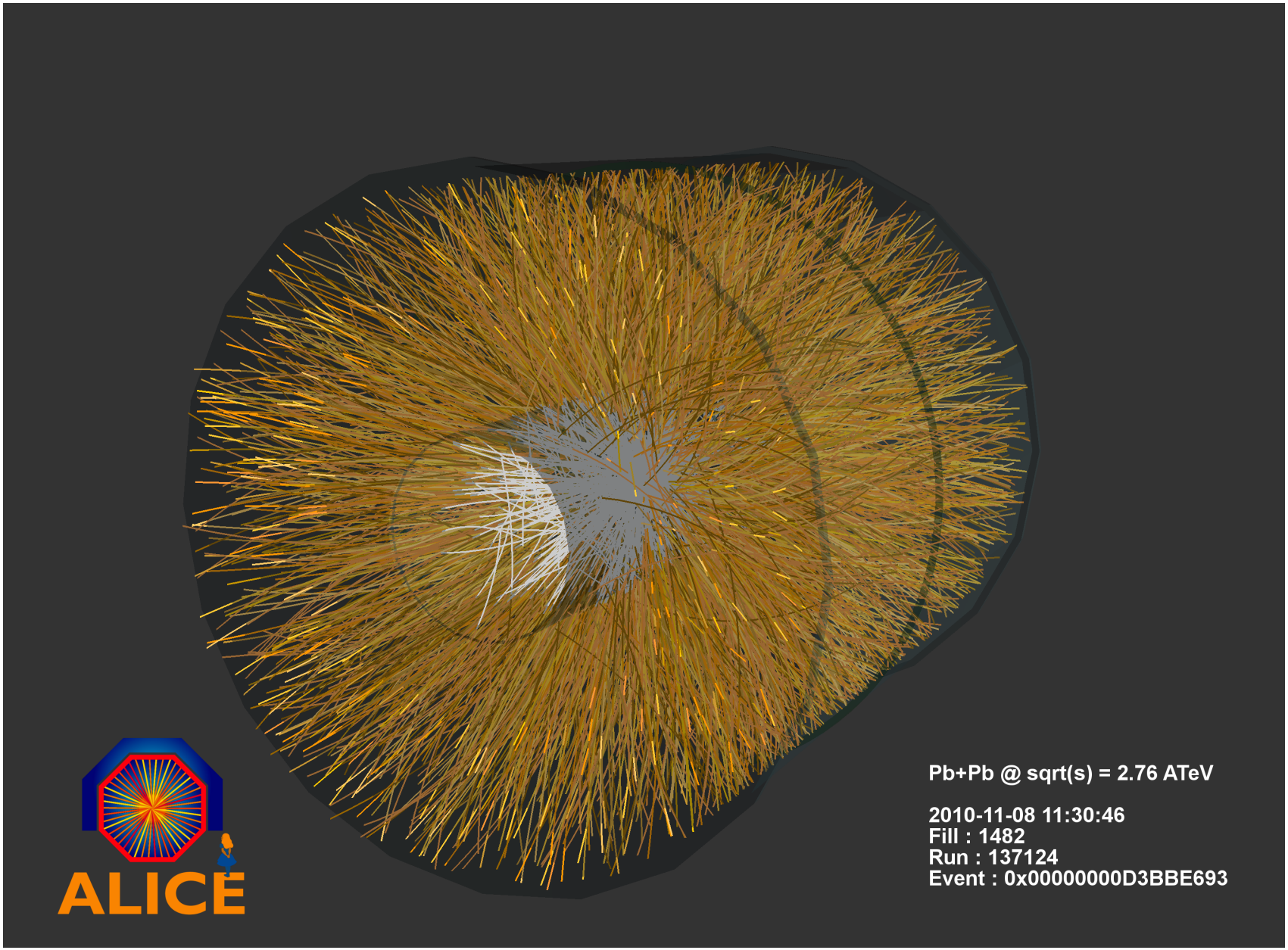
Work done in collaboration with Carlos Salgado and Manoel Rodríguez.
To be published soon.

# Outline

# Difficulties in heavy ion collisions



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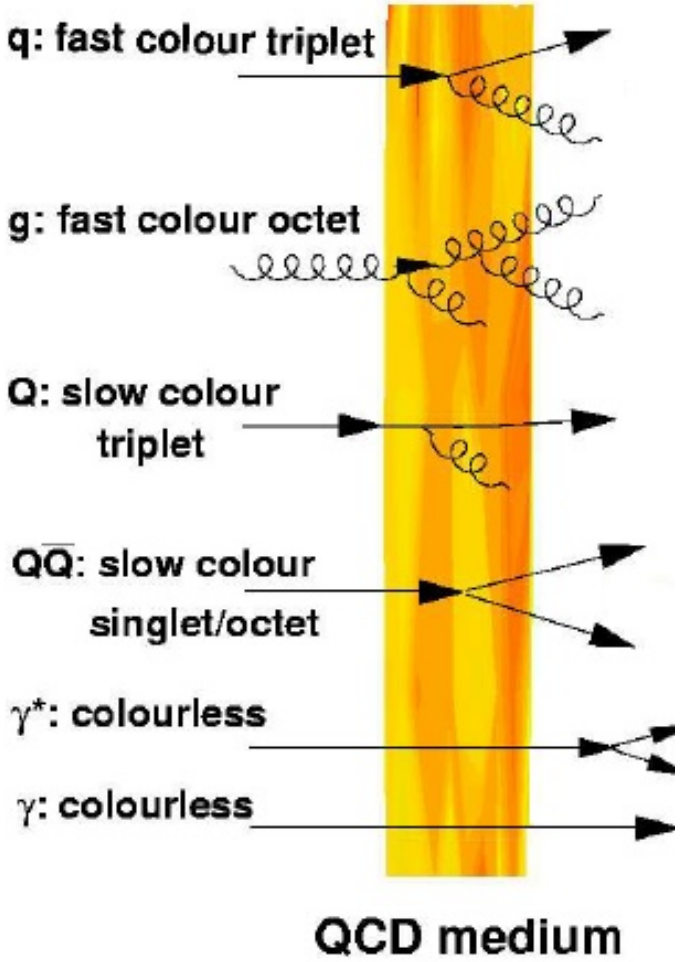
- This is just one of the many stages that takes place in heavy ion collisions.
- At the end what we see in the detectors is the same type of particles as in proton-proton collisions, but a lot of them.

# Hard probes in heavy ion collisions

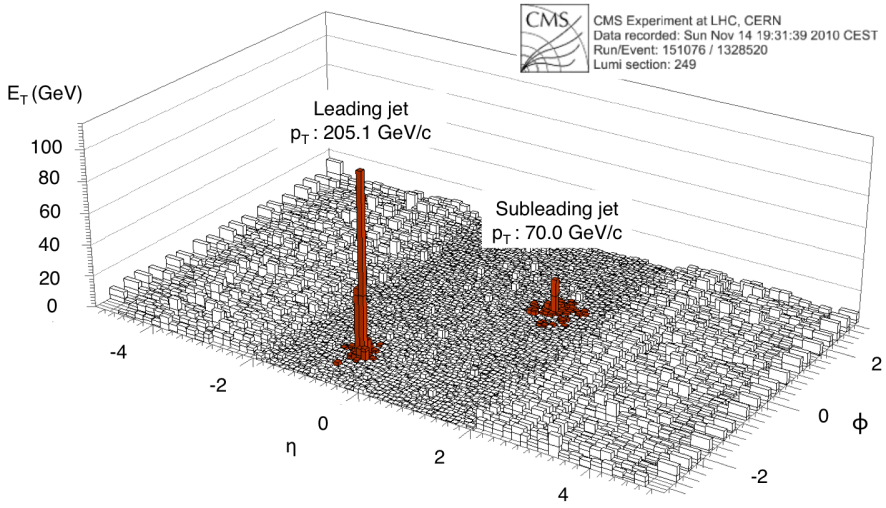



Probes that are created at the beginning of the collision (typically because its creation needs a high energy) that get modified in a substantial way and that are relatively easy to detect.

- Jet quenching.
- Heavy quark diffusion.
- Quarkonium suppression.
- Photon production.

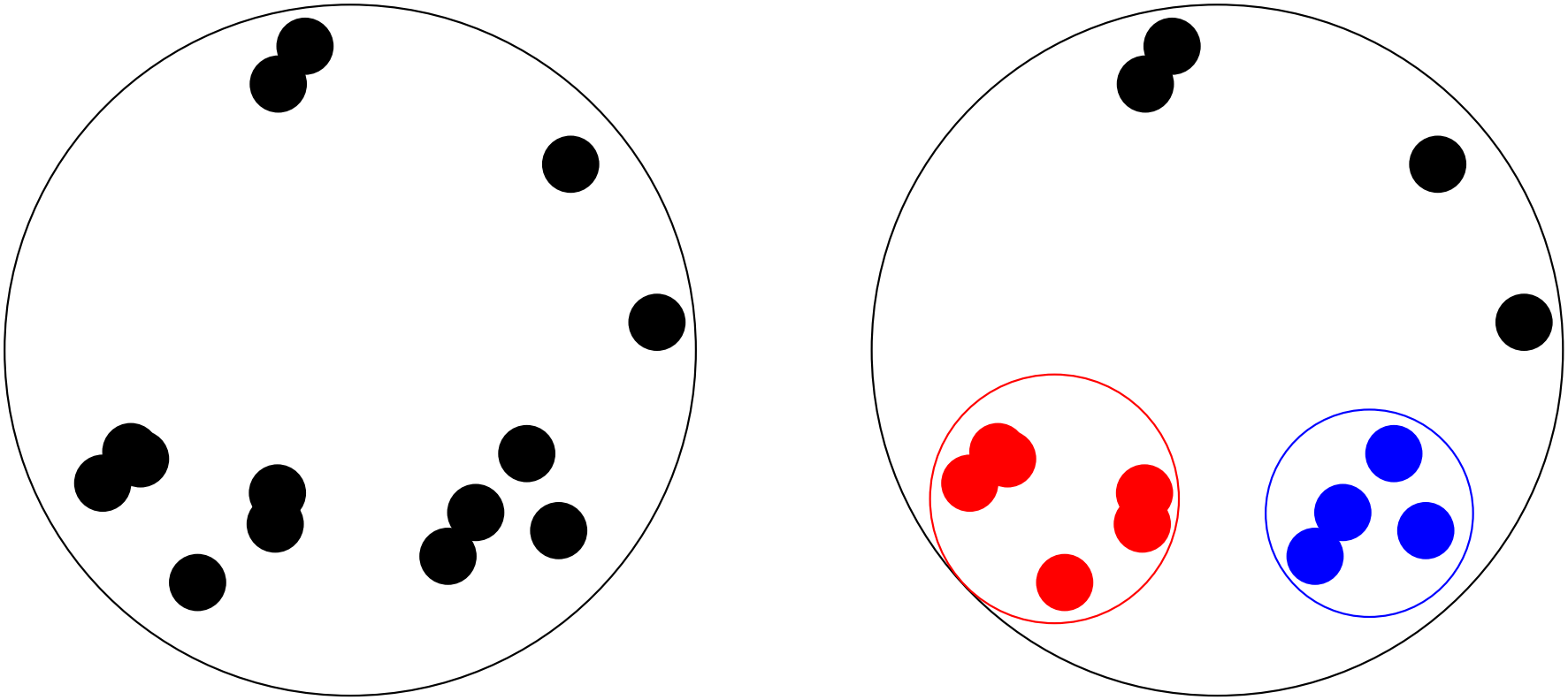
Picture taken from d'Enterria (2007)

# Jet quenching




- It is a measure of the opacity of the medium to boosted particles.
- Its strength depends on a transport coefficient called $\hat{q}$.

# Jet substructure



Recently developed techniques to characterize the internal structure of the jet. For example, re-cluster jets inside the jet.

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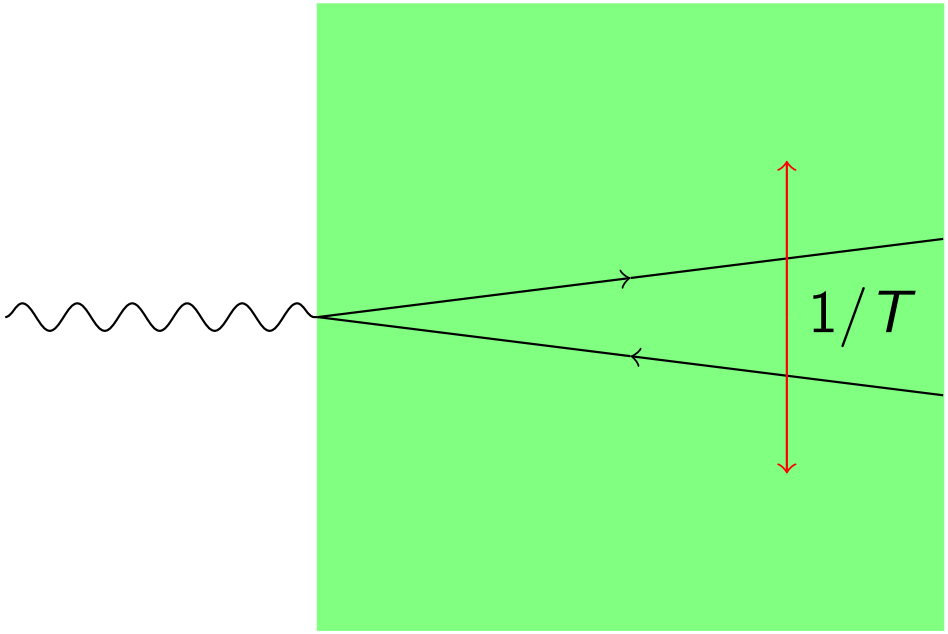
- We need to understand how jet quenching depends on the angle between high energy partons.
- In the vacuum we have angular ordering. What do we have when the jet is inside of a medium?
- The antenna configuration is a simple system in which we can try to understand these issues.

# Jet antenna, small angle

Mehtar-Tani, Salgado and Tywoniuk, 2011 and 2012; Calderrey-Solana and Iancu, 2011




- The separation between the quark and the antiquark is always smaller than the medium resolution scale.
- From the point the view of the medium, it if as if the photon never split. Therefore, medium effects are suppressed.

# Jet antenna, large angle

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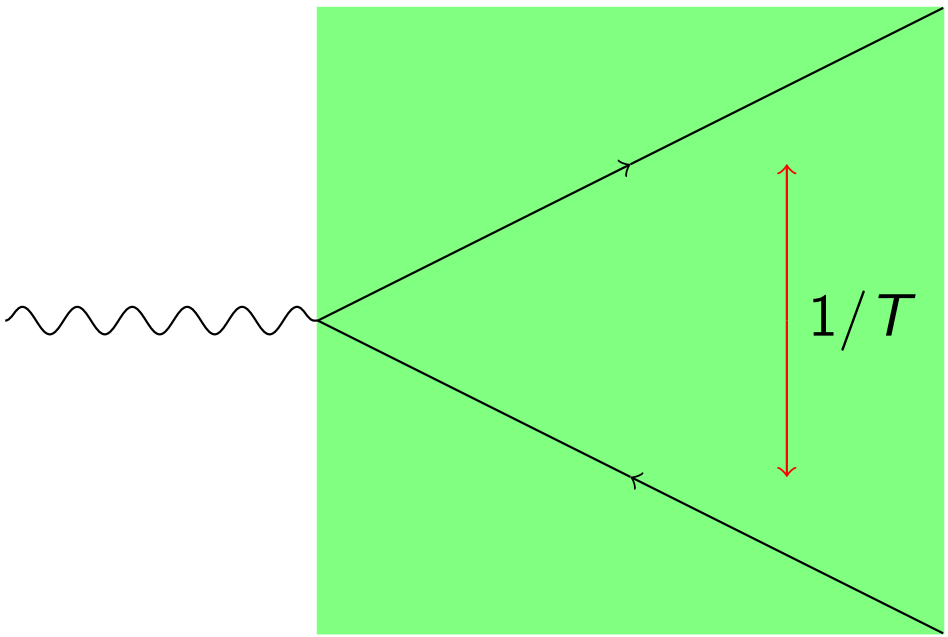


- Soon after the splitting the medium sees the quark and the antiquark as very far apart.
- They interact with the medium in a completely uncorrelated way. Loss of coherence $\rightarrow$ No angular ordering.

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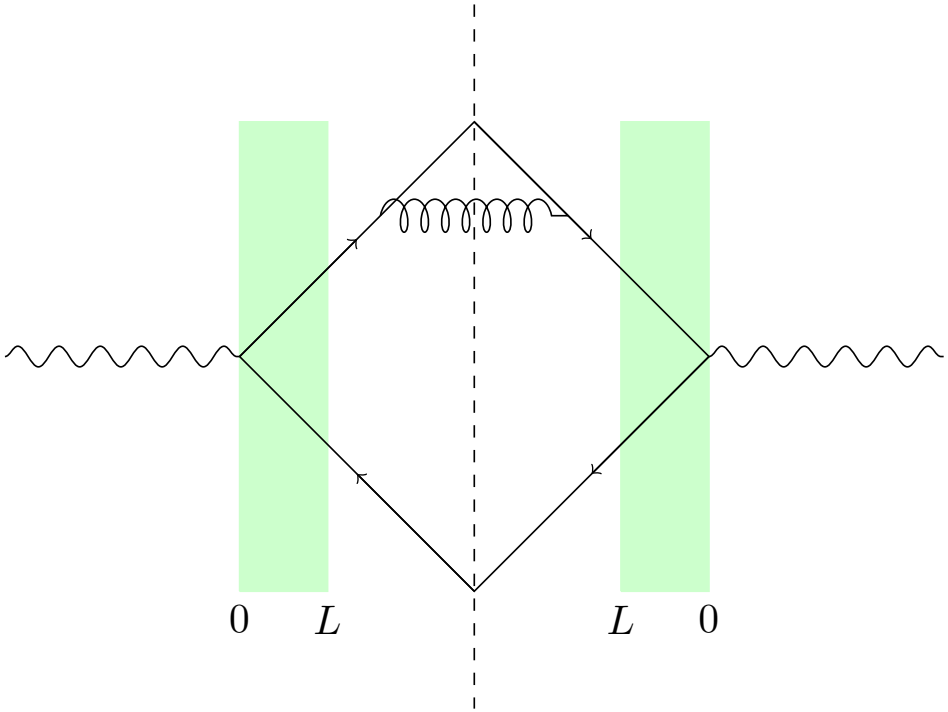
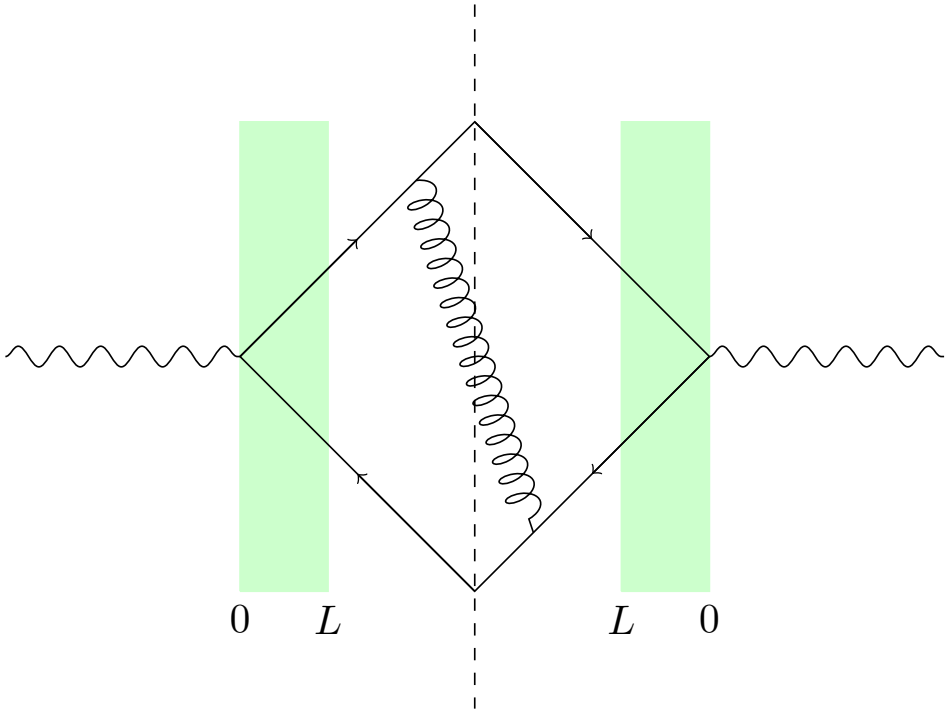
- Now we are going to consider that each resulting parton follows a different light-cone direction.

# Outline

# Emission after the medium

We focus on the simple scenario of a pair of quark-antiquark created from a photon that radiate a high energy gluon after traversing the medium.



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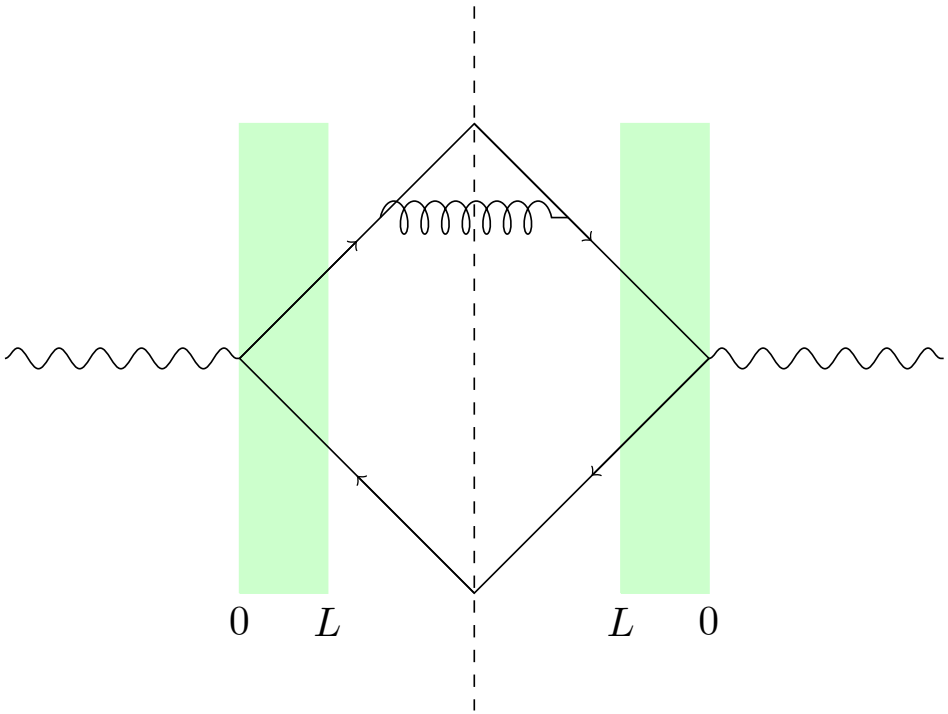
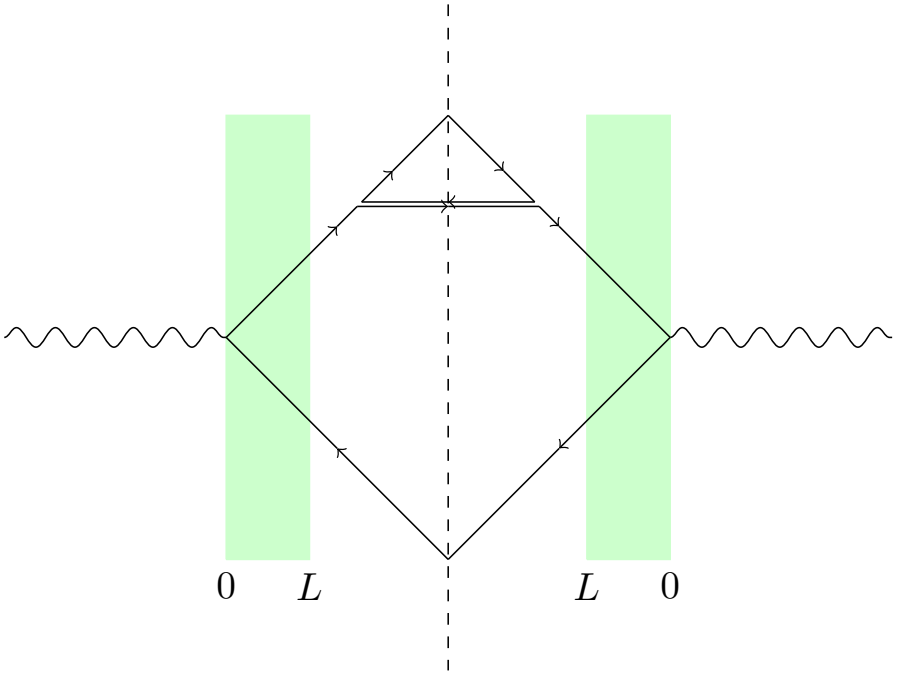
- The large $N_c$ limit.
- Classical limit. The gauge fields are equal in the amplitude and in the complex conjugate.
- The eikonal approximation.
- We focus on the medium influence and consider a static QCD brick. Chromoelectric and chromomagnetic fields outside of the medium vanish. Therefore, the gauge field is a pure gauge outside of the medium.

# Large $N_c$

## Self-energy




$\frac{1}{2}$



# Classical limit

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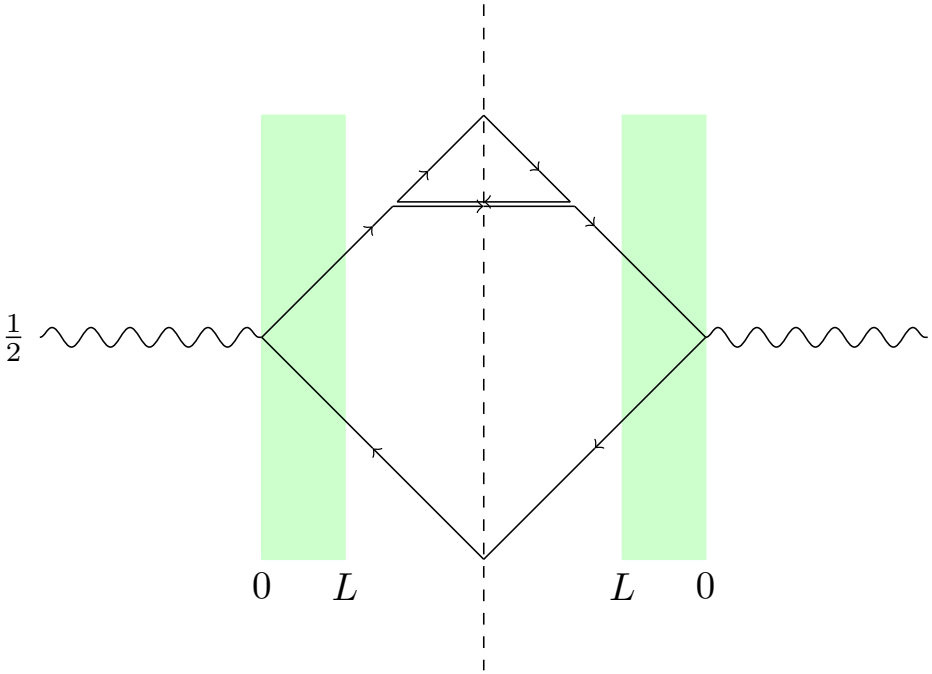
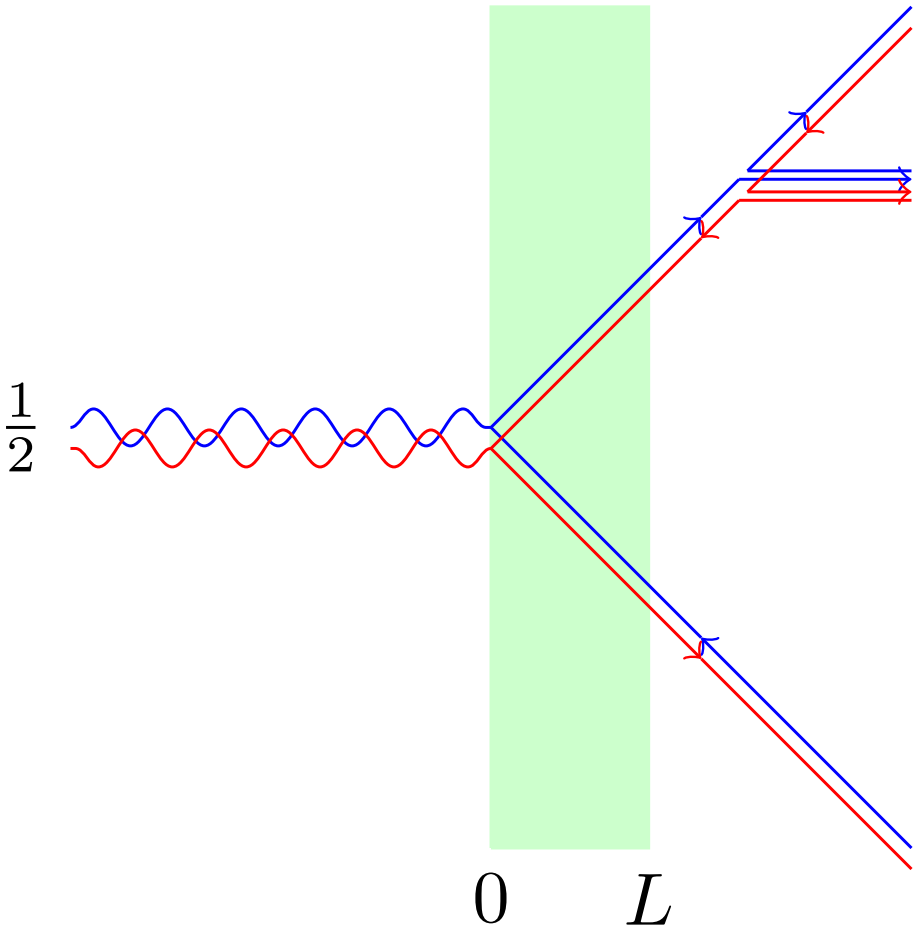
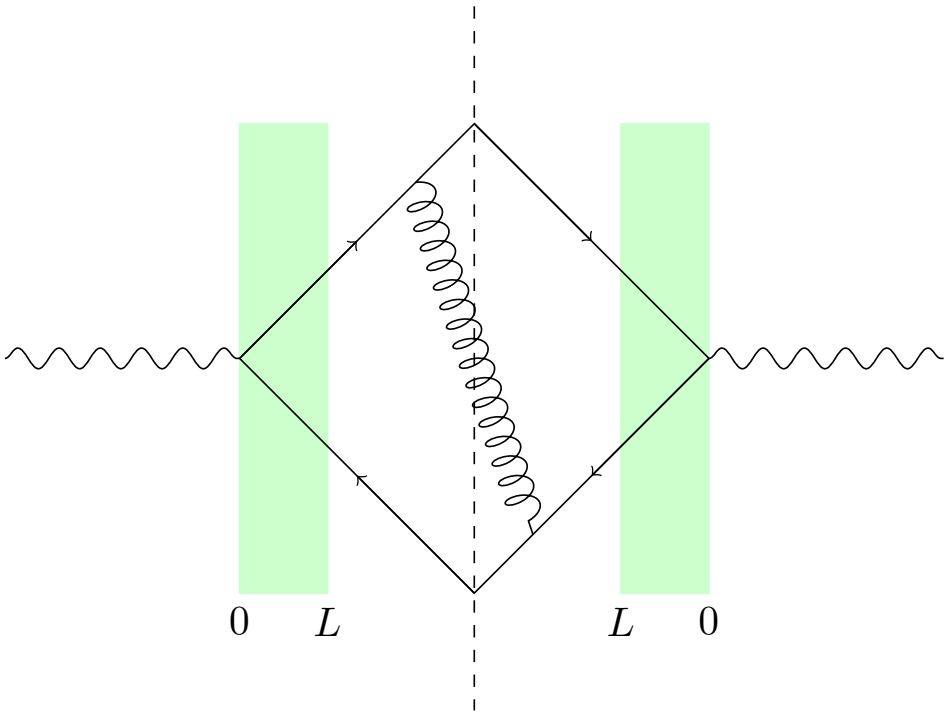



Wilson lines cancel out. No thermal effects.

# Large $N_c$

## Crossed term

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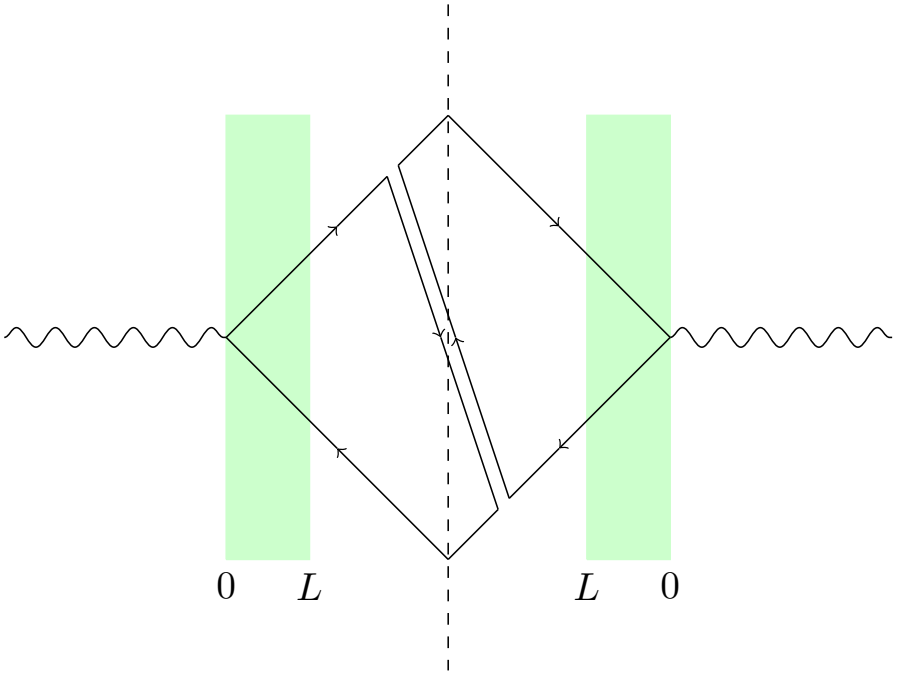

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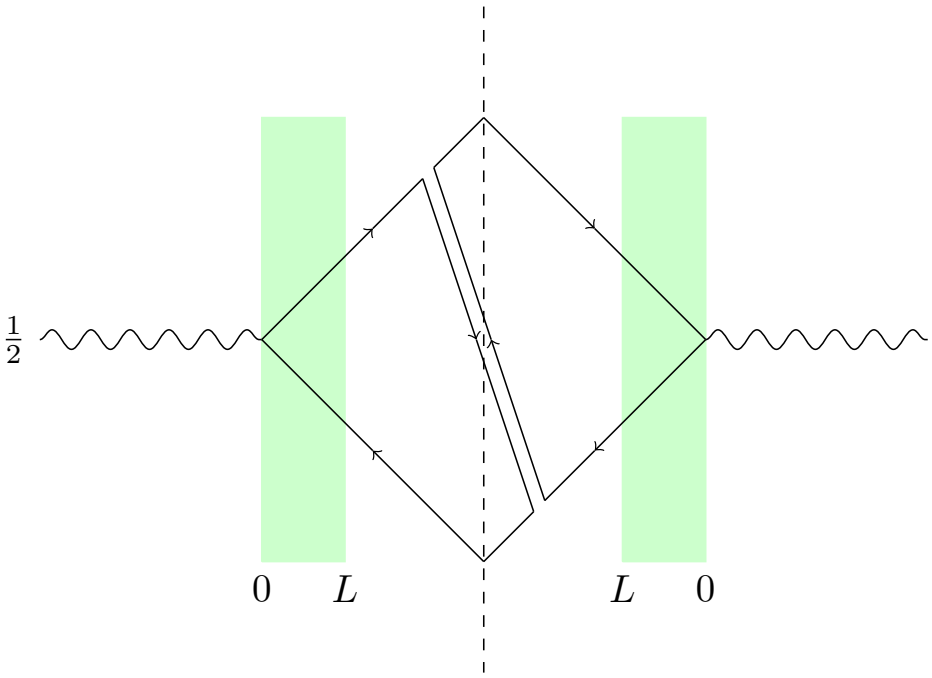
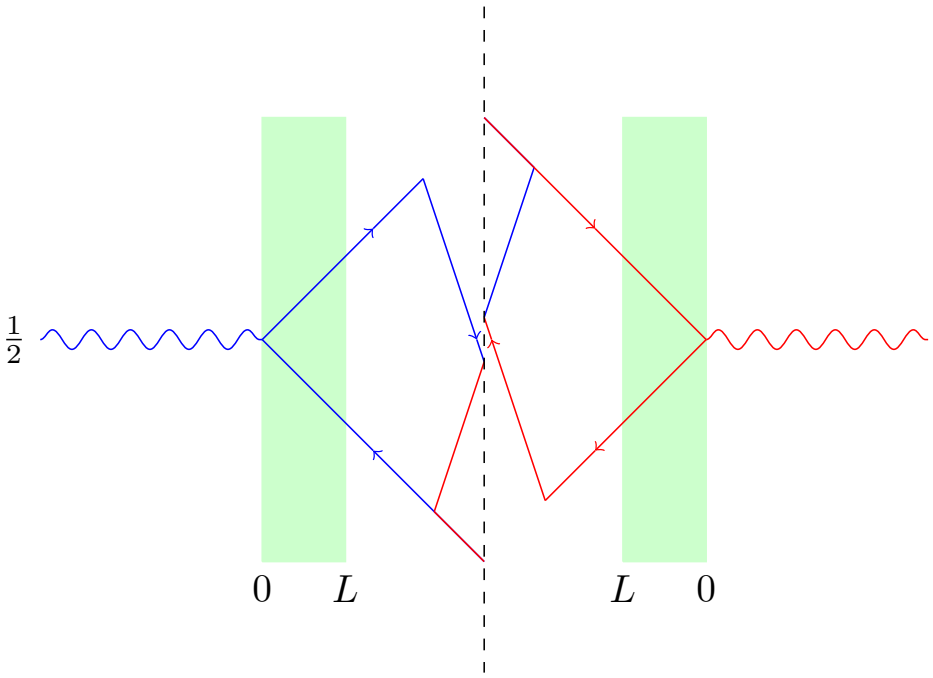
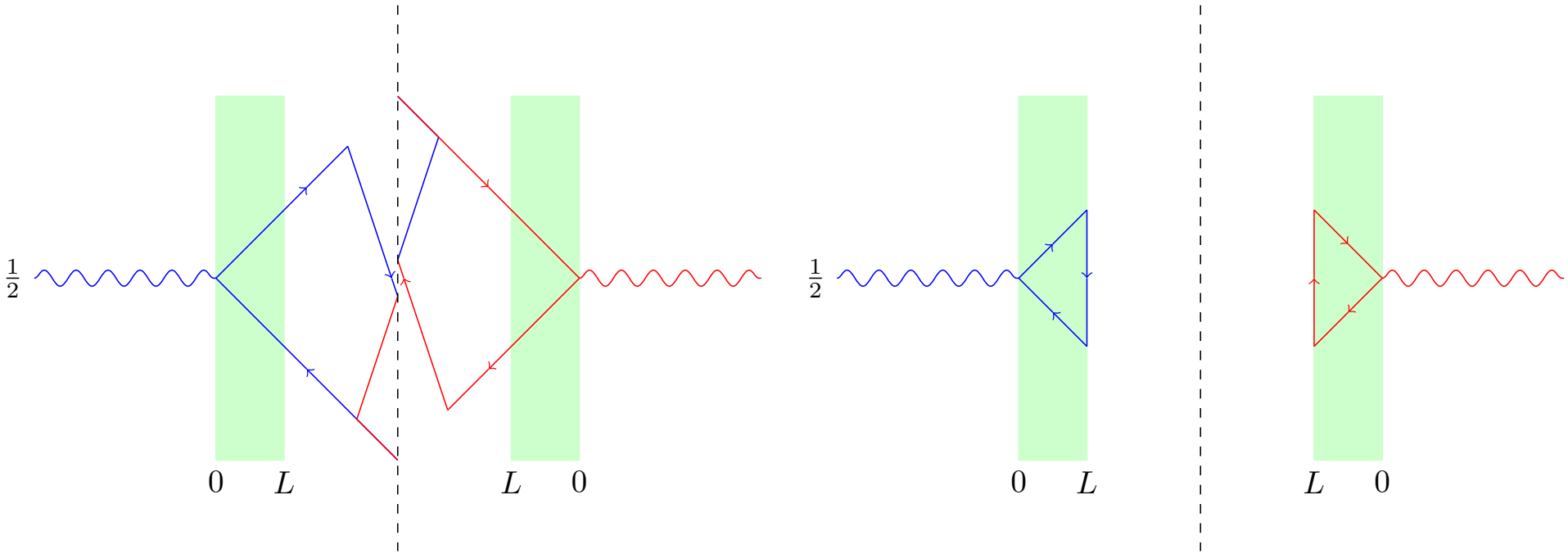


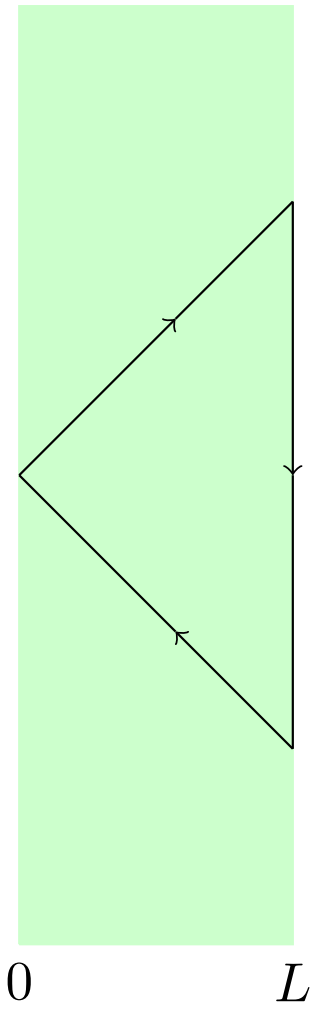
# $A_\mu$ is a pure gauge in the vacuum




Amplitude. Complex conjugate amplitude.

# In summary

Thermal effects are encoded (in the large $N_c$ limit) in the modulus square of



# In summary

The contribution from the interference term is multiplied by $\Delta_{med}$ and that contains all medium effects.

$$\Delta_{med} = 1 - \frac{1}{N_c^2} \langle \mathbf{Tr} \mathrm{e}^{ig \oint \, dl \cdot A} \rangle^2$$

where the contour is the previous triangle. Quark (antiquark) goes around light-cone direction $n_1$ ($n_2$).

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## Previous literature

$$\Delta_{med} = 1 - \frac{1}{N_c^2}\langle \mathbf{Tr} U_1(L^+, 0) U_2^\dagger(L^+, 0) \rangle^2 \,,$$

where

$$U_i(x^+, 0) = \mathcal{P}_+ \exp\left[ ig \int_0^{x^+} d\tau n \cdot A \left( n\tau + \frac{k_{i,\perp}\tau}{\sqrt{2}E_i} \right) \right]$$

$n$ is the light-cone direction of the parent photon.

# Differences with previous studies

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- We take into account the gauge field component perpendicular to $n$ (photon direction).
- We have an additional vertical line in the border of the medium. Analogous to Aharonov-Bohm effect.
- Manifestly gauge invariant.
- At LO in the small angle limit we get the same result.

# Eikonal approximation

We assume that the quark (antiquark) moves eikonally with direction $n_1$ ($n_2$).

$$n_1 n_2 = \frac{1}{2}(1 - \cos\theta) \sim \frac{\theta^2}{4} \,,$$

where $\theta \ll 1$. We can define

$$n = \frac{n_1 + n_2}{2}$$

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**Note that...**

- $n$ is not a light-cone vector, but almost. $n^2 \sim \theta^2$.
- $n\delta n = 0$, therefore $\delta n$ is a transverse vector with modulus of order $\theta$.

# Outline

# Perturbation theory

We evaluate the triangle in perturbation theory using the Coulomb gauge.

## Small $\theta$ limit

- Modes with energy $T$. First contribution enters at one loop.
- Modes with energy $m_D$. Computed using HTL.

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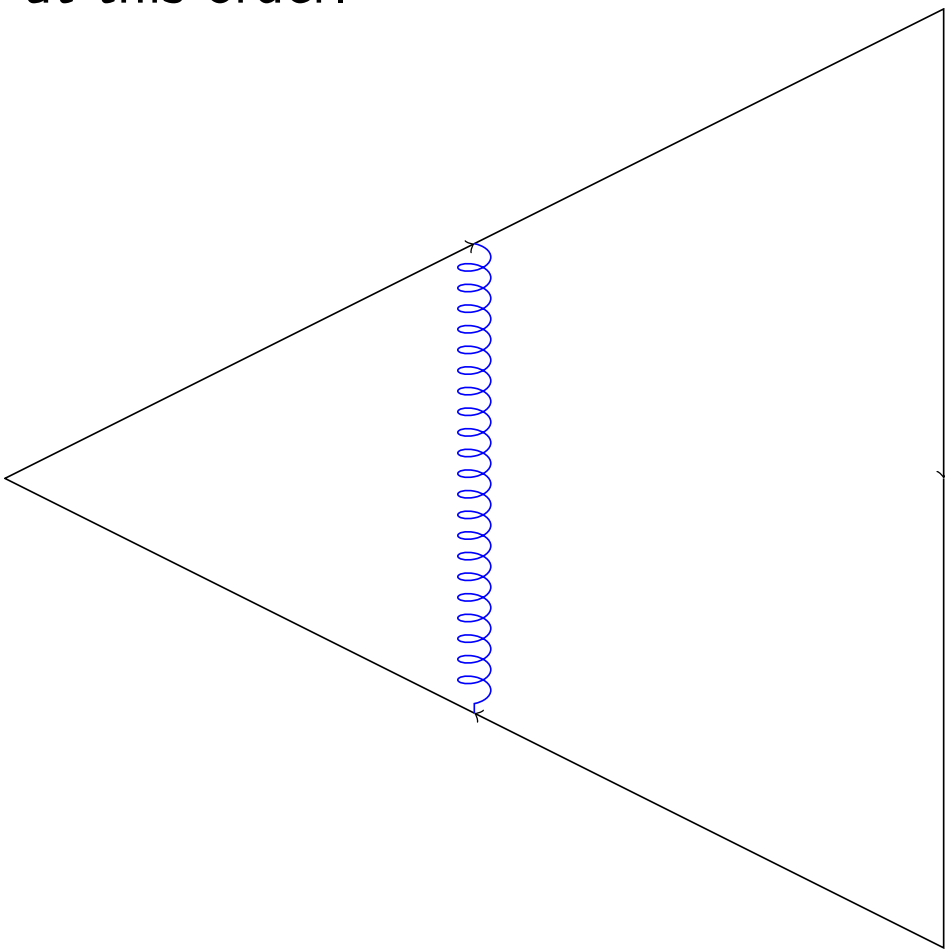
## First $\theta$ corrections

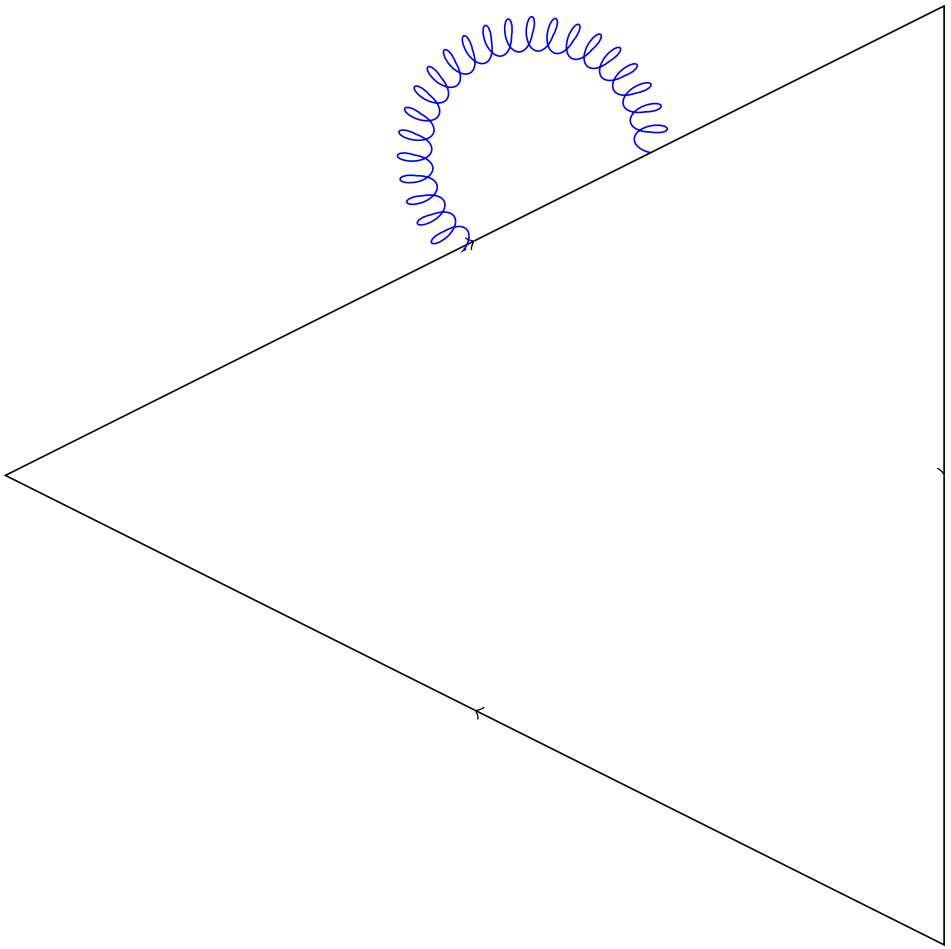
Modes with energy $T$ at tree level.

# Leading order

## $T$ modes. Tree level

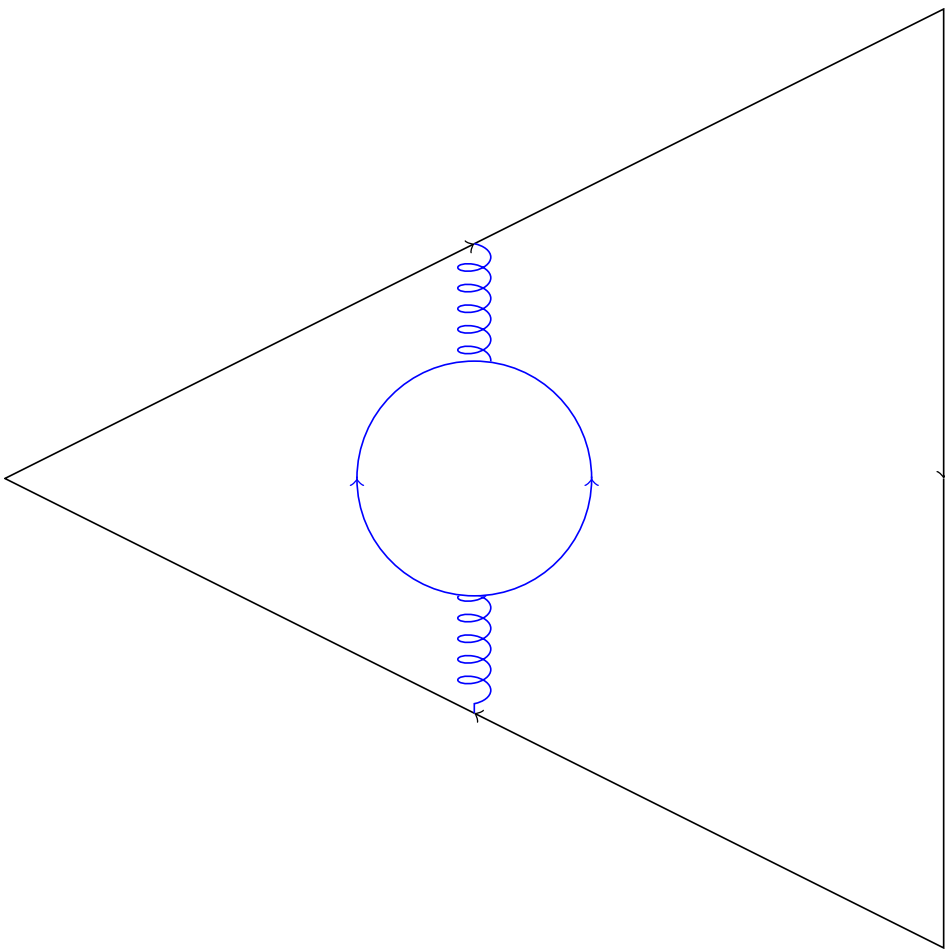
Leading contribution is enhanced by powers of $LT$. This enhancement is only achieved for off-shell gluons. Therefore, tree level does not contribute at this order.

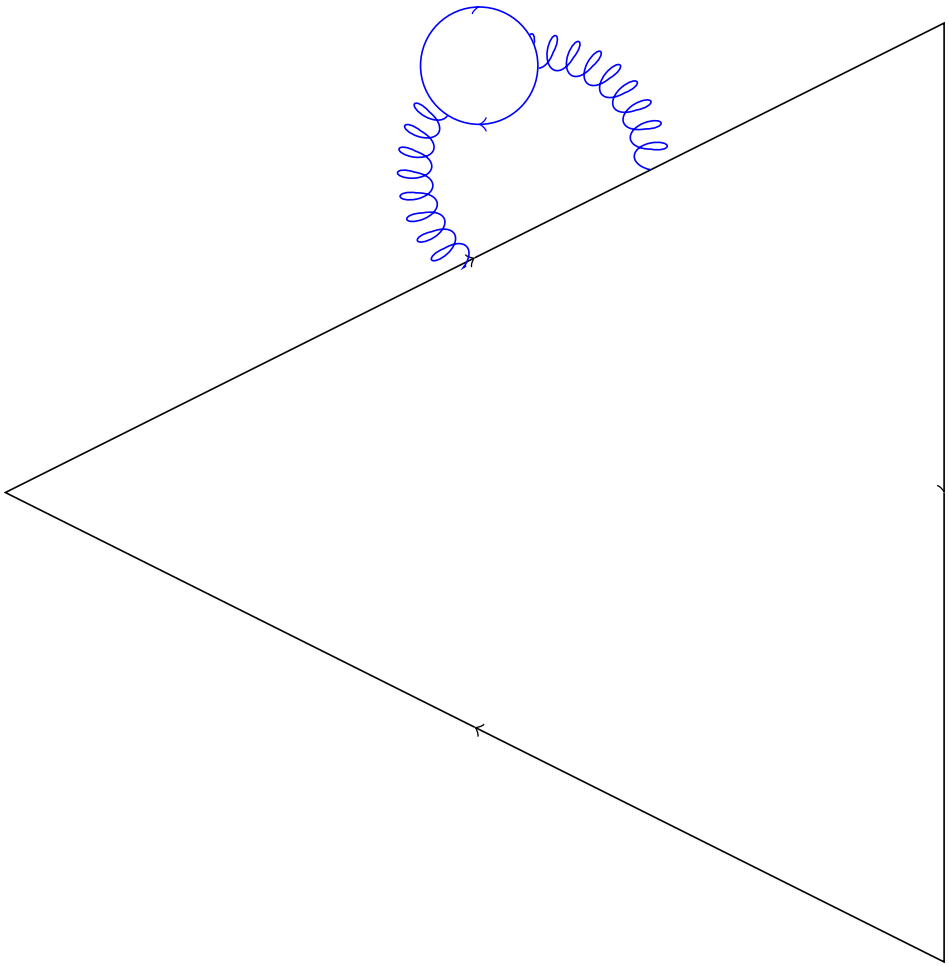




# Leading order

## $T$ modes. One loop

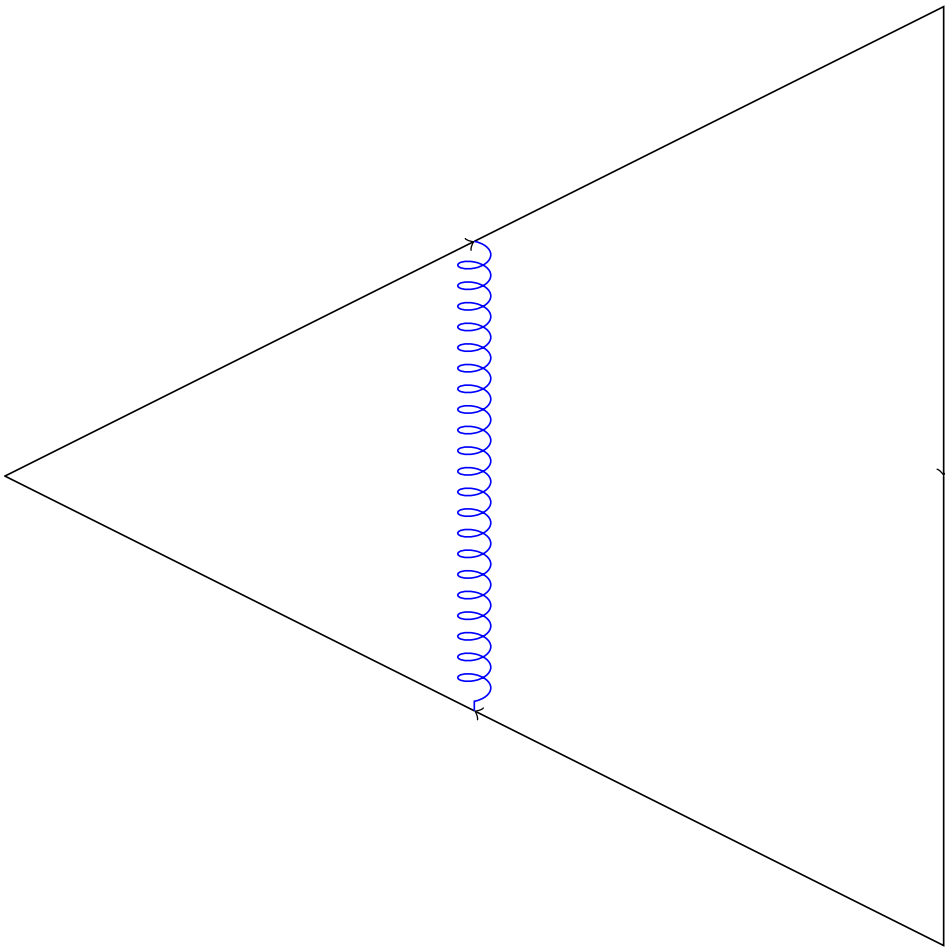


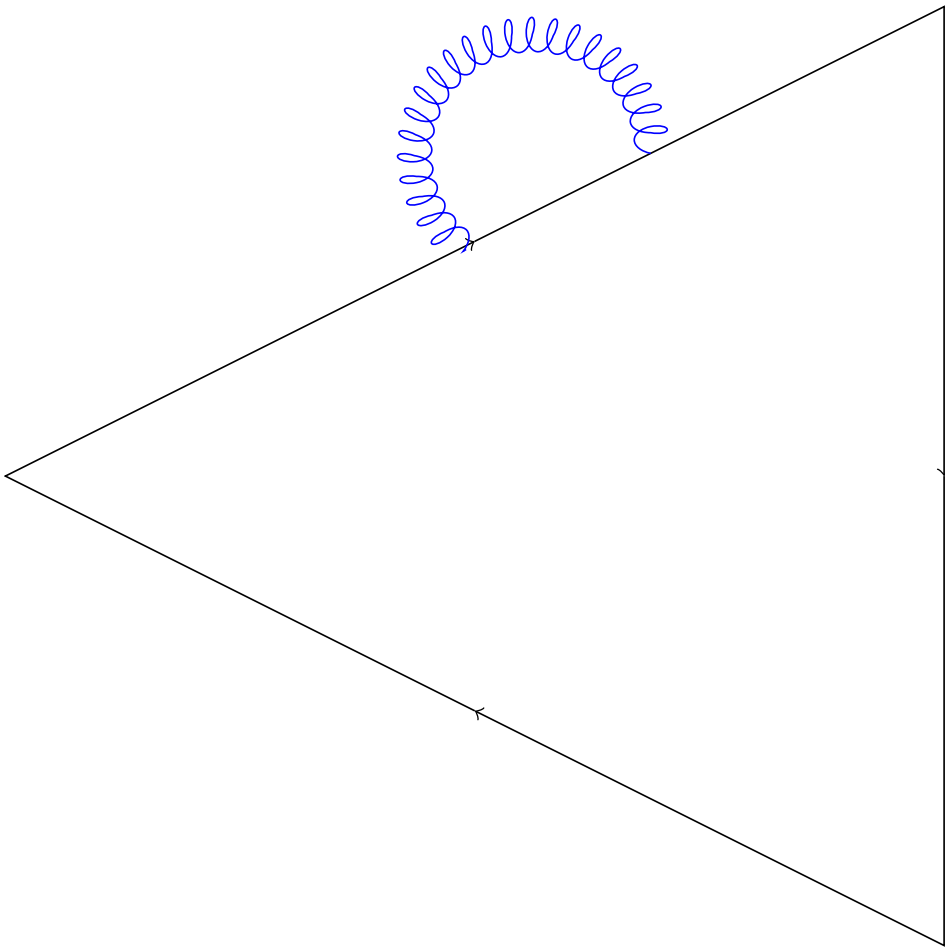


If $W = \mathbf{Tr}\mathrm{e}^{ig\oint dl\cdot A}$, then it gives a contribution to $\log\langle W\rangle$ of order $\alpha_s^2 T^3 L^3 \theta^2 = \alpha_s^2 TL(TL\theta)^2$.

# Leading order

## $m_D$ modes





It gives a contribution to $\log\langle W\rangle$ of order $\alpha_s T m_D^2 L^3 \theta^2 = \alpha_s^2 TL(TL\theta)^2$.

# Leading order

$T + m_D$

$$\log\langle W\rangle = -\frac{1}{4\sqrt{2}}\int_0^{L^+} d\tau_s \hat{q}(\delta n_\perp \tau_s)(\delta n_\perp \tau_s)^2$$

which is of order $\alpha_s^2 TL(TL\theta)^2$. Note that we consider that $\theta$ is small but $TL$ is large. Therefore, we consider for the power counting that $TL\theta \sim 1$.

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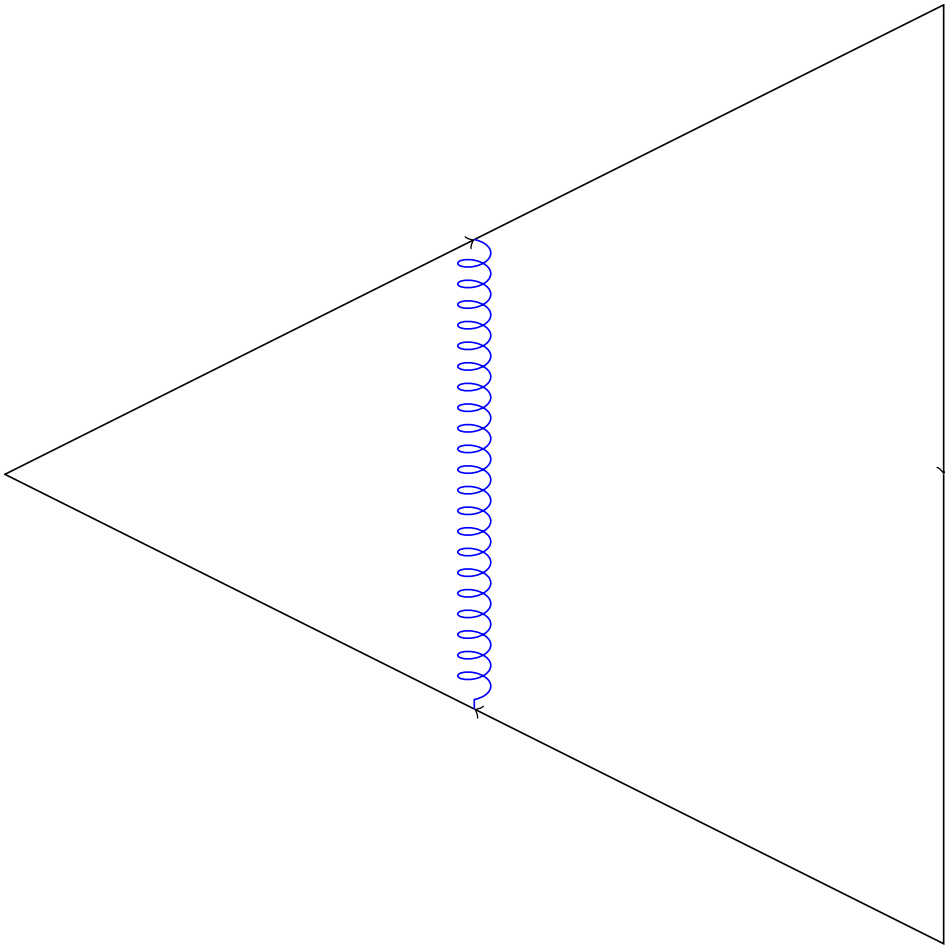
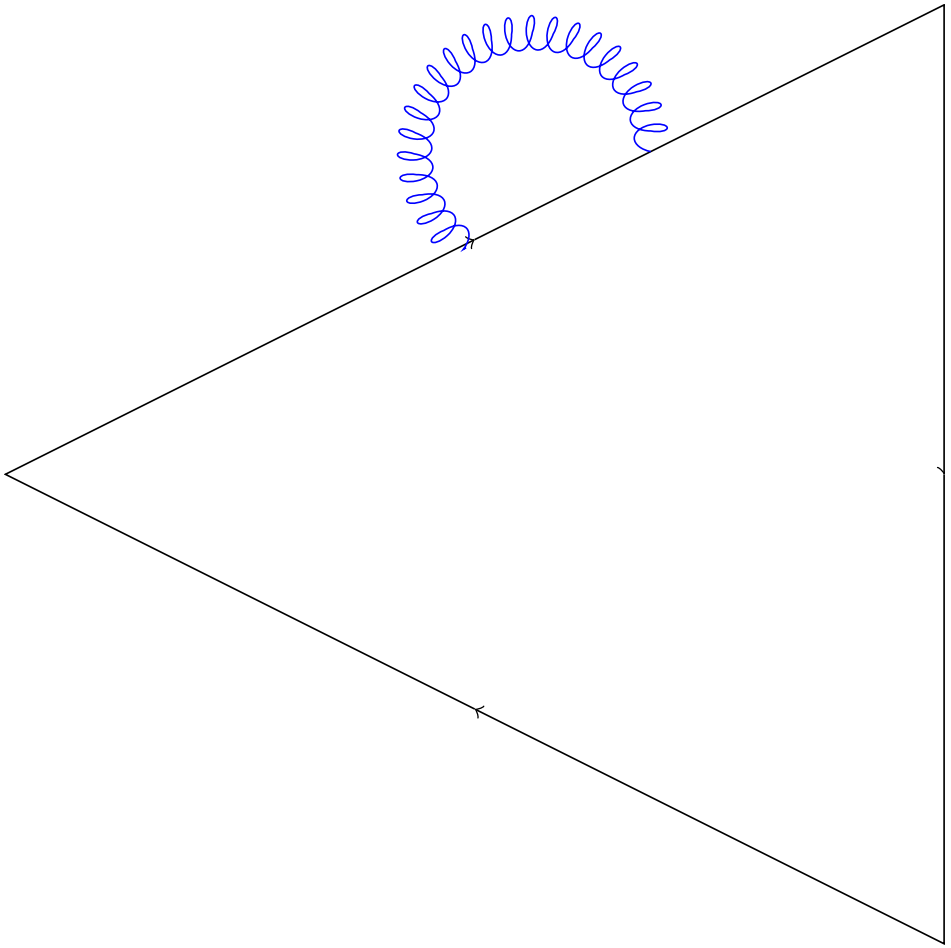
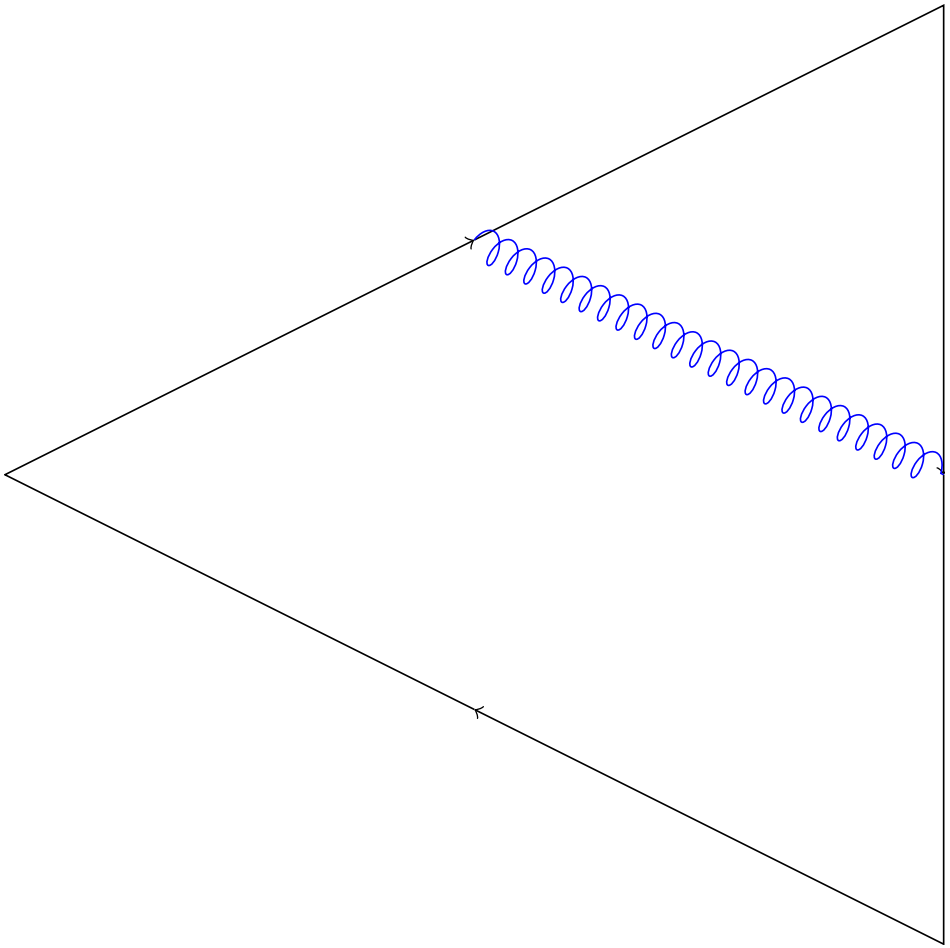
- The contribution from on-shell gluons is not enhanced by $TL$ but enters at tree level. It goes like $\alpha_s f(TL\theta)$.
- Sub-leading contribution of off-shell gluons can at most be of size $\alpha_s^2(TL\theta)^3 \ll \alpha_s$.

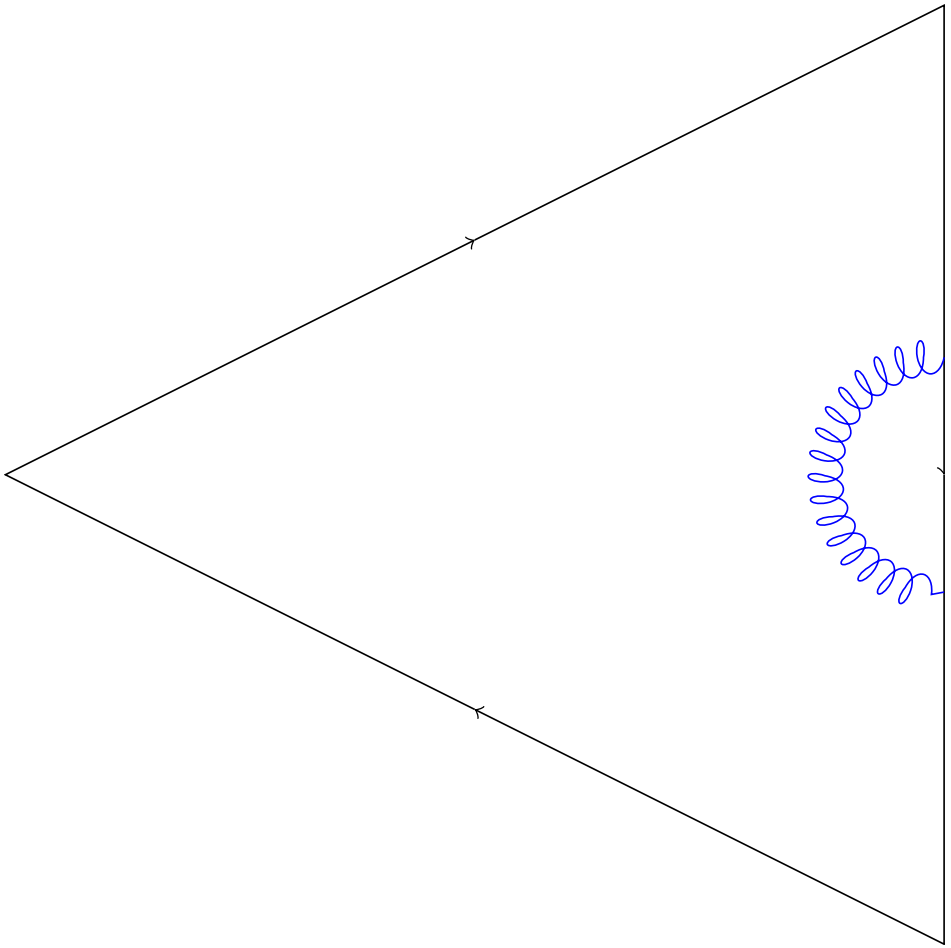
# First corrections. Diagrams I





# First corrections. Diagrams II





It happens that the diagram on the left vanishes in the Coulomb gauge.

# Outline

# Results

$$\Delta_{med} = 1 - |\langle W \rangle|^2$$

$$\log\langle W \rangle = g^2 C_F (W_2^{LO} + W_2^{NLO})$$

$$W_2^{LO} = -\frac{1}{4\sqrt{2} g^2 C_F} \int_0^{L^+} d\tau_s \hat{q}(\delta n_\perp \tau_s)(\delta n_\perp \tau_s)^2$$

We do not have a simple analytic expression for $W_2^{NLO}$. For large $\theta$

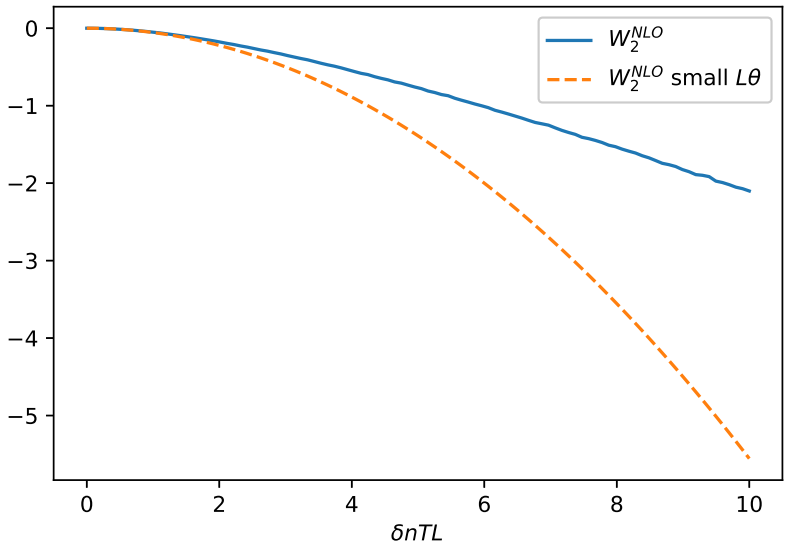
$$W_2^{NLO} \sim -\frac{TL^+\delta n}{4\pi} \left(\log(TL^+\delta n) + \gamma_E\right)$$

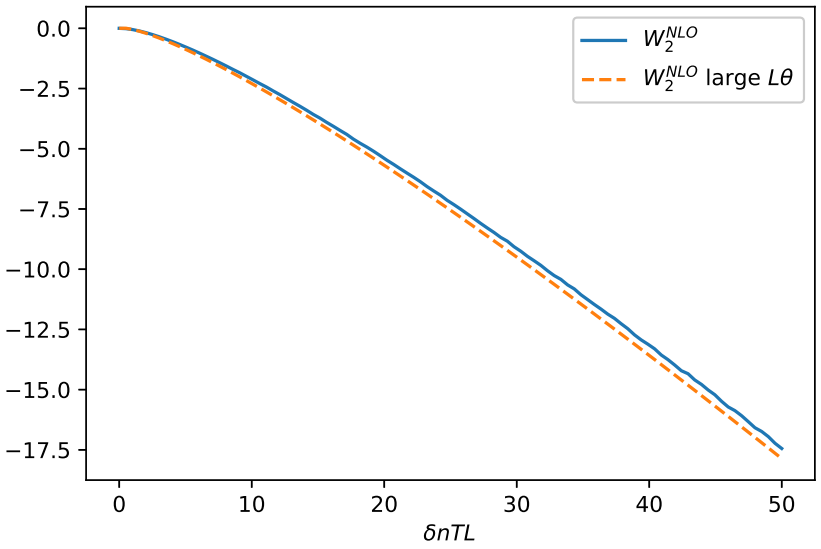
and for small $\theta$

$$W_2^{NLO} \sim -\frac{T^2 \delta n^2 L^{+2}}{18}$$

$\delta n = \theta/\sqrt{2}$ and $L^+ = \sqrt{2}L$.

# $W_2^{NLO}$ numerical



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- The interaction with the gauge fields after the medium ends up giving the vertical Wilson line. Can not be ignored at NLO. At this order, soft gluons after the medium matter.
- We can consistently add higher order corrections in $\theta$ in a manifestly gauge invariant way.